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RESEARCH

Optimal Design of a Cam Mechanism with Translating Flat-Face Follower using Genetic Algorithm

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A B S T R A C T

The optimum design of a cam mechanism is a time consuming task, due to the numerous alternatives considerations. In the present work, the problem of design parameters optimization of a cam mechanism with translating flat-face follower is investigated from a multi-objective point of view. The design parameters, just like the cam base circle radius, the follower face width and the follower offset can be determined considering as optimization criteria the minimization of the cam size, of the input torque and of the contact stress. During the optimization procedure, a number of constraints regarding the pressure angle, the contact stress, etcare taken into account. The optimization approach, based on genetic algorithm, is applied to find the optimal solutions with respect to the afore-mentioned objective function and to ensure the kinematic requirements. Finally, the dynamic behaviour of the designed cam mechanism is investigated considering the frictional forces.

1. INTRODUCTION

The optimal design of cam mechanism is handled in many publications [1-6], where various constraints and methods are considered. Parts applied in cam systems are often coated for increasing their superficial hardness and for reducing friction coefficient [7,8]. A non-linear programming technique with constraints, known as SUMT algorithm is used in [3] for optimum synthesis of a disk cam mechanism with swinging roller follower. In [4] the design parameters are determined by the minimization of the maximum compressive stress at the contact area of a cam-disk mechanism with translating roller follower, where the cam profile is described with the aid of cubic spline functions. Tsiafis et al. present in [5] a multi-objective procedure based on genetic algorithms to optimize the design parameters of a disk-cam mechanism with a roller follower.

In the present paper the problem of the design parameters optimization of a cam mechanism with a reciprocating flat-face follower is investigated, using multi-objective optimization with genetic algorithm. The design parameters for this type of mechanism are the radius of the cam base circle, the follower face width and the follower offset. The optimization is achieved by the development of programs using the high level computing language MATLAB with the GA (genetic algorithm) toolbox application. Furthermore, the dynamical analysis of the designed mechanism considering friction is investigated.

2. MATHEMATICAL FORMULATION

A cam mechanism with a translating flat-face follower is shown in Fig. 1. The cam is assumed to have constant angular velocity. The profile of the cam can be determined considering the kinematical and dynamical requirements of the mechanism.

The design parameters under optimization are the cam base circle R_b, the width follower face L and the follower offset e as shown in Fig. 1.

The optimization of the design parameters of the cam mechanism can be achieved by the minimisation of the cam size, of the torque required to drive the cam and the contact stress between the cam and the follower.

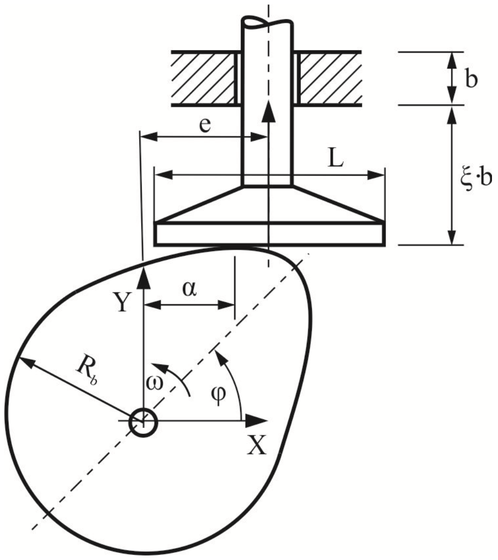

Fig. 1. Cam mechanism with translating flat-face follower.

Therefore, it could be formulated as an optimization problem, where the objective function (F) takes into account the cam size (F_1), the input torque (F_2) and the maximum contact stress (F_3):

$$F = \alpha \cdot F_1 + \beta \cdot F_2 + \gamma \cdot F_3 \tag{1}$$

with

$$F_1 = R_b + L \tag{2}$$

$$F_2 = T = \frac{(P \cdot v)}{\omega} \tag{3}$$

$$F_2 = \sigma_{max} = \left[\frac{P'}{\rho\left(\frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}\right)} \right]^{1/2} \tag{4}$$

where T is the input torque, P is the total normal load on the cam, v is the follower velocity, ω is the camshaft angular velocity, σ_{max} is the maximum contact stress between the follower and the cam, P' is the normal load per unit width of the contacting members, ρ is the radii of curvature of the cam, $\mu_1$and $\mu_2$are Poisson's ratio for the cam and the follower respectively and E_1, E_2 are the module of elasticity of the cam and the follower respectively.

