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homepage: [www.GrowingScience.com/ijiec](http://www.GrowingScience.com/ijiec)**A hybrid multiple attribute decision making method for solving problems of industrial environment****Dinesh Singh<sup>a</sup> and R. Venkata Rao<sup>a\*</sup>**<sup>a</sup>*Department of Mechanical Engineering, S.V. National Institute of Technology Ichchanath, Surat, Gujarat – 395 007, India***ARTICLE INFO***Article history:*

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**ABSTRACT**

The selection of appropriate alternative in the industrial environment is an important but, at the same time, a complex and difficult problem because of the availability of a wide range of alternatives and similarity among them. Therefore, there is a need for simple, systematic, and logical methods or mathematical tools to guide decision makers in considering a number of selection attributes and their interrelations. In this paper, a hybrid decision making method of graph theory and matrix approach (GTMA) and analytical hierarchy process (AHP) is proposed. Three examples are presented to illustrate the potential of the proposed GTMA-AHP method and the results are compared with the results obtained using other decision making methods.

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**1. Introduction**

During the past few years, fast-changing technologies on the product front have created fast response from the industries. The old, traditional model of ‘unfocused, short-term views and non-holistic vision’ is replaced by the enlightened approach of ‘focused, holistic and strategic vision’. To meet the challenges, industries have to select appropriate production strategies, product designs, production processes, work and tool materials, machinery and equipment, etc. Since decision-making is a complex process we need for simple, systematic, and logical methods or mathematical tools to guide decision makers in considering a number of selection attributes and their interrelations. The applications of quite a good number of multiple attribute decision making (MADM) methods for solving the deterministic decision making problems of the industrial environment have been reported in the literature (Rao, 2007). However, these methods have their own merits and demerits.

MADM is employed to solve problems involving selection from a finite number of alternatives. Each decision table in MADM methods consists of four main parts, namely: (a) alternatives, (b) attributes, (c) relative importance of each attribute, and (d) measures of performance of attributes for different alternatives. Given the decision table input data and a decision making method, we need to find the

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best alternative and/or to rank the entire set of alternatives. The aim of the present paper is to propose a hybrid MADM method named as “analytical hierarchy graph theory and matrix approach (AHGTMA)” to deal with the decision making situations of the industrial environment considering both qualitative and quantitative attributes. Three examples, electroplating system selection, robot selection, welding process selection, are included to illustrate the proposed method.

The next section describes the proposed methodology “analytical hierarchy graph theory and matrix approach (AHGTMA)”.

## 2. Analytical hierarchy graph theory and matrix approach (AHGTMA)

The proposed methodology “analytical hierarchy graph theory and matrix approach (AHGTMA)” is the integration of analytical hierarchy process “AHP” (Saaty, 1980, 2000) and graph theory and matrix approach (Rao, 2007). In the proposed method, the AHP is used to get the consistent relative importance of attributes. The consistent relative importance of attributes are used to form a matrix, known as “alternative selection attribute matrix” for each alternative using graph theory and matrix approach (Rao, 2007). The stepwise procedure of the proposed “analytical hierarchy graph theory and matrix approach (AHGTMA)” is given as follow:

### Step 1: Decision matrix

Decision matrix is the collection of attribute data for each alternative. First, attributes or criteria are identified and then we need to measure the performance of attributes to select the alternative from the available alternatives. For an MADM problem when there are ‘ $M$ ’ alternatives and ‘ $N$ ’ attributes, the  $i^{th}$  alternative can be expressed as  $Y_i = (y_{i1}, y_{i2}, \dots, y_{ij}, \dots, y_{iN})$  in decision matrix form, where  $y_{ij}$  is the performance value (or measure of performance) of attribute  $j$  ( $j = 1, 2, 3, \dots, N$ ) for alternative  $i$  ( $i = 1, 2, 3, \dots, M$ ). The general form of decision matrix  $D$  is given as follows,

$$D = \begin{bmatrix} y_{11} & \dots & y_{1j} & \dots & y_{1N} \\ \dots & \dots & \dots & \dots & \dots \\ y_{i1} & \dots & y_{ij} & \dots & y_{iN} \\ \dots & \dots & \dots & \dots & \dots \\ y_{M1} & \dots & y_{Mj} & \dots & y_{MN} \end{bmatrix} \quad (1)$$

The attributes may be objective, subjective or a combination of both. The subjective attributes are represented in linguistic terms and they need to be converted into corresponding crisp scores.

### Step 2: Normalization / Normalized decision matrix

Normalization is the procedure to set the attribute data in the same scale so that comparisons can be made easier. It makes all the attribute values dimensionless. There are many normalization procedures available in the literature. In the proposed method, the following normalization procedure is adopted. Let  $x_{ij}$  be the normalized value of  $y_{ij}$  for attribute  $j$  compared with alternative  $i$  as follows,

$$x_{ij} = \frac{y_{ij}}{\max_j (y_{ij})} \quad ; \text{ if } j^{th} \text{ attribute is beneficial} \quad (2)$$

$$x_{ij} = \frac{\min_j (y_{ij})}{y_{ij}} \quad ; \text{ if } j^{th} \text{ attribute is non-beneficial} \quad (3)$$

