AN EOQ MODEL FOR A DETERIORATING ITEM WITH NON-LINEAR DEMAND UNDER INFLATION AND A TRADE CREDIT POLICY

S. K. MANNA,

Department of Mathematics, Sovarani Memorial College, Jagatballavpur, Howrah, India
skmanna_5@hotmail.com

K. S. CHAUDHURI

Department of Mathematics, Jadavpur University, Kolkata, India
k_s_chaudhuri@yahoo.com

Abstract: This paper develops an infinite time-horizon deterministic economic order quantity (EOQ) inventory model with deterioration based on discounted cash flows (DCF) approach where demand rate is assumed to be non-linear over time. The effects of inflation and time-value of money are also taken into account under a trade-credit policy of type "α / T_1 net T". The results are illustrated with a numerical example. Sensitivity analysis of the optimal solution with respect to the parameters of the system is carried out.

Keywords: Infinite-time horizon, deterioration, non-linear demand, inflation, trade-credit policy.

1. INTRODUCTION

The effects of inflation and time-value of money were ignored in the classical inventory models. It was believed that inflation would not influence the cost and price components. The economic situation of most of the countries has changed considerably during the last 25 years due to large-scale inflation and sharp decline in the purchasing power of money. Buzacott (1975) and Misra (1975) were the first to develop EOQ models taking inflation into account. Both of them considered a constant inflation rate for all the associated costs and minimized the average annual cost to derive an expression for the economic lot size. Their work was extended by researchers like Chandra and Bahner (1985), Aggarwal (1981), Misra (1979), Bierman and Thomas (1977), Sarker and Pan (1994), etc. to cover considerations of time value of money, inflation rate, finite

replenishment rate, etc. All these researchers worked on a constant demand rate. Datta and Pal (1990) and Bose et al. (1995) introduced a linearly trended demand along with inflation, time-value of money and shortages in inventory.

In developing mathematical inventory model, it is assumed that payments will be made to the supplier for the goods immediately after receiving the consignment. In day-to-day dealings, it is observed that the suppliers offer different trade credit policies to the buyers. One such *trade credit* policy is "α / T_1 net *T*" which means that a α% *discount on sale price is granted if payments are made within T_1 days and the full sale price is due within $T (> T_1)$ days from the date of invoice if the discount is not taken.*

Ben-Horim and Levy (1982), Chung (1989), Aggarwal and Jaggi (1994), etc., discussed trade-credit policy of type "α / T_1 net *T*" in their models. Aggarwal et al. (1997) discussed an inventory model taking into consideration inflation, time- value of money and a trade credit policy "α / T_1 net *T*" with a constant demand rate. Lal and Staelin (1994) developed an optimal discounting pricing policy. Ladany and Sternlieb (1994) tried to make an interaction between economic order quantities and marketing policies.

In this paper, an attempt has been made to extend the model of Aggarwal et al. (1997) to an inventory of items deteriorating at a constant rate θ ($0 < \theta < 1$). Also the demand rate is taken to be non-linear instead of a constant demand rate. Sensitivity of the optimal solution is examined to see how far the output of the model is affected by changes in the values of its input parameters.

2. ASSUMPTIONS AND NOTATIONS

A deterministic order-level model with an infinite rate of replenishment is developed with the following assumptions and notations.

(i) $D(t) = a - b\rho^t$ ($a > b > 0, 0 < \rho < 1$) is the demand rate at any time t . It is an increasing concave function of *t*, but the rate of increase is diminishing.

Mathematically, $D'(t) = \dfrac{dD(t)}{dt} > 0$

and $D''(t) = \dfrac{d^2 D(t)}{dt^2} \le 0$.

Thus marginal demand is a decreasing function of time.

(ii)   *T* is the replenishment cycle length.
(iii) Q_i is the order quantity in the (i + 1)th cycle.
(iv)   Replenishment is instantaneous.
(v)   Lead time is zero.
(vi)   Inventory carrying charge is *I* per unit per unit time.
(vii)   Shortages are not allowed.
(viii) $C(0)$ and $A(0)$ are respectively the unit cost of the item and the ordering cost per order at time zero.
(ix)   *h* is the inflation rate per unit time .
(x)   *r* is the opportunity cost per unit time.

