UNIVERSITY OF WEST BOHEMIA
Applied and Computational Mechanics 2 (2008) 227–234
Applied and Computational Mechanics

# Input Shaping Control with Reentry Commands of Prescribed Duration

P. Beneš[a,*], M. Valášek[a]

[a] Department of Mechanics, Biomechanics and Mechatronics, Faculty of Mechanical Engineering, CTU in Prague, Karlovo nám. 13, 121 35 Praha 2, Czech Republic

Received 10 September 2008; received in revised form 21 October 2008

**Abstract**

Control of flexible mechanical structures often deals with the problem of unwanted vibration. The input shaping is a feedforward method based on modification of the input signal so that the output performs the demanded behaviour. The presented approach is based on a finite-time Laplace transform. It leads to no-vibration control signal without any limitations on its time duration because it is not strictly connected to the system resonant frequency. This idea used for synthesis of control input is extended to design of dynamical shaper with reentry property that transform an arbitrary input signal to the signal that cause no vibration. All these theoretical tasks are supported by the results of simulation experiments.



*Keywords:* input shaping, vibration suppression, reentry control, flexible mechanical structures

## 1. Introduction

Precise positioning is very common task form many mechatronical systems. But flexible mechanical systems have to deal with the problem of residual vibration. This unwanted performance can be more or less improved using materials of a very high stiffness in combination with powerful motors. But despite proper optimization of these design properties machines are still limited by their own dynamics and control actions cause vibration of the overall system.

The flexible structures can be basically controlled by two main approaches — feedback or feedforward. The method described in this paper — input shaping control belongs to the latter group. It is based on modification of the input signal in such way that it leads to zero residual vibration. This principle is shown in fig. 1 and has been effectively used in many applications such as robot manipulator [4], telescopic handler [6], antisway crane [8] etc.

We can find more than one approach to vibration suppression based on modification of the input signal. Singer and Seering [7] proposed a pre-shaping technique which consists of convolving a control input with a sequence of impulses. The duration of this sequence is determined by the system resonant frequencies and leads to time delays. Especially in the case of more complex systems this could be unacceptable. In 1993 Miu [5] published a theory that explains many other methods (including above mentioned convolution) using formulation of the point-to-point control problem in Laplace s-domain. The basic idea of almost all input shaping theories is that the system resonant frequency must not be excited by control signal. The strength of Miu's theory is that even resonant frequencies can be excited. The only condition

*Corresponding author. Tel.: +420 224 357 231, e-mail: petr.benes@fs.cvut.cz.

is that all the energy trapped in the elastic elements must be completely relieved at the end of rigid body travel.

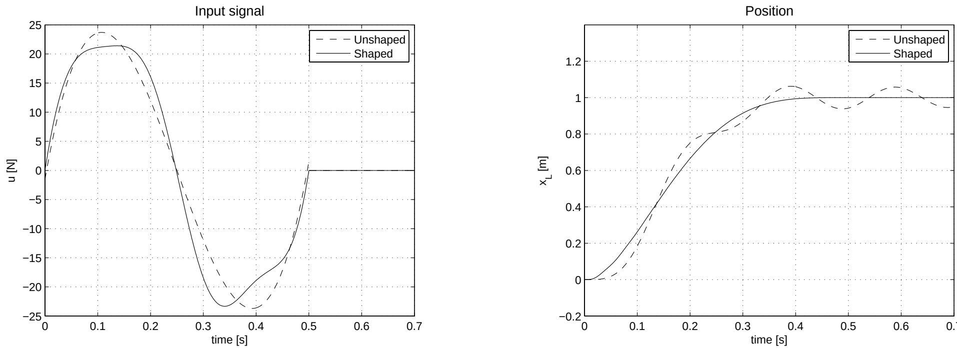


Fig. 1. Input shaping for vibration suppression

This paper is based on Miu's approach but instead of one single point-to-point maneuver it extends this idea to the development of the shaper that modifies any arbitrary input signal to the signal that causes no vibration. Similarly to Singer/Seeing method it leads to the time delay in the input signal but in this case we can choose this delay as a free parameter of the shaper. It is no longer connected to the system resonant frequencies.