After normalization, normalized decision matrix  $D'$  is given as follows:

$$D' = \begin{bmatrix} x_{11} & \dots & x_{1j} & \dots & x_{1N} \\ \dots & \dots & \dots & \dots & \dots \\ x_{i1} & \dots & x_{ij} & \dots & x_{iN} \\ \dots & \dots & \dots & \dots & \dots \\ x_{M1} & \dots & x_{Mj} & \dots & x_{MN} \end{bmatrix} \quad (4)$$

### Step 3: Relative importance of attributes

The relative importance of attribute is the judgment made by the decision maker(s) after analyzing different attributes (both subjective as well as objective attributes) with respect to the goal. A pair-wise comparison matrix is constructed using a scale of relative importance. The judgments are entered using the fundamental scale of the analytical hierarchy process (Saaty, 1980, 2000). The scale for pair wise comparison is given in Table 1.

**Table 1**  
Scale for pair wise comparison

Degree of importance	Definition
1	Equal (no preference)
2	Intermediate between 1 and 3
3	Moderately preferable
4	Intermediate between 3 and 5
5	Strongly preferable
6	Intermediate between 5 and 7
7	Very strongly preferable
8	Intermediate between 7 and 9
9	Extremely strongly preferable

Given ' $N$ ' attributes, the pair wise comparison of attribute  $i$  with attribute  $j$  yields a square matrix  $B_{N \times N}$  where  $b_{ij}$  denotes the comparative importance of attribute  $i$  with respect to attribute  $j$ . In the matrix,  $b_{ij} = 1$  when  $i = j$  and  $b_{ji} = 1/b_{ij}$ .

$$B_{N \times N} = \begin{bmatrix} 1 & b_{12} & b_{13} & \dots & \dots & b_{1N} \\ b_{21} & 1 & b_{23} & \dots & \dots & b_{2N} \\ b_{31} & b_{32} & 1 & \dots & \dots & b_{3N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{N1} & b_{N2} & b_{N3} & \dots & \dots & 1 \end{bmatrix} \quad (5)$$

Now, the consistency check is required for the relative importance of attributes. The consistency check is carried out similar to the AHP process (Saaty, 1980). It is required that the consistency ratio (CR) of the relative importance of attributes should not be more than 0.1, otherwise, the judgments of deciding the relative importance of attributes are not consistent.

### Step 4: Formation of alternative selection attribute matrix for each alternative

The alternative selection attribute matrix is framed by keeping the diagonal elements as the normalized values for attributes data for the respective alternative. The normalized value of attributes for an alternative are:  $[A_1, A_2, A_3, \dots, A_N] = [x_{i1}, x_{i2}, x_{i3}, \dots, x_{iN}]$ , where ' $i$ ' is the alternative number. This matrix is represented by ' $C$ ', as given by Eq. (6).

$$C = \begin{bmatrix} A_1 & b_{12} & b_{13} & \dots & \dots & b_{1N} \\ b_{21} & A_2 & b_{23} & \dots & \dots & b_{2N} \\ b_{31} & b_{32} & A_3 & \dots & \dots & b_{3N} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots \\ b_{N1} & b_{N2} & b_{N3} & \dots & \dots & A_N \end{bmatrix} \quad (6)$$

Step 5: Get the permanent value of the alternative selection attribute matrix for each alternative

The permanent value of the alternative selection attribute matrix ‘C’ for each alternative is calculated similar to the graph theory and matrix approach (Rao, 2007). It is represented by  $per(C)$  and also called as *index score* for the respective alternative. Eq. (7) is used to calculate  $per(C)$ .

$$\begin{aligned} per(C) = & \prod_{i=1}^N A_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{ji}) A_k A_l A_m A_n A_o \dots \dots A_t A_N \\ & + \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{ki} + b_{ik} b_{kj} b_{ji}) A_l A_m A_n A_o \dots \dots A_t A_N \\ & + \left( \sum_{i=1}^{N-3} \sum_{j=i+1}^N \sum_{k=i+1}^{N-1} \sum_{l=i+2}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{ji}) (b_{kl} b_{lk}) A_m A_n A_o \dots \dots A_t A_N \right. \\ & \left. + \sum_{i=1}^{N-3} \sum_{j=i+1}^{N-1} \sum_{k=i+1}^N \sum_{l=j+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{kl} b_{li} + b_{il} b_{lk} b_{kj} b_{ji}) A_m A_n A_o \dots \dots A_t A_N \right) \\ & + \left( \sum_{i=1}^{N-2} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^N \sum_{l=1}^{N-1} \sum_{m=l+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{ki} + b_{ik} b_{kj} b_{ji}) (b_{lm} b_{ml}) A_n A_o \dots \dots A_t A_N \right. \\ & \left. + \sum_{i=1}^{N-4} \sum_{j=i+1}^{N-1} \sum_{k=i+l=i+1}^N \sum_{m=j+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{kl} b_{lm} b_{mi} + b_{im} b_{ml} b_{lk} b_{kj} b_{ji}) A_n A_o \dots \dots A_t A_N \right) \\ & + \left( \sum_{i=1}^{N-3} \sum_{j=i+1}^{N-1} \sum_{k=i+l=j+1}^N \sum_{l=1}^{N-1} \sum_{m=1}^N \sum_{n=m+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{kl} b_{li} + b_{il} b_{lk} b_{kj} b_{ji}) (b_{mn} b_{nm}) A_o \dots \dots A_t A_N \right. \\ & + \sum_{i=1}^{N-5} \sum_{j=i+1}^{N-1} \sum_{k=j+1}^N \sum_{l=1}^{N-2} \sum_{m=l+1}^{N-1} \sum_{n=m+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{ki} + b_{ik} b_{kj} b_{ji}) (b_{lm} b_{mn} b_{nl} + b_{nl} b_{nm} b_{ml}) A_o \dots \dots A_t A_N \\ & + \sum_{i=1}^{N-5} \sum_{j=i+1}^N \sum_{k=i+l=i+2}^{N-3} \sum_{m=k+1}^{N-1} \sum_{n=k+2}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{ji}) (b_{kl} b_{lk}) (b_{mn} b_{nm}) A_o \dots \dots A_t A_N \\ & \left. + \sum_{i=1}^{N-5} \sum_{j=i+1}^{N-1} \sum_{k=i+l=i+1}^N \sum_{m=i+1}^N \sum_{n=j+1}^N \dots \dots \dots \sum_{N=t+1}^N (b_{ij} b_{jk} b_{kl} b_{lm} b_{mn} b_{ni} + b_{in} b_{nm} b_{ml} b_{lk} b_{kj} b_{ji}) A_o \dots \dots A_t A_N \right) \\ & + \dots \dots \dots \end{aligned} \quad (7)$$