(xi)   A constant fraction $\theta\,(0<\theta<1)$ of the on-hand inventory deteriorates per unit of time.

(xii) The time horizon of the inventory system is infinite.

(xiii) The term of credit policy is "α/M_1 net M", which means that a M_1 percent discount of sale price is granted if payments are made within M_1 days and the full sale price is due within M days from the date of invoice if the discount is not taken.

3. MATHEMATICAL FORMULATION

Let $A(t)$ and $C(t)$ be the ordering cost and unit cost of the item at any time t. Then $A(t) = A(0)e^{ht}$ and $C(t) = C(0)e^{ht}$, assuming continuous compounding of inflation.

Let $I_i(t)$ be the instantaneous inventory level at any time t in the $(i + 1)$th cycle. The differential equation governing the instantaneous states of $I_i(t)$ in the interval $[iT, (i+1)T]$ is

$$\frac{dI_i(t)}{dt} + \theta I_i(t) = -(a - b\rho^t), \quad iT \le t \le (i+1)T \tag{1}$$

where $I_i(iT) = Q_i$ and $I_i((i+1)T) = 0,\ i = 0, 1, 2, ...$ $\tag{2}$

The solution of equation (1) with boundary condition $I_i(iT) = Q_i$ (See Appendix I) is

$$I_i(t) = \frac{a}{\theta}(e^{(iT-t)\theta} - 1) - \frac{b}{\theta + \log\rho}(\rho^{iT} e^{(iT-t)\theta} - \rho^t) + Q_i e^{(iT-t)\theta},$$
$$iT \le t \le (i+1)T, \quad i = 0, 1, 2, ... \tag{3}$$

where $Q_i = -\dfrac{a}{\theta}(1 - e^{\theta T}) + \dfrac{b}{\theta + \log\rho}(\rho^{iT} - \rho^{(i+1)T} e^{\theta T}), \quad i = 0, 1, 2, ...$ $\tag{4}$

Let $t_0, t_1, t_2, ...$ be the replenishment points and $t_{i+1} - t_i = T$ so that $t_i = iT$, $i = 0, 1, 2,...$

(a) Case I: When discount is taken

Here purchases made at time t_i are paid after M_1 days and purchase price in real terms for Q_i units at time t_i in the $(i+1)$th cycle is

$$= Q_i(1 - \alpha)C(t_i)e^{-hM_1}.$$

The present worth of cash-flows for the $(i+1)$-th cycle is

$$PV_i^{(d)}(T) = [A(t_i) + Q_i C(t_i)(1 - \alpha)e^{-hM_1} + IC(t_i)(1 - \alpha)e^{-hM_1}\int_{iT}^{(i+1)T} I_i(t)e^{-rt}\,dt]e^{-rt_i}$$
$$= A_1 + A_2 + A_3, \quad \text{say}$$

where

$$A_1 = A(t_i)e^{-rt_i}$$
$$= A(0)e^{-iRT}, \quad \text{(assuming } R = r - h \text{ and using } t_i = iT),$$

$$A_2 = Q_i C(t_i)(1-\alpha)e^{-hM_1}e^{-rt_i}$$

$$= -\frac{aC(0)(1-\alpha)e^{-hM_1}}{\theta}(1-e^{\theta T})e^{-iRT} + \frac{bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}\rho^{iT}e^{-iRT}$$

$$-\frac{bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}e^{\theta T}\rho^{(i+1)T}e^{-iRT},$$

$$A_3 = [IC(t_i)(1-\alpha)e^{-hM_1}\int_{iT}^{(i+1)T}I_i(t)e^{-rt}dt]e^{-rt_i}.$$