## 2. Description of the theory

The behaviour of linear under-actuated and time-invariant dynamical system can be characterised by the matrix equation

$$\dot{\mathbf{y}}(t) = \mathbf{A}\mathbf{y}(t) + \mathbf{B}\mathbf{u}(t), \tag{1}$$

where $\mathbf{A}$ and $\mathbf{B}$ are constant system matrixes, vector $\mathbf{y}[m \times 1]$ denotes system states and $\mathbf{u}[n \times 1]$ is control input, where at least $m/2 > n$, i.e. the system includes at least $m/2$ degrees of freedom (DOF) the number of which is larger than the number $n$ of actuators.

Dealing with a point-to-point control problem we want to find input $\mathbf{u}(t)$ that transforms the initial state $\mathbf{y}(t_1)$ to the final state $\mathbf{y}(t_2)$. Than the solution of (1) is

$$\mathbf{y}(t_2) = e^{\mathbf{A}(t_2 - t_1)}\mathbf{y}(t_1) + \int_{t_1}^{t_2} e^{\mathbf{A}(t_2 - \tau)}\mathbf{B}\mathbf{u}(\tau)\,\mathrm{d}\tau. \tag{2}$$

Assuming full controllability, eq. (2) can be transformed by a unique transformation to the Jordan canonical form and with some rearrangement it can be re-written in the following form [3].

$$e^{-\mathbf{J}t_2}\mathbf{z}(t_2) - e^{-\mathbf{J}t_1}\mathbf{z}(t_1) = \int_{t_1}^{t_2} e^{-\mathbf{J}\tau}\mathbf{C}\mathbf{u}(\tau)\,\mathrm{d}\tau, \tag{3}$$

where

$$e^{\mathbf{J}t} = \mathrm{diag}\,\{e^{\mathbf{J_i}t}\}, \tag{4}$$

$$e^{\mathbf{J_i}t} = e^{p_i t} \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ t & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \dfrac{t^{r_i-1}}{(r_i-1)!} & \dfrac{t^{r_i-2}}{(r_i-2)!} & \cdots & 1 & 0 \\ \dfrac{t^{r_i}}{r_i!} & \dfrac{t^{r_i-1}}{(r_i-1)!} & \cdots & t & 1 \end{bmatrix}, \tag{5}$$

$p_i$ is the pole of the $i$-th Jordan block $\mathbf{J}_i$ with multiplicity of $r_i + 1$.

The right hand side of (3) can be rewritten as a sum of contributions from individual inputs $u_l$

$$e^{-\mathbf{J}t_2}\mathbf{z}(t_2) - e^{-\mathbf{J}t_1}\mathbf{z}(t_1) = \sum_{l=1}^{n} \int_{t_1}^{t_2} e^{-\mathbf{J}\tau}\mathbf{u}_l(\tau)\,\mathrm{d}\tau \cdot \mathbf{c}_l, \tag{6}$$

where $\mathbf{c}_l$ is a column of the matrix $\mathbf{C}$ corresponding to $\mathbf{u}_l$.

The expressions on the right-hand side of (6) resembles the finite time Laplace transform of the control input as defined in [5]

$$U(s) = \int_{t_1}^{t_2} e^{-s\tau}u(\tau), \mathrm{d}\tau. \tag{7}$$

Therefore we can connect Laplace transform of the control input with the states of the system at the beginning and at the end of the point-to-point maneuver

$$\sum_{l=1}^{n} U_l(s)|_{s=\mathbf{J}} \cdot \mathbf{c}_l = e^{-\mathbf{J}t_2}\mathbf{z}(t_2) - e^{-\mathbf{J}t_1}\mathbf{z}(t_1). \tag{8}$$

The algebraic equation (8) is equivalent with differential equation (1).

## 3. Zero residual vibration

The formulation of necessary conditions for zero residual vibration will be shown using simple two-mass spring damper model in fig 2. It has two DOFs $x_0$, $x_1$ and one input $f$.