Step 6: Rank of alternatives

The rank of alternatives are based on the permanent values of the alternative selection attribute matrix i.e.  $per(C)$ , also called as *index score*. The higher the *index score* value, higher is the rank of that alternative. In the next section, the proposed AHGTMA method is applied for three decision making situations of manufacturing environment.

### 3. Examples

Three examples are considered to demonstrate and to validate the application of the proposed methodology of selection problems for a given engineering applications of manufacturing environment.

#### 3.1 Example 1: Selection of electroplating system

The electroplating is generally used to alter the characteristics of a surface to provide improved appearance, ability to withstand corrosive agents, resistance to abrasion, or other desired properties.

Electroplating are basically classified under four main categories: (i) Electroplating performance characteristics, (ii) Electroplating application characteristics, (iii) Electroplating handling characteristics and (iv) Electroplating stability (adhesion) characteristics. Performance characteristics of an electroplating are involved with its capability for different applications in its adverse environment. The environment effect and resisting capacity determine the performance potential of electroplating. Application capability is involved with the feeling of a person in the environment of the electroplating product. The applications of this kind of problem mainly come from erosion or peeling of the plating, which may be induced by variety of sources including surface irregularities and environment factors. Handling characteristic of plating deals with its response to users' application and the environmental inputs affecting the performance of plating, such as acidic or basic. There are two kinds of problems in electroplating handling; one to control the plating erosion and the other to stabilize the plating against external disturbances. Stability of plating is the ability to stabilize its retention against environmental disturbance. Unstable plating gradually erodes from the original plating even after the disturbance is removed. The disturbance may appear from corroding environment, momentary factor like friction of mating parts, slight movement of joints and other causes (Kumar & Agrawal, 2009).

The electroplating is normally used for the characteristics of a surface to provide improved appearance, ability to withstand corrosive agents, resistance to abrasion, or other desired properties or a combination of them (Lowenheim, 1978). Electroplating with various capabilities are available for a wide range of applications such as surface finishing, thickness maintaining, avoiding rusting, restoring dimension of under size parts, aesthetic, etc. (SIRI, 2004). The use of an appropriate coating can increase the life expectancy of the component or entire machines (Kanani, 2006). There is a growing interest among the manufacturers of electroplating product to optimize the manufacturing strategies and attributes. There are five basic materials used for electroplating and metal finishing industries in India, which are chromium, nickel, zinc, cadmium and copper. The quality of these metals is controlled by tenting thickness and corrosion thickness (SIRI, 2004). Thus, the quality, the reliability, the maintainability, etc. are the major issues faced by the electroplaters in the global market. The ability of a new electroplating system to identify critical attributes plays an important in marketing this kind of facilities (Kumar & Agrawal, 2009).

The selection of the electroplating to suit a particular application of manufacturing environment, from the large number of platings available in the market today has become a difficult task. There have been many attempts in the past aimed at quality control of the different electroplating process for the performance characteristic attributes and for carrying out sensitivity analysis. These include growth characteristic of tungsten grown through cementation, vapor deposition and electroplating (Inal & Torma, 1979), development of coating system (Fisher & Fisher, 1981), gold and silver electrode for electroplating system (Busby & Creighton, 1982), optimized surface pre treatment for copper electroplating (Kim & Kim, 2001), etc. Bayati et al. (2005) designed an electroplating bath, for toxicity. Sombatsompop et al. (2004) also designed the electroplating experiment apparatus for improvement of efficiency by cathode rotating (CR) and anode circumference rotating (ACR), simultaneously. Janssen and Koene (2002) suggested the usefulness of different alternative electrodes for different applications and their effect on environment. Any electroplating design consists of important material selection as an actual contributor to the engineering qualities of the final product (Durney, 1996).

