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# Simple Stability Conditions of Linear Discrete Time Systems with Multiple Delay

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**Abstract:** In this paper we have established a new Lyapunov-Krasovskii method for linear discrete time systems with multiple time delay. Based on this method, two sufficient conditions for delay-independent asymptotic stability of the linear discrete time systems with multiple delays are derived in the shape of Lyapunov inequality. Numerical examples are presented to demonstrate the applicability of the present approach.

**Keywords:** Discrete time-delay systems, Lyapunov-Krasovskii method, Delay-independent stability, Liner matrix inequality.

## 1 Inotrduction

During the last three decades, the problem of stability analysis of time delay systems has received considerable attention and many papers dealing with this problem have appeared. In the literature, various stability techniques have been utilized to derive stability criteria for time delay systems by many researchers. The techniques can be grossly classified into two categories: frequency domain approach (which are suitable for systems with a small number of heterogeneous delays) and time-domain approach (for systems with a many heterogeneous delays).

The second approach is based on the comparison principle based techniques for functional differential equations [1, 2] or the Lyapunov stability approach with the Krasovskii and Razumikhin methods [3, 4]. In the past few years stability problems are thus reduced to one of finding solutions to Lyapunov [5] or Riccati equations [6] solving linear matrix inequalities (LMIs) [7] or analyzing eigenvalue distribution of appropriate finite-dimensional matrices [8].

It is well-known that the choice of an appropriate Lyapunov-Krasovskii functional is crucial for deriving stability conditions [9]. The general form of this functional leads to a complicated system of partial differential equations

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[10]. Special forms of Lyapunov–Krasovskii functional lead to simpler delay-independent [2, 7, 9] and less conservative delay-dependent conditions [9, 11, 12]. In the past few years, there have been various approaches to reduce the conservatism of delay-dependent conditions by using new bounding for cross terms (Park’s and similarly inequalities) [2, 13] or choosing new Lyapunov–Krasovskii functional and model transformation. However, the model transformation may introduce additional dynamics [14]. In [15] it is shown that descriptor transformation leads to a system which is equivalent to the original one, does not depend on additional assumptions for stability of the transformed system and requires bounding of fewer cross-terms.

Since most physical systems evolve in continuous time, it is natural that theories for stability analysis are mainly developed for continuous-time. However, it is more reasonable that one should use a discrete-time approach for that purpose because the controller is usually implemented digitally. Despite this significance mentioned, less attention has been paid to discrete-time systems with delays [16-23, 25, 26]. It is mainly due to the fact that the delay-difference equations with known delays can be converted into a higher-order delay less system by augmentation approach. However, for systems with large known delay amounts, this scheme will lead to large-dimensional systems. Furthermore, for systems with unknown delay the augmentation scheme is not applicable.

In this paper, new delay-independent asymptotic stability conditions are derived for discrete state-delayed systems with multiple delays. These conditions are derived using Lyapunov-Krasovskii method for discrete time-delay systems which is presented in [26].

Throughout this paper we use the following notation.  $\mathfrak{R}$  denote real vector space or the set of real numbers,  $Z^+$  denotes the set of all non-negative integers. The superscript T denotes transposition. For real matrix  $F$  the notation  $F > 0$  means that the matrix  $F$  is positive definite. I and 0 represent identity matrix and zero matrix. In symmetric block matrices or long matrix expressions, we use an asterisk (\*) to represent a term that is induced by symmetry. Blockdiag {·} stands for a block-diagonal matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.  $\|v\|$  and denote norm of vector  $v$  and matrix  $F$ , while  $\|v\|_2 = \sqrt{\sum_{i=1}^n v_i^2}$  and  $\|F\|_2 = \sqrt{\lambda_{\max}(F^T F)}$  denote their Euclid norms.  $\lambda(F)$  is the eigenvalue of matrix  $F$  and  $\sigma(F)$  denotes singular value of matrix  $F$  ( $\sigma_{\max}(F) = \|F\|_2$ ).

## 2 Model Description and Preliminaries

A linear, autonomous, multivariable discrete time-delay system can be represented by the difference equation

$$x(k+1) = A_0 x(k) + \sum_{j=1}^N A_j x(k-h_j), \quad 0 < h_1 < \dots < h_N, \quad (1)$$

with an associated function of initial state

$$x(\theta) = \psi(\theta), \quad \theta \in \{-h_N, -h_N+1, \dots, 0\} \triangleq \Delta, \quad (2)$$

$x(k) \in \mathfrak{R}^n$ ,  $A \in \mathfrak{R}^{n \times n}$  is a constant matrix of appropriate dimension and  $h \in Z^+$  is unknown time delay in general case.