The weighting factors α, β and γ are used in order to scale the contribution of the corresponding terms in the objective function value. The minimization of the objective function determines the optimum values of the unknown parameters. During the optimization procedure the following functional constraints are imposed:

a) The maximum value of the pressure angle must be smaller than the maximum permitted: $\delta_{max}<\delta_{per}$.

The pressure angle can be calculated by [1]:

$$\delta = atan\left(\frac{v-e}{s+\sqrt{R_b^2-e^2}}\right) \tag{5}$$

where s is the follower displacement.

b) The maximum value of the contact stress must be smaller than the material permissible strength: $\sigma_{max}<\sigma_{per}$.

c) The offset e must satisfy the constraints: $0<e<L/2$ and $e<s$.

d) In order to avoid the follower jamming the eccentricity a must fulfil the conditions [1]:

$$a < \frac{b}{2\mu} + \frac{b \cdot \mu_0(1+2\xi)}{2} \tag{6}$$

and $a<L/2$, where the dimensions a, b and the parameter ξ are explained in Fig. 1 and μ is the coefficient of friction between the follower stem and its guide.

The distance a is calculated with the following equation:

$$a = \left(r^2 - \left(R_b + s\right)^2\right)^{1/2} \quad (7)$$

with $r = \left(x^2 + y^2\right)^{1/2}$, where x and y are the cam profile coordinates.

3. PROPOSED ALGORITHM

In the present paper a multi-objective genetic algorithm (GA) method in MATLAB programming environment is used to find the optimal solution.

The input data are the cam mechanism type, the kinematic and functional requirements, the variables bounds and the algorithm parameters. In these parameters are included the initial parameters of the GA such as the population size, the crossover rate, the mutation rate, etc. and the number of the GA loops. Using the equation (1) the fitness function is defined, which is used in all steps of the algorithm.

During the genetic algorithm, starting populations are randomly generated to set variables values, which are used to calculate the fitness function value. Genetic algorithm [9] uses selection, elitism, crossover and mutation procedures to create new generations. The new generations converges towards a minimum that is not necessarily the global one. After some repetitions when the maximum generations' number is achieved, the variables values corresponding to the minimum fitness function value are selected as the optimum variables values of the genetic algorithm.

An important issue in genetic algorithms is the treatment of constraints. For each solution of the population, the objective fitness values are calculated. Furthermore, every solution is checked for constraints violation.

4. NUMERICAL APPLICATION

The introduced methodology is applied to find the design parameters of a cam mechanism with translating flat-face follower where the follower offset is set equal to zero (e=0).

Figure 2 shows the kinematic requirements per transient region of the indicated in this figure follower displacement diagram.

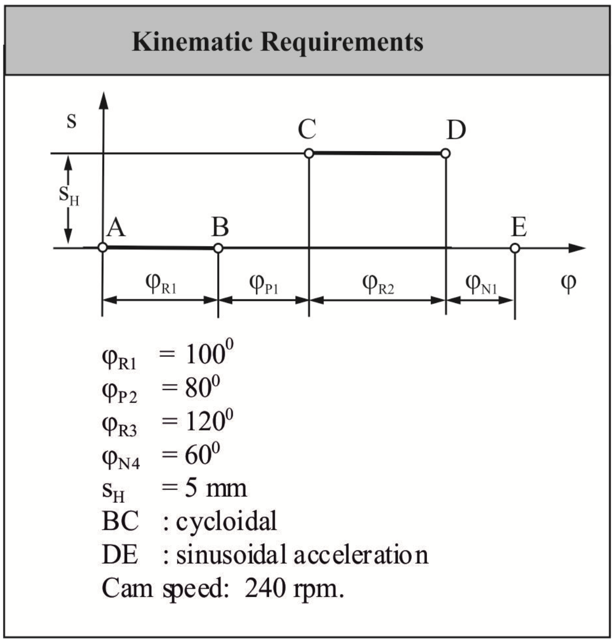

Fig. 2. Kinematic requirements.