The performance of the electroplating depends on various attributes, which affect the characteristics of the plating. The electroplating attributes are characterized into six categories i.e. design attributes, performance characteristic attributes, use characteristics attributes, handling characteristics attributes, stability characteristic attributes and general attributes. The design attributes affect all the characteristics of the electroplating. These are: type of process tank, breadth of electrode, length of electrode, distance between electrodes, electrolytic concentration, current value, voltage value, etc. Performance characteristic attributes affect the performance characteristics. These attributes include layer thickness, surface cleaning, corrosion resistance, dullness, roughness, environmental factor, etc.

These parameters will decide the life, durability, capability, grad ability and operating economy of the electroplating. The primary objective is to control the wear or erosion so that the user adaptability of product does not exceed the certain level. Some of the important attributes, which affect the user characteristics are: adhesions, cohesion, hardness of plating, coefficient of friction, surface tension, deposition rate, deposition time, wear resistance, etc. The attributes affecting the handling characteristics are known as handling characteristics attributes, which include part geometry, part irregularity, friction of plating parts, heat resistance, color of plating, impurities impingement, etc. The stability of the plating is defined by an adhesion or peeling index of the plating. The stability is affected by the over use of plated part and the attributes are: polarity at the time of plating, pH of electrolyte, stress (internal), current density, tensile strength, scratch resistance, specific conductivity, quality of plating, etc. Some general attributes affecting the electroplating quality and performance for the desired applications are cost of plating, ecological factor, reliability of plating, consistency and uniformity, appearance and aesthetic, safety of working personal, brightness, noise factor, maintenance, viscosity of electrolyte, recycling of wastes or waste disposal, degree of automation, etc. From the above attributes under different characteristic we can conclude that the plating characteristics are interdependent factors (Kumar & Agrawal, 2009).

In this example, the selection of electroplating system for ornamental purpose is taken from the case study conducted by Kumar and Agrawal (2009). Decision matrix is given in Table 2 which includes both quantitative and qualitative attributes.

**Table 2**

Decision matrix for Example 1 (Kumar & Agrawal, 2009)

Cost	Adhesion	Aesthetic	Thickness( $\mu\text{m}$ )	Hardness(HV)	Alternatives
Medium(2)	Good(4)	Good(4)	20	350	1-Silver
High(3)	Average(3)	Excellent(5)	25	250	2-Gold
Low(1)	Poor(1)	Average(3)	30	150	3-Lead
Medium(2)	Average(3)	Fair(2)	20	400	4-Rhodium
Low(1)	Fair(2)	Poor(1)	30	550	5-Nickel
Low(1)	Excellent(5)	Poor(1)	35	600	6-Chromium
High(3)	Good(4)	Good(4)	30	580	7-Platinum

The numerical values are assigned to qualitative attributes, which are given in bracket. The next step is the normalization of attribute data of decision matrix using Eq. (2) and Eq. (3). The normalized decision matrix is given in Table 3.

**Table 3**

Normalized decision matrix for Example 1

Alternatives	Hardness (HV)	Thickness( $\mu\text{m}$ )	Aesthetic	Adhesion	Cost
Silver	0.5833	0.5714	0.8	0.8	0.5
Gold	0.4167	0.7143	1	0.6	0.3333
Lead	0.25	0.8571	0.6	0.2	1
Rhodium	0.6667	0.5714	0.4	0.6	0.5
Nickel	0.9167	0.8571	0.2	0.4	1
Chromium	1	1	0.2	1	1
Platinum	0.9667	0.8571	0.8	0.8	0.3333

Now, the same relative importance of attributes given in Table 4 used by Kumar and Agrawal (2009) is considered to solve the problem. The next step is to form “alternative selection attribute matrix” for each alternative and to calculate the permanent value of the matrix formed for each alternative called the “index score” for the alternatives. The “alternative selection attribute matrix” for alternative 1, represented by  $C_1$  is given below.

**Table 4**  
Relative importance of attributes for Example 1(Kumar & Agrawal, 2009)

Attributes	Hardness (HV)	Thickness( $\mu\text{m}$ )	Aesthetic	Adhesion	Cost
Hardness (HV)	1	1	2	1/2	1/3
Thickness ( $\mu\text{m}$ )	1	1	1/2	2	2
Aesthetic	1/2	2	1	3	2
Adhesion	2	1/2	1/3	1	1/3
Cost	3	1/2	1/2	3	1

The permanent of matrix  $C_1$  (i.e. index score for  $C_1$ ) obtained is 271.8269. The index score values for all alternatives are arranged in decreasing order of their values. The higher the index score, the better is the rank of that alternative.

$$C_1 = \begin{bmatrix} 0.5833 & 1 & 2 & 0.5 & 0.3333 \\ 1 & 0.5714 & 0.5 & 2 & 2 \\ 0.5 & 2 & 0.8 & 3 & 2 \\ 2 & 0.5 & 0.3333 & 0.8 & 0.3333 \\ 3 & 0.5 & 0.5 & 3 & 0.5 \end{bmatrix}$$

The index score values for the alternatives obtained are:

Metal	Chromium	Platinum	Nickel	Silver	Gold	Lead	Rhodium
Rank	6	7	5	1	2	3	4
Score	319.0652	288.6697	275.1737	271.8269	264.5685	263.8824	250.8611