Let  $x_k \triangleq [x^T(k) \ x^T(k-1) \ \dots \ x^T(k-h_N)]^T$ , is state vector,  $D(\Delta, \mathfrak{R}^n)$  - space of continuous functions mapping the discrete interval  $\Delta$  into  $\mathfrak{R}^n$  and  $\|\phi\|_D = \sup_{\theta \in \Delta} \|\phi(\theta)\|$ ,  $\phi(\theta): \Delta \mapsto \mathfrak{R}^n$  - the norm of an element  $\phi$  in  $D$ . Further,  $D^\gamma = \{\phi \in D: \|\phi\|_D < \gamma, \gamma \in \mathfrak{R}\} \subset D$ . For initial state, the next condition is assumed

$$\|\psi\|_D \in D^\infty. \quad (3)$$

Evidentially,

$$x_k: \theta \mapsto x_k(\theta) \triangleq x(k+\theta) \in D \text{ and } x(k) = x(k, \psi).$$

**Definition 1.** The equilibrium state  $x=0$  of (1) is *asymptotically stable* if any function of initial state  $\psi(\theta)$  which satisfies

$$\psi(\theta) \in D^\infty, \quad (4)$$

holds

$$\lim_{k \rightarrow \infty} x(k, \psi) \rightarrow 0. \quad (5)$$

**Lemma 1.** [25] If there exist positive numbers  $\alpha$  and  $\beta$  and continuous functional  $V: D \rightarrow \mathfrak{R}$  such that

$$0 < V(x_k) \leq \alpha \|x_k\|_D^2, \quad \forall x_k \neq 0, \quad V(0) = 0, \quad (6)$$

$$\Delta V(x_k) \triangleq V(x_{k+1}) - V(x_k) \leq -\beta \|x(k)\|^2, \quad (7)$$

$\forall x_k \in D$  satisfying (1), then the equilibrium state  $x=0$  of (1) is global asymptotically stable.

**Definition 2.** Discrete system with time delay (1) is asymptotically stable if and only if it's the equilibrium state  $x = 0$  is asymptotically stable.

**Lemma 2.** For any two matrices  $F$  and  $G$  of dimension  $n \times m$  and for any square matrix  $P = P^T > 0$  of dimension  $n$ , the following statement is true

$$(F + G)^T P (F + G) \leq (1 + \varepsilon) F^T P F + (1 + \varepsilon^{-1}) G^T P G, \quad (8)$$

where  $\varepsilon$  is some positive constant.

**Lemma 3.** Thebyshev's inequality holds for any real vector  $v_i$

$$\left( \sum_{i=1}^m v_i \right)^T \left( \sum_{i=1}^m v_i \right) \leq m \sum_{i=1}^m v_i^T v_i. \quad (9)$$

### 3 Main Results

**Theorem 1.** The linear discrete time-delay system (1) with  $\|A_0\|_2 \neq 0$  is asymptotically stable if there exists real symmetric matrix  $P > 0$  such that

$$(1 + \varepsilon_m) A_0^T P A_0 + \frac{N(1 + \varepsilon_m)}{\varepsilon_m} \sum_{j=1}^N A_j^T P A_j - P < 0. \quad (10)$$

$$\varepsilon_m = \left( N \sum_{j=1}^N \|A_j\|_2^2 \right)^{\frac{1}{2}} \|A_0\|_2^{-1}.$$

**Proof.** Let the Lyapunov functional be

$$V(x_k) = x^T(k) P x(k) + \sum_{j=1}^N \sum_{l=1}^{h_j} x^T(k-l) S_j x(k-l), \quad (11)$$

$$P = P^T > 0, \quad S_j = S_j^T \geq 0,$$

where

$$x_k = x(k + \theta), \quad \theta \in \{-h_N, -h_N + 1, \dots, 0\}. \quad (12)$$

The forward difference along the solutions of system (1) is

$$\begin{aligned} \Delta V(x_k) = & \left[ A_0 x(k) + \sum_{j=1}^N A_j x(k-h_j) \right]^T P \cdot \left[ A_0 x(k) + \sum_{j=1}^N A_j x(k-h_j) \right] - \\ & - x^T(k) P x(k) + \sum_{j=1}^N \sum_{l=1}^{h_j} x^T(k+1-l) S_j x(k+1-l) - \\ & - \sum_{j=1}^N \sum_{l=1}^{h_j} x^T(k-l) S_j x(k-l). \end{aligned} \quad (13)$$

