The *abc* Conjecture of the Derived Logarithmic Functions of Euler's Function and Its Computer Verification

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Abstract

Regarding Euler's (totient) function, for an arbitrary number $n > 1$, there exists a k that possesses the characteristic where $\varphi^k(n) = 1$. In this case, if k is expressed as $L(n)$ for n, then L possesses the characteristic of being perfectly logarithmic. For this L, we (Yamashita, Miyata) have provided the following L version *abc* conjecture.

Conjecture: When a, b, and c are relatively prime, numbers are natural, and $a + b = c$, then

$$\max\{L(a), L(b), L(c)\} < 2 \cdot L(rad(abc))$$

is feasible.

This paper describes the properties of L and presents verification that this conjecture is correct up to 10^9 using a computer experiment. We also note that the *abc* conjecture recently considered solved by Prof. Mochizuki at Kyoto University is different from the conjecture presented here.

Introduction

Considering $\varphi^k(x) = \varphi(\varphi^{k-1}(x))$ $(k > 1)$ as $\varphi^1(x) = \varphi(x)$ with respect to Euler's function φ, when $x > 1$ then $\varphi(x) < x$. Therefore, there always exists a minimum k such that $\varphi^k(x) = 1$ for all $x > 1$. Heretofore, in regard to the properties of this k, Pilali ([1],[2]), Shapiro ([3],[9]), Murányi ([4]), et al have shown that k possesses (imperfect) logarithmic characteristics. Since then, a great deal of research on this has been conducted. Currently, it is known that by modifying this k (hereinafter, this k shall be indicated as $L(x)$), that the same becomes perfectly logarithmic.[*1]

In this paper, we describe the properties and the extensions of the logarithmic function $L(\mathrm{x})$ derived of Euler's function and note that the abc conjecture pertaining to $L(x)$ we provide holds even under appropriate conditions other than primitive φ-triple, and also cite ours proof of this conjecture.

1. Various Properties of $L(x)$

1.1 Perfect logarithms of $L(x)$ and the evaluation thereof

Definition. 1. (Yamashita, [5]) *L is defined for the natural number n as follows and is called* a derived logarithmic function *of Euler's function*.

$$L(n) = \begin{cases} 0 & (n = 1) \\ L(\varphi(n)) & (n\text{: odd number} > 1) \\ L(\varphi(n)) + 1 & (n\text{: even number}). \end{cases}$$

At this time,

Proposition. 2. *L is perfectly logarithmic for any natural number x, y, i.e.*,

$$L(xy) = L(x) + L(y).$$

Therefore, the following simple evaluation can be obtained for L.

Proposition. 3. *If* $L(x) = n$, *then*

$$2^n \leq x \leq 3^n.$$

Then, immediately from there:

Corollary. 4. (E1) *If* $x \leq 2^n$ *then* $L(x) \leq n$.
(E2) *If* $x \geq 3^n$ *then* $L(x) \geq n$.

Corollary. 5. *Let* $x = 2^t \cdot x_0$ (x_0 : *odd*). *If* $L(x) = n$, *then*

$$x \leq 2^t \cdot 3^{n-t}.$$

Corollary. 6. *Let* $x = 2^t \cdot x_0$ (x_0 : *odd*). *If* $x > 2^t \cdot 3^{n-t}$, *then*

$$L(x) > n.$$

etc. can be obtained, and the following evaluation formula can also be obtained.

Proposition. 7.

(E3) $\log_3 2\,(\min(L(x), L(y)) + 1) \leq L(x+y)$
$\leq \log_2 3\,(\max(L(x), L(y)) + 1)$

(E4) $L(x-y) \leq \log_2 3\,\max(L(x), L(y))$

Remark: $\log_2 3$=1.58496250..., $\log_3 2$ = 0.63092975...

As for this L, we have also obtained the following theorem as an extension form of Euler's function φ.