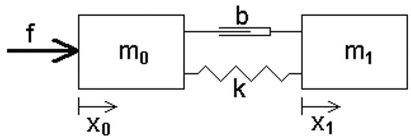


Fig. 2. Two-mass spring damper system

The system is described by matrix differential equation

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{B}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{F}(t), \tag{9}$$

where

$$\mathbf{x} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix}, \quad \mathbf{M} = \begin{bmatrix} m_0 & 0 \\ 0 & m_1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} b & -b \\ -b & b \end{bmatrix},$$
$$\mathbf{K} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}, \quad \mathbf{F} = \begin{bmatrix} f \\ 0 \end{bmatrix}. \tag{10}$$

The point-to-point maneuver without residual vibrations is prescribed using the following conditions for the final time $t_2$.

$$\mathbf{x}(t)|_{t=t_2} = \begin{bmatrix} X_0 \\ X_0 \end{bmatrix}, \quad \dot{\mathbf{x}}(t)|_{t=t_2} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{11}$$

Using modal coordinates and transformation to the Jordan canonical form equation (9) can be rewritten as

$$\underbrace{\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix}}_{\dot{\mathbf{z}}} = \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p^* \end{bmatrix}}_{\mathbf{J}} \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix}}_{\mathbf{z}} + \underbrace{\begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}}_{\mathbf{c}} u, \tag{12}$$

where $p$ and $p^*$ are complex conjugated poles of the flexible mode. The system has two poles at zero as well. Now it is described as by (1).

Using above mentioned theory and assuming that $t_1 = 0$, $t_2 = T$ and zero initial conditions it can be derived [2, 5] that necessary conditions for zero residual vibration are

$$U(s)|_{s=0} = 0, \quad \frac{\mathrm{d}U(s)}{\mathrm{d}s}|_{s=0} = X_0, \quad U(s)|_{s=p^\cdot} = 0, \quad U(s)|_{s=p^*} = 0. \tag{13}$$

A physical interpretation of these conditions is that for point-to-point control without residual vibrations the time-bounded input signal has to contain zero resultant energy at the poles of the flexible modes. If the system is un-damped and so has poles along the imaginary axis this condition means that the Fourier transform of the input signal has zero amplitude at the system resonant frequency.

## 4. Control input synthesis

There is an infinite number of signals $u(t)$ that satisfy the condition (13). Now we can use so called s-domain synthesis technique [5] to find a unique solution. Assuming that the control input can be written as a linear combination of functions from linear independent basis it can be written

$$u(t) = \sum_{i=1}^{2n+2} \lambda_i \phi_i(t), \tag{14}$$

where $\phi_i(t)$ is the basis function, $\lambda_i$ is weighting coefficient, $n$ is number of flexible modes.

Then according to conditions (13)

$$\mathbf{S}\boldsymbol{\lambda} = [0, X_0, 0, \ldots, 0]^T, \quad \mathbf{S} = \begin{bmatrix} \Phi_1(0) & \cdots & \Phi_{2n+2}(0) \\ \dfrac{\mathrm{d}\Phi_1(0)}{\mathrm{d}s} & \cdots & \dfrac{\mathrm{d}\Phi_{2n+2}(0)}{\mathrm{d}s} \\ \Phi_1(p_1) & \cdots & \Phi_{2n+2}(p_1) \\ \vdots & \ddots & \vdots \\ \Phi_1(p_n^*) & \cdots & \Phi_{2n+2}(p_n^*) \end{bmatrix}, \tag{15}$$

where $\Phi_i(s)$ is a finite time Laplace transform of $\phi_i(t)$ and $\boldsymbol{\lambda} = [\lambda_1, \lambda_2, \ldots, \lambda_{2n+2}]^T$.

From Eq. (15) definition for $\boldsymbol{\lambda}$ can be derived

$$\boldsymbol{\lambda} = \mathbf{S}^{-1} [0, X_0, 0, \ldots, 0]^T. \tag{16}$$

The crucial point of this method is the selection of basis functions. In [1] examples using polynomials can be found. In order to achieve simple reentry model we can choose control input in the following form of damped sinusoids

$$u(t) = \lambda_1 e^{at} \sin \frac{2\pi}{T} t + \lambda_2 e^{at} \sin \frac{4\pi}{T} t + \lambda_3 e^{at} \cos \frac{2\pi}{T} t + \lambda_4 e^{at} \cos \frac{4\pi}{T} t, \qquad (17)$$

where $a < 0$ and $T$ are parameters of the shaper. Speaking about single point-to-point maneuver, parameter $T$ represents its time duration. In the case of shaper it is the resulting time delay of the input signal.