Applying *Lemma 2* on (13), one can get

$$\begin{aligned} \Delta V(x_k) \leq & (1+\varepsilon) x^T(k) A_0^T P A_0 x(k) + \\ & + (1+\varepsilon^{-1}) \sum_{j=1}^N x^T(k-h_j) A_j^T P \sum_{j=1}^N A_j x(k-h_j) \\ & - x^T(k) P x(k) + x^T(k) \sum_{j=1}^N S_j x(k) - \\ & - \sum_{j=1}^N x^T(k-h_j) S_j x(k-h_j). \end{aligned} \quad (14)$$

Based on *Lemma 3* follows

$$\begin{aligned} \Delta V(x_k) \leq & x^T(k) \left[ (1+\varepsilon) A_0^T P A_0 + \sum_{j=1}^N S_j - P \right] x(k) + \\ & + (1+\varepsilon^{-1}) N \sum_{j=1}^N x^T(k-h_j) A_j^T P A_j x(k-h_j) \\ & - \sum_{j=1}^N x^T(k-h_j) S_j x(k-h_j), \end{aligned} \quad (15)$$

$$\begin{aligned} \Delta V(x_k) \leq & x^T(k) \left[ (1+\varepsilon) A_0^T P A_0 + \sum_{j=1}^N S_j - P \right] x(k) + \\ & + \sum_{j=1}^N x^T(k-h_j) \left[ N(1+\varepsilon^{-1}) A_j^T P A_j - S_j \right] x(k-h). \end{aligned} \quad (16)$$

If one adopt

$$S_j = N(1+\varepsilon^{-1}) A_j^T P A_j, \quad (17)$$

then

$$\Delta V(\mathbf{x}_k) \leq \mathbf{x}^T(k) \left[ (1 + \varepsilon) A_0^T P A_0 + N(1 + \varepsilon^{-1}) \sum_{j=1}^N A_j^T P A_j - P \right] \mathbf{x}(k). \quad (18)$$

Let us define the following function

$$f(\varepsilon, \mathbf{x}(k)) = \mathbf{x}^T(k) \left[ (1 + \varepsilon) A_0^T P A_0 + N(1 + \varepsilon^{-1}) \sum_{j=1}^N A_j^T P A_j \right] \mathbf{x}(k). \quad (19)$$

Since matrices  $A_0^T P A_0$  and  $A_j^T P A_j$ ,  $j=1, 2, \dots, N$  are symmetric and positive semidefinite then, based on *Rayleigh* and *Amir-Moez* inequalities [23, 24]

$$\begin{aligned} f(\varepsilon, \mathbf{x}(k)) &\leq \\ &\leq \mathbf{x}^T(k) \left[ (1 + \varepsilon) \lambda_{\max}(A_0^T P A_0) + N(1 + \varepsilon^{-1}) \sum_{j=1}^N \lambda_{\max}(A_j^T P A_j) \right] \mathbf{x}(k) \quad (20) \\ &= g(\varepsilon) \lambda_{\max}(P) \|\mathbf{x}(k)\|_2^2, \end{aligned}$$

where

$$g(\varepsilon) = (1 + \varepsilon) \sigma_{\max}^2(A_0) + N(1 + \varepsilon^{-1}) \sum_{j=1}^N \sigma_{\max}^2(A_j). \quad (21)$$

Minimum of the scalar function  $g(\varepsilon)$  is obtained from condition

$$\frac{d}{d\varepsilon} g(\varepsilon) = 0 \Rightarrow \sigma_{\max}^2(A_0) - \frac{N}{\varepsilon^2} \sum_{j=1}^N \sigma_{\max}^2(A_j) = 0, \quad (22)$$

$$\varepsilon = \varepsilon_m = \left( N \sum_{j=1}^N \sigma_{\max}^2(A_j) \right)^{\frac{1}{2}} \sigma_{\max}^{-1}(A_0) = \left( N \sum_{j=1}^N \|A_j\|_2^2 \right)^{\frac{1}{2}} \|A_0\|_2^{-1}, \quad (23)$$

where from follows

$$\Delta V(\mathbf{x}_k) \leq f(\varepsilon_m, \mathbf{x}(k)) \leq f(\varepsilon, \mathbf{x}(k)). \quad (24)$$