Theorem. 8. (*Miyata–Yamashita*, [11], [12]) *Let* **P** *be a set of prime numbers and* **P** $\to$ **N** (*natural numbers*) *be a function such that* $1 \leq f(p) < \mathrm{p} \in \mathbf{P}$. *If*

$$\varphi_f(x) = x\prod_{i=1}^{r}\frac{f(p_i)}{p_i},\ x = p_1^{e_1}p_2^{e_2}...p_r^{e_r}$$

and

$$L_{\varphi f}(1) = 0$$
$$L_{\varphi f}(x) = L(\varphi_{\varphi f}(x)) + \#\{p \in f^{-1}(1) : p|x\}.$$

then

$$L_{\varphi f}(xy) = L_{\varphi f}(x) + L_{\varphi f}(y).$$

The φ_f in the above theorem is a formal generalization of Euler's function by f. Also, according to the symbol of this theorem, $L(x) = L_\varphi(x)$.

1.2 Extensibility of $L(x)$

L defined on the natural numbers can naturally be extended on $\mathbf{Z}\setminus\{0\}$ via $L(-1) = 0$, $L(-x) = L(x)$. For $L(0)$, if we define, for example, $L(0) = \infty$, it can also be is defined on **Z**. Therefore, if we define $L\left(\frac{x}{y}\right) = L(x) - L(y)$ for $\frac{x}{y} \in \mathbf{Q}^\times = \mathbf{Q}\setminus\{0\}$, then we have a natural extension to **Q**. In other words, the following holds:

Proposition. 9. *The L in Definition* 1 *can be naturally expanded on rational* *numbers* **Q** *and the properties of Proposition* 2 *are also inherited.*

Can this L (here is where we part ways with the world of Euler's function φ) be expanded to a number $Q[\sqrt{-1}]$ which is obtained by adding $\sqrt{-1}$ to real numbers **R** and **Q**, or complex number **C**, while maintaining the properties of Proposition 2?

Let us calculate by assuming the properties of Proposition 2. If we do some calculations with irrational numbers then,

- $L(\sqrt{2}) = L(2^{1/2}) = \frac{1}{2}L(2) = \frac{1}{2}$
- $L\left(\frac{1}{\sqrt{2}}\right) = L(2^{-1/2}) = -\frac{1}{2}L(2) = -\frac{1}{2}$
- $L(2^{\sqrt{2}}) = \sqrt{2}L(2) = \sqrt{2}$
- $L(2^{\pi}) = \pi L(2) = \pi$

When observing this situation, in order for L to be welldefined even on **R**, the range must be at least **R**.

In addition, let us continue to observe **C** as well.
If

$$\omega = \cos\left(\frac{2\pi}{n}\right) + \sqrt{-1}\sin\left(\frac{2\pi}{n}\right) \quad (n \in \mathbf{N})$$

then from

$$nL(\omega) = L(\omega^n) = L(1) = 0$$

we obtain

$$L(\omega) = 0.$$

In addition, if we let $\zeta = \cos\alpha + \sqrt{-1}\sin\alpha)$, and then take β as $\alpha\beta = 2\pi$, then

$$\beta L(\zeta) = L(\zeta^{\beta}) = L(1) = 0 \quad \text{より} \quad L(\zeta) = 0$$

Then, it will be $L(w) = 0$ for the point w on the unit circle of a complex plane, and the arbitrary z of **C** has the form $z = |z|w$. Therefore, we can obtain

$$L(z) = L(|z|w) = L(|z|) + L(w) = L(|z|).$$

However, there remain issues as to whether L can continue or extend in a well-defined manner from **Q** to **R** and **R** to **C** (including handling of transcendental numbers).

2. *abc* Conjecture for the Derived Logarithmic Function *L*

2.1 On the *abc* conjecture for *L*

Regarding this L, we have provided an *abc* conjecture (L version *abc* conjecture) pertaining to this derived logarithmic function L of Euler's function.

Conjecture. (Yamashita–Miyata [14])

Let a, b, c be coprime. If $a + b = c$, then

$$\max\{L(a), L(b), L(c)\} < 2 \cdot L(\mathrm{rad}(abc)).$$

Regarding this conjecture, we confirmed the correctness up to $c < 2^{30}$ by computer verification (Miyata-Yamashita [16]), and by touching lightly on the proof of the polynomial version *abc* conjecture by Stothers ([8]). The results we obtained were as follows.

