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Heat transfer effects on flow past an exponentially accelerated vertical plate with variable temperature

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Abstract

An exact solution to the problem of flow past an exponentially accelerated infinite vertical plate with variable temperature is analyzed. The temperature of the plate is raised linearly with time t. The dimensionless governing equations are solved using Laplace-transform technique. The velocity and temperature profiles are studied for different physical parameters like thermal Grashof number Gr, time and an accelerating parameter a. It is observed that the velocity increases with increasing values of a or Gr.

Keywords: exponential, accelerated, vertical plate, heat transfer.

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List of notations

A, a', a	constants
C_p	specific heat at constant pressure $J.kg^{-1}K$
Gr	thermal Grashof number
g	acceleration due to gravity $m.s^{-2}$
k	thermal conductivity $W.m^{-1}.K$
Pr	Prandtl number
T	temperature of the fluid near the plate K
t'	time s
t	dimensionless time
u	velocity of the fluid in the x'-direction $m.s^{-1}$
u_0	velocity of the plate $m.s^{-1}$
u	dimensionless velocity
y'	coordinate axis normal to the plate m
y	dimensionless coordinate axis normal to the plate

Greek symbols

α	thermal diffusivity $m^2.s^{-1}$
β	volumetric coefficient of thermal expansion K^{-1}
μ	coefficient of viscosity $Pa.s$
ν	kinematic viscosity $m^2.s^{-1}$
ρ	density of the fluid
θ	dimensionless temperature
η	similarity parameter
$erfc$	complementary error function

Subscripts

w	conditions at the wall
∞	conditions in the free stream

1 Introduction

A few representative fields of interest in which combined heat and mass transfer plays an important role, are design of chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields and groves of fruit trees, damage of crops due to freezing, and pollution of the environment.

Sakiadis [1,2] studied the growth of the two-dimensional velocity boundary layer over a continuously moving horizontal plate, emerging from a wide slot, at uniform velocity. Soundalgekar [3] was the first to present an exact solution to the flow of a viscous fluid past an impulsively started infinite isothermal vertical plate. The solution was derived by the usual Laplace-transform technique and the effects of heating or cooling of the plate on the flow-field were discussed through Grashof number. Free convection effects on flow past an exponentially accelerated vertical plate was studied by Singh and Naveen Kumar [4]. The skin friction for accelerated vertical plate has been studied analytically by Hossain and Shayo [5].

The object of the present paper is to study the flow past an exponentially accelerated infinite vertical plate in the presence of variable surface temperature. The dimensionless governing equations are solved using the Laplace-transform technique. The solutions are in terms of exponential and complementary error function.

2 Analysis

Here the flow of a viscous incompressible fluid past an infinite vertical plate with variable temperature is considered. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. At time $t' \leq 0$, the plate and fluid are at the same temperature T_∞ . At time $t' > 0$, the plate is exponentially accelerated with a velocity $u = u_0 \exp(a't')$ in its own plane and the plate temperature is raised linearly with time t. It is assumed that the effect of viscous dissipation is negligible. Then by usual Boussinesq's and boundary layer approximation. The unsteady flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho C_p \frac{\partial T}{\partial t'} = k\,\frac{\partial^2 T}{\partial y^2} \tag{2}$$

with the following initial and boundary conditions:

$$\begin{array}{lllll} & u = 0, & T = T_\infty, & \text{for all} & y, t' \leq 0 \\ t' > 0: & u = u_0 \exp(a't'), & T = T_\infty + (T_w - T_\infty)\ A\ t', & \text{at} & y = 0 \\ & u \to 0 & T \to T_\infty, & \text{as} & y \to \infty \end{array} \tag{3}$$

where $A = \frac{u_0^2}{\nu}$.