Putting  $\varepsilon_m$  instead of  $\varepsilon$  into (18) we obtain

$$\Delta V(\mathbf{x}_k) \leq \mathbf{x}^T(k) \left[ (1 + \varepsilon_m) A_0^T P A_0 - P + N(1 + \varepsilon_m^{-1}) \sum_{j=1}^N A_j^T P A_j \right] \mathbf{x}(k) \quad (25)$$

If the condition (10) is satisfied then

$$\Delta V(\mathbf{x}_k) \leq -\lambda_{\min}\{Q\} \|\mathbf{x}(k)\|_2^2 = -\beta \|\mathbf{x}(k)\|_2^2 < 0, \quad \beta \triangleq \lambda_{\min}\{Q\} > 0. \quad (26)$$

Likewise, for  $x_k \neq 0$  holds

$$\begin{aligned}
 & 0 < V(\mathbf{x}_k) \leq \\
 & \leq \max \left\{ \mathbf{x}^T(k) P \mathbf{x}(k) + N(1 + \varepsilon_m^{-1}) \sum_{j=1}^N \sum_{l=1}^{h_j} \mathbf{x}^T(k-l) A_j^T P A_j \mathbf{x}(k-l) \right\} \\
 & \leq \lambda_{\max} \{P\} \|x(k)\|_2^2 + N(1 + \varepsilon_m^{-1}) \sum_{j=1}^N h_j \lambda_{\max} \{A_j^T P A_j\} \|x(k)\|_D^2 \quad (27) \\
 & \leq \left[ \lambda_{\max} \{P\} + N(1 + \varepsilon_m^{-1}) \sum_{j=1}^N h_j \lambda_{\max} \{A_j^T P A_j\} \right] \|x(k)\|_D^2 \\
 & = \alpha \|x(k)\|_D^2,
 \end{aligned}$$

where

$$\alpha \triangleq \lambda_{\max} \{P\} + N(1 + \varepsilon_m^{-1}) \sum_{j=1}^N h_j \lambda_{\max} \{A_j^T P A_j\} > 0. \quad (28)$$

So, based on *Lemma 1*, system (1) is asymptotically stable.

**Corollary 1.** The linear discrete time-delay system (1) is asymptotically stable if there exist real symmetric matrix  $P > 0$  and scalar  $\varepsilon > 0$  such that

$$\begin{bmatrix} N(1 + \varepsilon)P & (*) & (*) & \cdots & (*) \\ N(1 + \varepsilon)P A_0 & NP & (*) & \cdots & (*) \\ N(1 + \varepsilon)P A_1 & 0 & \varepsilon P & \cdots & (*) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ N(1 + \varepsilon)P A_N & 0 & 0 & \cdots & \varepsilon P \end{bmatrix} > 0. \quad (29)$$

**Proof.** From (10), for

$$\hat{A}_0(\varepsilon) = A_0 \sqrt{1 + \varepsilon}, \quad \hat{A}_j(\varepsilon) = A_j \sqrt{N(1 + \varepsilon) / \varepsilon} \quad (30)$$

follows

$$P - \sum_{j=1}^N \hat{A}_j^T P \hat{A}_j - \hat{A}_0^T (P^{-1})^{-1} \hat{A}_0 > 0, \quad P > 0. \quad (31)$$

Using Schur complements [7] it is easy to see that the condition (31) is equivalent to

$$\begin{bmatrix} P - \sum_{j=1}^N \hat{A}_j^T P \hat{A}_j & \hat{A}_0^T \\ \hat{A}_0 & P^{-1} \end{bmatrix} > 0, \quad P > 0. \quad (32)$$

Similarly, the condition (32) is equivalent to

$$\begin{bmatrix} P - \sum_{j=2}^N \hat{A}_j^T P \hat{A}_j & \hat{A}_0^T \\ \hat{A}_0 & P^{-1} \end{bmatrix} - \begin{bmatrix} \hat{A}_1^T \\ 0 \end{bmatrix} (P^{-1})^{-1} \begin{bmatrix} \hat{A}_1 & 0 \end{bmatrix} > 0 \Leftrightarrow$$

$$\Leftrightarrow \begin{bmatrix} P - \sum_{j=2}^N \hat{A}_j^T P \hat{A}_j & \hat{A}_0^T & \hat{A}_1^T \\ \hat{A}_0 & P^{-1} & 0 \\ \hat{A}_1 & 0 & P^{-1} \end{bmatrix} > 0. \quad (33)$$

