The Cryosphere, 6, 763–770, 2012
www.the-cryosphere.net/6/763/2012/
doi:10.5194/tc-6-763-2012

The Cryosphere

Significant contribution to total mass from very small glaciers

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Received: 27 January 2012 – Published in The Cryosphere Discuss.: 17 February 2012
Revised: 28 May 2012 – Accepted: 13 June 2012 – Published: 12 July 2012

Abstract. A single large glacier can contain tens of millions of times the mass of a small glacier. Nevertheless, very small glaciers (with area $\leq 1\ \text{km}^2$) are so numerous that their contribution to the world's total ice volume is significant and may be a notable source of error if excluded. With current glacier inventories, total global volume errors on the order of 10 % are possible. However, to reduce errors to below 1 % requires the inclusion of glaciers that are smaller than those recorded in most inventories. At the global scale, 1 % accuracy requires a list of all glaciers and ice caps (GIC, exclusive of the ice sheets) larger than $1\ \text{km}^2$, and for regional estimates requires a complete list of all glaciers down to the smallest possible size. For this reason, sea-level rise estimates and other total mass and total volume analyses should not omit the world's smallest glaciers. In particular, upscaling GIC inventories has been common practice in sea level estimates, but downscaling may also be necessary to include the smallest glaciers.

1 Introduction

The world's largest glaciers dwarf the world's smallest glaciers by five or more orders of magnitude, and one large glacier (circa $10^4\ \text{km}^2$) can contain up to 10 million times more ice mass than the smallest glacier (circa $10^{-1}\ \text{km}^2$). Although such an overwhelming ratio suggests that a few of the world's largest glaciers contain the bulk of the world's ice mass (exclusive of the ice sheets), it is equally reasonable to ask if the rest of the glaciers are so numerous that they contain as much or more total ice. After all, for each single large glacier there are tens of thousands of smaller glaciers.

For the purposes of sea-level rise estimates and other analyses that depend on glacier inventories, this question of mass distribution is more than academic and in a warming climate could have important engineering and political consequences. We might ask, for example, if the estimated one-quarter to one-third of the total sea-level rise due to melting glaciers and ice caps (cf., Radić and Hock, 2011, 2010; Bahr et al., 2009; Meier et al., 2007) will be dominated by the few largest glaciers, or if sea level will rise faster in response to many smaller glaciers. While the world's glacier inventories have become increasingly thorough and accurate (Haeberli et al., 1989; NSIDC, 1999; Cogley, 2009), the very smallest glaciers are still the ones that are most likely to be overlooked (cf., Radić and Hock, 2010, Table 3). It is entirely possible that the smallest glaciers' sea-level contribution could be underestimated, in large part because of practical reasons which make a catalog of the smallest glaciers expensive, time consuming, and error prone due to difficulties in separating small glaciers from snow patches (Bolch et al., 2010). As an inventory's size threshold is lowered, relative errors may rise, but with the smallest glaciers rapidly melting and possibly disappearing over the next few decades (Mernild et al., 2011; Radić and Hock, 2011), the potentially rapid sea-level contribution of these smallest glaciers should be considered, or systematic errors due to their exclusion should be estimated.

Most calculations of sea-level rise from glaciers and ice caps rely on an estimate of the total volume of land ice, either on a region by region or on a global basis (e.g., Mernild et al., 2011; Radić and Hock, 2011; Bahr et al., 2009). For glaciers and ice caps (GIC), the most recent and complete calculation found a total volume of 0.60 ± 0.07 m sea level equivalent, but by necessity this estimate must be scaled up from incomplete inventories (Radić and Hock, 2010). An

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upscaling generally assumes that the estimated total mass is biased most significantly by missing large glaciers (for example, Radić and Hock (2010) found that the largest glacier was excluded from each of the regional inventories of Alaska, Arctic Canada, and Greenland), but it is also possible that improved recognition of smaller glaciers could alter the total sea level equivalent while simultaneously shifting the mass distribution towards smaller glaciers. In other words, a downscaling may also be necessary. At the very least, any future upscaling or downscaling of incomplete inventories would benefit from knowledge about the theoretical distribution of glacier mass at the smallest glacier sizes.

