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Problem of the gyroscopic stabilizer damping

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Abstract

The gyroscopic stabilization of the vibro-isolation system of an ambulance couch is analyzed. This paper follows several previous papers, which concern the derivation of the complete system of appropriate differential equations and some analyses were provided there, as well. It was supposed that mass matrix, stiffness matrix and gyroscope impulse-moment remain constant and the stability of equilibrium state was solved according to different alternatives of the damping and of the radial correction. Little known theorems of the stability were used there. With respect to these theorems, vibro-isolation systems can be classified according to odd or even number of generalized coordinates.

Keywords: gyroscopic stabilization, impulse-moment of the gyroscope, radial correction forces, Cardan frame, precession frame, Hurwitz conditions, complete dissipation, stability areas

1. Introduction

The vibro-isolation of the couch for a terrain ambulance car (see fig. 1) is realized by pneumatic springs and controlled by hydraulic dampers. These vibro-isolation elements are placed between the members of conducting space-mechanism (see fig. 2), which consists of the parallelogram (for the elimination of vertical kinematic excitation), on the upper base of which the Cardan suspension is placed (for the elimination of horizontal rotations). The complete system of appropriate nonlinear differential equations was derived in [1], where the linearised case is also mentioned. Further, the problem of the dependency of characteristic equation roots on the general position of human body was solved in [2]. The velocity characteristics of dampers were analysed in [3] and [4]. In [5] the authors solved possibilities of single and double internal resonance. The main problem of human body vibro-isolation system lays in a decreasing of suspension stiffness (see [6]). The comprehensive overview of all possibilities of the gyroscopic stabilization using Cardan suspension was given in [7]. The dependencies of characteristic equation roots on the gyroscope impulse-moment were solved there.

Fig. 1. Sprung ambulance couch

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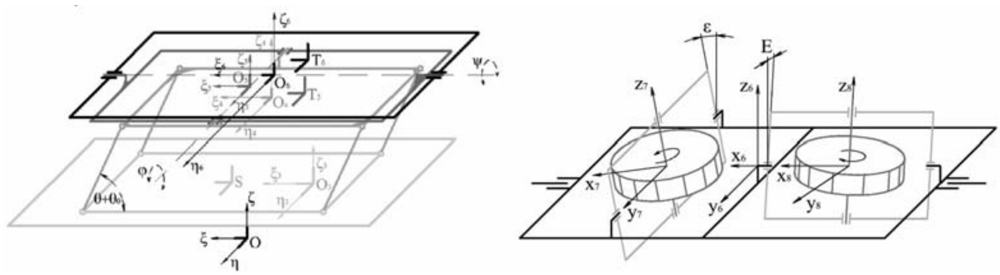


Fig. 2. Scheme of conducting mechanism

The verification of possible applications of theorems for gyroscopic systems and also for vibro-isolation systems with gyroscopic stabilizer are main aims of this paper.

2. Formulation of the problem

First of all, we identify a certain destabilizing effect of radial correction forces by the dynamic analysis of the vibro-isolation system with two gyroscopic stabilizers (see fig. 2). It will be necessary to consider this effect in the relation to damping forces and to define area of the stability.

Equations describing system in fig. 1 were derived in [7] with angle coordinates: θ – the angle of parallelogram arms, φ and ψ – angles of Cardan frames, ε and E – angles of precession frames, γ and Γ – angles of self rotation of gyroscopes. Then we eliminate cyclic coordinates γ and Γ and we denote $\vec{q} = \vec{q}(\varphi, \psi, \theta, \varepsilon, E)$ as the vector of position coordinates.