For the same relative importance of attributes, the ranking of alternative electroplating systems obtained using proposed AHGTMA method is: 6 – 7 – 5 – 1 – 2 – 3 – 4, whereas ranking suggested by Kumar and Agrawal (2009) using TOPSIS method was: 1 – 7 – 2 – 6 – 3 – 5 – 4. The proposed AHGTMA method suggests alternative 6 i.e. Chromium as the best alternative, whereas Kumar and Agrawal (2009) suggests alternative 1 i.e. Silver as the best choice. When we compare the attribute data for the alternative 6 and alternative 1, we find that four attributes (i.e. hardness, thickness, adhesion and cost) are in favor of alternative 6, while only one attribute (i.e. aesthetic) is in favor of alternative 1. Therefore, alternative 6 should be preferred to alternative 1, which is the same as obtained using proposed AHGTMA method. The second best alternative obtained using proposed AHGTMA method is 7 i.e. Platinum, which is same as given by Kumar and Agrawal (2009) using TOPSIS method. However, we have found that the relative importance of attributes used by Kumar and Agrawal (2009) are not consistent. They have used the relative importance of attributes with consistency ratio (CR) as 0.215, which is not acceptable.

**Table 5**  
New relative importance of attributes for Example 1

Attributes	Hardness (HV)	Thickness( $\mu\text{m}$ )	Aesthetic	Adhesion	Cost
Hardness (HV)	1	3	2	1	1/5
Thickness ( $\mu\text{m}$ )	1/3	1	1/2	1/3	1/5
Aesthetic	1/2	2	1	1	1/3
Adhesion	1	3	1	1	1/2
Cost	5	5	3	2	1

Now, the new relative importance of attributes are decided by the decision makers given in Table 4 where the consistency ratio (CR) is 0.029.

The new index score values for the alternatives obtained are as follows.

Metal	Chromium	Platinum	Silver	Nickel	Gold	Lead	Rhodium
Rank	6	7	1	5	2	3	4
Score	127.4245	119.2953	109.0518	108.3139	105.6910	100.5556	98.3936

Therefore, the ranking of alternative electroplating systems obtained using proposed AHGTMA method is: 6 – 7 – 1 – 5 – 2 – 3 – 4. This shows that the Chromium is the best alternative for electroplating and Platinum is the second best alternative. Hence, the proposed HAGTMA method is validated.

### 3.2 Example 2: Robot Selection

The robots are used extensively by many advanced manufacturing organizations to perform various dangerous/non-dangerous tasks. Therefore, the selection of robot becomes an important problem, because improper selection of the robots may adversely affect the profitability, significantly.

The robot selection problem has become more difficult in recent years due to increasing complexity, availability of different features and facilities offered by different robotic products. Several factors (criteria or attributes) have to be considered while selecting a robot, effectively. The factors may be objective or subjective in nature. The objective criteria refer to those attributes of robots, which can be measured and assigned by numerical values. There are different examples for the attributes such as velocity of the robot arm, load capacity of the robot, repeatability, robot cost, memory capacity, manipulator reach, types of drives (actuators) and degree of freedom, etc. The subjective criteria are qualitative in nature, e.g. vendor's service quality, robot's programming flexibility, etc.

Liang and Wang (1993) suggested a fuzzy multi-criteria decision-making approach for industrial robot selection. They suggested group decision making for the selection of robots. Zhao et al. (1996) introduced genetic algorithm (GA) for optimal Robot Selection problem in a CIM system. Goh (1997) used AHP method for robot selection incorporating inputs from multiple decision makers and considered both the subjective and the objective criteria. Chu and Lin (2003) used a fuzzy TOPSIS Method for robot selection with subjective as well as objective criteria. Bhangale et al. (2004) used TOPSIS and graphical method for the selection of a robot for some pick-n-place operation. Shih (2008) suggested an incremental analysis method with group TOPSIS for the selection of industrial robots. Chatterjee et al. (2010) applied 'VIsekriterijumsko KOMPromisno Rangiranje' (VIKOR) and 'ELimination and Et Choice Translating REality' (ELECTRE) methods for the selection of robot for some industrial application.

Now an example is taken to further validate the proposed analytical hierarchy graph theory and matrix approach (AHGTMA) method for the problems in manufacturing situations. In this example, a jointed-arm robot is selected for supporting numerical control machines and manufacturers provide the actual data. The decision matrix with twelve alternative robots and five attributes are given in Table 6. The attribute are PC: Purchasing cost in USD, HC: Handling coefficient, LC: Load capacity in kg, RE: 1/Repeatability in  $\text{mm}^{-1}$  and VE: Velocity (m/s). The purchasing cost of robots is the input attribute, which is a non-beneficial type, and other attributes are output attributes for which higher value is desirable. The attribute handling coefficient is derived from the six different factors: diameter (mm), elevation (mm), basic rotation (degrees), roll (degrees) pitch (degrees) and yaw (degrees). The diameter, elevation and basic rotation, which are work area-related measures of the robot's arm and the roll, pitch and yaw which are associated with the wrist rotation angles around the three principal axes. The attribute load capacity is the robot's maximum transportable weight, repeatability is the measure of the accuracy where the robot permits the end effector to return to a specific point and the velocity is the end effector's maximum attainable speed. Since the lower value of repeatability indicate better performance, the reciprocal value of repeatability are used in the computations for enabling the treatment of 'repeatability' attribute as output (Braglia & Petroni, 1999; Karsak & Ahiska, 2005, 2008; Wang & Chin, 2009).