Theorem. 10. (Yamashita–Miyata [14]) *Let a, b, c be coprime. If $a + b = c$, then*

$$\max\{L(a), L(b), L(c)\} < 2 \cdot L(\mathrm{rad}(abc)).$$

The condition of Theorem 10 where $\varphi(a) + \varphi(b) = \varphi(c)$ is feasible, (a, b, c), is called **primitive φ–triple** (Miyata–Yamashita [17]). Yamashita–Miyata have argued regarding the feasibility status of primitive φ–triple, and it is predicted to exist infinitely many times, and it is also known that the probability of existence of primitive φ–triple differs greatly due to the even/odd of c (Yamashita–Miyata [17]).

2.2 Cases other than primitive φ–triple

In Theorem 10 we asserted that our conjecture is correct in the case of primitive φ–triple (Yamashita–Miyata [11]). However, what about cases other than primitive φ–triple?

Let p and q be coprime, and assume

$$\frac{q}{p} = \frac{\varphi(a) + \varphi(b)}{\varphi(c)}$$

If so, then the following theorem holds.

Theorem. 11. *Let a, b, c be coprime. If $a + b = c > 2$, then under the following* *condition* (*)

$$(*)\quad \max\left(L(p), L(q)\right) \le \left(2 - \log_2 3\right) L\left(\mathrm{rad}\left(abc\right)\right)$$

we obtain

$$\max\{L(a), L(b), L(c)\} < 2 \cdot L\left(\mathrm{rad}\left(abc\right)\right).$$

Proof. When simultaneously both $a + b = c$ and $p\varphi(a) + p\varphi(b) = q\varphi(c)$, then we obtain

$$ac\left(q\frac{\varphi(c)}{c} - p\frac{\varphi(a)}{a}\right) = bc\left(p\frac{\varphi(b)}{b} - q\frac{\varphi(c)}{c}\right)$$

Then if we assume

$$q\frac{\varphi(c)}{c} - p\frac{\varphi(a)}{a} = 0$$

then

$$aq\varphi(c) = cp\varphi(a) = (a+b)\varphi(a)$$

On the other hand, $q\varphi(c) = p\varphi(a) + p\varphi(b)$ results via

$$(\#\ 1)\quad a\varphi(b) = b\varphi(a)$$

However it must be $a|\varphi(a)$ because $(a,\ b) = 1$ and it must be $a = 1$. Meanwhile, if $a = 1$, then it is $\varphi(b) = b$ via (# 1), therefore $b = 1$, resulting in a contradiction in $2 < c = a + b = 1 + 1 = 2$. Therefore,

$$q\frac{\varphi(c)}{c} - p\frac{\varphi(a)}{a} \neq 0.$$

From which follows:

$$\frac{a}{b} = \frac{p\frac{\varphi(b)}{b} - q\frac{\varphi(c)}{c}}{q\frac{\varphi(c)}{c} - p\frac{\varphi(a)}{a}}$$

$$= \frac{\mathrm{rad}\,(abc)\left(p\frac{\varphi(b)}{b} - q\frac{\varphi(c)}{c}\right)}{\mathrm{rad}\,(abc)\left(q\frac{\varphi(c)}{c} - p\frac{\varphi(a)}{a}\right)}$$

$$= \frac{p\left(\mathrm{rad}\,(abc)\frac{\varphi(b)}{b}\right)-q\left(\mathrm{rad}\,(abc)\frac{\varphi(c)}{c}\right)}{q\left(\mathrm{rad}\,(abc)\,q\frac{\varphi(c)}{c}\right)-p\left(\mathrm{rad}\,(abc)\frac{\varphi(a)}{a}\right)}$$

Then, if we note the fact that with $k = a,\ b,\ c$ then

$$\mathrm{rad}\,(abc)\frac{\varphi(k)}{k}\in\mathbf{N}$$

(hereinafter, rad (abc) will be denoted as rad*), then

$$a\,|\,p\left(\mathrm{rad}^*\frac{\varphi(b)}{b}\right)-q\left(\mathrm{rad}^*\frac{\varphi(c)}{c}\right).$$