To estimate any bias from an incomplete inventory, we calculate the potential error in total volume associated with any lower-bound threshold in glacier size. For example, we can estimate the extent to which total volume may be underestimated when excluding glaciers smaller than 0.05 km^2 (e.g., Bolch et al., 2010), 0.1 km^2 (e.g., Schiefer et al., 2008), 2 km^2 (e.g., Jiskoot et al., 2012), or for any other cutoff in the many inventories that collectively comprise the World Glacier Inventory (Haeberli et al., 1989; NSIDC, 1999; Cogley, 2009). The following shows that 1 % and sometimes even 10 % errors in the total volume necessitate inventories with surprisingly small ice masses, in some cases pushing the semantic boundary between glaciers and snow patches.

2 Assessing the mass distribution

2.1 Scaling relationships

Our calculation of total volume (or, equivalently, mass) uses two power-law scaling relationships. Number–size scaling gives the number of glaciers that have any particular area. Volume–area scaling converts each glacier's area to its volume. Combined, these power laws give the total volume of glaciers that happen to have a particular area (for example, the total volume of all glaciers of size 100 km^2). Integrating can then give the total mass of any range in glacier sizes, such as the total mass of the glaciers from 1000 to 10 000 km^2. Scaling relationships for ice caps are considered separately at the end of the analysis, and obviously this study is not discussing the massive Greenland and Antarctic ice sheets whose volumes, kinematics, dynamics, and contributions to sea level are always calculated separately (e.g., Pfeffer, 2011; Rignot et al., 2011).

Let $V(S)$ be the volume of a glacier of size or surface area S. Data and theory support a power-law relationship of the form

$$V(S) = cS^{\gamma} \tag{1}$$

where $\gamma = 1.36$ is derived from data (Chen and Ohmura, 1990), and $\gamma = 1.375$ is derived from theory (Bahr et al., 1997). The scaling constant c will disappear and become irrelevant.

Let $N(S)$ be the number of glaciers of size S. Data and theory (Bahr, 1997; Bahr and Meier, 2000) support a power law of the form

$$N(S) = bS^{-\beta} \tag{2}$$

and data shows $\beta = 2.10 \pm 0.09$, as derived from the world glacier inventory (Haeberli et al., 1989; NSIDC, 1999; Cogley, 2009) sampled for 10 different regions (Figs. 1 and 2). The exponent $\beta = 2.10$ is also in good agreement with the theoretically predicted value of 2.05 derived from percolation theory (Bahr and Meier, 2000). Refinements of this scaling exponent would change the final mass totals and the error estimates but not the general conclusions. The scaling constant b will disappear and become irrelevant.

A theoretical analysis shows that the $N(S)$ power-law relationship should be multiplied by an exponential tail, or in other words, a more rapid decrease than a power law at the largest glacier sizes (Bahr and Meier, 2000). This exponential decay occurs only when the largest glaciers in a region are so big that they are bumping up against the size of the region being considered. In effect, the largest glaciers are limited in size by the area of the region or mountains in which they can grow. Figure 1 suggests the existence of such a tail, but our goal is to produce a reasonable approximation to the total volume error, and the exponential tail is a correction to a trend which is reasonably estimated by a power law. For this reason, and for clarity in the mathematics, we first assume that the tail is irrelevant and derive a closed form solution. If anything, this assumption will overestimate the total mass of the largest glaciers and make the following arguments and conclusions stronger. However, for completeness, we then derive a correction factor due to the exponential tail. The correction factor would need to be evaluated numerically, but (to a low order) the factor is close to unity and should not change an order-of-magnitude estimate of the total volume error.

At the smallest sizes, the data in Fig. 1 show a deviation from the power law (Eq. 2) suggesting fewer glaciers than predicted by the power law number–size distribution. The following analysis includes an adjustment for this contingency, but several considerations suggest that these smallest glaciers may be underrepresented in the inventory data. Certainly the smallest glaciers are the hardest to count. In many regions, these smallest glaciers are blurring the distinction between snow patches and glaciers, which makes their numbers particularly difficult to assess (Bolch et al., 2010). Data also show that melting snow patches have a power-law distribution (Shook and Gray, 1996), making it unlikely that small glaciers should deviate from a power law but then resume power-law behavior for only slightly smaller snow patches. In addition, theory suggests that the scaling exponents for glaciers and snow patches should be the same (Bahr and Meier, 2000). Furthermore, we applied a modified automated "flowshed" algorithm (Schwanghart and Kuhn, 2010) on the most recently compiled glacier mask for Western Canada (Bolch et al., 2010). This model splits contiguous ice cover