## 5. Shaper with reentry property

Using knowledge from previous chapters it is possible to design single control input in the form of eq. (17) that ensures translation $X_0$ of masses $m_0$, $m_1$ in the chosen time period $T$ without residual vibration. Coefficients $\lambda_i$ are from eq. (16). The last parameter $a$ can be set arbitrary in order to achieve better properties of control input.

Described approach is suitable for the design of one single point-to-point maneuver with predefined travel length and time duration. But this concept is not usable for continuous operating of the system where the new input to the system starts before the previous one ends, e.g. manual operating of the crane. In such case a system with reentry property is needed. This means that the system accepts new commands even during performing of previously set commands, i.e. the shaper is re-entered before it has ended. And it has to satisfy zero vibration condition as well. The continuous input to the shaper is the desired final position $X_0$.

To design the shaper with reentry property the control input (17) cannot be directly constructed as time function but the solution is to realize the time function as dynamic blocks. If the function $\phi_i(t)$ is created by the output of a dynamical block then this output is created to any input even a continuous function exactly as the solution of differential equation. Thus the solution is that the basis functions from (14) are represented as dynamical blocks, for example the sinus pulse in fig. 3. If a Dirac pulse is an input to the scheme than the output is one sinus wave with the period $T$. Damped sinusoids from eq. (17) are a bit more complex. Final shaper is a summa of all dynamically represented basis functions multiplied by corresponding coefficients $\lambda_i$ from eq. (16) giving the control input (17) from the basis functions $\phi_i(t)$.

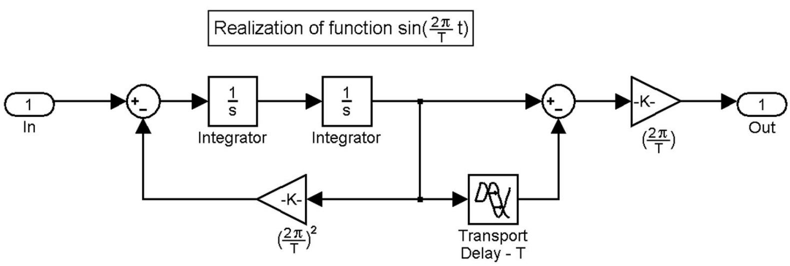


Fig. 3. Dynamical block — sinus

## 6. Simulation experiments

### *6.1. Experiments with time period of the shaper*

Simulation experiments were performed using two-mass model from chapter 3.. Natural frequency of the system was set to 1 Hz, travel distance $X_0$ is 1 m. Input signal was in the form of eq. (17).

In the first set of experiments the time duration of the input signal (the period of sinusoids) was set to 1 second, so it exactly matches system resonant frequency. In the first experiment the damping parameter of sinusoids $a = -5$ and in the second one it was set to $-20$. The results are in fig. 4.

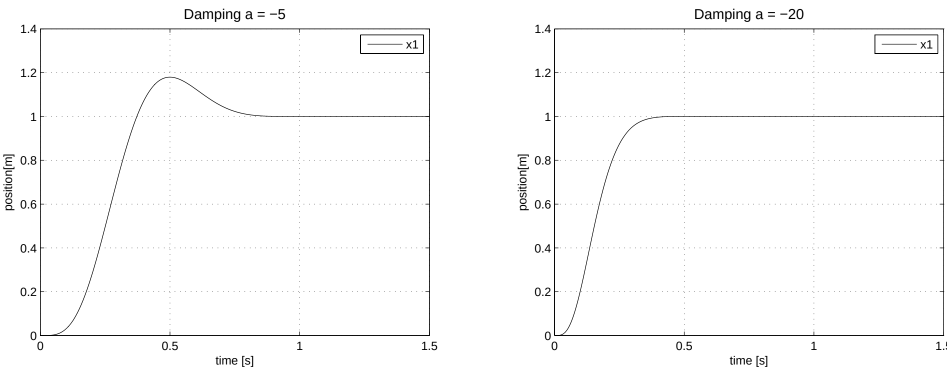


Fig. 4. Shaper period equals system resonant period

Both cases the final position was reached in predefined time of 1 second without residual vibration. However during maneuver with slightly damped input the final position was "overshoot". In the second experiment the overshoot vanished and stabilization was faster. On the other hand realization of this input needs more power from actuator but that could be improved using additional conditions for input signal.

