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ON-LINE BLIND SEPARATION OF NON-STATIONARY SIGNALS

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Abstract: This paper addresses the problem of blind separation of non-stationary signals. We introduce an on-line separating algorithm for estimation of independent source signals using the assumption of non-stationarity of sources. As a separating model, we apply a self-organizing neural network with lateral connections, and define a contrast function based on correlation of the network outputs. A separating algorithm for adaptation of the network weights is derived using the state-space model of the network dynamics, and the extended Kalman filter. Simulation results obtained in blind separation of artificial and real-world signals from their artificial mixtures have shown that separating algorithm based on the extended Kalman filter outperforms stochastic gradient based algorithm both in convergence speed and estimation accuracy.

Keywords: Blind source separation, decorrelaton, neural networks, extended Kalman filter.

1. INTRODUCTION

Blind separation of sources refers to the problem of recovering source signals from their instantaneous mixtures using only the observed mixtures. The separation is called blind, because it assumes very weak assumptions on source signals and the mixing process. The key assumption is the statistical independence of source signals. A goal is to obtain output signals that are as independent as possible using the observed mixture signals.

In the last few years, the problem of blind source separation has received considerable attention. Since 1985, when blind source separation was initially proposed by Jutten and Herault to explain some phenomena in human brain due to simultaneous excitation of biological sensors, various approaches have been proposed [10]. These approaches include independent component analysis - ICA [7], information maximization [2], the natural gradient approach [5,6], etc. Most of the approaches use the independence property either directly, through optimization of criteria based on the Kullback-Leibler divergence, or indirectly, through minimization of criteria based on the cumulants. Having in mind the independence property of sources, the task of blind separation is to recover independence of the estimated output signals. Since the independence of sources implies that cumulants of all orders should be equal to zero, the problem is obviously related to higher-order statistics (HOS). It has been shown that the fourth-order statistics are enough to achieve independence, and therefore most of the algorithms based on HOS use fourth-order cumulants [4]. However, application of HOS is limited to non-Gaussian signals, because for Gaussian signals, cumulants of order higher than two vanish. If the source signals are stationary, Gaussian processes, it has been shown that blind separation is impossible in a certain sense.

In this paper, we consider blind separation of non-stationary signals using second-order statistics. In [11,12], it has been shown that, using the additional assumption on non-stationarity of sources, blind source separation of Gaussian or non-Gaussian signals can be achieved using only second-order statistics (SOS). Mainly, we are interested in second-order non-stationarity in the sense that source variances vary with time. We base our algorithm on diagonalization of the output correlation matrix in order to achieve decorrelation of the estimated output signals. As a mixing model, we consider instantaneous linear mixture of non-stationary, statistically independent sources. In order to blindly separate source signals from the observed mixtures, we apply a self-organizing neural network with lateral connections, which uses the observed mixtures as inputs, and provides the estimated source signals as outputs. Throughout the learning process, the network weights are adapted in a direction that reduces correlation between outputs. As an optimization algorithm that minimizes cross-correlations between output signals, we propose an on-line algorithm derived from the Extended Kalman Filter (EKF) equations. In our experiments with real-world signals, the EKF based algorithm has shown superior convergence properties compared to the stochastic gradient separating algorithm.

The paper is organized as follows. In Section 2, we formulate the problem of blind source separation. In Section 3, we briefly describe a stochastic gradient based method for blind separation of non-stationary sources which uses a neural network with lateral connections as a demixing model. In section 4, we propose a separating algorithm based on the contrast function derived using only the second-order statistics, and apply EKF as an optimization algorithm in order to estimate neural network weights and recover non-stationary sources. Section 5 contains the simulation results obtained in separation of non-stationary artificial and real-world source signals. In Section 6, we give the concluding remarks.

2. PROBLEM FORMULATION

Let $s=[s_1\ s_2\ ...\ s_N]^T$ represent N zero-mean random source signals whose exact probability distributions are unknown. Suppose that M sensors receive linear mixtures $\mathbf{x}=[x_1\ x_2\ ...\ x_M]^T$ of source signals. If we ignore delays in signal propagation, this can be expressed in the matrix form:

$$\mathbf{x}=\mathbf{A}\,\mathbf{s} \tag{1}$$

where **A** is the unknown $M\times N$ linear combination matrix, and x is the vector of the observed mixtures. In a demixing system, source signals have to be recovered using the observed mixtures as inputs. As a result, we get generally an N-dimensional ($N\leq M$) random vector **y** of separated components:

$$\mathbf{y}=\mathbf{Bx}=\mathbf{BAs}=\mathbf{Gs} \tag{2}$$

where **B** is an $N\times M$ matrix, and **G** is an $N\times N$ global system matrix. Since it is of interest to obtain separated components that represent possibly scaled and permuted versions of source signals, the matrix **G** has to represent a *generalized* permutation matrix [3]. Ideally, if **G** is an identity matrix, the set of sources is completely separable. Therefore, the problem is to obtain, if possible, a matrix **B** such that each row and each column of **G** contains only one nonzero element. It should be noted that the problem has inherent indeterminacies in terms of ordering and scaling of the estimated output signals. Due to the lack of prior information, the matrix **A** can not be identified from the observed signals even if it should be possible to extract all source signals, because their ordering remains unknown. The magnitudes of the source signals are also not recoverable, because a scalar multiple of s_j, ks_j, can not be distinguished from multiplication of the j-th column of **A** by the same scalar k. Therefore, we can obtain at best $\mathbf{y}=\mathbf{DPs}$, where P is a permutation matrix, and D is a nonsingular diagonal scaling matrix. This means that only permuted and rescaled source signals can be recovered from mixture signals. In most cases, such solution is satisfactory, because the signal waveform is preserved. In our further considerations we assume for simplicity that $\mathbf{DP}=\mathbf{I}$ without loss of generality.