Functional requirements
Permitted max. pressure angle: $\delta_{per}=30^0$ Cam base circle radius: $20<R_b<40$ mm Follower length: $30<L<60$ mm Permitted contact stress for the cam: $\sigma_{per}=1750$ N/mm^2
Materials properties
Cam Poisson's ratio: $\mu_1=0.3$ Follower Poisson's ratio: $\mu_2=0.26$ Cam modulus of elasticity: $E_1=2.1\text{x}10^5$ N/mm^2, Follower modulus of elasticity: $E_2=1.15\text{x}10^5$ N/mm^2.

Fig. 3. Materials properties and functional requirements.

The functional requirements and the material properties used in this investigation are inserted in Fig. 3.

The parameters involved in all tests, mainly in GA procedure, are the same and selected as optimums through many applied tests: population of individuals = 20, cross probability = 80 %, elite count = 2 and the maximum number of generations is 100.

Considering kinematic requirements the displacement, velocity and acceleration of the follower are determined (Fig. 4).

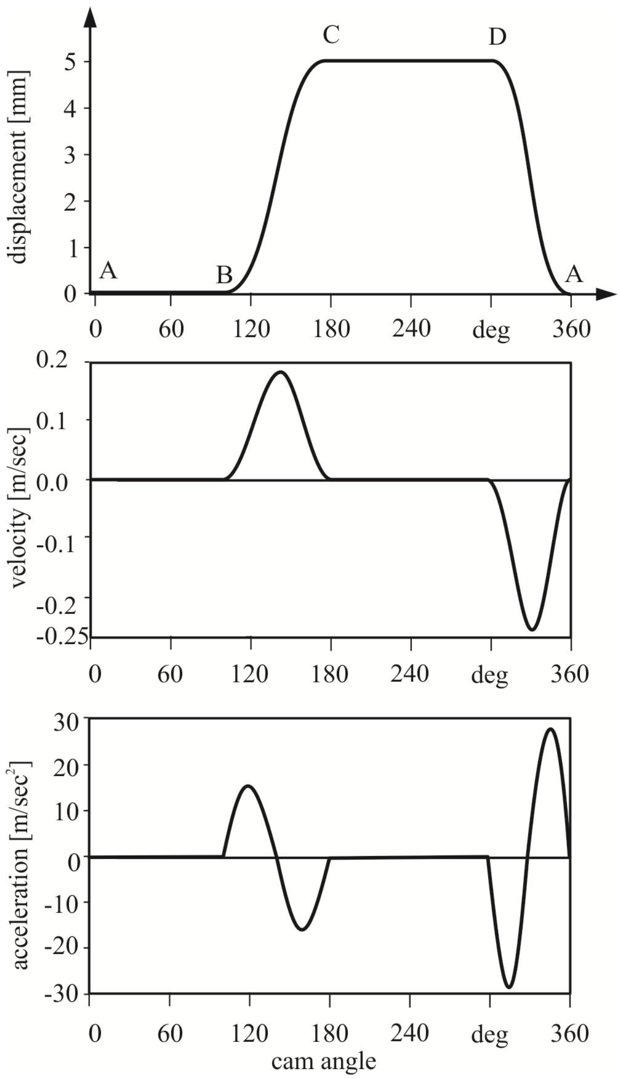

Fig. 4. The follower motion diagrams.

In general the weighting factors α, β and γ of the fitness function (1) are selected considering the importance of the objectives that must be achieved by the mechanism. A high value of the weighting factor α increases the importance of first part of the objective function (F_1) that is to obtain a small cam size. After several tests the following weighting factors are chosen: α=0.1, β=0.1 and γ=0.8. Running the MATLAB codes with above mentioned parameters, the following design parameters are obtained: R_b=32.67 mm and L=53.21 mm. For constructed mechanism these parameters are finally set: R_b=35 mm and L=50 mm.

The cam profile is shown in Fig. 5. The 3D model of the designed cam mechanism is illustrated in Fig. 6.