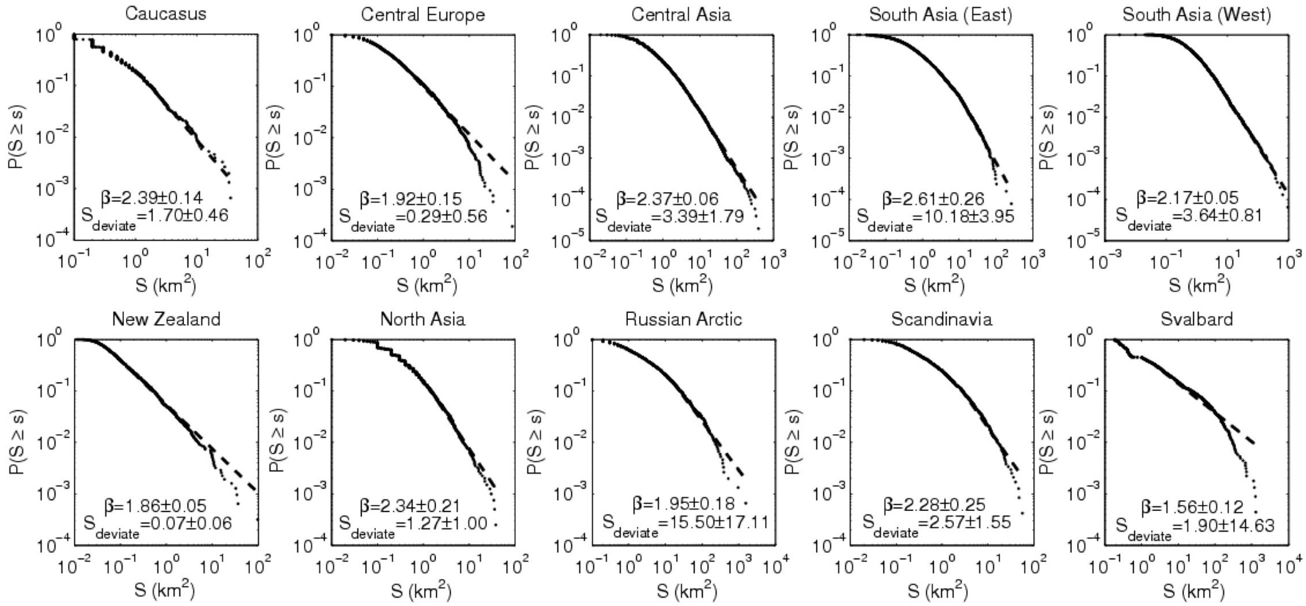

Fig. 1. The cumulative number–size distribution for 10 regions around the world. Only regions with greater than a 90 % complete glacier inventory from Cogley (2009) are included. The location of each region is mapped in Fig. 2. The dashed line shows the power-law fit. The power-law scaling exponent β is shown for each region, as is the lower bound S_{deviate} below which a power-law is not guaranteed by the data. Both β and S_{deviate} are calculated using the statistical techniques outlined in Clauset et al. (2009). A weighted average of the regions gives $\beta = 2.10 \pm 0.09$ and $S_{\text{deviate}} = 0.79 \pm 4.98$. Weights are the inverse of the error for each value of β and S_{deviate}.

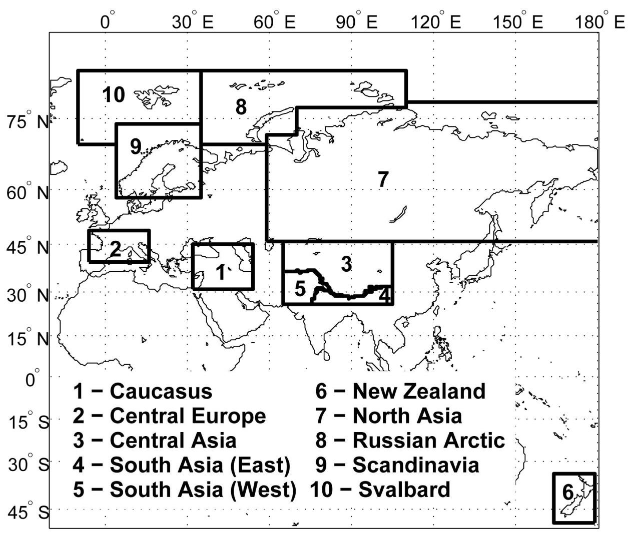

Fig. 2. Locations of the 10 glacierized regions whose glacier distributions are presented in Fig. 1 and Table 1.

into separate glaciers as defined by flow to derive the sizes and numbers of glaciers in ten different subregions of British Columbia (Figs. 3 and 4). The model predicts a mean scaling exponent of $\beta = 2.18 \pm 0.11$ in substantial agreement with data and theory. This agreement seems unlikely if the theoretically derived power law is incorrect. At the very least, a power-law fit is a reasonable approximation to the data and is sufficient to estimate total volume errors in the following analysis.