On introducing the following non-dimensional quantities:

$$U = \frac{u}{u_0},\ t = \frac{t' u_0^2}{\nu},\ Y = \frac{y u_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ Pr = \frac{\mu C_p}{k},\ a = \frac{a'\nu}{u_0^2} \tag{4}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\theta + \frac{\partial^2 U}{\partial Y^2} \tag{5}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\,\frac{\partial^2 \theta}{\partial Y^2} \tag{6}$$

The initial and boundary conditions in non-dimensional quantities are

$$\begin{array}{llllll} & U = 0, & \theta = 0, & C = 0 & \text{for all} & Y, t \leq 0 \\ t > 0: & U = exp(at), & \theta = t & \text{at} & y = 0 & \\ & U \to 0, & \theta \to 0 & \text{as} & y \to \infty & \end{array} \tag{7}$$

where $A = \frac{u_0^2}{\nu}$.

All the physical variables are defined in the nomenclature. The dimensionless governing equations (5) and (6), subject to the boundary conditions (7), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\theta = t\left[(1+2\,\eta^2\ Pr)\ erfc(\eta\sqrt{Pr}) - \frac{2}{\sqrt{\pi}}\,\eta\,\sqrt{Pr}\ \exp(-\eta^2 Pr)\right] \tag{8}$$

$$\begin{aligned} U = {} & \frac{\exp(at)}{2}\left[\exp(2\eta\sqrt{at})erfc(\eta+\sqrt{at}) + \exp(-2\eta\sqrt{at})erfc(\eta-\sqrt{at})\right] \\ & + \frac{Gr\ t^2}{6(Pr-1)}\left[(3+12\eta^2+4\eta^4)erfc(\eta) - \frac{\eta}{\sqrt{\pi}}(10+4\eta^2)\exp(-\eta^2)\right. \\ & \left. -(3+12\eta^2 Pr+4\eta^4(Pr)^2)erfc(\eta\sqrt{Pr}) + \frac{\eta\sqrt{Pr}}{\sqrt{\pi}}(10+4\eta^2 Pr)\exp(-\eta^2 Pr)\right] \end{aligned} \tag{9}$$

where, $\eta = Y/2\sqrt{t}$.

The purpose of the calculations given here is to assess the effects of the parameters a and Gr upon the nature of the flow and transport. The numerical values of the velocity are computed for different parameters like a, thermal Grashof number and time.

The velocity profiles for different values of time ($t = 0.2.0.4, 0.5$), $a = 0.5, Gr = 5$ and $Pr = 0.71$ are shown in figure 1. It is clear that the velocity increases with decreasing values of the time. The effect of velocity for different values of $a = 0, 2, 5, Gr = 5, Pr = 0.71$ at $t = 0.2$ are studied and presented in Figure 2. It is observed that the velocity increases with increasing values of a.

Figure 3. demonstrates the effects of the thermal Grashof number on the velocity when ($Gr = 2, 5, 10$), $a = 0.5, Pr = 0.71$ and $t = 0.2$. It is observed that the velocity increases with increasing values of the thermal Grashof number.

The temperature profiles are calculated for different values of time are shown in Figure 4. for water (Pr= 7.0) and air ($Pr = 0.71$). The effect of Prandtl number is important in temperature profiles. It is observed that the temperature increases with increasing time t. The effect of heat transfer is more in the presence of air than in water.

From the velocity field, the effect of heat transfer on the skin-friction is studied and is given in dimensionless form as

$$\tau = -\left(\frac{\partial U}{\partial Y}\right)_{Y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{\partial U}{\partial \eta}\right)_{\eta=0} \tag{10}$$