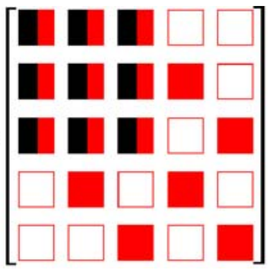
If we suppose small angles and small angle velocities, we can make a trigononometric and power linearization and we can write corresponding system of equation in the matrix form

$$\mathbf{A}\ddot{\vec{q}} + (\mathbf{B}_0 + \mathbf{B}_1(t) + \mathbf{G})\dot{\vec{q}} + (\mathbf{C}_0 + \mathbf{C}_1(t) + \mathbf{K})\vec{q} = \vec{E}_0 + \vec{E}_1(t), \tag{1}$$

where $\mathbf{A}$ is the symmetrical matrix of mass, $\mathbf{B}_0$ the diagonal matrix of damping, $\mathbf{B}_1(t)$ the damping matrix of parametric excitation, $\mathbf{C}_0$ the diagonal matrix of stiffness, $\mathbf{C}_1(t)$ the stiffness matrix of parametric excitation, $\vec{E}_0$ the vector of static gravity forces and static moment of pneumatic springs, $\vec{E}_1(t)$ the vector of external kinematic excitation, $\mathbf{G}$ the matrix of gyroscopic effect and $\mathbf{K}$is the matrix of radial correction[1].

The elements of the symmetrical mass matrix are functions of masses m_j, inertia moments J_{ij}, length of parallelogram arm R etc. The elements of the diagonal stiffness matrix are functions of pressure P_j, gravity forces, parameters of effective area S_{0i}, S_{1i} of the pneumatic springs and of arms r_{pi} of their moments. The elements of the diagonal damping matrix are functions of steepness b_{1j}, of damper velocity characteristics and of arms r_{Ti} of dampers moment.

[1]G. Ziegler made the classification of forces of different structure in [10]: Linear non-conservative forces with antisymmetrical matrix of their coefficients were called "circulation forces". However, Merkin in [9] proposed to remain by a name "forces of radial correction", used in the applied theory of gyroscopes. This term is used from the first half of 20$^{\text{th}}$ century and has a physical substance: a gyroscope moves by their influence to its equilibrium state by the shortest way.

A = 

$$
\begin{aligned}
A_{11} &= (m_R + m_4 + m_5 + m_6)R^2 + 4J_{Ry},\\
A_{22} &= J_{5y} + J_{6y} + m_6(x_{56}^2 + z_{56}^2 + 2x_{56}x_{T6} + 2z_{56}z_{T6}),\\
A_{12} &= R[m_5(-x_{T5}\cos\vartheta_0 - z_{T5}\sin\vartheta_0) +\\
&\quad + m_6((-x_{T6} - x_{56})\cos\vartheta_0 + (-z_{T6} - z_{56})\sin\vartheta_0)],\\
A_{33} &= J_{6x}, \qquad A_{13} = m_6 R\cos\vartheta_0 y_{T6},\\
A_{23} &= -D_{6xy} - m_6 x_{56} y_{T6}.
\end{aligned}
\tag{2}
$$

Elements $A_{i,j}$ correspond to original mass matrix (without gyroscopes) and they are denoted in black.

Elements of extended matrix (with gyroscopes) were derived in [8] and the are denoted in grey. They are functions of equatorial and axial inertia moments of the gyroscope and of the precession frames and of its placement in the second Cardan frame.

Damping and stiffness matrices are diagonal:

$$
B_{011} = 4b_{1i}r_{T\vartheta i}^2\cos^2\vartheta_0, \qquad B_{022} = \sum_{j=1}^{2} b_{1j}r_{T\varphi j}^2, \qquad B_{033} = \sum_{j=1}^{2} b_{1j}r_{T\psi j}^2, \tag{3}
$$

$$
\begin{aligned}
C_{011} &= -(2m_R + m_4 + m_5 + m_6)gR\sin\vartheta_0 +\\
&\quad + 4r_{p\vartheta}^2\left(\frac{n(p_a + p_4)S_{04}^2}{V_4} + p_4 S_{14}\right) + 4p_4 S_{04} r_{p\vartheta}\sin\vartheta_0,\\
C_{022} &= -(m_5 z_{T5} + m_6 z_{56} + m_6 z_{T6})g + \sum_{i=1}^{2} r_{p\varphi 1}^2\left(\frac{n(p_a + p_{5i})S_{05}^2}{V_5} + p_{5i}S_{15}\right),\\
C_{033} &= -m_6 g z_{T6} + \sum_{i=1}^{2} r_{p\psi i}^2\left(\frac{n(p_a + p_{6i})S_{06}^2}{V_6} + p_{6i}S_{16}\right).
\end{aligned}
\tag{4}
$$