However, instead of assuming an appropriate distribution of mass at the smallest sizes, the following derivations give two bounding cases for the estimates of GIC mass distribution. One case assumes a power-law distribution for small glacier sizes. This gives a lower bound on the size of glaciers necessary to assess the total GIC mass. The second case gives a defensible upper bound under the assumption that power-law scaling fails for glaciers smaller than approximately 1 km^2. In this case, glaciers smaller than ~ 1 km^2 are ignored and considered irrelevant to the total GIC mass. The correct value lies somewhere in-between the upper and lower bounds.

2.2 Mass contribution of smaller glaciers (lower bound)

The following establishes a lower bound for the smallest glaciers needed in both global and regional inventories. In other words, this section identifies the size below which small glaciers make no significant contribution to the total ice volume (either global or regional) assuming that a power-law scaling applies to $N(S)$ across all glacier scales. (The next section adds a correction to account for any deviations from power-law scaling at the smallest glacier sizes.)

Let $V_{\text{total}_S}(S)$ be the total volume of glaciers of size or area S. This is not the total volume of all glaciers; it is just the total volume of all of the glaciers that happen to have size S. This

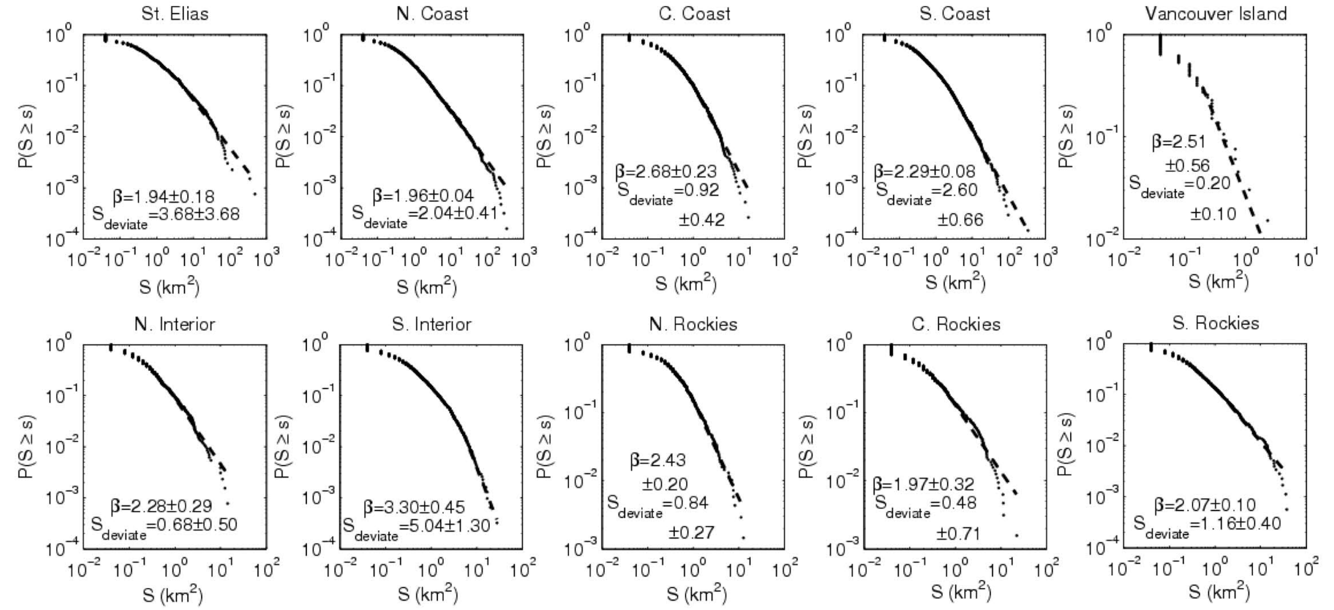

Fig. 3. The cumulative number–size distribution derived from a numerical flowshed algorithm for 10 different subregions of British Columbia. The location of each subregion is identified in Fig. 4. The dashed line shows the power-law fit. The power-law scaling exponent β is shown for each subregion, as is the lower bound S_{deviate} below which a power-law is not guaranteed by the numerical data. Both β and S_{deviate} are calculated using the statistical techniques outlined in Clauset et al. (2009). A weighted average of the regions gives $\beta = 2.18 \pm 0.11$ and $S_{\text{deviate}} = 0.97 \pm 1.29$. Weights are the inverse of the error for each value of β and S_{deviate}.