Therefore,

$$L(a)\le L\left(p\left(\mathrm{rad}^*\frac{\varphi(b)}{b}\right)-q\left(\mathrm{rad}^*\frac{\varphi(c)}{c}\right)\right).$$

If the domain of L is expanded **Q** and we note that

$$L\left(\frac{\varphi(k)}{k}\right)=\begin{cases}-1 & (k\text{: even})\\ 0 & (k\text{: odd})\end{cases}$$

regarding $k = a, b, c$, we can then use Proposition 7 (E4), which results in the following right side of the above equation:

$$L\left(p\left(\mathrm{rad}^*\frac{\varphi(b)}{b}\right)-q\left(\mathrm{rad}^*\frac{\varphi(c)}{c}\right)\right).$$

Simply, if we denote as rad* $\frac{\varphi(k)}{k} = C(k)\ (k = a, b, c)$ then

$$\begin{aligned}
&L(a)\le \log_2 3\cdot\max\left(L(pC(b)),L(qC(c))\right)\\
&\le \log_2 3\left(\left(\frac{2}{\log_2 3}-1\right)L(\mathrm{rad}^*)+\max(L(C(b)),L(C(c)))\right)\\
&\le \log_2 3\left(\left(\frac{2}{\log_2 3}-1\right)L(\mathrm{rad}^*)+L(\mathrm{rad}^*)\right)\\
&=2L(\mathrm{rad}^*)=2L(\mathrm{rad}\,(abc))
\end{aligned}$$

In the case of primitive φ–triple, since $L(p) = L(q) = L(1) = 0$, then the conditions of Theorem 11 are satisfied and it can then be obtained as a corollary.

Corollary. 12. (Theorem 10) *If a* primitive φ–triple *then*

$$\max\{L(a), L(b), L(c)\} < 2 \cdot L(\text{rad}\,(abc))\ /$$

3. Computer Verification of the Conjecture

3.1 The difficulty of computer verification for $c \leq N = 10^{10}$

For our conjecture, computer experiments have confirmed that the conjecture is true for $c < 2^{30}$ (Miyata–Yamashita [15]), but with $c \geq 2^{30}$ and above it is difficult to verify using a typical PC environment.

Generally, the problem of finding $\varphi(x)$ for x is called an RSA problem, and if $\varphi(x)$ is easily obtained, the RSA public key encryption problem terminates, hence this is a very challenging problem.

Since it is necessary to repeatedly calculate $\varphi(x)$ to calculate $L(x)$, finding $L(x)$ involves more difficulty than the RSA problem.

On the other hand, if $L(x)$ is found for all x where $O(N \log \log N)$, it is known that time complexity $O(N \log \log N)$ can be used [15]. With that method, $L(x)$ can be obtained with $O(\log \log N)$ per each case.

However, this method requires a storage area for $O(N)$ which amounts to 4 GB of memory for $N = 10^9$.

In order to execute $N = 10^{10}$ in the same way (since the integers to be handled exceed 32 bits, it would mean using a 64-bit integer type), 80 GB of memory is required, which is impossible to execute on a typical PC.

As follows, verification was performed at $c \leq N = 10^{10}$ for $(1, b, c)$. The verification results are shown in Table 1, and $q(1, b, c)$ in the Table is called a quality of $(1, b, c)$, expressed as

$$q(1,b,c) = \frac{L(c)}{L(\text{rad}\,(bc))}.$$

The verification environment was as follows:

- PC: Acer Veriton X4620G
- OS: Windows 8.1 Pro
- CPU: Intel Core *i*5-3340 CPU (3.10GHz)
- RAM: 12.0GB
- Language: Java 9.0.1 (64-bit) Java (TM) SE Develoment Kit 9.0.1 (64-bit)
- Software: Eclipse

Execution time: 36 minutes 54 seconds

3.2 Computer verification for $c < 10^{10}$ regarding $(1, b, c)$

3.2.1 memoization

In the verification of $(1, b, c)$, $L(x)$ and $L(\mathrm{rad}\,(x))$ were calculated in advance for $x \le 10^8$ using the Miyata–Yamashita method ([15]). As necessary, for reference, memoization was implemented. The storage area required for this is about 800 MB.

