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Exact solution of thermal radiation on vertical oscillating plate with variable temperature and mass flux

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Abstract

Thermal radiation effects on unsteady flow past an infinite vertical oscillating plate in the presence of variable temperature and uniform mass flux is considered. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The plate temperature is raised linearly with time and the mass is diffused from the plate to the fluid at an uniform rate. The dimensionless governing equations are solved using the Laplace transform technique. The velocity, concentration and temperature are studied for different physical parameters like the phase angle, radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing phase angle ωt.

Keywords: vertical plate, oscillating, radiation, mass flux.

Nomenclature

a^*	absorption coefficient
C'	species concentration in the fluid
C'_w	concentration of the plate
C'_∞	concentration in the fluid far away from the plate
C	dimensionless concentration

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C_p	specific heat at constant pressure
D	mass diffusion coefficient
g	acceleration due to gravity
Gr	thermal Grashof number
Gc	mass Grashof number
j''	mass flux per unit area at the plate
k	thermal conductivity of the fluid
Pr	Prandtl number
q_r	radiative heat flux in the y-direction
R	radiation parameter
Sc	Schmidt number
T_∞	temperature of the fluid far away from the plate
T_w	temperature of the plate
T	temperature of the fluid near the plate
t'	time
t	dimensionless time
u	velocity of the fluid in the x-direction
u_0	velocity of the plate
U	dimensionless velocity
y	coordinate axis normal to the plate
Y	dimensionless coordinate axis normal to the plate
β	volumetric coefficient of thermal expansion
β^*	volumetric coefficient of expansion with concentration
η	similarity parameter
$erfc$	complementary error function
μ	coefficient of viscosity
ν	kinematic viscosity
ρ	density
τ	dimensionless skin-friction
θ	dimensionless temperature
ωt	phase angle

1 Introduction

The interaction of convection and radiation in adsorbing-emitting media occurs in many practical cases. Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

England and Emery[1] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar[2] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[3]. In all above studies, the stationary vertical plate is considered. Das *et al*[4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Raptis and Perdikis[5] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Thermal radiation on unsteady free convection and mass transfer boundary layer over a vertical moving plate were analyzed by Raptis and perdikis[6].

The flow of a viscous, incompressible fluid past an infinite isothermal vertical plate, oscillating in its own plane, was solved by Soundalgekar [7]. The effect on the flow past a vertical oscillating plate due to a combination of concentration and temperature differences was studied extensively by Soundalgekar and Akolkar [8]. The effect of mass transfer on the flow past an infinite vertical oscillating plate in the presence of constant heat flux has been studied by Soundalgekar et al. [9].

However the thermal radiation effects on moving infinite vertical plate in the presence variable temperature and uniform mass flux is not studied in the literature. It is proposed to study thermal radiation effects on flow past an impulsively started infinite vertical plate with variable temperature and uniform mass flux. The dimensionless governing equations are solved using the Laplace transform technique.

2 Mathematical Analysis

Thermal radiation effects on unsteady flow of a viscous incompressible fluid past an impulsively started infinite vertical oscillating plate with variable temperature and uniform mass flux is studied. Here the x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate starts oscillating in its own plane with frequency ω' and the temperature of the plate is raised linearly with time and the mass diffused from the plate to the fluid at an uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Under the above assumptions the flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu\frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho\ C_p\ \frac{\partial T}{\partial t'} = k\ \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\ \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

Where, $\frac{\partial q_r}{\partial y}$ represents the change in the radiative flux with distance normal to the plate. The initial and boundary conditions may be expressed as:

$$\begin{array}{lll} t' \leq 0: & u = 0, \quad T = T_\infty, & C' = C'_\infty \ \text{ for all } \ y \\ t' > 0: & u = u_0, \quad T = T_\infty + (T_w - T_\infty)\ A\ t', & \dfrac{\partial C'}{\partial y} = -\dfrac{j''}{D} \ \text{at} \ \ y = 0 \\ & u = 0, \quad T \to T_\infty, & C' \to C'_\infty \quad \text{as} \quad y \to \infty \end{array} \tag{4}$$

where $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^* \ \sigma \left(T_\infty^4 - T^4\right) \tag{5}$$