In the following, we assume that the sources are non-stationary, mutually independent zero-mean random signals, the mixing process is linear, time invariant and instantaneous, and the number of observed mixtures N is equal to the number of sources and number of separated components, N=M. In practice, the number of sources is usually unknown, and may be less, equal, or greater than the number of mixtures, i.e. sensors. Most of the approaches to the blind separation are based on the prior assumption that the number of mixtures is equal or greater than the number of sources. However, the underdetermined case, i.e. the case when the number of sources is greater than the number of sources, has also been examined [5].

3. STOCHASTIC GRADIENT BASED ALGORITHM FOR BLIND SOURCE SEPARATION

Blind source separation using the additional assumption on non-stationarity of sources was initially proposed in [12]. It was shown that non-stationary signals can be separated from their mixtures using SOS if signal variances change with time, and fluctuate independently of each other during the observation. In order to separate non-stationary source signals from their instantaneous mixtures, a linear self-organizing neural network with lateral connections was applied as a demixing model [12].

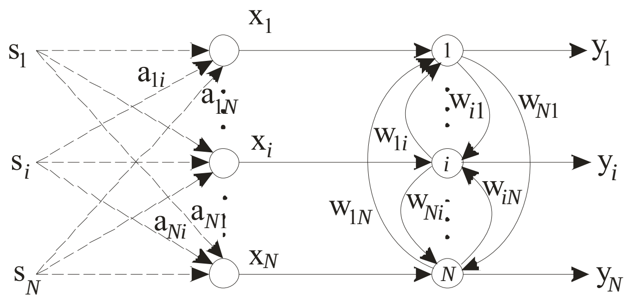


Figure 1: Self-organizing linear neural network with lateral connection for blind source separation

According to Figure 1, unknown source signals $s_1, s_2, ..., s_N$ generated by N independent sources, are mixed in an unknown mixing process, and picked up by N sensors. The network receives observed sensor signals $\mathbf{x}_t$ which represent mixtures of source signals as inputs and provides estimates of the original source signals $\mathbf{y}_t$ as outputs. In matrix notation, the dynamics of each output unit is given by the first-order linear differential equation:

$$\tau \frac{d\mathbf{y}_t}{dt} + \mathbf{y}_t = \mathbf{x}_t - \mathbf{W}\mathbf{y}_t \tag{3}$$

where the matrix $\mathbf{W} = [w_{ij}]$ denotes the mutual lateral connections between the output units. The output units have no self-connections, and therefore w_{ii}=0. In the steady state, the equation (3) becomes:

$$\mathbf{y}_t = (\mathbf{I} + \mathbf{W})^{-1} \mathbf{x}_t \tag{4}$$

Using the self-organized neural network (Fig. 1) as a demixing model, Matsuoka et al. [12] have derived an on-line stochastic gradient (SG) based algorithm for blind separation of non-stationary sources. The algorithm was obtained by minimization of the following contrast function [12]:

$$Q\left(\mathbf{W},\mathbf{R}_{y,t}\right)=\frac{1}{2}\left\{\sum_{i}\log<y_{i,t}^{2}>-\log|<\mathbf{y}_{t}\mathbf{y}_{t}^{\mathrm{T}}>|\right\} \quad (5)$$

where $\mathbf{R}_{y,t}$ is the output correlation matrix, and $<\cdot>$ denotes expectation. It should be noted that in the case of zero-mean signals, correlation matrix is equal to covariance matrix. In discrete-time *k*, the SG based separating algorithm is given by the following equations for adaptation of the network weights $w_{ij,k}$, $i, j = 1,...,N$ [12]:

$$w_{ij,k} = w_{ij,k} + \beta \frac{y_{i,k}\, y_{j,k}}{\phi_{i,k}} \quad (6a)$$

$$\phi_{i,k} = \alpha\phi_{i,k} + (1-\alpha) y_{i,k}^{2} \quad (6b)$$

In (6a), the learning rate β is assumed to be a very small positive constant, and the constant α in (6b) is a forgetting factor, $0<\alpha<1$. The learning algorithm (6a)-(6b) uses moving average $\phi_{i,k}$ in order to estimate $<y_{i,k}^{2}>$ in real time. In practice, expected values are not available, and time-averaged or instantaneous values can be used instead of them.

4. EXTENDED KALMAN FILTER BASED ALGORITHM FOR BLIND SOURCE SEPARATION

Separating algorithms based on stochastic gradient suffer from slow convergence. In order to improve convergence speed and estimation accuracy, we propose an application of the extended Kalman filter to the problem of blind source separation. Kalman filter [9] is well-known for its good properties in state estimation [8] and on-line learning [13]. Our approach to non-stationary blind signal separation is based on the assumption that cross-correlations of the output signals should be equal to zero. The problem of blind separation is formulated as minimization of the instantaneous contrast function [14]:

$$J(\mathbf{w}_k) = \mathbf{r}_k(\mathbf{w}_k)^T \mathbf{r}_k(\mathbf{w}_k)\,. \quad (7)$$