To obtain $L(x)$ for $x > 10^8$, the function was first factorized into prime numbers by trial division to obtain $\varphi(x)$, and then the memo could be referenced with values less than 10^8. When $x > 10^8$, we sought to the greatest extent possible not to evaluate $L(x)$.

3.2.2 Finding the maximum prime of *c*

In the verification algorithm, for $S = 10^6$, the calculation was performed by dividing 10^{10} into segments of size S.

For example, for the k–th segment, calculation is performed for $c = kS + 1, kS + 2, \dots, (k+1)\,S$, and then, using the segmented sieve algorithm, the largest prime factor of each c was sought.

If $S \le \sqrt{N}$, N(i.e.,$O(S\ \log N)$ per each one), $O(S\ \log N)$ is sufficient for the calculation amount. Also, the storage area was $O(S)$ (in reality, $16S$ bytes = 16 MB required).

Let p be the largest prime factor of and $c = xp$. If $p < 10^8$, $L(c)$ and $L(\mathrm{rad}(c))$ can be computed at high speed.

The reason being, if $p > 100$ then $x < 10^8$, it is therefore sufficient to merely reference the memo for both $L(x)$ and $L(p)$ because $L(c) = L(x) + L(p)$.

On the other hand, if $p < 100$, c can be factorized into prime numbers at high speed because it is rendered with the product of small prime numbers less than 100.

3.3 $(1, b, c)$–triple determination

Definition. 13. *Let* $1 + b = c$. $(1, b, c)$ *that satisfy*

$$\frac{L(c)}{L(\mathrm{rad}(abc))} \geq 1.25$$

is called $(1, b, c)$–**triple**.

In this verification, we will judge whether each c is

$$\frac{L(c)}{L(\mathrm{rad}(p-1))+L(\mathrm{rad}(c))} \geq 1.25$$

Determination is conducted as follows, with p being the maximum prime factor of c, and q being the maximum prime factor of $c - 1$.

3.3.1 Case $p \geq 10^8$

If $c = p$, then $L(p) \,/\, (L(\mathrm{rad}\,(c-1)) + L(p)) < 1$.

If $c = xp$, then $1 < x < 100$. Therefore,

$$\begin{aligned}\frac{L(c)}{L(\mathrm{rad}(c-1))+L(\mathrm{rad}(c))} &\leq \frac{L(x)+L(p)}{2+L(p)} \\ &= 1+\frac{-2+L(x)}{2+L(p)} \\ &\leq 1+\frac{-2+\log_2 10^2}{2+\log_3 10^8} < 1.248.\end{aligned}$$

Therefore, in this case, $(1, c-1, c)$ does not become a triple.

3.3.2 Case $p < 10^8$ and $q < 10^8$

In this case, $L(c)$, $L(\mathrm{rad}\,(c))$, $L(c-1)$, $L(\mathrm{rad}\,(c-1))$ can be calculated at high speed. So we actually calculate as

$$\frac{L(c)}{L(\mathrm{rad}(c-1))+L(\mathrm{rad}(c))}$$

and then we simply need to investigate whether it is 1.25 or higher or not.

3.3.3 Case $p < 10^8$ and $q \geq 10^8$

Since $L(q) \geq \log_3 10^8$, then we first seek out

$$\frac{L(c)}{L(\mathrm{rad}(c))+1+\log_3 10^8}$$

If this is not 1.25 or higher, then we can determine that is not a triple.

If not, then we calculate $L(c-1)$ and $L(\mathrm{rad}\,(c-1))$ while factorizing into prime numbers, then determine whether it is

$$\frac{L(c)}{L(\mathrm{rad}\,(c-1))+L(\mathrm{rad}\,(c))} \geq 1.25$$

or not.