Vectors $\vec{E}_0$, $\vec{E}_1(t)$ and matrices $\mathbf{B}_1(t)$, $\mathbf{C}_1(t)$ were derived in [8]. The matrix $\mathbf{G}$ is antisymmetrical and the matrix $\mathbf{K}$ is unsymmetrical. We denote H as the impulse moment of the gyroscope; ($H = J_{ax}\cdot\Omega$; J_{ax} is the axial inertia moment, Ω is the angle velocity of the gyroscope).

$$
\mathbf{G} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -H\\ 0 & 0 & 0 & H & 0\\ 0 & 0 & -H & 0 & 0\\ 0 & H & 0 & 0 & 0 \end{bmatrix}, \qquad
\mathbf{K} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & -k_{2,5}\\ 0 & 0 & 0 & k_{3,4} & 0\\ 0 & 0 & -k_{4,3} & 0 & 0\\ 0 & k_{5,2} & 0 & 0 & 0 \end{bmatrix}. \tag{5}
$$

We suppose that the both moment motors on axes of precession frames, as well as the both relieving motors on axes of Cardan frames, have identical characteristics. Then there hold the equalities

$$
k_{3,4} = k_{2,5} = k_1, \qquad k_{4,3} = k_{5,2} = k_2. \tag{6}
$$

3. General considerations

The system disturbed from equilibrium state, relevant to the model (1), is ($\delta\vec{q}$ is the disturbance of $\vec{q}$)

$$\mathbf{A}\delta\ddot{\vec{q}} + (\mathbf{B}_0 + \mathbf{G})\delta\dot{\vec{q}} + (\mathbf{C}_0 + \mathbf{K})\delta\vec{q} = \vec{0}. \tag{1'}$$

When we want to solve the stability of the equilibrium state of our system, it is useful to transform this system to the standard form (see [9]):

$$\ddot{x}_k + b_k\dot{x}_k + \sum_{j=1}^{s} [Hg_{kj} \cdot \dot{x}_j + (c_{kj} + e_{kj})x_j] = X_k, \tag{7}$$

where the kinetic energy and the Rayleigh dissipative function were transformed into quadratic forms with unit matrices; the parameter H was factored out from the gyroscopic terms. The transformed stiffness matrix (conservative forces) $[c_{kj}]$ is symmetrical, transformed matrices $[g_{kj}]$ and $[e_{kj}]$ of gyroscopic and radial correction forces, respectively, are anti-symmetrical. Terms on the right side of (7) denote nonlinearities.

If the axis of the Cardan frame (see fig. 2) is loaded by external failure moment, the relevant precession frame deflects and this deflection is input value for the relieving motor on the Cardan frame axis. This relieving motor realizes a moment, which is opposite to the failure moment and the precession frame returns to its equilibrium state. Analogously, the deflection of the Cardan frame (registered by water level) is eliminated by the correction moment of motor, which is placed on the axis of precession frame. The terms "moment motors" and "relieving motors" are useful also in applied theory of the gyroscopes.