In (7), $\mathbf{r}_k$ is the vector formed of the non-diagonal elements of the output correlation matrix, i.e. the cross-correlations $\left\langle y_i(\mathbf{w}_k)\, y_j(\mathbf{w}_k)\right\rangle$ of the network outputs $\mathbf{y}_k$ at time step *k*, parameterized by the unknown mixing weigths $\mathbf{w}_k$. As a demixing model, we have applied a neural network with lateral connections (Fig. 1). The network outputs, which represent the recovered source signals, are calculated at every time step according to:

$$y_{i,k} = x_{i,k} - \sum_{\substack{j=1 \\ j\neq i}}^{N} w_{ij,k}\, y_{j,k}, \quad i, j = 1,...N\,. \quad (8)$$

Since the averaged values $\langle y_i y_j \rangle$ are not available in blind signal processing, the cross-correlations of the network outputs $\langle y_i(\mathbf{w}_k) y_j(\mathbf{w}_k) \rangle$ are estimated as time-averaged values using the following moving average:

$$r_{ij,k} = \alpha\, r_{ij,k-1} + (1-\alpha) y_{i,k} y_{j,k}, \quad i, j = 1, \ldots, N \; . \tag{9}$$

To derive the extended Kalman filter equations which will minimize the contrast function (7), we have defined the following state space model in the observed-error form [14]:

$$\mathbf{w}_k = \mathbf{w}_{k-1} + \mathbf{d}_{k-1}, \quad \mathbf{d}_{k-1} \sim N(0, \mathbf{Q}_{k-1}) \tag{10a}$$

$$\mathbf{z}_k = -\mathbf{r}_k(\mathbf{w}_k) + \mathbf{v}_k, \quad \mathbf{v}_k \sim N(0, \mathbf{R}_k) \; . \tag{10b}$$

Note that the observations $\mathbf{z}_k$ of cross-correlations $\mathbf{r}_k(\mathbf{w}_k)$ are equal to zero at every time step *k*. The process noise $\mathbf{d}_{k-1}$ and the observation noise $\mathbf{v}_k$ are assumed mutually independent, white and Gaussian and with variances equal to $\mathbf{Q}_{k-1}$ and $\mathbf{R}_k$, respectively. The estimate of the network weights and its associated covariance $\mathbf{P}_k$ at time step *k*, are given by [14]:

$$\hat{\mathbf{w}}_k = \hat{\mathbf{w}}_{k-1} + \mathbf{K}_k \mathbf{r}_k(\hat{\mathbf{w}}_{k-1}) \tag{11a}$$

$$\mathbf{P}_k = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \cdot (\mathbf{P}_{k-1} + \mathbf{Q}_{k-1}) \, , \tag{12b}$$

where K_k is the Kalman gain:

$$\mathbf{K}_k = (\mathbf{P}_{k-1} + \mathbf{Q}_{k-1}) \mathbf{H}_k^T (\mathbf{R}_k + \mathbf{H}_k (\mathbf{P}_{k-1} + \mathbf{Q}_{k-1}) \mathbf{H}_k^T)^{-1}, \; (12)$$

and

$$\mathbf{H}_k = \partial \mathbf{r}_k(\mathbf{w}_k) \big/ \partial w_k \Big|_{w_k = \hat{w}_{k-1}} \; . \tag{13}$$

Recursions (11a) and (11b) represent the basic equations of the extended Kalman filter for the problem defined by the state space model (10).

5. SIMULATION RESULTS

In order to demonstrate performances of our EKF-based algorithm in blind source separation, we have compared it with the stochastic gradient separating algorithm proposed in [12]. We give here two examples.

Example 1. In this example we apply EKF and SG algorithms to separate two non-stationary artificial source signals from the same number of their observed mixtures. The sources are given by [12]:

$$\begin{aligned} s_{1,k} &= \sin(\pi k/400) \cdot n_{1,k}, \quad n_{1,k} \sim N(0.1) \\ s_{2,k} &= \sin(\pi k/200) \cdot n_{2,k}, \quad n_{2,k} \sim N(0.1) \end{aligned} \tag{14}$$

The waveforms of the source signals are represented in Fig. 2. Mixture signals (Fig. 3) used in this example are obtained artificially according to (1) using the following mixing matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 0.9 \\ 0.5 & 1 \end{bmatrix} \tag{15}$$

In this framework, we can measure the performance of the algorithm in terms of the performance index PI defined by [5]:

$$PI = \frac{1}{n(n-1)} \sum_{i=1}^{n} \left[\left(\sum_{k=1}^{n} \frac{|g_{ik}|}{\max_j |g_{ij}|} - 1 \right) + \left(\sum_{k=1}^{n} \frac{|g_{ki}|}{\max_j |g_{ji}|} - 1 \right) \right] \tag{16}$$

where g_{ij} is the (i, j)-th element of the global system matrix $\mathbf{G} = (\mathbf{I} + \mathbf{W})^{-1}\mathbf{A}$. The performance index indicates how far the global system matrix **G** is from a generalized permutation matrix. When perfect signal separation is achieved, the performance index is zero.