It is assumed that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3 \ T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho \ C_p \ \frac{\partial T}{\partial t'} = k \ \frac{\partial^2 T}{\partial y^2} + 16a^* \sigma T_\infty^3 \ (T_\infty - T) \tag{7}$$

On introducing the following dimensionless quantities:

$$\begin{aligned} & U = \frac{u}{u_0}, \ t = \frac{t' u_0^2}{\nu}, \ Y = \frac{y u_0}{\nu}, \ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \\ & Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3}, C = \frac{C' - C'_\infty}{\left(\frac{j''\nu}{D u_0}\right)}, \ Gc = \frac{\nu g \beta^* \left(\frac{j''\nu}{D u_0}\right)}{u_0^3}, \\ & Pr = \frac{\mu C_p}{k}, \ Sc = \frac{\nu}{D}, \ R = \frac{16 a^* \nu^2 \sigma T_\infty^3}{k u_0^2} \end{aligned} \tag{8}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr \ \theta + Gc \ C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr} \ \theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllll} & U = 0, & \theta = 0, & C = 0, & \text{for all} \quad Y, t \leq 0 \\ t > 0: & U = 1, & \theta = t, & \frac{\partial C}{\partial Y} = -1, & \text{at} \quad Y = 0 \\ & U = 0, & \theta \to 0, & C \to 0 & \text{as} \quad Y \to \infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned}\theta = \frac{t}{2}\Big[&\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at})\\ &+\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})\Big]\\ &-\frac{\eta Pr\sqrt{t}}{2\sqrt{R}}\Big[\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})\\ &-\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at})\Big]\end{aligned} \tag{13}$$

$$C = 2\sqrt{t}\left[\frac{\exp(-\eta^2 Sc)}{\sqrt{\pi}\ \sqrt{Sc}} - \ \eta\ erfc(\eta\sqrt{Sc})\right] \tag{14}$$

$$\begin{aligned}U = \frac{\exp(i\omega t)}{4}\Big[&\exp(2\eta\sqrt{i\omega t})\ erfc(\eta+\sqrt{i\omega t})\\ &+\exp(-2\eta\sqrt{i\omega t})\ erfc(\eta-\sqrt{i\omega t})\Big]\\ &+\frac{\exp(-i\omega t)}{4}\Big[\exp(2\eta\sqrt{-i\omega t})\ erfc(\eta+\sqrt{-i\omega t})\\ &+\exp(-2\eta\sqrt{-i\omega t})\ erfc(\eta-\sqrt{-i\omega t})\Big]\\ &-\frac{c\exp(bt)}{2}\Big[\exp(2\eta\sqrt{bt})\ erfc(\eta+\sqrt{bt})\\ &+\exp(-2\eta\sqrt{bt})\ erfc(\eta-\sqrt{bt})\Big]\\ &-\frac{Gc\ t\ \sqrt{t}}{3(1-Sc)\sqrt{Sc}}\Big[\frac{4}{\sqrt{\pi}}(1+\eta^2)\exp(-\eta^2)\\ &-\frac{4}{\sqrt{\pi}}(1+\eta^2 Sc)\ \exp(-\eta^2 Sc)\end{aligned} \tag{15}$$

$$-\eta\ (6+4\eta^2)\ erfc(\eta)+\eta\ \sqrt{Sc}\ (6+4\ \eta^2\ Sc)\ erfc(\eta\ \sqrt{Sc})\Big]$$
$$-\frac{c(1+bt)}{2}\Big[\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}$$
$$+\sqrt{at})+\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})\Big]$$
$$+\frac{bc\eta Pr\sqrt{t}}{2\sqrt{R}}\Big[\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})$$
$$-\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at})\Big]$$
$$+\frac{c\exp(bt)}{2}\ \Big[\exp(2\eta\sqrt{Pr(a+b)t})\ erfc(\eta\ \sqrt{Pr}+\sqrt{(a+b)t})+$$
$$+\exp(-2\eta\sqrt{Pr(a+b)t})\ erfc(\eta\ \sqrt{Pr}-\sqrt{(a+b)t})\Big],$$

where, $\eta = Y/2\sqrt{t},\ a=\dfrac{R}{Pr},\ b=\dfrac{R}{1-Pr}, c=\dfrac{Gr}{b^2(1-Pr)}$.