The second experiment was performed with the time period set up to 0.5 second that is in accordance with one half of system resonant period, fig. 5.

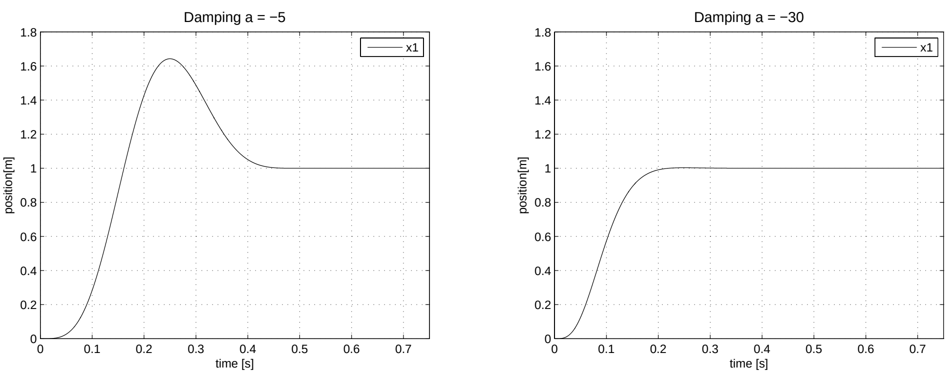


Fig. 5. Shaper period equals one half of system resonant period

Again there is an overshoot for smaller damping parameter. But the most important is the fact that the period of the shaper was shortened bellow the value of system resonant frequency and no vibration appears. It illustrates that shaper's period could be arbitrary shortened without lost of no-vibration property.

The third experiment was performed with the time period set up to 2 seconds. Results are in fig. 6.

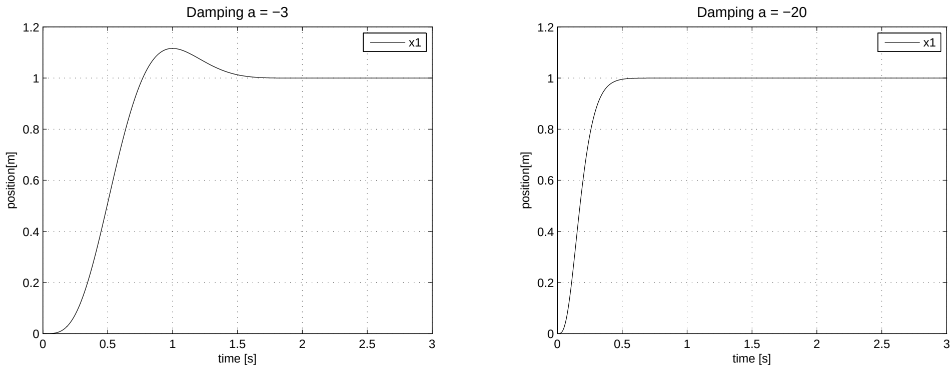


Fig. 6. Shaper period equals double of system resonant period

### *6.2. Experiment with reentry property*

The last experiment shows the modification of arbitrary input signal (here represented as a ramp signal) by the shaper, fig. 7. In fig. 8 there is a two-mass system response to this input. The shaper function was used in the following form

$$u(t) = \lambda_1 e^{at} \sin \frac{2\pi}{T} t + \lambda_2 e^{at} \sin \frac{6\pi}{T} t + \lambda_3 e^{at} \sin \frac{8\pi}{T} t + \lambda_4 e^{at} \sin \frac{4\pi}{T} t \tag{18}$$

that ensures zero initial and final value in prescribed period $T$ by itself without any further restriction. Time period was set to 1 second.