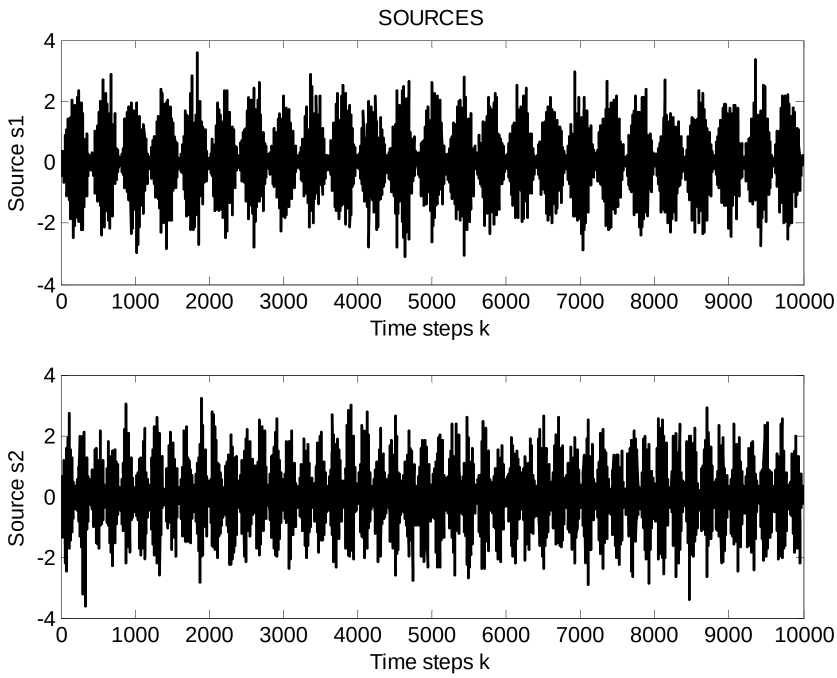


Figure 2: Source signals

In this example, the learning rate β and forgetting factor α for the SG-based algorithm were set to $\beta = 0.001$ and $\alpha = 0.8$. The observation noise covariance $\mathbf{R}_k$ for the EKF-based algorithm was fixed to identity matrix, $\mathbf{R}_k = \mathbf{I}$, and the process noise covariance $\mathbf{Q}_k$ was a diagonal matrix which entries were exponentially decayed in the range $\left(10^{-3} - 10^{-10}\right)$ in order to achieve fast convergence at the beginning of the learning process, and to retain good tracking abilities. For both algorithms, initial values of w_{12} and w_{21}, as well as the initial cross-correlations in moving average, were set to zeros, and the initial value of estimation error covariance $\mathbf{P}$ in EKF-based algorithm was set to $\mathbf{P}_k = 0.1 \cdot \mathbf{I}$.

The results obtained using EKF and SG algorithms in the estimation of source signals are shown in Figures 4-6.

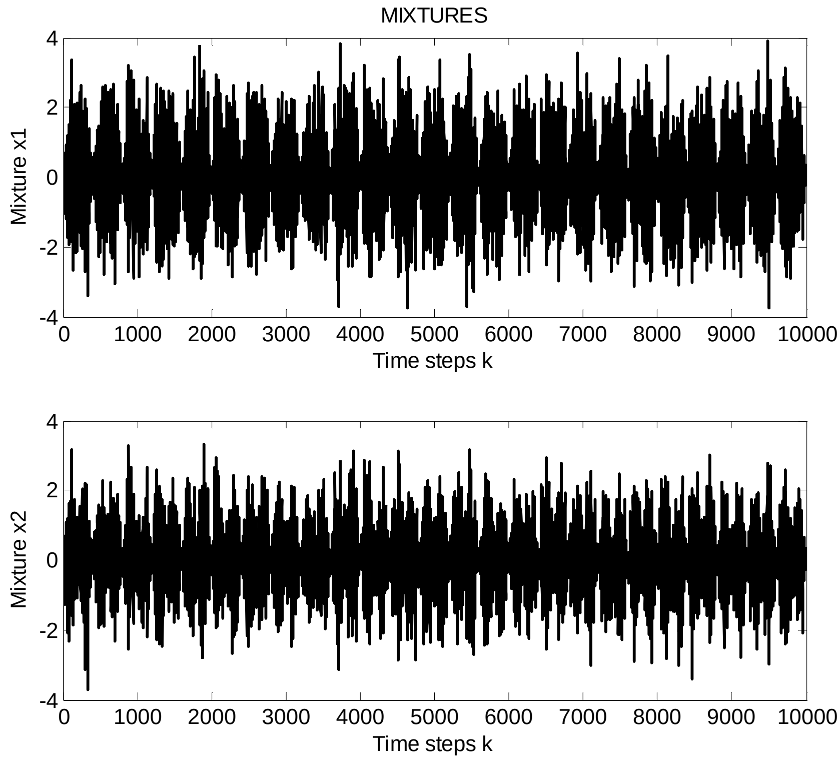
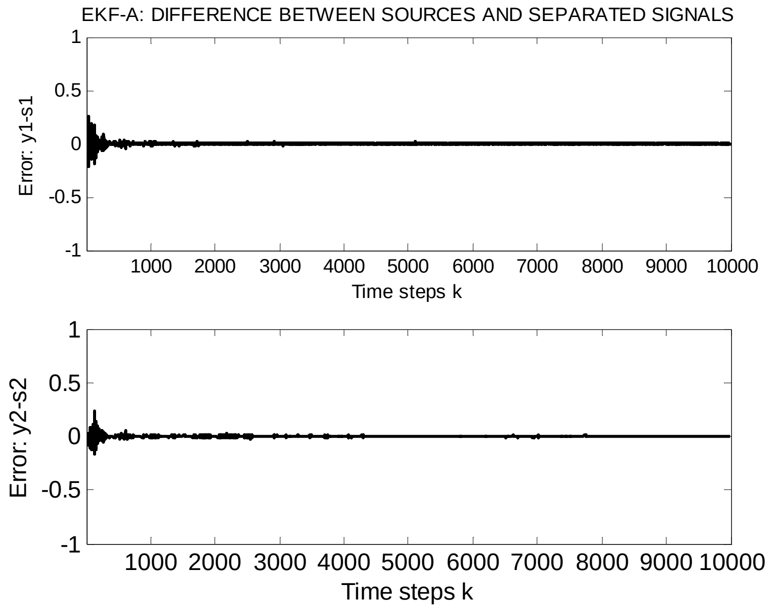
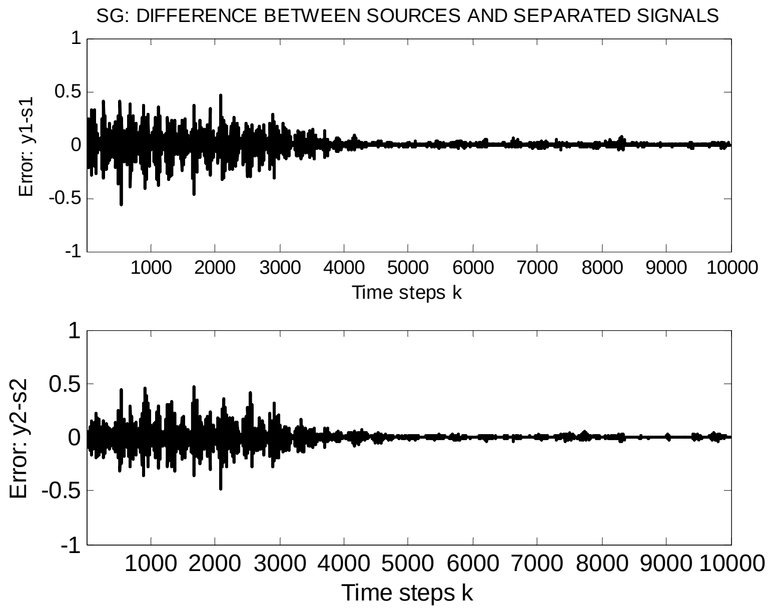