In order to get the physical insight into the problem, the numerical values of U have been computed from (15). While evaluating this expression, it is observed that the argument of the error function is complex and, hence, we have separated it into real and imaginary parts by using the following formula:

$$erf(a+ib)=erf(a)+\frac{\exp(-a^2)}{2a\pi}\ [1-cos(2ab)+isin(2ab)]$$
$$+\frac{2\exp(-a^2)}{\pi}\ \sum_{n=1}^{\infty}\ \frac{\exp(-n^2/4)}{n^2+4a^2}\ [f_n(a,b)+ig_n(a,b)]+\epsilon(a,b)$$

where $|\epsilon(a,b)|\ \approx\ 10^{-16}\ |erf(a+ib)|$,

$f_n = 2a-2a\ cosh(nb)\ cos(2ab)+n\ sinh(nb)\ sin(2ab)$

and

$g_n = 2a\ cosh(nb)\ sin(2ab)+n\ sinh(nb)\ cos(2ab).$

3 Discussion of Results

In order to get a physical insight of the problem numerical calculations are carried out for different values of the phase angle, radiation parameter, Schmidt number, thermal Grashof number and mass Grashof number and time. The purpose of the calculations given here is to assess the effects of the parameters $\omega t, R, Gr, Gc, Sc$ and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of exponential and complementary error function.

The temperature profiles are calculated for different values of thermal radiation parameter($R = 0.2, 2, 5, 10$) from Equation (13) and these are shown in Figure 1. for air (Pr=0.71) at $t = 1$. The effect of thermal radiation parameter is important in temperature profiles. It is observed that the temperature increases with decreasing radiation parameter. Figure 2 is a graphical representation which depicts the temperature profiles for different values of the time ($t = 0.2, 0.4, 0.6, 1$) and $Pr = 0.71$ in the presence of thermal radiation $R = 0.2$. It is clear that the temperature increases with increasing values of the time t.

Figure 3 represents the effect of concentration profiles at time $t = 0.2$ for different Schmidt number($Sc = 0.16, 0.3, 0.6, 2.01$). The effect of concentration is important in concentration field. The profiles have the common feature that the concentration decreases in a monotone fashion from the surface to a zero value far away in the free stream. It is observed that the wall concentration increases with decreasing values of the Schmidt number. Figure 4 is a graphical representation which depicts the concentration profiles for different values of the time ($t = 0.2, 0.4, 0.6, 1$) and $Sc = 0.6$. It is clear that the plate concentration increases with increasing values of the time t.

The velocity profiles for different phase angles ($\omega t = 0, \pi/6, \pi/3, \pi/2$), $Gr = 2, Gc = 2, R = 5, Sc = 0.6, Pr = 0.71$ and $t = 0.2$ are shown in figure 5. It is observed that the velocity increases with decreasing phase angle ωt. The velocity profiles for different values of the radiation parameter($R = 0.2, 5, 20$), $\omega t = \pi/6, Gr = Gc = 5, Sc = 0.6, Pr = 0.71$ and $t = 0.4$ are shown in figure 6. The trend shows that the velocity increases with decreasing radiation parameter. This shows that the velocity decreases in the presence of high thermal radiation.