At the same time the new terminology is introduced: The forces with positive coefficients b_k are called "dissipative" and with negative coefficients b_k are "accelerating". If dissipative forces exceed the accelerating ones, then $\sum_{k=1}^{s} b_k > 0$ and vice versa. If all the $b_k > 0$, we say that the dissipation is complete or full. The linearised system corresponding to (7) is

$$\ddot{x}_k + b_k\dot{x}_k + \sum_{j=1}^{s} [Hg_{kj} \cdot \dot{x}_j + (c_{kj} + e_{kj})x_j] = 0. \tag{7'}$$

The presence of dissipative and radial correction forces determines, if the system is conservative or not, because gyroscopic forces do not perform a work and therefore they do not have any influence on the energy integral. It is useful to remember some theorems of the gyroscopic system stability (see [9]):

1. If the conservative system has an unstable equilibrium state and this instability is of the odd degree ($|c_{kj}| < 0$), it is impossible to reach a stability by the help of gyroscopic forces.
2. If the equilibrium state is stable, then it is possible to reach an asymptotic stability by help of gyroscopic forces and dissipative forces with a full dissipation.

And for non-conservative systems we have these following theorems:

3. If $\sum_{k=1}^{s} b_k < 0$, the system (7) is unstable by arbitrary gyroscopic, radial correction and conservative forces and by arbitrary structure of nonlinear terms.
4. If the system does not contain conservative forces ($c_{kj} = 0$) or nonlinear terms ($X_k = 0$), then

(a) in the case of odd number of position coordinates, it is impossible to reach an asymptotic stability by the help of gyroscopic, dissipative and accelerating forces.
(b) in the case of even number of position coordinates (if we want to reach the asymptotic stability), it is necessary to attach gyroscopic forces and forces with full dissipation at the same time.

5. If the potential energy of a conservative system has an isolated maximum, then:

   (a) in the case of odd number of position coordinates and by arbitrary nonlinear terms it is impossible to reach a stability by means of any gyroscopic, dissipative and radial correction forces;
   (b) in the case of even number of position coordinates and if the dissipation is complete (full), then it is necessary to attach gyroscopic and radial correction forces (independently of nonlinear terms) for reaching the stability.

If we apply all above introduced theorems at the system analysed, we can say that our system has an odd number of position coordinates (unfavourable for reaching the stability).

Before the activation of gyroscopes, it is necessary to reach a stable equilibrium state. All elements of the stiffness matrix must be positive. We can fulfil this requirement by means of a suitable spring and by its displacement.

The decrease of spring stiffness is limited — to keep the stability.

1. According to the validity of important theorems, stated in [9], it is necessary to reach the stability of equilibrium state before the activation of gyroscopes.
2. On the basis of numerical experiments, we record a destabilizing effect of radial correction forces.
3. The characteristic equation

$$\mathrm{Det}\,[\mathbf{A}\cdot\lambda^2 + (\mathbf{B_0}+\mathbf{G})\cdot\lambda + (\mathbf{C_0}+\mathbf{K})] = \mathrm{Det}\,[\mathbf{A}]\cdot\sum_{i=0}^{10} a_i\lambda^i = 0,$$

$$a_{10} = 1, \qquad a_9 = \sum_{j=1}^{5} b_{0j}\bar{A}_j^{(4)}\cdot(\mathrm{Det}\,\mathbf{A})^{-1}, \tag{8}$$

is of the tenth degree. $\bar{A}_j^{(4)}$ are diagonal minors of the fourth degree. To reach an asymptotic stability, it is necessary that $b_{0i} > 0$ for at least one i.

4. Solution of the stability

Let us denote $D_i (i = 1, \ldots, 10)$ Hurwitz determinants corresponding to the equation (8). The dependencies of characteristic equation coefficients a_i and Hurwitz determinants D_i on the parameters k_1 and k_2 are presented in fig. 3. We will change:

(a) parameters of radial correction of the Cardan frames or precession frames k_1 or k_2, respectively;
(b) coefficients b_i of linear damping (b_1 is the damping of parallelogram, b_2, b_3 represent the damping of Cardan frames. Let us remind that this damping will be realized with controlled magnetorheologic dampers. Coefficients b_4, b_5 represent the damping of the precession frames and it is supposed to be very small. This damping is uncontrolled.)