Figure 3: Linear mixtures of sources



a)



b)

Figure 4: Difference between the original source signals and recovered source signals obtained using: a) EKF-based separating algorithm; b) SG-based separating algorithm

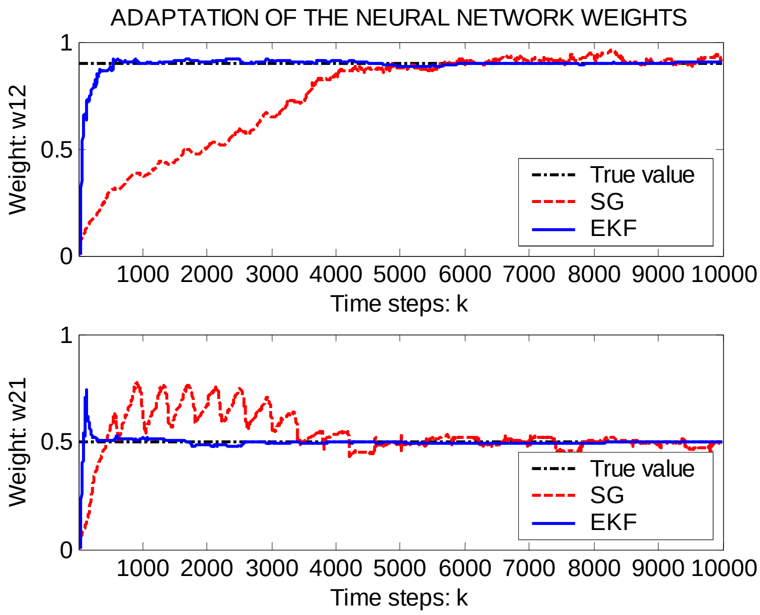


Figure 5: Adaptation of the neural network weights through iterations

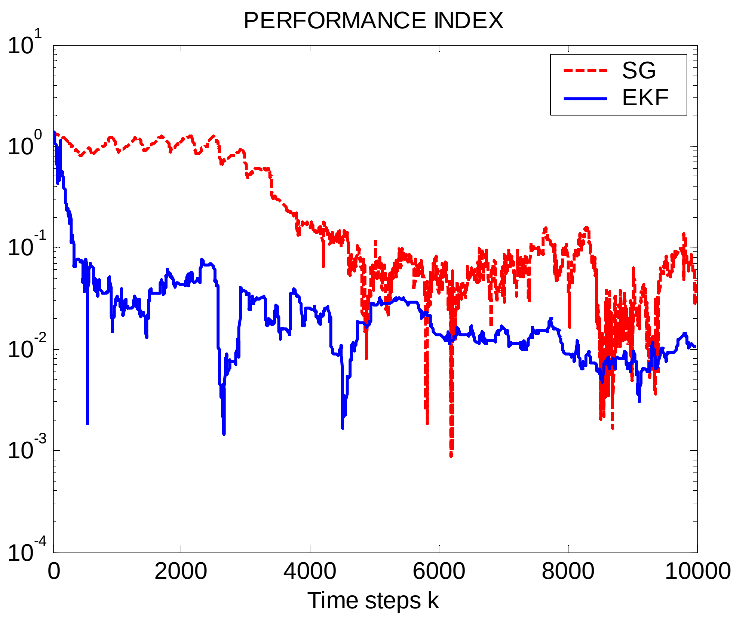
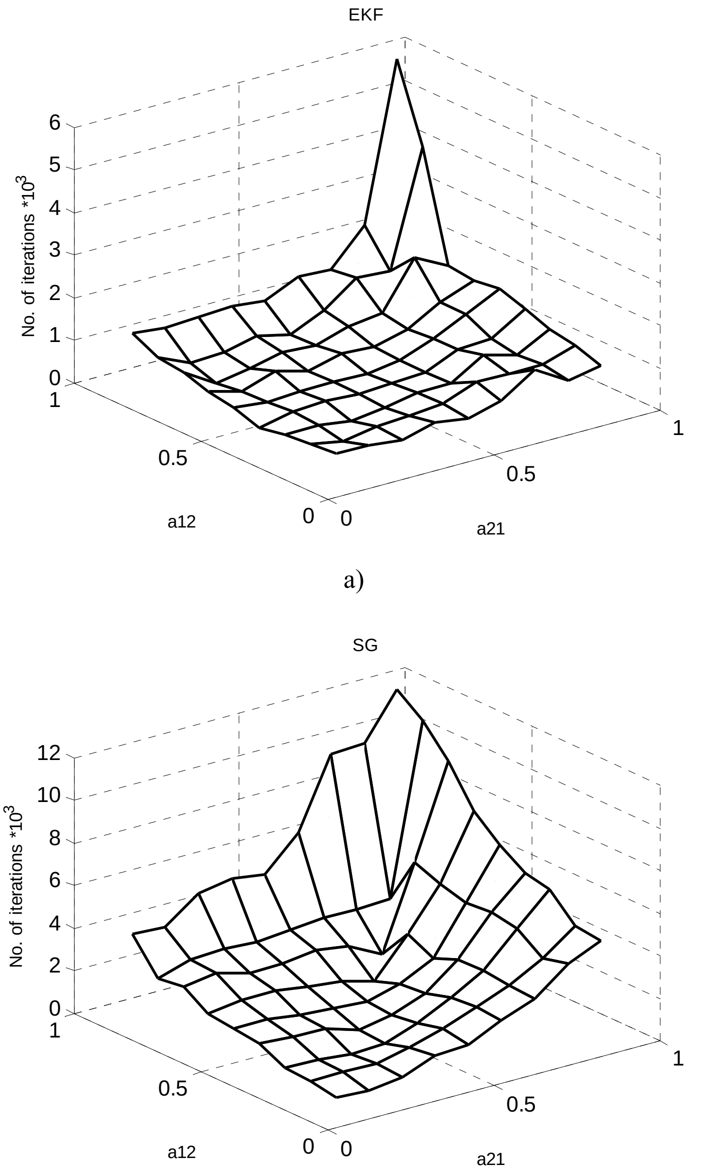


Figure 6: Evolution of the performance index through iterations



b)

Figure 7: Averaged numbers of iterations versus mixing coefficients a_{12} and a_{21} obtained in blind separation of two sources using: a) EKF-based separating algorithm; b) SG-based separating algorithm

In Fig. 7, we give the average numbers of iterations with respect to mixing coefficients a_{12} and a_{21}, required to obtain performance index less than 0.1. For both EKF and SG based algorithms, the task was to recover two source signals, given by (14), from their artificial mixtures. The results shown in Fig. 7 were obtained by averaging 20 independent trials for every pair of the mixing coefficients $(a_{12,}a_{21})$ on the interval $[0.1, 0.9]$ with the step 0.1. It is evident that the convergence of both algorithms is slower when the mixing matrix is near to become singular (the case $a_{12} = a_{21} = 0.9$), and when the signals are badly scaled (for example, the case $a_{12} = 0.1 \wedge a_{21} = 0.9$). However, according to obtained results, the EKF based algorithm is more robust with respect to mixing coefficient values, and outperforms SG-based algorithm both in convergence speed and estimation accuracy.