By the way, the number of cases where it was necessary to calculate $L(c-1)$ and $L(\mathrm{rad}\,(c-1))$ by prime factorization was only several hundred times out of $c \leq 10^{10}$. Of those, those that were 1.25 or higher more were 0 times.

3.3.4 The reason for a threshold of 1.25

There are three reasons why we used 1.25 as the sieving threshold for the verification algorithm.

Reason1: The lower the threshold, the higher the number of corresponding triples. And, for this study, we were not interested in small triples.

Reason2: Our conjecture was

$$\frac{\max\{L(c-1), L(c)\}}{L(\mathrm{rad}\,(c-1))+L(\mathrm{rad}\,(c))} < 2$$

but the enumeration is

$$\frac{L(c)}{L(\mathrm{rad}\,(c-1))+L(\mathrm{rad}\,(c))} \geq 1.25.$$

If there is a c such that

$$\frac{\max\{L(c-1), L(c)\}}{L(\mathrm{rad}\,(c-1))+L(\mathrm{rad}\,(c))} \geq 2,$$

then it will be

$$\frac{L(c)}{L(\mathrm{rad}(c-1))+L(\mathrm{rad}(c))} \geq 2\log_3 2 > 1.26$$

which means that it will always be included in this enumeration. Therefore, setting the threshold to 1.25 makes it possible to verify that there is no counterexample.

Reason3: As a practical reason, we tried memoization for 10^8 or less as we wanted to be able to be execute this verification using a personal computer of ordinary specifications. (10^9 is impossible to do without a slightly high-performance personal computer.)

4. Summary and Future Issues

In this study, we examined the domain extensibility of $L(x)$, further improved results using primitive φ–triple, and showed our conjecture is correct for non-primitive φ–triples as long as certain conditions were met.

In terms of verifying our conjecture, we focused on $(1, b, c)$ and verified that our conjecture is correct for $C \leq 10^{10}$.

In terms of future issues, we still need a proof for our conjecture's feasibility, but we also need to verify our conjecture. For the time being, we will further increase N until $c \leq N$. However, of note are the following:

- In the case of $(1, b, c)$, we will increase the evaluation accuracy of the inequality in Section 4.5.1 and lower the sieve threshold to below 1.248.
- We will optimally apply the inequality condition of Theorem 11 to the verification algorithm.

This paper is a partial addition to [19].

Notes

*1. Yamashita showed that k according to a different definition from theirs was completely logarithmic in his high school days ([5]). After that, in 1977, during correspondence with Professor Saburo Uchiyama (Tsukuba University) (Yamashita-Uchiyama, Uchiyama-Yamashita [6],[7]) he learned for the first time of Pilali ([1],[2]), Shapiro ([3]), and Murányi's work ([4]). However, at this timing, facts in a perfect logarithmic form were not known in academic circles. It was not perfectly logarithmic in the first edition of Shapiro's textbook in 1983 ([9]). The first time it become known that it was perfectly logarithmic in academic circles was in the note made by Prasad, et al. ([10]).

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Calculation Results

b	c	$q(1, b, c)$
$2 \cdot 3^7$	$5^4 \cdot 7$	1.667
$19 \cdot 509^3$	$2^{19} \cdot 3^4 \cdot 59$	1.647
$3 \cdot 5^5 \cdot 47^2$	$2^{18} \cdot 79$	1.643
$3^9 \cdot 7^2 \cdot 197$	$2^7 \cdot 5^7 \cdot 19$	1.6
$3^{16} \cdot 7$	$2^3 \cdot 11 \cdot 23 \cdot 53^3$	1.563
$2^4 \cdot 3^7 \cdot 547$	$5^8 \cdot 7^2$	1.538
$3^2 \cdot 7$	2^6	1.5
$3^3 \cdot 7 \cdot 19 \cdot 73$	2^{18}	1.5
$11^4 \cdot 47$	$2^{15} \cdot 3 \cdot 7$	1.5
$2^{11} \cdot 3^3 \cdot 19$	$5^4 \cdot 41^2$	1.5
$5^4 \cdot 367$	$2^{15} \cdot 7$	1.417
$31 \cdot 127^2$	$2^5 \cdot 5^6$	1.417
$7^2 \cdot 127 \cdot 337$	2^{21}	1.4
$2^6 \cdot 3 \cdot 5 \cdot 7 \cdot 13^4 \cdot 17$	239^4	1.4
$3^7 \cdot 13 \cdot 23^2$	$2^9 \cdot 5^4 \cdot 47$	1.375
$7^2 \cdot 43^4$	$2 \cdot 5^4 \cdot 13^3 \cdot 61$	1.353