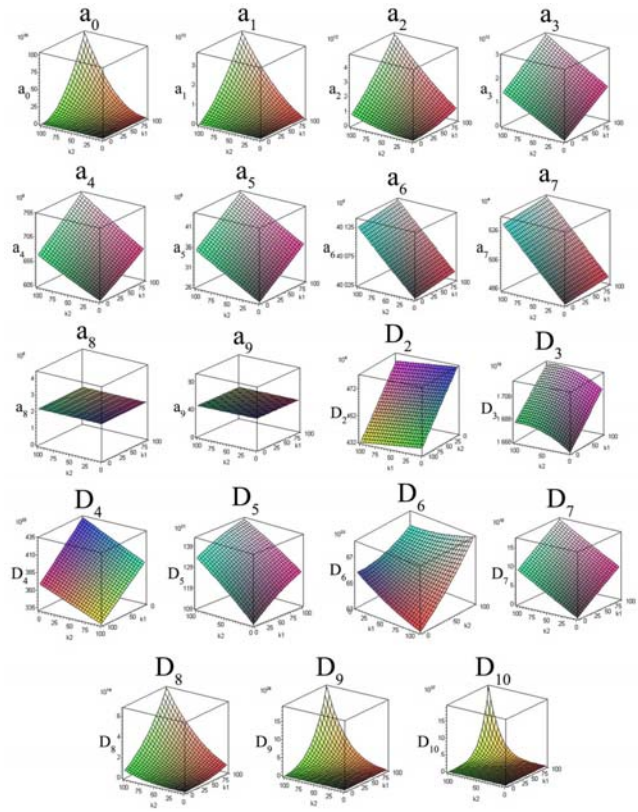


Fig. 3. Hurwitz conditions

From the testing of the validity of all the 19 Hurwitz conditions (the example is in fig. 3), we can determine the limits of stability in the plane of parameters (k_1, k_2).

The results of systematic stability mapping in the plane of radial correction parameters (k_1, k_2) at different alternatives (b_1, b_2, b_3) are demonstrated in fig. 4. Grey points correspond to the state when all the 19 Hurwitz stability conditions are fulfilled and black points represent the opposite case. Passing through the stability limit, we can denote, which Hurwitz condition is not fulfilled and which root changes the sign of its real part. The change of b_1 is in the vertical direction, the change of b_2, b_3 is in the horizontal direction.

The change of $b_4 = b_5 = b_p$ by analogical mapping is presented in fig. 5.

5. Conclusion

The application of general theorems from gyroscopic system area allows the principal orientation in dynamic analysis of vibro-isolation system with gyroscopic stabilizer. Our system has seven degrees of freedom and is described by five position coordinates and two cyclic coordinates. The general theory of gyroscopic systems distinguishes even (favourable) and odd (unfavourable) number of position coordinates.

1. Owing to the validity of relevant theorems (stated in the references), it is necessary to reach the stable equilibrium state before the activation of gyroscopes.