Example 2. Blind separation of sound sources is an important task in blind signal processing [5, 6, 15]. In this example, the task was to separate two real-world sound signals from their artificial mixtures. The source signals used in this example were two speech recordings obtained from http://medi.uni-oldenburg.de/members/ane/pub/demo_asa99/. The signals were recorded in anechoic chamber at sampling rate 12kHz and each recording consisted of 60000 samples. The speech signal waveforms are given in Fig. 8. Mixture signals (Fig. 9) are obtained artificially using the mixing matrix:

$$\mathbf{A} = \begin{bmatrix} 1 & 0.5 \\ 0.9 & 1 \end{bmatrix} \tag{17}$$

In this example, the learning rate β and forgetting factor α for the SG-based algorithm were set to $\beta = 0.0001$ and $\alpha = 0.8$, the observation noise covariance $\mathbf{R}_k$ for the EKF-based algorithm was fixed to $\mathbf{R}_k = \mathbf{I}$, and the entries of the diagonal process noise covariance matrix $\mathbf{Q}_k$ were exponentially decayed in the range $(10^{-1} - 10^{-6})$. As in the first example, initial values of w_{12}, w_{21}, and the initial cross-correlation in moving average were set to zeros, and the initial value of estimation error covariance $\mathbf{P}$ in EKF-based algorithm was set to identity matrix.

The results obtained using EKF and SG algorithms in separation of real-world speech signals are shown in Figures 10-12.