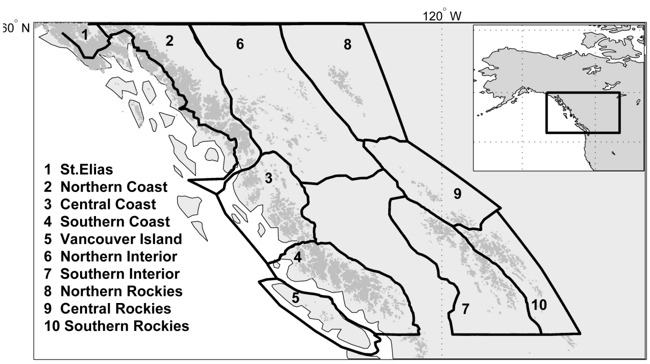

Fig. 4. Locations of the 10 subregions of British Columbia whose glacier distributions are plotted in Fig. 3. Gray dots on the map correspond to the locations of glaciers within each region.

total volume can be written as

$$V_{\text{total}_S}(S) = N(S)V(S) = bcS^{\gamma-\beta}. \tag{3}$$

Integrating gives the total mass for any range of sizes. For example, let S_{smallest} be the smallest size of glaciers that could make a relevant contribution to the total volume of all glaciers V_{total}. If all glaciers are relevant, then S_{smallest} will be the smallest existing glacier. Let $S_{\max}$ be the largest glacier. Although $S_{\max}$ could be reasonably given a value on the order of 10 000 km^2, $S_{\max}$ can remain arbitrary for the moment. It follows that the total volume of all glaciers (those in the range S_{smallest} to $S_{\max}$) is

$$V_{\text{total}} = \int_{S_{\text{smallest}}}^{S_{\max}} bcS^{\gamma-\beta}\,dS \tag{4}$$

$$= bc\left(\frac{1}{\eta}\right)S^{\eta}\Big|_{S_{\text{smallest}}}^{S_{\max}} \tag{5}$$

$$= bc\left(\frac{1}{\eta}\right)\left({S_{\max}}^{\eta} - {S_{\text{smallest}}}^{\eta}\right) \tag{6}$$

$$= bc\left(\frac{1}{\eta}\right){S_{\max}}^{\eta} \tag{7}$$

where $\eta = \gamma - \beta + 1$. The last line follows because $S_{\max} \gg S_{\text{smallest}}$ (by many orders of magnitude). Also note that the derivation remains unchanged whether or not power-law scaling of $N(S)$ applies to the smallest glaciers – in either case, $S_{\max}$ is far greater than S_{smallest}. Although the final estimate of total volume depends only on $S_{\max}$, all of the smaller glaciers have still been included in the calculation as part of the integration.

Consider the total volume of all glaciers V_{total} and an approximation of the total volume V_{approx} that ignores all glaciers below some size $S_{\min}$. Integrating as before, the relative error θ between the volume approximation and the actual volume is

$$\theta = \frac{V_{\text{total}} - V_{\text{approx}}}{V_{\text{total}}} \tag{8}$$

$$= 1 - \frac{\int_{S_{\min}}^{S_{\max}} bcS^{\gamma-\beta}\,dS}{V_{\text{total}}} \tag{9}$$

$$= 1 - \frac{{S_{\max}}^{\eta} - {S_{\min}}^{\eta}}{{S_{\max}}^{\eta}} \tag{10}$$

$$= \left(\frac{S_{\min}}{S_{\max}}\right)^{\eta}. \tag{11}$$

If the previously discussed exponential tail is included, then we must integrate $N(S)V(S) = bcS^{\eta-1}e^{-kS}$ for some unknown constant k that would need to be established from a fit to the number–size data. The integral is not possible in closed form. Instead, for limits of integration from 0 to S, the integral of $S^{\eta-1}e^{-kS}$ is commonly defined as a special function called the "lower incomplete gamma function", $\Gamma(\eta, kS)$. Although this function is usually notated as a lower-case gamma, we use Γ to distinguish it from the common notation for the volume–area scaling exponent γ. Following the same process as before,