Now,

$$\int_{iT}^{(i+1)T}I_i(t)e^{-rt}dt$$

$$= [-\frac{a}{\theta(r+\theta)}(e^{-(r+\theta)T}-1)e^{-irT} + \frac{a}{r\theta}(e^{-rT}-1)e^{-irT} + \frac{b(e^{-(r+\theta)T}-1)}{(\theta+\log\rho)(r+\theta)}\rho^{iT}e^{-irT}$$

$$+\frac{b(\rho^T e^{-rT}-1)}{(\theta+\log\rho)(\log\rho-r)}\rho^{iT}e^{-irT} + \frac{a}{\theta(r+\theta)}(1-e^{\theta T})(e^{-(r+\theta)T}-1)e^{-irT}$$ (5)

$$-\frac{b(e^{-(r+\theta)T}-1)}{(\theta+\log\rho)(r+\theta)}\rho^{iT}e^{-irT} + \frac{b(e^{-rT}-e^{-\theta T})}{(\theta+\log\rho)(r+\theta)}\rho^{(i+1)T}e^{-irT}]$$

Therefore,

$$PV_i^{(d)}(T) = [A(0) - \frac{aC(0)(1-\alpha)e^{-hM_1}}{\theta}(1-e^{\theta T})]e^{-iRT} + \frac{bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}\rho^{iT}e^{-iRT}$$

$$-\frac{bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}e^{\theta T}\rho^{(i+1)T}e^{-iRT} + IC(0)(1-\alpha)e^{-hM_1}[\frac{a}{r\theta}(e^{-rT}-1)$$

$$-\frac{a}{\theta(r+\theta)}(e^{-rT}-e^{\theta T})]e^{-iPT} + [\frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(\log\rho-r)}(\rho^T e^{-rT}-1)]\rho^{iT}e^{-iPT}$$ (6)

$$+[\frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(r+\theta)}(e^{-rT}-e^{\theta T})]\rho^{(i+1)T}e^{-iPT}, \quad \text{(taking } P = 2r-h).$$

The present worth of all future cash flows is

$$PV_\infty^{(d)}(T) = \sum_{i=0}^{\infty}PV_i^{(d)}(T)$$

$$= [A(0) - \frac{aC(0)(1-\alpha)e^{-hM_1}}{\theta}(1-e^{\theta T})]\frac{1}{(1-e^{-RT})} + \frac{bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(1-\rho^T e^{-RT})}$$

$$-\frac{\rho bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}\frac{e^{\theta T}}{(1-\rho^T e^{-RT})} + IC(0)(1-\alpha)e^{-hM_1}[\frac{a}{r\theta}(e^{-rT}-1)$$ (7)

$$-\frac{a}{\theta(r+\theta)}(e^{-rT}-e^{\theta T})]\frac{1}{(1-e^{-PT})} + \frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(\log\rho-r)}\frac{(\rho^T e^{-rT}-1)}{(1-\rho^T e^{-PT})}$$

$$+\frac{b\rho IC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(r+\theta)}\frac{(e^{-rT}-e^{\theta T})}{(1-\rho^T e^{-PT})}, \quad \text{(using Appendix II)}.$$

The solution of the equation $\dfrac{dPV_\infty^{(d)}(T)}{dT}=0$ gives the optimum value of T provided it satisfies the condition