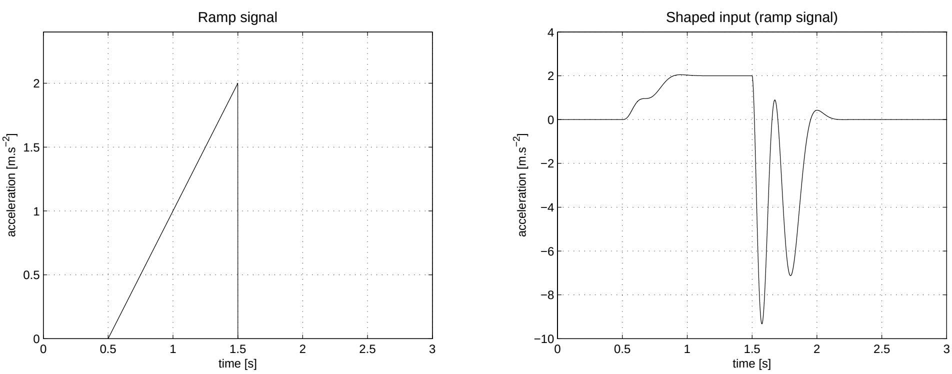


Fig. 7. Original and shaped signal

In fig. 7 both original and shaped signal start at the time 0.5 second. But when the ramp signal ends at the time 1.5 second shaper has to release energy trapped in flexible modes. So it performs additional sequence with prescribed duration 1 second (period of the shaper).

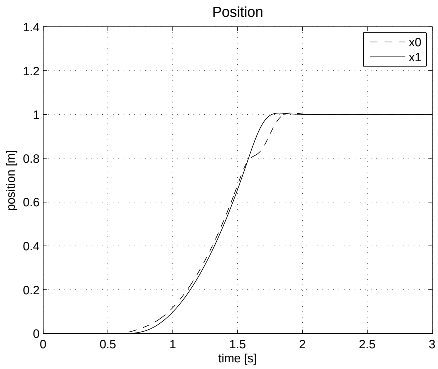


Fig. 8. System response

## 7. Conclusion

The description of the input shaper design was made. The basic principle is based on formulation of point-to-point control problem in Laplace domain [5]. Its advantage over other theories is in unrestricted length of the signal that is not connected to system resonant frequency. This approach was further extended to creation of the dynamical shaper with reentry property. This shaper is able to modify any input signal to signal that cause no residual vibration in prescribed time.

The results were proved by simulation experiments. Further development of the method will be focused on improvement of shaped control properties. One possible way is to put additional conditions to s-domain synthesis technique [2, 5]. The other way is searching for advanced basis functions. The criteria for these improvements could be e.g. suppression of position “overshoots” or the energy saving.

## References

[1] P. Beneš, M. Valášek, Application of Input Shaping Control to Drives of Machine Tools, Proc. Interaction and Feedbacks ’2006, Institute of Thermomechanics AS CR, Prague, 2006, pp. 5–12.

[2] P. Beneš, M. Valášek, Derivation of conditions for residual vibration suppression by input shaping control, Proc. of 7th Workshop of Applied Mechanics, CTU in Prague, Prague, 2007, pp. 7–12.

[3] S. P. Bhat, D. K. Miu, Solutions to Point-to-Point Control Problems Using Laplace Transform Technique, Journal of Dynamics Systems, Measurement and Control 113 (1991) 425–431.

[4] P. H. Chang, J. Y. Park, Time-varying Input Shaping Technique Applied to Vibration Reduction of an Industrial Robot, Control Engineering Practice 13 (2005) 121–130.

[5] D. K. Miu, Mechatronics, Electromechanics and Contromechanics. Springer-Verlag, New York, 1993.

[6] J. Y. Park, P. H. Chang, Vibration Control of a Telescopic Handler Using Time Delay Control and Commandless Input Shaping Technique. Control Engineering Practice 12 (2004) 769–780.

[7] N. C. Singer, W. P. Seering, Preshaping Command Inputs to Reduce System Vibration. Journal of Dynamics Systems, Measurement and Control 112 (1990) 76–82.

[8] M. Valášek, Input Shaping Control of Mechatronical Systems, Proc. of 9th World Congress, IFToMM, Politecnico di Milano, Milano, 1995, pp. 3 049–3 052.