In figure 7, the effect of velocity for different values of the time ($t =$

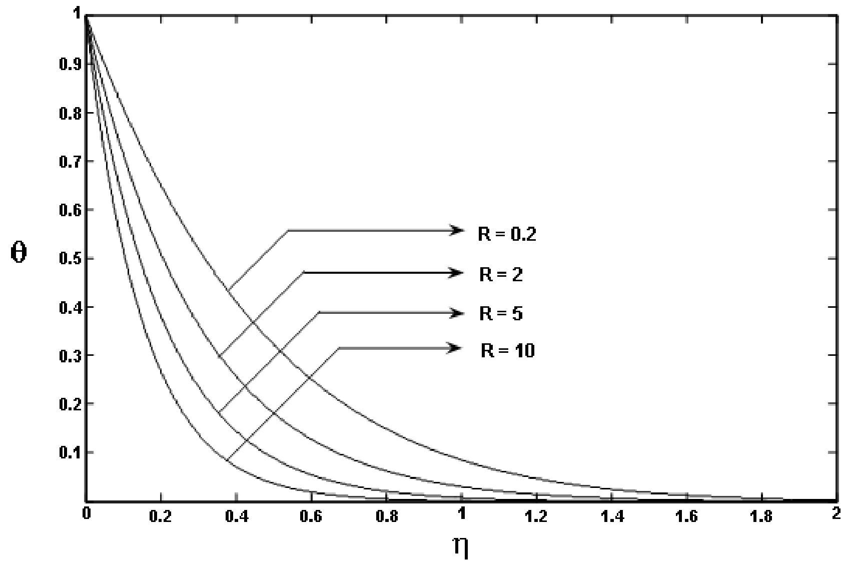

Figure 1: Temperature profiles for different values of R

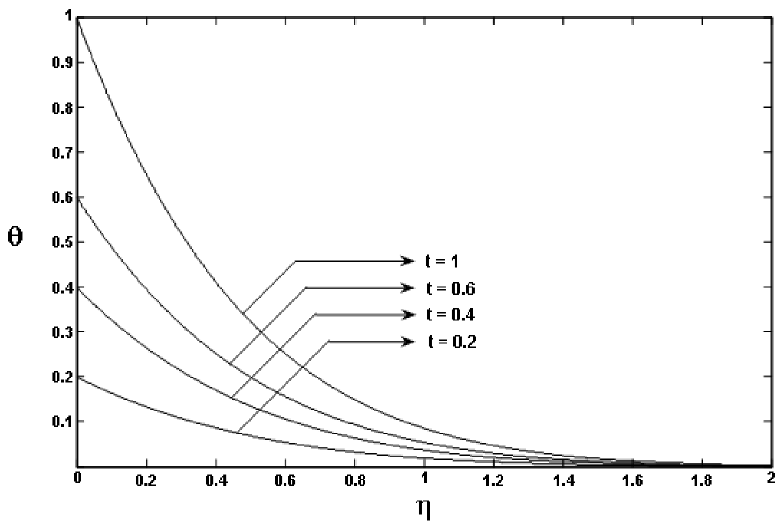

Figure 2: Temperature profiles for different values of t

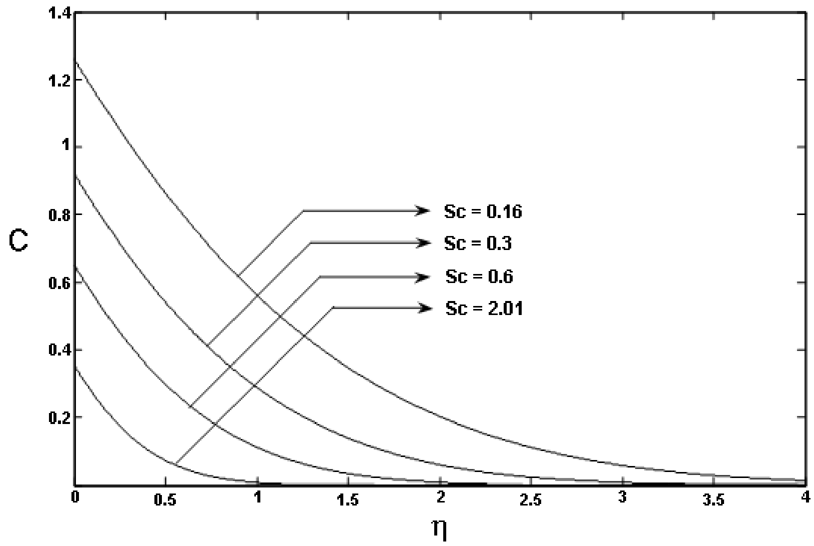

Figure 3: Concentration profiles for different values of Sc

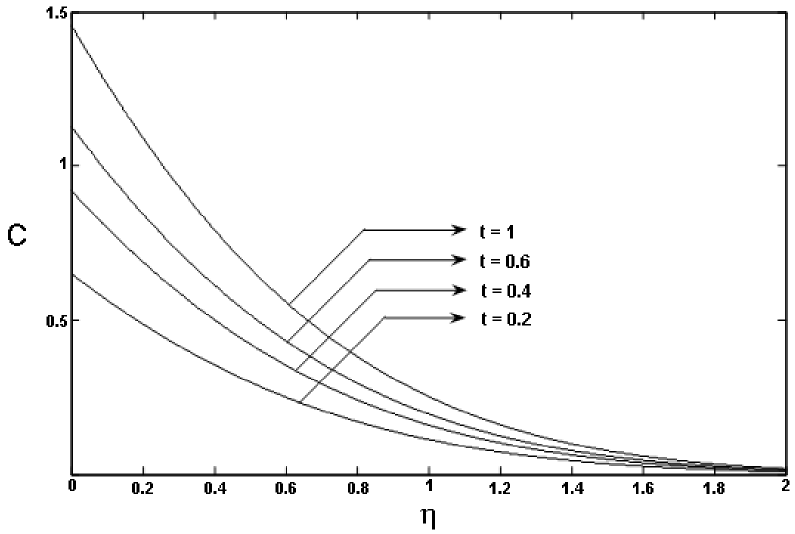

Figure 4: Concentration profiles for different values of t

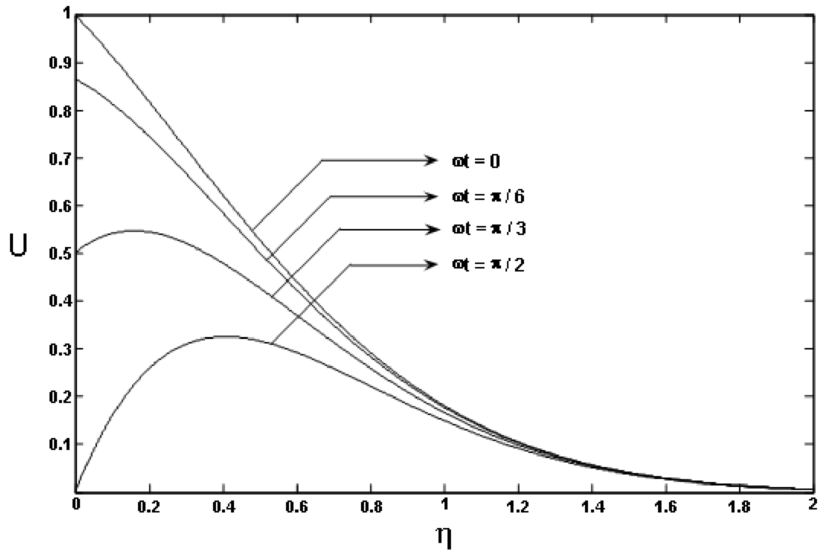

Figure 5: Velocity profiles for different values of t

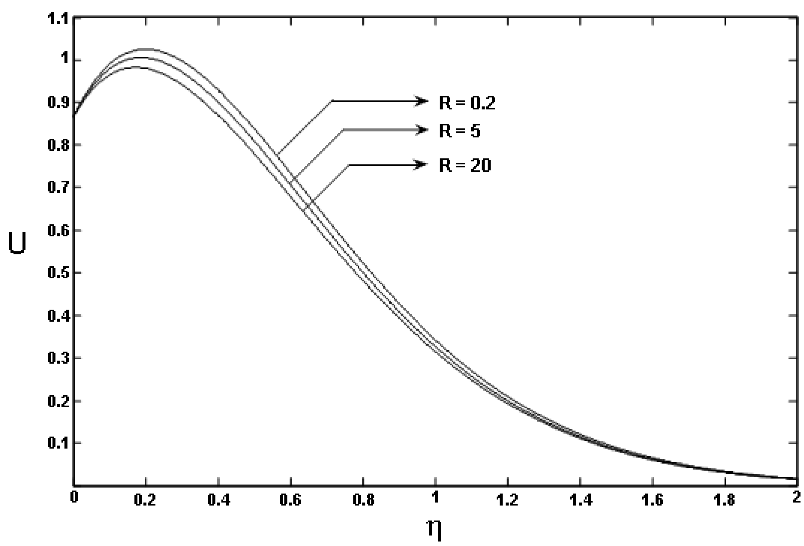

Figure 6: Velocity profiles for different values of R

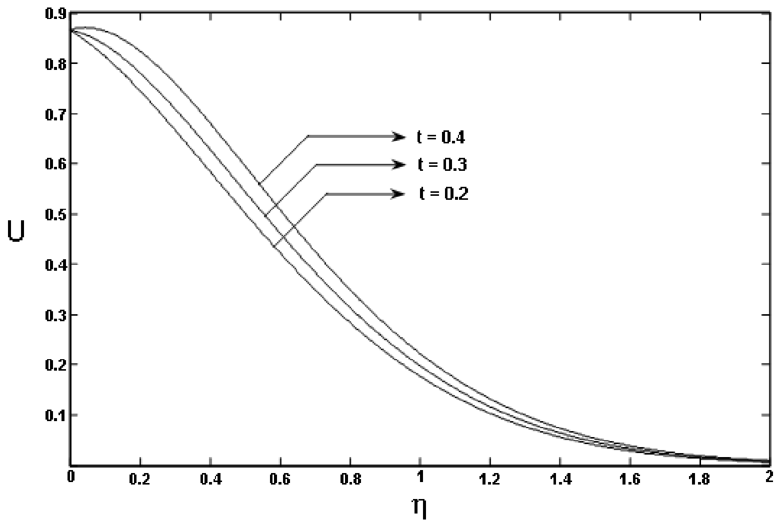

Figure 7: Velocity profiles for different values of t

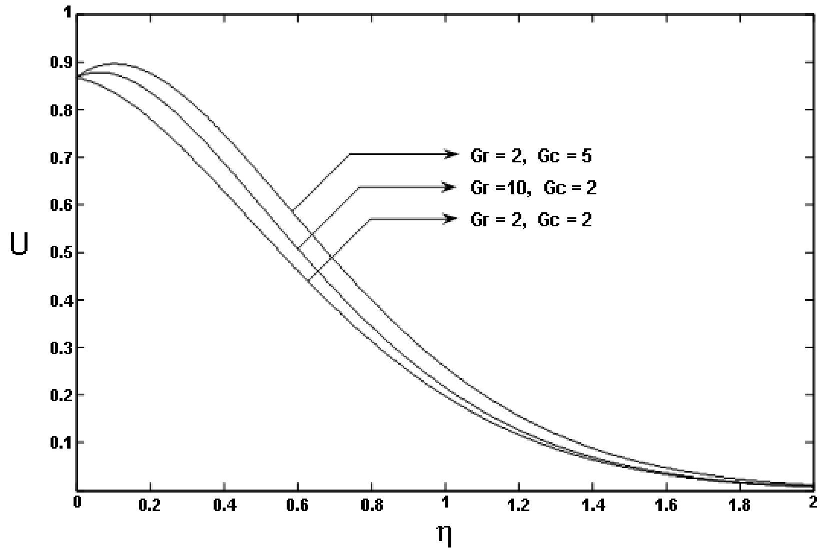

Figure 8: Velocity profiles for different values of Gr and Gc

$0.2, 0.3, 0.4), \omega t = \pi/6, R = 5, Sc = 0.6, Gr = Gc = 2$ and $Pr = 0.71$ are presented. It is observed that the velocity increases with increasing t. The velocity profiles for different values of thermal Grashof number($Gr = 2, 10$), mass Grashof number($Gc = 2, 5$), $\omega t = \pi/6, R = 5, Sc = 0.6, Pr = 0.71$ and time $t = 0.3$ are shown in figure 8. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

4 Conclusion

The problem of flow past an oscillating infinite vertical plate, in the presence of variable temperature and uniform mass flux is studied. The dimensionless equations are solved using Laplace transform technique. The effect of velocity, temperature and concentration for different parameters like $\omega t, R, Gr, Gc, Sc$ and t are studied. The study concludes the following results:

(i) The velocity increases with decreasing phase angle ωt and radiation parameter R. The trend is just reversed with respect to time t.

(ii) The temperature decreases due to high thermal radiation.

(iii) It is observed that the concentration increases with decreasing Schmidt number.

References

[1] England W.G. and Emery A.F., Thermal radiation effects on the laminar free convection boundary layer of an absorbing gas., *Journal of Heat Transfer*,**91** (1969),37-44.

[2] Soundalgekar V.M. and Takhar H.S., Radiation effects on free convection flow past a semi-infinite vertical plate., *Modelling, Measurement & Control*, **B51** (1993), 31-40.

[3] Hossain M.A. and Takhar H.S., *Radiation effect on mixed convection along a vertical plate with uniform surface temperature.*, Heat and Mass Transfer **31** (1996), 243-248.

[4] Das U.N., Deka R.K. and Soundalgekar V.M., *Radiation effects on flow past an impulsively started vertical infinite plate.*, J.Theoretical Mechanics **1** (1996), 111-115.

[5] Raptis A. and Perdikis C., *Radiation and free convection flow past a moving plate.*, Int. J. of App. Mech. and Engg.,**4** (1999), 817-821.

[6] Raptis A. and Perdikis C., *Thermal radiation of an optically thin gray gas.*, Int. J. of Applied Mechanics and Engineering, **8** (2003), 131-134.

[7] Soundalgekar V.M. , *Free Convection Effects on the Flow Past a Vertical Oscillating Plate.*, Astrophysics Space Science **64**(1979), 165-172.

[8] Soundalgekar V.M. and Akolkar S.P., *Effects of Free Convection Currents and Mass Transfer on the Flow Past a Vertical Oscillating Plate.*, Astrophysics Space Science **89** (1983), 241-254.

[9] Soundalgekar V.M., Lahurikar R.M., Pohanerkar S.G. and Birajdar N.S. , *Effects of Mass Transfer on the Flow Past an Oscillating Infinite Vertical Plate with Constant Heat Flux.*, Thermophysics and AeroMechanics **1** (1994),119-124.

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Tačno rešenje termičkog zračenja na vertikalnoj oscilujućoj ploči sa promenljivim temperaturnim i masenim fluksom

Posmatraju se uticaji termičkog zračenja na nestacionarno tečenje preko neke beskonačne vertikalne oscilujuće ploče u prisustvu promenljivog temperaturskog i uniformnog masenog fluksa. Izučavani fluid je siv, apsorbuje i emituje zračenje, ali ne raspršava. Temperatura ploče raste linearno sa vremenom dok se masa razliva po ploči uniformnom brzinom. Bezdimenzione jednačine problema su rešene tehnikom Laplace-ove transformacije. Brzina, koncentracija i temperatura su proučene pri različitim vrednostima sledećih fizičkih parametara: fazni ugao, parametar zračenja, Schmidt-ov broj, termički Grashof-ov broj, maseni Grashof-ov broj i vreme. Uočava se porast brzine pri smanjivanju faznog ugla ωt.

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