$$\frac{d^2 PV_\infty^{(d)}(T)}{dT^2}>0 \tag{8}$$

Now $\dfrac{dPV_\infty^{(d)}(T)}{dT}=0$ yields the equation

$$
-[A(0)-\frac{aC(0)(1-\alpha)e^{-hM_1}}{\theta}(1-e^{\theta T})]\frac{\mathrm{Re}^{-RT}}{(1-e^{-RT})^2}+aC(0)(1-\alpha)e^{-hM_1}\frac{e^{\theta T}}{(1-e^{-RT})}
$$
$$
+[\frac{bC(0)(1-\alpha)(\log\rho-R)e^{-hM_1}}{(\theta+\log\rho)}-\frac{\rho bC(0)(1-\alpha)(\log\rho-R)e^{-hM_1}e^{\theta T}}{(\theta+\log\rho)}]\frac{\rho^T e^{-RT}}{(1-\rho^T e^{-RT})^2}
$$
$$
-\frac{\rho\theta bC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)}\frac{e^{\theta T}}{(1-\rho^T e^{-RT})}-PIC(0)(1-\alpha)e^{-hM_1}[-\frac{a}{\theta(r+\theta)}(e^{-rT}-e^{\theta T})
$$
$$
+\frac{a}{r\theta}(e^{-rT}-1)]\frac{e^{-PT}}{(1-e^{-PT})^2}+[-\frac{a}{\theta}e^{-rT}+\frac{a}{\theta(r+\theta)}(re^{-rT}+\theta e^{\theta T})]\frac{IC(0)(1-\alpha)e^{-hM_1}}{(1-e^{-PT})}
$$
$$
+\frac{bIC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(\log\rho-r)}\frac{(1-\rho^T e^{-PT})(\log\rho-r)\rho^T e^{-rT}+(\rho^T e^{-rT}-1)(\log\rho-P)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2}
$$
$$
-\frac{b\rho IC(0)(1-\alpha)e^{-hM_1}}{(\theta+\log\rho)(r+\theta)}\frac{(1-\rho^T e^{-PT})(re^{-rT}+\theta e^{\theta T})+(e^{-rT}-e^{\theta T})(P-\log\rho)\rho^T e^{-PT}}{(1-\rho^T e^{-PT})^2}=0. \tag{9}
$$

This equation being highly nonlinear can not be solved analytically. It can be solved numerically for given parameter values. Its solution gives the optimum value $T*$ of the replenishment cycle time T. Once $T*$ is obtained, we can get optimum order quantities Q_i^* $(i=0,1,2,...)$ from (4) and $PV_\infty^{(d)}(T)^*$, the optimum present value of all future cash flows from (7).

Case II: When discount is not taken

Here purchases made at time t_i are paid after M days and purchase price in real terms for Q_i units at time t_i in the $(i+1)$th cycle is

$$= Q_i C(t_i)e^{-hM} .$$

The present worth of cash flows for the (i+1)th cycle is

$$PV_i^{(wd)}(T) = [A(t_i) + Q_i C(t_i)e^{-hM} + IC(t_i)e^{-hM} \int_{iT}^{(i+1)T} I_i(t)e^{-rt}\,dt]e^{-rt_i}$$

$$= [A(0) - \frac{aC(0)e^{-hM}}{\theta}(1-e^{\theta T})]e^{-iRT} + \frac{bC(0)e^{-hM}}{(\theta + \log\rho)}\rho^{iT}e^{-iRT}$$

$$- \frac{bC(0)e^{-hM}}{(\theta + \log\rho)}e^{\theta T}\rho^{(i+1)T}e^{-iRT} + IC(0)e^{-hM}[\frac{a}{r\theta}(e^{-rT}-1) \qquad (10)$$

$$- \frac{a}{\theta(r+\theta)}(e^{-rT}-e^{\theta T})]e^{-iPT} + [\frac{bIC(0)e^{-hM}}{(\theta + \log\rho)(\log\rho - r)}(\rho^T e^{-rT}-1)]\rho^{iT}e^{-iPT}$$

$$+[\frac{bIC(0)e^{-hM}}{(\theta + \log\rho)(r+\theta)}(e^{-rT}-e^{\theta T})]\rho^{(i+1)T}e^{-iPT}, \quad \text{(taking } P = 2r-h\text{)}.$$

$$i = 0,1,2,...$$

The present worth of all future cash flows is

$$PV_\infty^{(wd)}(T) = \sum_{i=0}^\infty PV_i^{(wd)}(T)$$

$$= [A(0) - \frac{aC(0)e^{-hM}}{\theta}(1-e^{\theta T})]\frac{1}{(1-e^{-RT})} + \frac{bC(0)e^{-hM}}{(\theta + \log\rho)}\frac{1}{(1-\rho^T e^{-RT})}$$

$$- \frac{\rho bC(0)e^{-hM}}{(\theta + \log\rho)}\frac{e^{\theta T}}{(1-\rho^T e^{-RT})} + IC(0)e^{-hM}[\frac{a}{r\theta}(e^{-rT}-1) \qquad (11)$$

$$- \frac{a}{\theta(r+\theta)}(e^{-rT}-e^{\theta T})]\frac{1}{(1-e^{-PT})} + \frac{bIC(0)e^{-hM}}{(\theta + \log\rho)(\log\rho - r)}\frac{(\rho^T e^{-rT}-1)}{(1-\rho^T e^{-PT})}$$

$$+\frac{b\rho IC(0)e^{-hM}}{(\theta + \log\rho)(r+\theta)}\frac{(e^{-rT}-e^{\theta T})}{(1-\rho^T e^{-PT})}, \quad \text{(using Appendix II)}.$$