$$\theta = \frac{V_{\text{total}} - V_{\text{approx}}}{V_{\text{total}}} \tag{12}$$

$$= 1 - \frac{\int_{S_{\min}}^{S_{\max}} bcS^{\eta-1}e^{-kS}\,dS}{V_{\text{total}}} \tag{13}$$

$$= 1 - \frac{\Gamma(\eta, kS_{\max}) - \Gamma(\eta, kS_{\min})}{\Gamma(\eta, kS_{\max})} \tag{14}$$

$$= \frac{\Gamma(\eta, kS_{\min})}{\Gamma(\eta, kS_{\max})}. \tag{15}$$

This is an exact solution but must be estimated numerically or from a table of values for Γ. However, by substituting a Taylor series for the exponential tail, the lower incomplete gamma function can be integrated term-by-term to give

$$\Gamma(\eta, kS) = (kS)^{\eta} \sum_{n=0}^{\infty} \frac{(-1)^n (kS)^n}{n!\,(\eta+n)}. \tag{16}$$

Substituting gives

$$\theta = \left(\frac{S_{\min}}{S_{\max}}\right)^{\eta} \left(\frac{\sum\limits_{n=0}^{\infty} \frac{(-1)^n (kS_{\min})^n}{n!(\eta+n)}}{\sum\limits_{n=0}^{\infty} \frac{(-1)^n (kS_{\max})^n}{n!(\eta+n)}} \right). \tag{17}$$

The rightmost factor is a correction due to the exponential tail. To a low-order approximation, this term is unity. Therefore, as a reasonable order-of-magnitude approximation to the error θ, we ignore this term and avoid complications from the unknown constant k. It follows that for any specified relative error we can solve for the smallest glaciers needed in an inventory:

$$S_{\min} = S_{\max}\theta^{1/(\gamma-\beta+1)}. \tag{18}$$

Without the exponential tail, this is exact. With the exponential tail, this simplification overestimates the volume of the largest glaciers and makes $S_{\min}$ an upper bound.

The relative error θ (Eq. 8) is not sensitive to small changes in $S_{\min}$. For example, an order of magnitude change in $S_{\min}$ results in only a factor of 2 change in the relative error when using typical values of $\gamma = 1.36$ and $\beta = 2.1$. On the other hand, an order of magnitude reduction in the error would require a 6900-fold decrease in the size of the smallest glaciers used in an inventory. For example, with the world's largest glaciers on the order of $S_{\max} = 10\,000\ \text{km}^2$, keeping $\theta \leq 10\,\%$ means $S_{\min} = 1.43\ \text{km}^2$. In other words, all glaciers less than $1.43\ \text{km}^2$ can be excluded from the inventory because they contribute less than 10 % of the total volume. However, keeping $\theta \leq 1\,\%$ means $S_{\min} = 0.0002\ \text{km}^2$, effectively implying that all glaciers must be included.

The exact results are sensitive to the scaling exponents. With $\gamma = 1.36$ and the theoretically derived value of $\beta = 2.05$, keeping $\theta \leq 10\,\%$ means $S_{\min} = 8.36\ \text{km}^2$. The very small reduction in β from 2.1 to 2.05 results in nearly an order of magnitude increase in the size of the smallest glacier that is necessary to acheive a given error. However, for most reasonable choices of scaling exponents, we can conclude that 10 % and smaller errors in the total volume will require surprisingly small glaciers that fall at or near the lower limits of many inventories.

For many regions of the world, the largest glaciers may be orders of magnitude smaller than the globally relevant order of $10\,000\ \text{km}^2$. The largest glaciers in the Alps, for example, are on the order of $100\ \text{km}^2$, and the largest glaciers in the Brooks Range (Alaska) are on the order of $10\ \text{km}^2$. To keep relative errors at 10 % or at any other value, the regional $S_{\min}$ must diminish in proportion with the regional $S_{\max}$. For example, in the Alps, all glaciers larger than $1.43 \times 10^{-2}\ \text{km}^2$ must be included to keep errors at 10 % or less (using $\gamma = 1.36$, $\beta = 2.1$). For the Brooks Range, all glaciers larger than $1.43 \times 10^{-3}\ \text{km}^2$ must be included, effectively implying that all glaciers must be inventoried to keep errors in the region's total volume below 10 %.