Finally, the condition (31) is equivalent to

$$\begin{bmatrix} P & \hat{A}_0^T & \hat{A}_1^T & \dots & \hat{A}_N^T \\ \hat{A}_0 & P^{-1} & 0 & \dots & 0 \\ \hat{A}_1 & 0 & P^{-1} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \hat{A}_N & 0 & 0 & \dots & P^{-1} \end{bmatrix} > 0. \quad (34)$$

Pre and post multiply (34) with  $blockdiag\{I, P, \dots, P\}$  we obtain

$$\begin{bmatrix} P & \hat{A}_0^T P & \hat{A}_1^T P & \dots & \hat{A}_N^T P \\ P \hat{A}_0 & P & 0 & \dots & 0 \\ P \hat{A}_1 & 0 & P & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ P \hat{A}_N & 0 & 0 & \dots & P \end{bmatrix} > 0. \quad (35)$$

Using (30) and pre and post multiply (35) with  $blockdiag\{I, I\sqrt{1/(1+\varepsilon)}, I\sqrt{\varepsilon/N(1+\varepsilon)}, \dots, I\sqrt{\varepsilon/N(1+\varepsilon)}\}$  we obtain



$$\begin{bmatrix} P & A_0^T P & A_1^T P & \dots & A_N^T P \\ PA_0 & \frac{1}{1+\varepsilon} P & 0 & \dots & 0 \\ PA_1 & 0 & \frac{\varepsilon}{N(1+\varepsilon)} P & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ PA_N & 0 & 0 & \dots & \frac{\varepsilon}{N(1+\varepsilon)} P \end{bmatrix} > 0. \quad (36)$$

With  $P / (N(1 + \varepsilon))$  replaced by  $P$  we obtain (29).

#### 4 Numerical Example

**Example 1.** Let us consider a discrete delay system described by

$$\mathbf{x}(k+1) = A_0 \mathbf{x}(k) + A_1 \mathbf{x}(k-h_1) + A_2 \mathbf{x}(k-h_2),$$

$$A_0 = \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & \alpha \end{bmatrix}, \quad A_1 = \gamma \begin{bmatrix} 0.3 & 0 \\ 0.2 & 0.1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.01 & 0.05 \\ 0.03 & 0.02 \end{bmatrix},$$

where  $\gamma$  is adjustable parameter and system scalar parameter  $\alpha$  takes the following values:  $-0.15$  and  $0.5$ .

To determined the largest parameter  $\gamma$  for various values of  $\varepsilon$  by *Corollary 1*, the feasibility of equation (36) with  $\gamma$  as a variable can be cast into a generalized eigenvalue problem

$$\min_{P>0} \alpha, \quad \gamma = 1 / \alpha,$$

$$\begin{bmatrix} 0 & (*) & (*) & (*) \\ 0 & 0 & (*) & (*) \\ -N(1+\varepsilon)PA_1 & 0 & 0 & (*) \\ 0 & 0 & 0 & 0 \end{bmatrix} < \alpha \begin{bmatrix} N(1+\varepsilon)P & (*) & (*) & (*) \\ N(1+\varepsilon)PA_0 & NP & (*) & (*) \\ 0 & 0 & \varepsilon P & (*) \\ N(1+\varepsilon)PA_2 & 0 & 0 & \varepsilon P \end{bmatrix}.$$

The delay-independent asymptotic stability conditions are characterized by means of range of parameter  $\gamma$  and are summarized in **Table 1**.

**Table 1**  
Stability Conditions.

Conditions	Parameter $\alpha$			
	- 0.15		+ 0.50	
Theorem 1	$ \gamma  < 1.370$	$\varepsilon = 1$	$ \gamma  < 1.022$	$\varepsilon = 1$
Theorem 1	$ \gamma  < 1.468$	$\varepsilon = \varepsilon_m = 2.066$	$ \gamma  < 1.023$	$\varepsilon = \varepsilon_m = 0.886$
Corollary 1	$ \gamma  < 1.469$	$\varepsilon = \varepsilon_{op} = 2.144$	$ \gamma  < 1.050$	$\varepsilon = \varepsilon_{op} = 0.767$

## 5 Conclusion

In this paper we have established a new Lyapunov-Krasovskii method for linear discrete time systems with multiple time delay. Based on this method, two sufficient conditions for delay-independent asymptotic stability of the linear time systems with multiple delays are derived. These conditions stabilities have been expressed in the shape of Lyapunov inequality. Numerical examples are presented to demonstrate the applicability of the present approach.

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