The solution of the equation $\dfrac{dPV_\infty^{(wd)}(T)}{dT} = 0$ gives the optimum value of T provided it satisfies the condition

$$\frac{d^2 PV_\infty^{(wd)}(T)}{dT^2} > 0. \qquad (12)$$

Now $\dfrac{dPV_\infty^{(wd)}(T)}{dT} = 0$ yields the equation

$$-[A(0) - \frac{aC(0)e^{-hM}}{\theta}(1 - e^{\theta T})]\frac{Re^{-RT}}{(1 - e^{-RT})^2} + aC(0)e^{-hM}\frac{e^{\theta T}}{(1 - e^{-RT})}$$

$$+ \frac{bC(0)(\log\rho - R)e^{-hM}}{(\theta + \log\rho)}\frac{\rho^T e^{-RT}}{(1 - \rho^T e^{-RT})^2} - \frac{\rho bC(0)(\log\rho - R)e^{-hM}}{(\theta + \log\rho)}\frac{\rho^T e^{(\theta - R)T}}{(1 - \rho^T e^{-RT})^2}$$

$$- \frac{\rho\theta bC(0)e^{-hM}}{(\theta + \log\rho)}\frac{e^{\theta T}}{(1 - \rho^T e^{-RT})} - PIC(0)e^{-hM}[\frac{a}{r\theta}(e^{-rT} - 1)$$

$$- \frac{a}{\theta(r + \theta)}(e^{-rT} - e^{\theta T})]\frac{e^{-PT}}{(1 - e^{-PT})^2} + [-\frac{a}{\theta}e^{-rT} + \frac{a}{\theta(r + \theta)}(re^{-rT} + \theta e^{\theta T})]\frac{IC(0)e^{-hM}}{(1 - e^{-PT})}$$

$$+ \frac{bIC(0)e^{-hM}}{(\theta + \log\rho)(\log\rho - r)}\frac{(1 - \rho^T e^{-PT})(\log\rho - r)\rho^T e^{-rT} + (\rho^T e^{-rT} - 1)(\log\rho - P)\rho^T e^{-PT}}{(1 - \rho^T e^{-PT})^2}$$

$$- \frac{b\rho IC(0)e^{-hM}}{(\theta + \log\rho)(r + \theta)}\frac{(1 - \rho^T e^{-PT})(re^{-rT} + \theta e^{\theta T}) + (e^{-rT} - e^{\theta T})(P - \log\rho)\rho^T e^{-PT}}{(1 - \rho^T e^{-PT})^2} = 0.$$

$$(13)$$

As commented before, this equation also needs to be solved numerically.

4. NUMERICAL EXAMPLES

(a) Example 1

Let $a = 50$, $b = 5$, $C(0) = 10$, $r = 0.04$, $I = 0.02$, $A(0) = 2000$, $M_1 = 30$, $h = 0.02$, $\alpha = 0.1$, $\theta = 0.01$, $\rho = 0.5$ in appropriate units. Solving the highly non-linear equation (9) for Case I by Bisection Method, we get the optimum value of T as $T^* = 17.899$. Substituting T^* in equations (4) and (7), we get the optimum value of Q_i and $PV_\infty^{(d)}(T)$ as $Q_0^* = 972.725$, $Q_i^* = 980.044$, $(i = 1, 2, 3, ...)$ and $PV_\infty^{(d)}(T)^* = 23746.070$.