For regions dominated by ice caps, the volume–area curve is less steep, with data supporting a scaling exponent of $\gamma = 1.22$ (Meier and Bahr, 1996) and a theoretical value of $\gamma = 1.25$ (Bahr et al., 1997). As with glaciers, theory predicts $\beta = 2.05$. Using the theoretically derived exponents and $S_{\max}$ on the order of $10\,000\ \text{km}^2$, all ice caps larger than $0.1\ \text{km}^2$ must be included to keep errors at or below 10 %. Errors below 1 % require inventories to include all ice caps as small as $10^{-6}\ \text{km}^2$. Clearly, the smallest ice caps are always significant when calculating total ice cap volume in any region.

For global analyses or regions that contain both ice caps and glaciers, separate scaling analyses can be applied to each (e.g., Bahr et al., 2009). Ideally, regions should be selected so that glaciers and ice caps are not mixed. However, when that is not possible, a revised scaling exponent γ could be estimated from a combination of glacier and ice

Table 1. A calculation of the smallest glaciers in an inventory that would be necessary for relative errors in total regional volume that are less than or equal to 1 %, 5 %, and 10 %. Calculations are for 10 different glacierized regions around the world (Fig. 2) with inventories (Cogley, 2009) that are greater than 90 % complete. For regions dominated by ice caps (Svalbard and Russian Arctic), calculations use the theoretically derived values of $\gamma = 1.25$ and $\beta = 2.05$. For regions dominated by glaciers, calculations use $\gamma = 1.36$ and $\beta = 2.1$. The maximum size glacier is estimated by order of magnitude. Results are also presented as order of magnitude estimates in km^2.

Region	Order of magnitude of largest glacier in inventory (km^2)	Minimum size S_{min} necessary for 1 % error in total volume (km^2)	S_{min} for 5 % error (km^2)	S_{min} for 10 % error (km^2)
Caucasus	10^2	10^{-6}	10^{-3}	10^{-2}
Central Europe	10^2	10^{-6}	10^{-3}	10^{-2}
Central Asia	10^3	10^{-5}	10^{-2}	10^{-1}
South Asia (East)	10^3	10^{-5}	10^{-2}	10^{-1}
South Asia (West)	10^3	10^{-5}	10^{-2}	10^{-1}
New Zealand	10^2	10^{-6}	10^{-3}	10^{-2}
North Asia	10^2	10^{-6}	10^{-3}	10^{-2}
Russian Arctic	10^4	10^{-6}	10^{-3}	10^{-1}
Scandinavia	10^2	10^{-6}	10^{-3}	10^{-2}
Svalbard	10^3	10^{-7}	10^{-4}	10^{-2}

cap volume–area data. If the number of ice caps (or glaciers) in a region is small compared to the number of glaciers (or ice caps), then γ is unlikely to change significantly. On the other hand, if the numbers of ice caps and glaciers in a region are similar, then, because most regions have many glaciers, the preferred solution of constructing separate distributions may be reasonable.

Table 1 shows the size of the smallest glaciers that are necessary for inventories of 10 different regions around the world. For 10 % errors in total regional volume (rather than global volume), many regions do not have sufficiently small glaciers in their inventories. None of the inventories have sufficiently small glaciers for regional volume errors at 5 % or less.

2.3 Mass contribution of smaller glaciers (upper bound)

If the smallest glaciers deviate from power-law scaling, then we can apply a correction to the previous estimates. This correction assumes that all glaciers smaller than S_{deviate} do not contribute to the total volume, in which case V_{total} becomes an integral from S_{deviate} to S_{max}. Because every region contains glaciers smaller than S_{deviate}, this gives an upper bound on the smallest glaciers S_{min} that need to be included when calculating the total ice volume.

Integrating for the total volume gives

$$V_{\text{total}} = bc\left(\frac{1}{\eta}\right)\left(S_{\text{max}}{}^{\eta} - S_{\text{deviate}}{}^{\eta}\right). \tag{19}$$

And the relative error becomes

$$\theta = 1 - \frac{S_{\text{max}}{}^{\eta} - S_{\text{min}}{}^{\eta}}{S_{\text{max}}{}^{\eta} - S_{\text{deviate}}{}^{\eta}}. \tag{20}$$

Solving for S_{min},

$$S_{\text{min}} = \left(\theta S_{\text{max}}^{\eta} + (1-\theta)\, S_{\text{deviate}}{}^{\eta}\right)^{1/\eta}. \tag{21}$$

The second term is a correction to the original equation that assumes power-law scaling across all glacier sizes. The correction term becomes small and irrelevant for large relative errors θ (Fig. 5).