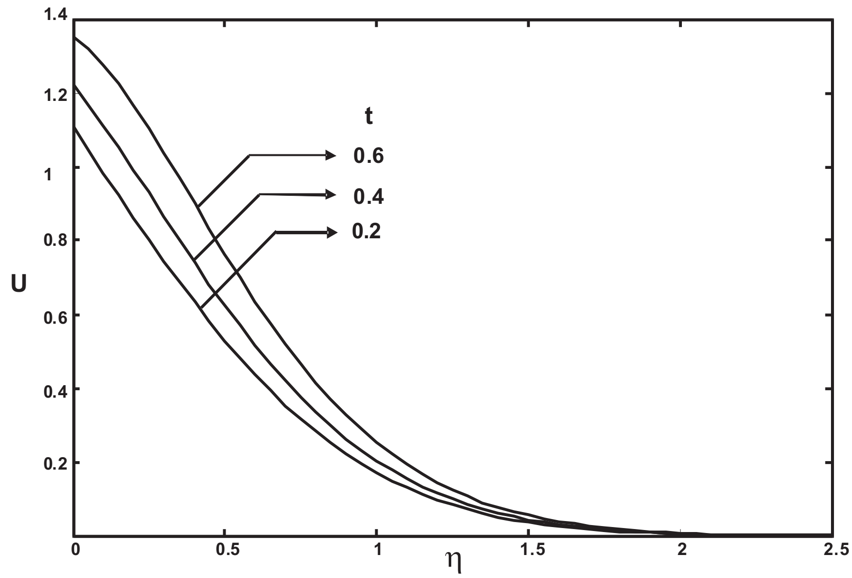

Figure 1: Velocity profiles for different t

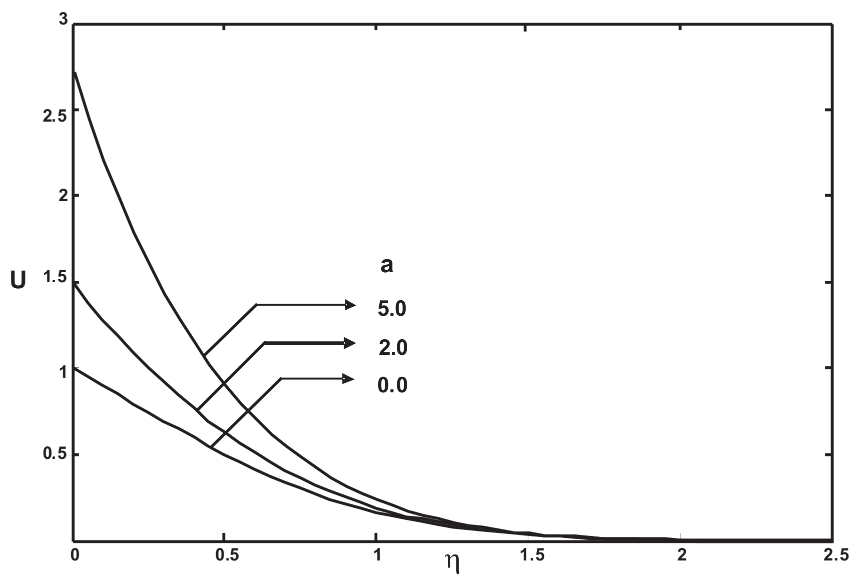

Figure 2: Velocity profiles for different a

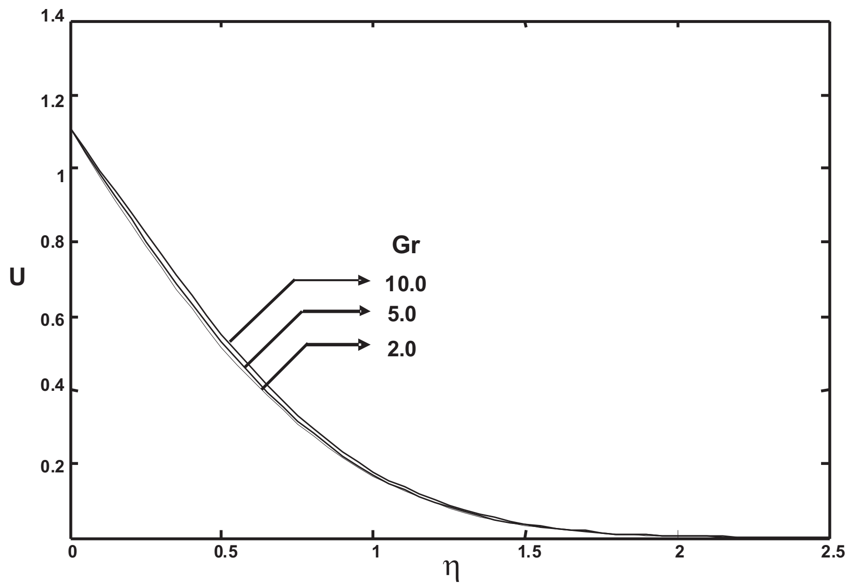

Figure 3: Velocity profiles for different Gr

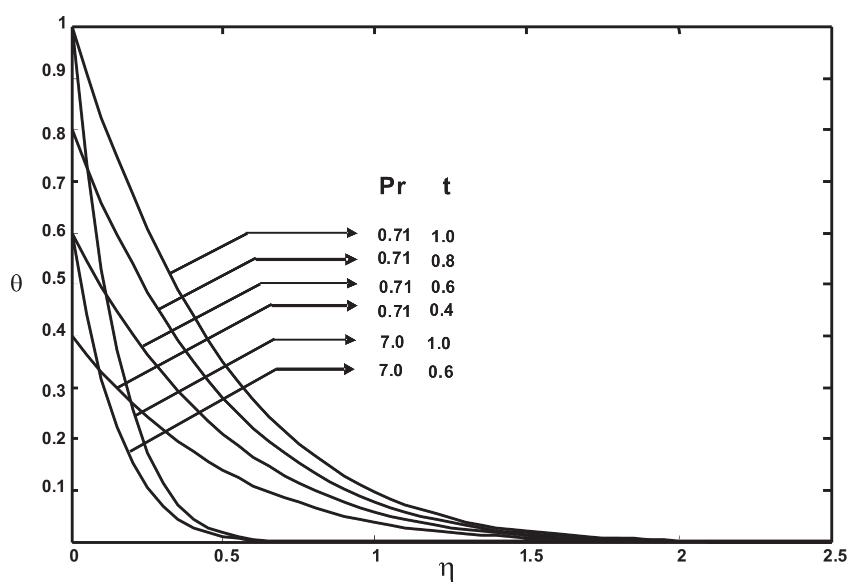

Figure 4: Temperature profiles

Hence, from equation (9) and (10),

$$\tau = \frac{1}{\sqrt{\pi t}} \left[\exp(at)(1 + \sqrt{\pi at}\ erf(\sqrt{at})) - \frac{4Grt^2}{3(\sqrt{Pr}+1)} \right] \qquad (11)$$

The numerical values of τ are presented in the following table. It is clear from this table, that an increase in the thermal Grashof number leads to an decrease in the value of the skin-friction but the trend is just reversed with respect a and time t. This shows that the skin friction increases with increasing a or t.

Gr	a	t	Pr=0.71	Pr=7.0
2	2	0.2	6.3447	6.3808
5	2	0.2	6.2352	6.3255
10	2	0.2	6.0526	6.2332
2	2	0.4	8.7627	8.8649
2	0	0.2	2.4501	2.4862
2	5	0.2	17.0299	17.0660

3 Concluding Remarks

The theoretical solution of flow past an exponentially accelerated infinite vertical plate in the presence of variable temperature is considered. The dimensionless governing equations are solved by the usual Laplace-transform technique. The effect of different parameters like thermal Grashof number, a and t are studied. It is observed that the velocity increases with increasing values of the thermal Grashof number,a and t.

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Uticaji prenosa mase na tečenje preko eksponencijalo ubrzavajuće vertikalne ploče sa promenljivom temperaturom

Analizira se jedno egzaktno rešenje problema tečenje preko eksponencijalo ubrzavajuće beskonačne vertikalne ploče sa promenljivom temperaturom. Temperatura ploče raste linearno sa vremenom. Bezdimenzione jednačine problema su rešene metodom Laplasove transformacije. Profili brzine i temperature se proučavaju za razli;ite fizičke parametre kao: termički Grashofov broj, vreme i parametar ubrzanja. Uočeno je da brzina raste sa porastom ubrzanja i Grashofovog broja.

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