(b) Example 2

Let $a = 50$, $b = 5$, $C(0) = 10$, $r = 0.04$, $I = 0.02$, $A(0) = 2000$, $M = 35$, $h = 0.02$, $\theta = 0.01$, $\rho = 0.5$ in appropriate units. Solving the equation (13) for Case II by the same Method as in Case I, we get the optimum value of T as $T^* = 17.856$. Substituting T^* in equations (4) and (11), we get the optimum value of Q_i and $PV_\infty^{(wd)}(T)$ as $Q_0^* = 970.154$, $Q_i^* = 977.473$, $(i = 1, 2, 3, ...)$ and $PV_\infty^{(wd)}(T)^* = 23837.940$.

5. SENSITIVITY ANALYSIS

Based on the numerical examples considered above, a sensitivity analysis of T^*, $PV_\infty^{(d)}(T)^*$, $PV_\infty^{(wd)}(T)^*$ is performed by changing (increasing or decreasing) the parameters by 25% and 50% and taking one parameter at a time, keeping the remaining parameters at their original values. In Table 1, it is seen that the percentage change in $PV_\infty^{(d)}(T)^*$ is almost equal for both positive and negative changes of all the parameters

except r and h. It is somewhat more sensitive for a negative change than an equal positive change of parameter r; but it is more sensitive for a positive change than an equal negative change in h. Due to positive and negative percentage changes in parameters a, $C(0)$, I, $A(0)$, h, θ and ρ, $PV_\infty^{(d)}(T)^*$ increases and decreases respectively. But this trend is reversed for the parameters b, r, M_1 and α. From Table 1, it is clear that r is highly sensitive among all the parameters. a, $C(0)$, M_1, h are moderately sensitive. I, $A(0)$, α, θ have little sensitivity while b and ρ are insensitive. Behaviours of the parameters in Table 2 ($\alpha = 0$) are nearly the same as in Table 1.

Table 1: Discount case

Changing parameters	(%) change	T^*	$PV_\infty^{(d)}(T)^*$	(%) change in T^*	(%) change in $PV_\infty^{(d)}(T)^*$
a	+50	14.994	32060.270	-16.231	+35.031
	+25	16.300	27790.990	-8.933	+17.034
	-25	20.130	19561.300	+12.466	-17.623
	-50	23.996	14782.000	+34.064	-37.750
b	+50	17.893	23741.920	-0.033	-0.017
	+25	17.897	23743.980	-0.013	-0.009
	-25	17.900	23748.160	+0.006	+0.009
	-50	17.904	23750.240	+0.026	+0.018
$C(0)$	+50	14.990	32054.410	-16.251	+34.988
	+25	16.534	27122.420	-7.628	+14.219
	-25	19.698	20272.870	+10.054	-14.626
	-50	24.010	17785.870	+34.143	-37.733
r	+50	13.918	13827.010	-22.244	-41.771
	+25	15.539	17193.860	-13.185	-27.593
	-25	22.007	42687.370	+22.949	+79.766
	-50	27.127	97354.160	+51.556	+309.980
$\underline{I}$	+50	17.043	24243.470	-4.780	+2.095
	+25	17.454	23998.700	-2.486	+1.064
	-25	18.389	23484.750	+2.736	-1.100
	-50	18.930	23213.690	+5.762	-2.242
$A(0)$	+50	21.284	26827.750	+18.914	+12.978
	+25	19.698	25341.090	+10.054	+6.717
	-25	15.790	21998.650	-11.781	-7.359
	-50	13.192	20012.860	-26.298	-15.721
M_1	+50	20.357	19221.390	+13.732	-19.054
	+25	19.012	21490.310	+6.217	-9.500
	-25	16.845	26290.540	-5.887	+10.715
	-50	15.705	29603.310	-12.256	+24.666
h	+50	25.525	33816.640	+42.608	+42.409
	+25	21.086	26913.440	+17.806	+13.338
	-25	15.454	22129.880	-13.660	-6.806
	-50	13.507	21334.920	-24.538	-10.154

Table 1: Discount case continue...