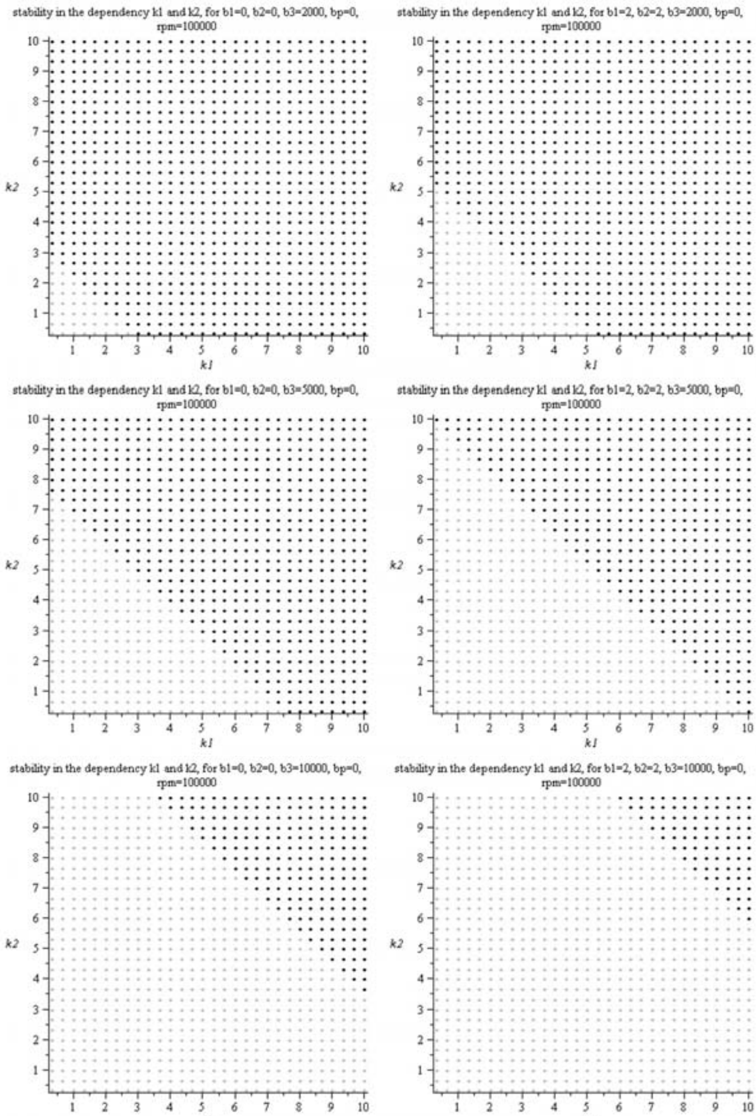


Fig. 4. The areas of stability in the plane k_1, k_2

2. On the base of the fulfilling of Hurwitz conditions, it is possible to map areas of the stability in the plane of parameters of the radial correction by different alternatives of damping parameters.
3. The area of the stability, defined in the plane of the radial correction, increases with the damping. This fact is valid as for damping of the parallelogram, as for damping of Cardan frames and even for damping of precession frames.

The proposed method of stability mapping was verified in other cases of system tuning and it can be used with small expense of machine time.

Acknowledgements

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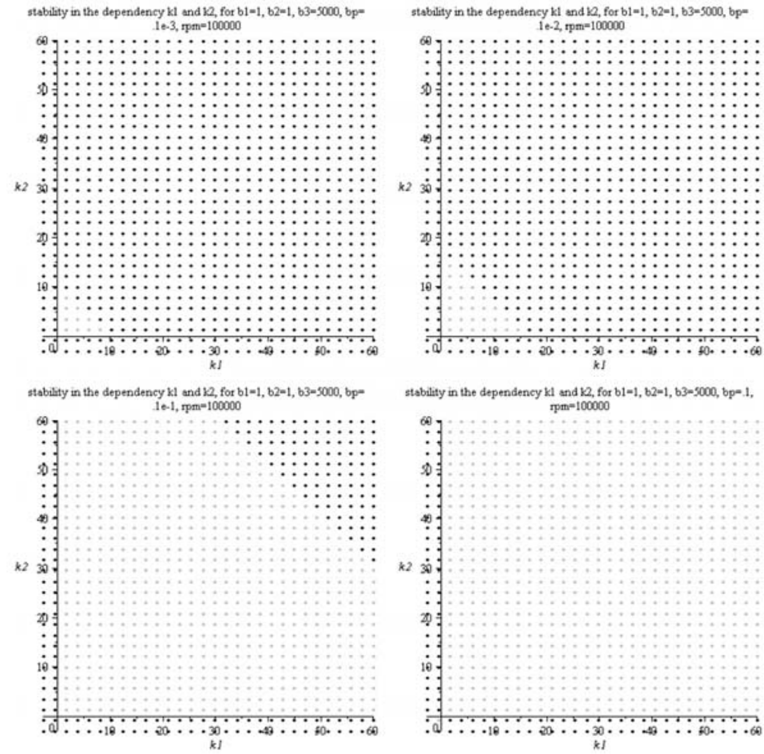

Fig. 5. The areas of stability in the plane k_1, k_2

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