b	c	$q(1, b, c)$
$5^4 \cdot 19 \cdot 15541$	$2^{24} \cdot 11$	1.35
$3^{13} \cdot 1277$	$2^3 \cdot 19^2 \cdot 89^3$	1.35
$2^3 \cdot 7^5 \cdot 13^2 \cdot 109$	$3^3 \cdot 11^3 \cdot 41^3$	1.35
$2^5 \cdot 3^2$	17^2	1.333
$2^5 \cdot 3 \cdot 5^2$	7^4	1.333
$3^2 \cdot 5 \cdot 7 \cdot 13$	2^{12}	1.333
$3^4 \cdot 79$	$2^8 \cdot 5^2$	1.333
$5 \cdot 11^3$	$2^9 \cdot 13$	1.333
$2^6 \cdot 3^2 \cdot 5 \cdot 29$	17^4	1.333
$2^5 \cdot 3 \cdot 5 \cdot 7 \cdot 29^2$	41^4	1.333
$5^3 \cdot 7^4 \cdot 11$	$2^{13} \cdot 13 \cdot 31$	1.333
$7^4 \cdot 2399$	$2^{10} \cdot 3^2 \cdot 5^4$	1.333
$2^5 \cdot 3^3 \cdot 7 \cdot 13 \cdot 307$	17^6	1.333
$3^7 \cdot 53 \cdot 131^2$	$2^{20} \cdot 7 \cdot 271$	1.333
$3^9 \cdot 5^4 \cdot 709$	$2^6 \cdot 53 \cdot 137^3$	1.333
$2^6 \cdot 3^{10} \cdot 331$	$17^5 \cdot 881$	1.318
$7^2 \cdot 71^2 \cdot 223$	$2^{15} \cdot 41^2$	1.316
$2^{14} \cdot 8111$	$3^5 \cdot 5^7 \cdot 7$	1.313
$7^3 \cdot 487$	$2 \cdot 17^4$	1.308
$7^2 \cdot 13^2 \cdot 186391$	$2^{26} \cdot 23$	1.304
$19^3 \cdot 23^2 \cdot 1613$	$2^{19} \cdot 3 \cdot 61^2$	1.304
$2^{12} \cdot 5^3$	$3^5 \cdot 7^2 \cdot 43$	1.3
$31^3 \cdot 79^2$	$2^{16} \cdot 2837$	1.3
$3^7 \cdot 11 \cdot 19^2 \cdot 31$	$2^{18} \cdot 13 \cdot 79$	1.3
$3^2 \cdot 7^3 \cdot 19^4$	$2^{13} \cdot 49109$	1.3
$7^4 \cdot 13 \cdot 23^2 \cdot 59$	$2^4 \cdot 3^6 \cdot 17^4$	1.3
$5 \cdot 139^3$	$2^7 \cdot 3 \cdot 11^2 \cdot 17^2$	1.294
$2^7 \cdot 3 \cdot 5^2 \cdot 7^4$	4801^2	1.294
$2^3 \cdot 3^7 \cdot 5^4 \cdot 7$	$13^2 \cdot 673^2$	1.294
$2 \cdot 3 \cdot 281^3$	$7^5 \cdot 89^2$	1.294