α	+50	18.346	22792.310	+2.498	-4.017
	+25	18.120	23270.160	+1.233	-2.004
	-25	17.688	24220.150	-1.180	+1.996
	-50	17.482	24692.460	-2.327	+3.985
θ	+50	16.576	24504.890	-7.391	+3.196
	+25	17.206	24129.810	-3.870	+1.616
	-25	18.665	23352.940	+4.278	-1.656
	-50	19.514	22949.540	+9.025	-3.354
ρ	+50	17.868	23760.750	-0.172	+0.062
	+25	17.890	23750.300	-0.053	+0.018
	-25	17.904	23744.650	+0.026	-0.006
	-50	17.907	23744.910	+0.046	-0.005

Table 2: Without discount case

Changing parameters	(%) change	T^*	$PV_\infty^{(wd)}(T)^*$	(%) change in T^*	(%) change in $PV_\infty^{(wd)}(T)^*$
a	+50	14.958	32191.060	-16.228	+35.041
	+25	16.265	27901.690	-8.912	+17.047
	-25	20.088	19633.970	+12.499	-17.636
	-50	23.939	14833.230	+34.070	-37.775
b	+50	17.851	23833.760	-0.030	-0.018
	+25	17.854	23835.860	-0.011	-0.009
	-25	17.858	23840.040	+0.009	+0.009
	-50	17.861	23842.140	+0.029	+0.018
$C(0)$	+50	14.955	32185.170	-16.248	+35.017
	+25	16.498	27230.000	-7.604	+14.230
	-25	19.656	20348.790	+10.081	-14.637
	-50	23.954	14837.130	+34.149	-37.758
r	+50	13.886	13876.120	-22.235	-41.790
	+25	15.504	17257.390	-13.175	-27.605
	-25	21.957	42862.340	+22.967	+79.807
	-50	27.062	97772.830	+51.556	+310.156
I	+50	17.005	24336.520	-4.769	+2.092
	+25	17.412	24091.170	-2.489	+1.062
	-25	11.346	23576.000	+2.745	-1.099
	-50	18.884	23304.320	+5.759	-2.239
$A(0)$	+50	21.238	26925.400	+18.943	+12.952
	+25	19.656	25435.980	+10.081	+6.704
	-25	15.751	22087.080	-11.787	-7.345
	-50	13.160	20097.230	-26.299	-15.692
M	+50	20.654	18769.370	+15.671	-21.263
	+25	19.132	21269.650	+7.147	-10.774
	-25	16.658	26785.360	-6.712	+12.364
	-50	15.394	30629.790	-13.789	+28.492

Table 2: Without discount case continue...

h	+50	26.021	32724.570	+45.727	+37.279
	+25	21.270	26532.950	+19.121	+11.306
	-25	15.252	22618.630	-14.582	-5.115
	-50	13.185	22204.270	-26.160	-6.853
θ	+50	16.541	24598.470	-7.366	+3.190
	+25	17.167	24222.550	-3.857	+1.613
	-25	18.622	23443.910	+4.292	-1.653
	-50	19.472	23039.580	+9.050	-3.349
ρ	+50	17.829	23852.650	-0.149	-0.062
	+25	17.847	23842.190	-0.050	+0.018
	-25	17.865	23836.520	+0.049	-0.006
	-50	17.865	23836.800	+0.049	-0.005

6. CONCLUDING REMARKS

The model developed here deals with the optimum replenishment policy of a deteriorating item in the presence of inflation and a trade credit policy. It is different from the existing models in that the demand rate is taken to be time-dependent in contrast to a constant demand rate in the other models. Time dependent demand is applicable to the goods whose demand changes steadily over time. Demand of a consumer goods changes steadily along with a steady change in the population density. A time-dependent demand rate is certainly more realistic than a constant demand rate. We can make a comparative study between the results of the discount case and without discount case. In the numerical examples, it is found that the optimum present worth of all future cash flows $PV_{\infty}^{(d)}(T)^*$ in case I is 0.385% less than that of case II. Hence the discount case is considered to be better economically. There is no appreciable change in the optimum value of the present worth of all future cash flows for changes made in the value of θ either in the discount case or in the no-discount case.