The next step is to normalize the attribute data given in decision matrix given in Table 6, so that values of all attributes become dimensionless and on the same scale.

**Table 6**

Decision matrix for Example 2 (Braglia & Petroni, 1999; Karsak & Ahiska, 2005, 2008; Wang & Chin, 2009)

Alternative robots	PC	HC	LC	RE	VE
R1	100,000	0.995	85	1.7	3.00
R2	75,000	0.933	45	2.5	3.60
R3	56,250	0.875	18	5.0	2.20
R4	28,125	0.409	16	1.7	1.50
R5	46,875	0.818	20	5.0	1.10
R6	78,125	0.664	60	2.5	1.35
R7	87,500	0.880	90	2.0	1.40
R8	56,250	0.633	10	8.0	2.50
R9	56,250	0.653	25	4.0	2.50
R10	87,500	0.747	100	2.0	2.50
R11	68,750	0.880	100	4.0	1.50
R12	43,750	0.633	70	5.0	3.00

The normalization of attribute data is carried out using Eq. (2) and Eq.(3) and the normalized decision matrix is given in Table 7.

**Table 6**

Decision matrix for Example 2 (Braglia & Petroni, 1999; Karsak & Ahiska, 2005, 2008; Wang & Chin, 2009)

Alternative robots	PC	HC	LC	RE	VE
R1	0.2813	1	0.85	0.2125	0.8333
R2	0.375	0.9377	0.45	0.3125	1
R3	0.5	0.8794	0.18	0.625	0.6111
R4	1	0.4111	0.16	0.2125	0.4167
R5	0.6	0.8221	0.2	0.625	0.3056
R6	0.36	0.6673	0.6	0.3125	0.375
R7	0.3214	0.8844	0.9	0.25	0.3889
R8	0.5	0.6362	0.1	1	0.6944
R9	0.5	0.6563	0.25	0.5	0.6944
R10	0.3214	0.7508	1	0.25	0.6944
R11	0.4091	0.8844	1	0.5	0.4167
R12	0.6429	0.6362	0.7	0.625	0.8333

The next step is to get the relative importance of attributes. The judgments made by decision makers on relative importance of attributes are given in Table 8.

**Table 8**

Relative importance of attributes for Example 2

Attributes	PC	HC	LC	RE	VE
PC	1	5	5	1	3
HC	1/5	1	3	1/2	1/3
LC	1/5	1/3	1	1/6	1/7
RE	1	2	6	1	1/2
VE	1/3	3	7	2	1

We have found that the judgments of deciding the relative importance of attributes are consistent and the consistency ratio (CR) equals to 0.0754. Now, the next step is to form the “alternative selection attribute matrix” for each alternative and to calculate the permanent value of the matrix formed for

each alternative, which is also called the “index score” for the alternatives. The “alternative selection attribute matrix” for alternative 1, represented by  $C_1$  is given below.

The permanent of matrix  $C_1$  (i.e. index score for  $C_1$ ) obtained is 134.9183. The index score values for all alternatives are calculated and these values are arranged in decreasing order to get the ranking of alternatives.

$$C_1 = \begin{bmatrix} 0.2813 & 5 & 5 & 1 & 3 \\ 0.2 & 1 & 3 & 0.5 & 0.3333 \\ 0.2 & 0.3333 & 0.85 & 0.1667 & 0.1428 \\ 1 & 2 & 6 & 0.2125 & 0.5 \\ 0.3333 & 3 & 7 & 2 & 0.8333 \end{bmatrix}$$

The index score values for the alternative robots obtained are:

R12	R11	R1	R10	R2	R7	R8	R3	R9	R5	R6	R4
134.92	133.55	131.57	127.62	126.86	122.99	122.43	120.83	115.99	115.90	112.61	104.41

The rank order of alternative obtained using proposed AHGTMA method is: R12 - R11 - R1 - R10 - R2 - R7 - R8 - R3 - R9 - R5 - R6 - R4. Braglai and Petroni (1999) suggested R12 as the best alternative using DEA models. The rank order given by Karsak and Ahiska (2005) using common weight multi-criteria decision making (MCDM) method is: R12 - R11 - R5 - R4 - R3 - R2 - R7 - R1 - R10 - R9 - (R6 - R8). Karsak and Ahiska (2008) also suggested R12 as the best alternative robot using improved MCDM method. Wang and Chin (2009) gave the following rank order of alternative robots using DEA approach with double frontiers: R12 - R11 - R4 - (R5 - R8) - R3 - R2 - R9 - R10 - R7 - R1 - R6. The best alternative obtained using proposed AHGTMA method is R12, which is the same as the one suggested by previous researchers. The second best alternative obtained is R11, which was also suggested as the second best by Karsak and Ahiska (2005) using common weight MCDM method and Wang and Chin (2009) using DEA approach with double frontiers. This validates the proposed AHGTMA method for the decision making situation of manufacturing environment.