b	c	$q(1,b,c)$
$3 \cdot 157 \cdot 3323^2$	$2^{25} \cdot 5 \cdot 31$	1.292
$3^5 \cdot 5$	$2^6 \cdot 19$	1.286
$3^7 \cdot 13 \cdot 17$	$2^{13} \cdot 59$	1.286
$2^3 \cdot 3^3 \cdot 5 \cdot 7^3 \cdot 127$	19^6	1.286
$3^6 \cdot 5^3 \cdot 4003$	$2^{17} \cdot 11^2 \cdot 23$	1.286
$3^{14} \cdot 311$	$2^{15} \cdot 5 \cdot 7 \cdot 1297$	1.286
$3^6 \cdot 5 \cdot 493291$	$2^{18} \cdot 19^3$	1.286
$2^{10} \cdot 3 \cdot 5^2 \cdot 43 \cdot 1321$	257^4	1.28
$2^7 \cdot 3^2 \cdot 5 \cdot 29 \cdot 41761$	17^8	1.28
$23^2 \cdot 109 \cdot 491$	$2^{20} \cdot 3^3$	1.278
$3 \cdot 43 \cdot 127$	2^{14}	1.273
$3^5 \cdot 5 \cdot 7^2$	$2^4 \cdot 61^2$	1.273
$2 \cdot 3^3 \cdot 11^3$	$5^5 \cdot 23$	1.273
$7^2 \cdot 17^3 \cdot 2143$	$2^{22} \cdot 3 \cdot 41$	1.273
$47^3 \cdot 53 \cdot 109$	$2^{22} \cdot 11 \cdot 13$	1.273
$19 \cdot 37^3 \cdot 937$	$2^{22} \cdot 5 \cdot 43$	1.273
$5^3 \cdot 71^2 \cdot 2971$	$2^{17} \cdot 3^3 \cdot 23^2$	1.273
$13^3 \cdot 43 \cdot 163^2$	$2^7 \cdot 5^7 \cdot 251$	1.273
$2^4 \cdot 3^2 \cdot 5^3 \cdot 7 \cdot 17^2 \cdot 109$	251^4	1.273
524287	2^{19}	1.267
$3 \cdot 7^3 \cdot 11^3$	$2^9 \cdot 5^2 \cdot 107$	1.267
$3^4 \cdot 37 \cdot 79 \cdot 173$	$2^{16} \cdot 5^4$	1.263
$3^8 \cdot 7 \cdot 937$	$2^{10} \cdot 5^2 \cdot 41^2$	1.263
$3 \cdot 5 \cdot 7 \cdot 11^3 \cdot 317$	$2^{18} \cdot 13^2$	1.263
$2^4 \cdot 3^2 \cdot 7 \cdot 11 \cdot 13^2 \cdot 79$	23^6	1.263
$3^6 \cdot 17^3 \cdot 71$	$2^{12} \cdot 7^3 \cdot 181$	1.263
$3^3 \cdot 5^2 \cdot 7^3 \cdot 5779$	$2^{22} \cdot 11 \cdot 29$	1.261
$2^{14} \cdot 7^4 \cdot 13^2$	$17^3 \cdot 29^2 \cdot 1609$	1.261
19^3	$2^2 \cdot 5 \cdot 7^3$	1.25
$3^4 \cdot 7 \cdot 11^2$	$2^{10} \cdot 67$	1.25

b	c	$q(1, b, c)$
$3^5 \cdot 643$	$2 \cdot 5^7$	1.25
$5 \cdot 7^4 \cdot 19$	$2^8 \cdot 3^4 \cdot 11$	1.25
$3 \cdot 5^2 \cdot 11 \cdot 31 \cdot 41$	2^{20}	1.25
$5 \cdot 29 \cdot 47^3$	$2^9 \cdot 3^5 \cdot 11^2$	1.25
$2^4 \cdot 5^2 \cdot 7^2 \cdot 13^2 \cdot 29$	$3^8 \cdot 11^4$	1.25
$5^9 \cdot 163$	$2^4 \cdot 3^3 \cdot 23 \cdot 179^2$	1.25
$3^2 \cdot 7 \cdot 11 \cdot 31 \cdot 151 \cdot 331$	2^{30}	1.25
$3^8 \cdot 13^2 \cdot 2311$	$2^{18} \cdot 5^2 \cdot 17 \cdot 23$	1.25
$2^2 \cdot 5^4 \cdot 17^3 \cdot 211$	$3^3 \cdot 7^3 \cdot 23^4$	1.25