REFERENCES

[1]   Aggarwal, K.K., Aggarwal, S.P., and Jaggi, C.K., "Impact of inflation and credit policies on economic ordering", *Bull. Pure Appl. Sci.,* 16(1) (1997) 93-100.
[2]   Aggarwal, S.C., "Purchase-inventory decision models for inflationary conditions", *Interfaces*, 11 (1981) 18-23.
[3]   Aggarwal, S.P., and Jaggi, C.K., "Credit financing in economic ordering policies of deteriorating items", *International Journal of Production Economics*, 34 (1994) 151-155.
[4]   Ben-Horim, M., and Levy, H., "Inflation and the trade credit period", *Management Science*, 28(6) (1982) 646-651.
[5]   Bierman, H., and Thomas, J., "Inventory decisions under inflationary conditions", *Decision Sciences*, 8(1) (1977) 151-155.

[6]   Bose, S., Goswami, A., and Chaudhuri, K.S., "An EOQ model for deteriorating items with linear time-dependent demand rate and shortages under inflation and time discounting", *Journal of Operational Research Society*, 46 (1995) 771-782.

[7]   Buzacott, J.A., "Economic order quantities with inflation", *Operational Research Quarterly,* 26(3) (1975) 553-558.

[8]   Chandra, M., and Bahner, M.L., "The effects of inflation and the time-value of money on some inventory systems", *International Journal of Production Research*, 2314 (1985) 723-730.

[9]   Chung, K.H., "Inventory control and trade-credit revisited", *Journal of Operational Research Society*, 4(5) (1989) 495-498.

[10] Datta, T.K., and Pal, A.K., "A note on an inventory model with inventory-level dependent demand rate", *Journal of Operational Research Society*, 41(10) (1990) 971-975.

[11] Ladany, S., and Sternlieb, A., "The interaction of economic ordering quantities and marketing policies", *AIIE Transactions*, 6 (1994) 35-40.

[12] Lal, R., and Staelin, R., An approach for developing an optimal discount pricing policy, *Management Science*, 30 (1994) 1524-1539.

[13] Misra, R.B., "A study of inflationary effects on inventory systems", *Logistics Spectrum,* 9(3) (1975) 260-268.

[14] Misra, R.B., "A note on optimal inventory management under inflation", *Naval Research Logistics Quarterly,* 26 (1979) 161-165.

[15] Sarker, R.B., and Pan, H., "Effects of inflation and the time value of money on order quantity and allowable shortage", *International Journal of Production Economics*, 34 (1994) 65-72.

APPENDIX I

From equation (1), we have

$$I_i(t) = -\frac{a}{\theta} + \frac{b}{\theta + \log \rho}\rho^t + c_i e^{-\theta t}, \ c_i \text{ being a constant.}$$

From the boundary condition (2), we have

$$c_i = (Q_i + \frac{a}{\theta} - \frac{b}{\theta + \log \rho}\rho^{iT})e^{i\theta T}.$$

Therefore,

$$I_i(t) = \frac{a}{\theta}(e^{(iT-t)\theta} - 1) - \frac{b}{\theta + \log \rho}(\rho^{iT}e^{(iT-t)\theta} - \rho^t) + Q_i e^{(iT-t)\theta}$$

where $Q_i = -\frac{a}{\theta}(1 - e^{\theta T}) + \frac{b}{\theta + \log \rho}(\rho^{iT} - \rho^{(i+1)T}e^{\theta T})$.

APPENDIX II

Assuming $r > h$, we have

$$\sum_{i=0}^{\infty} e^{-iRT} = \frac{1}{1 - e^{-RT}}, \quad \sum_{i=0}^{\infty} e^{-iPT} = \frac{1}{1 - e^{-PT}},$$

$$\sum_{i=0}^{\infty} \rho^{iT} e^{-iRT} = \frac{1}{(1 - \rho^T e^{-RT})}, \quad \sum_{i=0}^{\infty} \rho^{iT} e^{-iPT} = \frac{1}{(1 - \rho^T e^{-PT})},$$

$$\sum_{i=0}^{\infty} \rho^{(i+1)T} e^{-iRT} = \frac{\rho}{(1 - \rho^T e^{-RT})}, \quad \sum_{i=0}^{\infty} \rho^{(i+1)T} e^{-iPT} = \frac{\rho}{(1 - \rho^T e^{-PT})}.$$