Data suggest that the deviation from a power law happens at approximately $S_{\text{deviate}} = 1$ km^2 in most regions of the world (Fig. 1). For typical values of $\gamma = 1.36$, $\beta = 2.1$, and $S_{\text{max}} = 10\,000$ km^2, Fig. 5 shows that the correction to the error is smaller than an order of magnitude for glaciers larger than 1.3 km^2. In general, for reasonable choices of scaling exponents, the correction term is less than an order of magnitude for glaciers only slightly larger in size than S_{deviate}.

3 Conclusions

Glacier and ice cap areas span six or more orders of magnitude, but the smallest of these glaciers are much more numerous than the largest. As a result, the vast numbers of the smallest glaciers can have a significant total mass. When assessing the relative contributions of different-sized glaciers and ice caps to sea-level rise (or to any other analysis), a seemingly small cutoff in glacier sizes could have a surprisingly large impact. As an example, the dynamic response time of the smallest glaciers can be a hundred times faster than that of the largest glaciers (Jóhannesson et al., 1989). So faster rates of sea-level rise could be expected if the total mass of the small glaciers is deemed significant, as is suggested here.

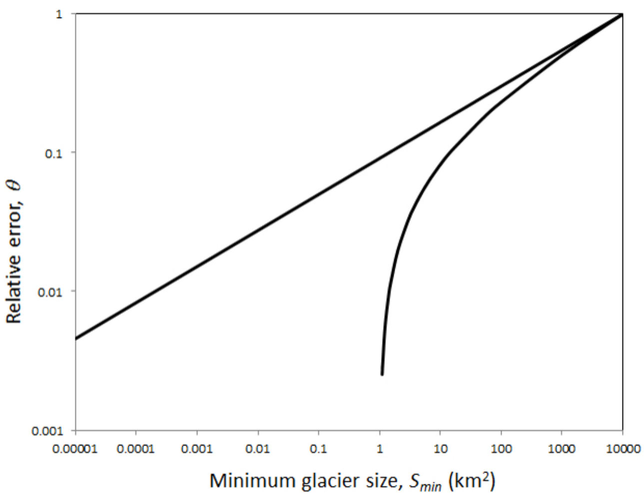

Fig. 5. The relative error in total volume calculated as a function of the minimum glacier size. The top curve shows the error when power-law scaling applies to $N(S)$ at all glacier sizes. The bottom curve shows the error when power-law scaling does not apply to glaciers smaller than $S_{\text{deviate}} = 1\,\text{km}^2$. Note that the difference in relative error is small (a fraction of an order of magnitude) for sizes larger than roughly $10\,\text{km}^2$ and is less than an order of magnitude for almost all sizes larger than $S_{\text{deviate}} = 1\,\text{km}^2$. Calculations for this plot use $\gamma = 1.36$, $\beta = 2.1$, and $S_{\max} = 10\,000\,\text{km}^2$.

The total volume of all the world's glaciers can be estimated to within any specified tolerance as long as glacier inventories are sufficiently complete at both the largest and the smallest sizes. If 10 % errors are acceptable, then Fig. 5 suggests current inventories are adequate. However, for primarily practical reasons many regional glacier inventories are incomplete at the smallest glacier sizes, and these small glaciers are notably relevant when the context calls for small errors. For sea level estimates, upscaling of inventories has been common. These results suggest that downscaling may also be important.

Most regions need to include even smaller glaciers to obtain an accurate estimate of the total regional volume (Table 1). Without especially large glaciers to bias and shift the mass distribution upwards, the Alps, for example, need inventories that are complete down to the smallest objects that could conceivably be called glaciers. Errors of less than 10 % in the Alps (certainly relevant for regional water resource planning) would require an inventory of all ice masses down to $0.01\,\text{km}^2$. At these scales, the difference between glaciers and snow patches becomes blurred, and the inventory must be effectively 100 % complete.

As a practical measure, the relative error θ in total ice volume can be estimated easily from the largest and smallest glaciers used in an analysis (Eq. 11). If power-law scaling does not apply to the world's smallest-sized glaciers, then the relative error should be modified with a correction term (Eq. 21). However, the differences at an order of magnitude scale are generally irrelevant, and Eq. (11) should be a reasonable estimate for errors under most circumstances.

Acknowledgements. We thank Garry Clarke for kindly providing the code for the flowshed algorithm. This work was supported by a grant from the Natural Sciences and Engineering Research Council of Canada.

Edited by: A. Klein

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