### 3.3 Example 3: Welding process selection

Welding is one of the necessary manufacturing processes for joining different materials. The welding process is different from one material to another and choosing an appropriate method for welding is an essential task. Therefore, the selection of a welding process to accomplish a joint of desired specifications and quality is imperative before undertaking the fabrication task. The selection of the respective processes can be attributed to the fact that the desired weld quality is accomplished at the least cost and cost is therefore the main selection criterion. There are many cases where different processes can be effective in achieving the end product (Ravisankar et al., 2006).

The selection of the welding process is usually based on economic considerations and the welded joint properties. The economic factors include equipment, consumable (filler material, shielding gas, etc.), labor and overhead costs. The weld joint properties are mechanical and metallurgical properties of the joint. The mechanical properties are strength, hardness, impact strength, fatigue strength, residual stress level, etc. In addition, metallurgical analysis of the welded joints also include chemical composition analysis, micro-structural analysis and fracture surface analysis of the base metal, weld metal and heat affected zone, which will provide a correlation with the changes in mechanical properties. All these are the quantitative factors of the selection process. When different welding process alternatives are available to accomplish a particular joint, it is essential to base the final decision on quantitative as well as qualitative analysis (Muralidharan et al., 1999; Mohanty & Deshmukh, 1993). Unlike quantitative factors which are easily tractable, the analysis of qualitative factors requires a more meaningful approach.

This example is to select a welding technique to fabricate the butt joints of high strength aluminium alloy of AA 7075 grade, based on the qualitative factors of welding processes and it is taken from the research paper of Ravisankar et al. (2006) to further validate the proposed AHGTMA method. Many of the structural components in machines, pressure vessels, transport vehicles, earthmoving equipment, spacecraft, etc., are made of welded joints. The butt welds are the most common ones in the fabrication and construction of many structures. The following three welding processes from arc welding family are commonly employed to fabricate the butt welds of high strength aluminium alloys: (i) gas metal arc welding (GMAW), (ii) gas tungsten arc welding (GTAW), and (iii) plasma arc welding (PAW) (Little, 1990). Description of process attributes considered are as follows,

1. Initial preparation required (IPR): Clamping joints in fixtures, setting welding parameters (voltage, current, welding speed, gas flow rate, wire feed, etc.), electrode/filler metal preparation, cleaning the base metal;
2. Availability of consumables (AC): Electrodes, filler wires, shielding gases;
3. Welder skill requirements (WSR): Fully skilled welder, semiskilled welder, ordinary welder;
4. Welding procedures (WP): Pre-heating requirements, root pass requirements, number of passes required, inter-pass temperature maintenance, post-heating requirements;
5. Weld quality (WQ): Weld bead appearance, percentage of rejects due to welding defects (e.g. distortion, misalignment, porosity, lack of penetration, etc.);
6. Operator fatigue (OF): Arc glare, smoke and fumes, electrode changing, nozzle cleaning;
7. Post-weld cleaning (PC): Slag removal, spatter removal;
8. Ease of automation (EA): Manual, semi-automatic, fully automatic;
9. Positional welding capability (PWC): Horizontal welding, vertical welding, overhead welding, root pass welding;
10. Cost of welding (COW): Labor, consumable and power costs.

The pair wise comparison of welding processes with respect to each attribute is given in Table 9 and the relative importance of attributes is given in Table 10.

**Table 9**

Pair wise comparison of welding processes with respect to each attribute for Example 3 (Ravisankar, et al., 2006)

Comparison of processes on IPR				Comparison of processes on AC			
Process	GMAW	GTAM	PAW	Process	GMAW	GTAM	PAW
GMAW	1	1/7	3	GMAW	1	1/5	1/3
GTAM	7	1	5	GTAM	5	1	2
PAW	1/3	1/5	1	PAW	3	1/2	1
Comparison of processes on WSR				Comparison of processes on WP			
Process	GMAW	GTAM	PAW	Process	GMAW	GTAM	PAW
GMAW	1	1/5	1/3	GMAW	1	1/7	1/5
GTAM	5	1	3	GTAM	7	1	3
PAW	3	1/3	1	PAW	5	1/3	1
Comparison of processes on WQ				Comparison of processes on OF			
Process	GMAW	GTAM	PAW	Process	GMAW	GTAM	PAW
GMAW	1	1/9	1/5	GMAW	1	3	5
GTAM	9	1	7	GTAM	1/3	1	3
PAW	1/3	1/5	1	PAW	1/5	1/3	1
Comparison of processes on PC				Comparison of processes on EA			
Process	GMAW	GTAM	PAW	Process	GMAW	GTAM	PAW
GMAW	1	1/9	1/5	GMAW	1	6	3
GTAM	9	1	4	GTAM	1/6	1	1/3
PAW	5	1/4	1	PAW	1/3	3	1
Comparison of processes on PWC				Comparison of processes on COW			
Process	GMAW	GTAM	PAW	Process	GMAW	GTAM	PAW
GMAW	1	5	7	GMAW	1	1/3	5
GTAM	1/5	1	5	GTAM	3	1	4
PAW	1/7	1/5	1	PAW	1/5	1/4	1

The priority weights for the alternative welding processes with respect to each attribute are calculated using AHP process. The priority weights are given in Table 11.