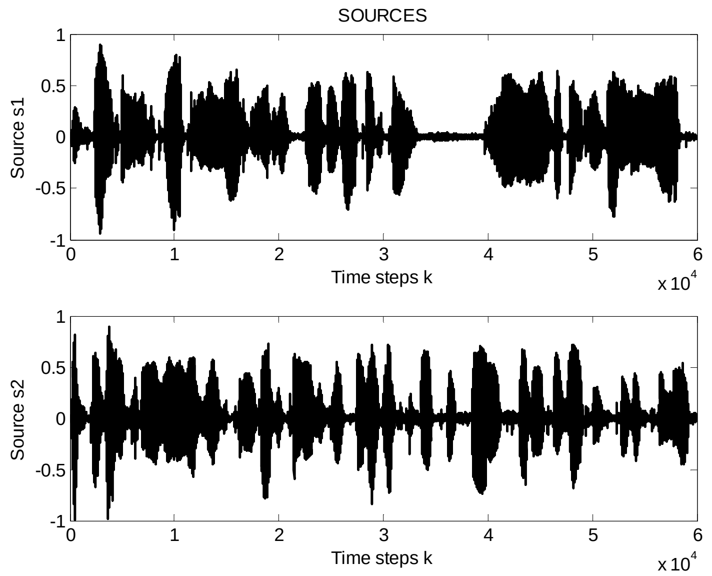


Figure 8: Source signals – speech recordings

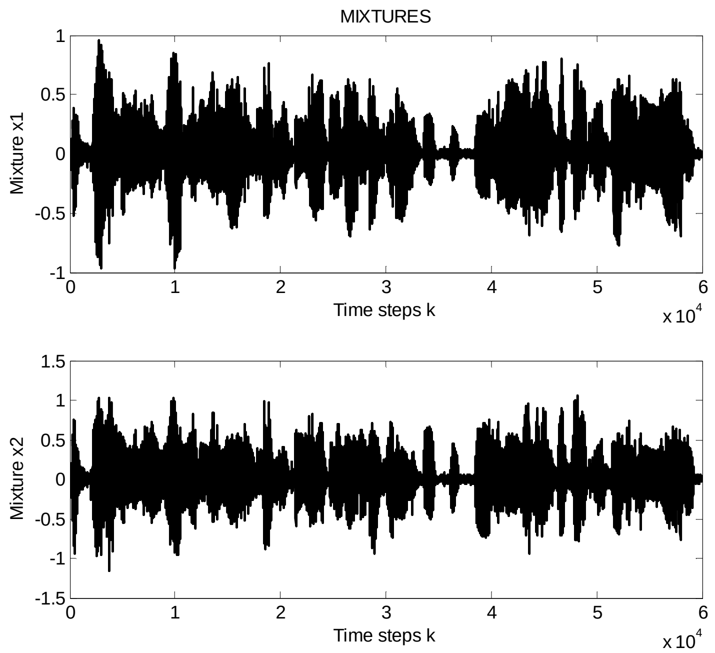


Figure 9: Linear mixtures of sources

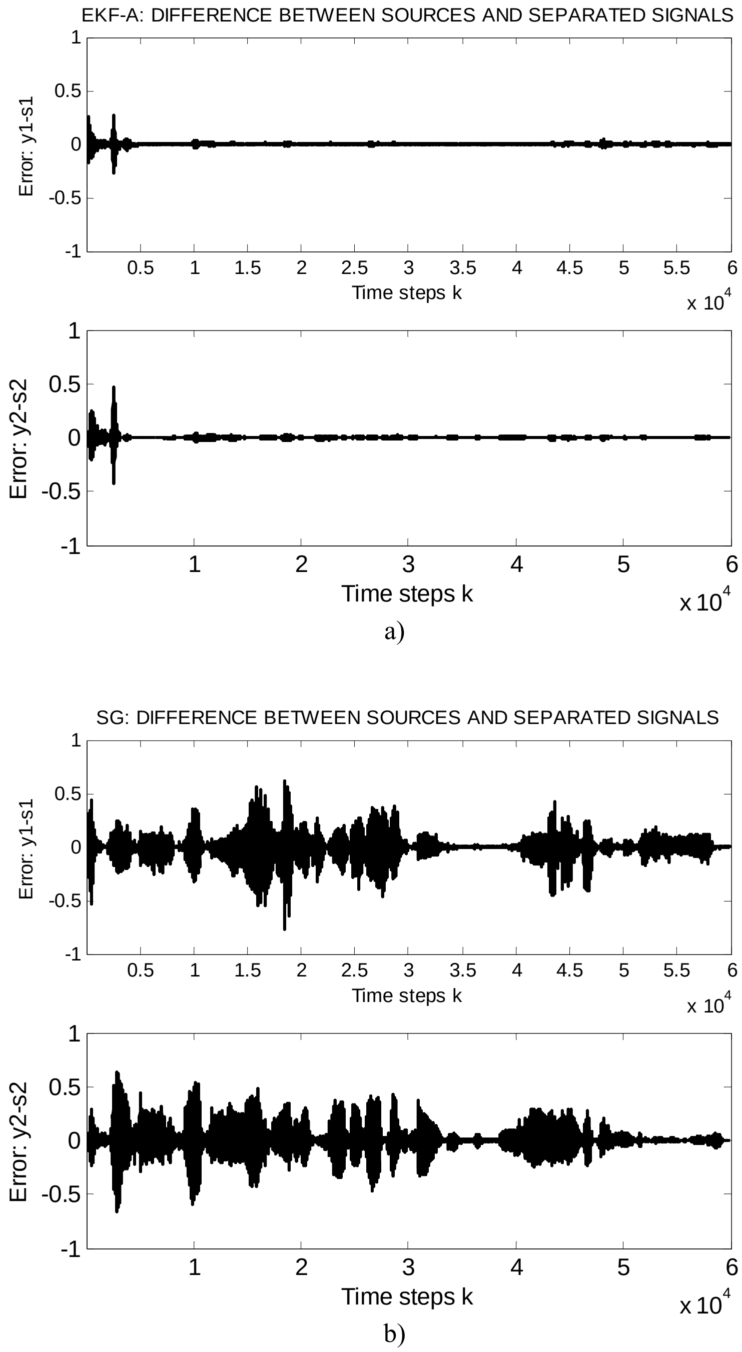


Figure 10: Difference between the original speech signals and recovered speech signals obtained using: a) EKF-based separating algorithm; b) SG-based separating algorithm

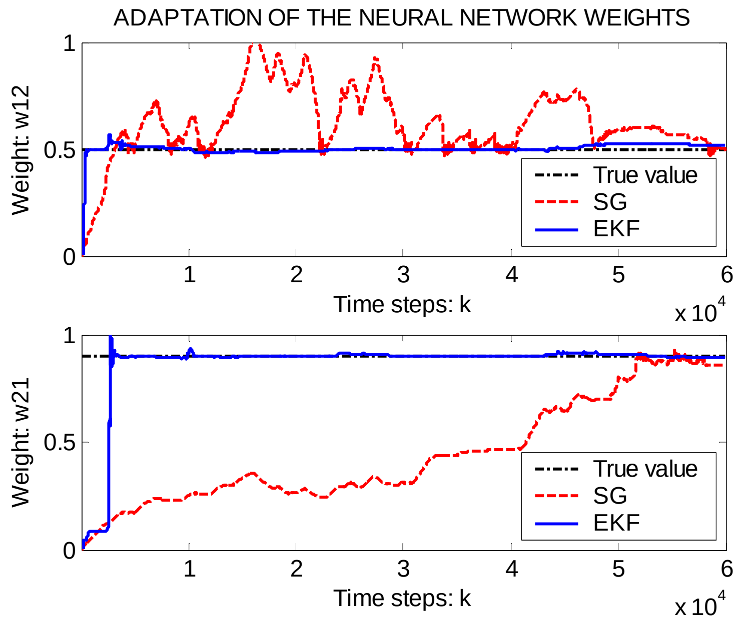


Figure 11: Blind separation of speech signals: adaptation of the neural network weights through iterations

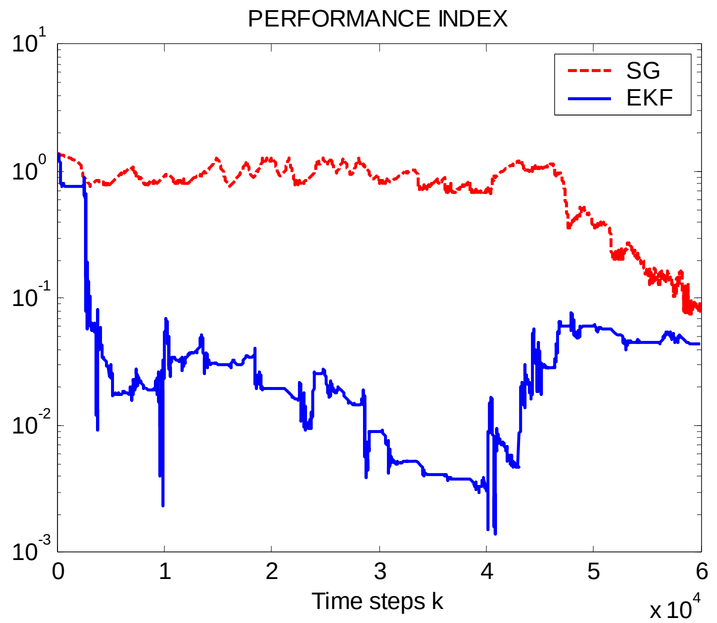


Figure 12: Blind separation of speech signals: evolution of the performance index through iterations

It is interesting to note that in this example better results in separation using EKF-based algorithm are obtained using instantaneous, instead of time-averaged values of output cross-correlations.

6. CONCLUSION

Blind source separation is an important task in signal processing that appears in many areas, such as biomedical signal processing, speech and image processing, and environmental engineering. So far, various approaches and methods were proposed to this problem. In this contribution, we have considered blind separation of instantaneously mixed non-stationary sources using second-order statistics. As a demixing model, we have applied a linear neural network with lateral connections. Using the additional assumption of non-stationarity of sources, we have defined a simple contrast function as the measure of decorrelation of output signals. In order to improve convergence properties of the existing on-line separating algorithm, we have proposed the application of the extended Kalman filter in optimization of the proposed contrast function. We have defined a state-space model of the mixing coefficient dynamics and applied extended Kalman filter to estimate the network weights. The proposed algorithm has shown better convergence properties compared to the algorithm based on the stochastic gradient descent. The algorithm can be applied in blind separation of non-stationary sources if the number of sources is equal or less than the number of mixtures. Since the proposed algorithm requires only second-order statistics of signals, it does not require any additional assumption on the source signal distributions, and can be applied to both Gaussian and non-Gaussian signals. In contrast to other approaches, the methods based on non-stationarity of sources allow the separation of colored Gaussian sources with identical power spectra shapes, but they do not allow separation of sources with identical non-stationarity properties.

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