It is found that the judgments of deciding the relative importance of attributes (Table 10) are consistent and the consistency ratio (CR) is equal to 0.0923.

**Table 10**

Relative importance of attributes for Example 3 (Ravisankar et al., 2006)

Attributes	IPR	AC	WSR	WP	WQ	OF	PC	EA	PWC	COW
IPR	1	1/2	1/9	1/7	1/9	1/7	1/3	1/5	1/5	1/9
AC	2	1	1/5	1/5	1/7	1/5	1/2	1/3	1/5	1/7
WSR	9	5	1	1/3	1/5	1/3	4	1/2	1/3	1/5
WP	7	5	3	1	1/2	2	9	6	1/2	1/2
WQ	9	7	5	2	1	3	6	4	5	1
OF	7	5	3	1/2	1/3	1	5	3	1/3	1/3
PC	3	2	1/4	1/9	1/6	1/5	1	1/7	1/7	1/6
EA	5	3	2	1/6	1/4	1/3	7	1	1/4	1/4
PWC	5	5	3	2	1/5	3	7	4	1	1/5
COW	9	7	5	2	1	3	6	4	5	1

**Table 11**

Priority weights for the alternative welding processes for Example 3

Alternative	IPR	AC	WSR	WP	WQ	OF	PC	EA	PWC	COW
1- GMAW	0.1702	0.1094	0.1047	0.0719	0.0545	0.6370	0.0603	0.6548	0.7147	0.3085
2- GTAW	0.7383	0.5816	0.6370	0.6491	0.7720	0.2583	0.7085	0.0953	0.2185	0.5957
3- PAW	0.0915	0.3090	0.2583	0.2789	0.1734	0.1047	0.2311	0.2499	0.0668	0.0958

Now, the next step is to form the “alternative selection attribute matrix” for each alternative and calculates the permanent value of the matrix formed for each alternative, i.e. the “index score” for the alternatives. The “alternative selection attribute matrix” for alternative 1, represented by  $C_1$  is shown below. The permanent of matrix  $C_1$  (i.e. index score for  $C_1$ ) obtained is 271.8269. The index score values for all alternatives are as follows,

$$C_1 = \begin{bmatrix} 0.1702 & 0.5 & 0.1111 & 0.1429 & 0.1111 & 0.1429 & 0.3333 & 0.2 & 0.2 & 0.1111 \\ 2 & 0.1094 & 0.2 & 0.2 & 0.1429 & 0.2 & 0.5 & 0.3333 & 0.2 & 0.1429 \\ 9 & 5 & 0.1047 & 0.3333 & 0.2 & 0.3333 & 4 & 0.5 & 0.3333 & 0.2 \\ 7 & 5 & 3 & 0.0719 & 0.5 & 2 & 9 & 6 & 0.5 & 0.5 \\ 9 & 7 & 5 & 2 & 0.0545 & 3 & 6 & 4 & 5 & 1 \\ 7 & 5 & 3 & 0.5 & 0.3333 & 0.6370 & 5 & 3 & 0.3333 & 0.3333 \\ 3 & 2 & 0.25 & 0.1111 & 0.1667 & 0.2 & 0.0603 & 0.1429 & 0.1429 & 0.1667 \\ 5 & 3 & 2 & 0.1667 & 0.25 & 0.3333 & 7 & 0.6548 & 0.25 & 0.25 \\ 5 & 5 & 3 & 2 & 0.2 & 3 & 7 & 4 & 0.7147 & 0.2 \\ 9 & 7 & 5 & 2 & 1 & 3 & 6 & 4 & 5 & 0.3085 \end{bmatrix}$$

The index score values for the alternatives are:

- 2- GTAW: 8826691.7728
- 1- GMAW: 7055994.4119
- 3- PAW: 6561441.9855

The ranking of alternative welding process obtained using proposed AHGTMA method is: GTAW > GMAW > PAW, which is exactly the same as suggested by Ravisankar, et. al. (2006) using AHP. The method suggests “GTAW” as the best alternative welding process to fabricate butt joints of high strength aluminium alloy of AA 7075 grade. Hence, the proposed AHGTMA method is again validated for solving the multiple attribute decision making problem of manufacturing environment.

#### 4. Conclusion

Selecting the best alternative is an important problem in the industrial environment considering various multiple performance attributes. This paper presents an integrated MADM method, analytical hierarchy graph theory and matrix approach (AHGTMA), for solving decision making problems of industrial situations. The proposed method of this paper allows the decision maker to systematically assign the values of relative importance to the attributes using AHP method.

Three MADM problems are solved using the proposed AHGTMA method and comparisons are made. The first problem was the electroplating system selection, the second problem was the robot selection, and the third problem was associated with the welding process selection. The results obtained by AHGTMA method for these problems are more logical as compared with the results of previous researchers.

The proposed AHGTMA method considers the values of the attributes and their relative importance together, and hence it provides a better accurate evaluation of the alternatives. The proposed AHGTMA method is a general method and can consider any number of quantitative and qualitative selection attributes, simultaneously. The suggested methodology can be extended to other decision making situations involving any number of alternatives and the selection attributes.

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