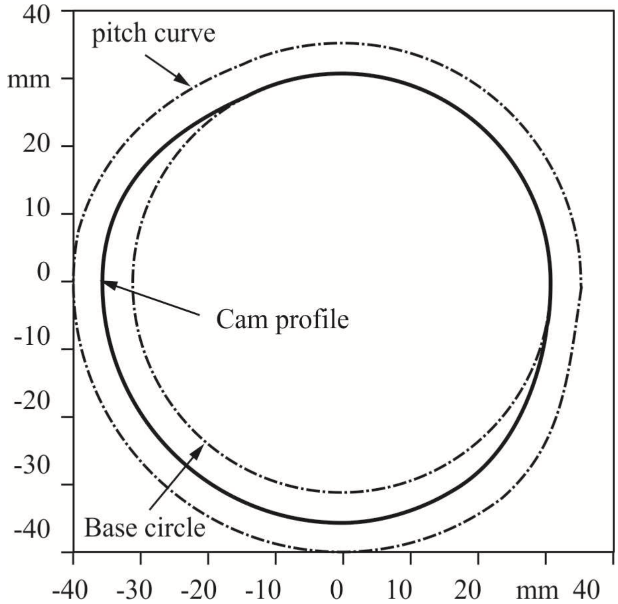

Fig. 5. Cam profile.

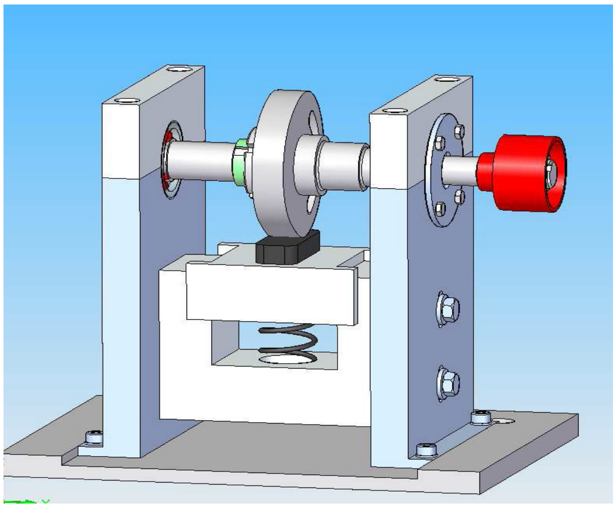

Fig. 6. 3D model of the cam mechanism.

5. FORCE ANALYSIS OF CAM MECHANISM CONSIDERING FRICTION FORCES

In this section the dynamic force analysis of the designed mechanism considering the friction force between follower and its guide and the friction force between cam and flat face follower is investigated.

The force transmission of a radial cam with a reciprocating flat-faced follower is shown in Fig. 7, where P is the external load on the follower, μ is the coefficient of friction between the follower stem and its guide, μ_o the coefficient of friction between the cam and the flat face follower and d is the guide diameter.

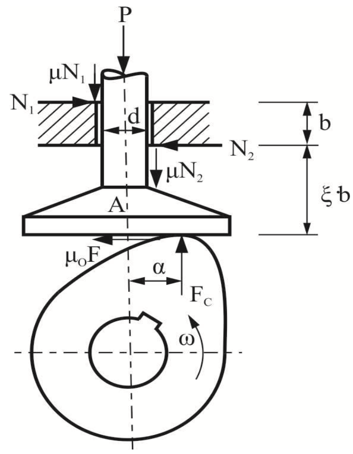

Fig. 7. Force transmission of cam mechanism with translating flat-face follower.

From the equilibrium equations of horizontal and vertical forces and moments about the point A and assuming that difference the between $\mu\frac{d}{2}N_1$ and $\mu\frac{d}{2}N_2$ is negligible, the forces F_c, N_1 and N_2 are determined [1]:

$$F_C = \frac{bP}{\Gamma} \tag{8}$$

$$N_1 = \frac{\left(a-\mu_0\xi b\right)P}{\Gamma} \tag{9}$$

$$N_2 = \frac{\left[a-\mu_0 b\left(1+\xi\right)\right]P}{\Gamma} \tag{10}$$

with

$$\Gamma = b - 2a\mu + \mu\mu_0 b\left(1+2\xi\right)$$

and

$$P = m\ddot{s} + c\dot{s} + k\left(s+s_0\right) + F_b$$

where m is the follower mass, s, $\dot{s}$ and $\ddot{s}$ are the displacement, velocity and acceleration of the follower respectively, c is the damping coefficient, k is the spring constant, s_0 is the initial compression of the spring and F_b is the follower weight.
Furthermore, the friction forces are written as:

$$Q_0 = \mu_0 F \tag{11}$$

$$Q_1 = \mu N_1 \tag{12}$$

$$Q_2 = \mu N_2 \tag{13}$$

and the cam shaft torque due to the friction is given by:

$$T_f = Q_0\left(R_b + s\right) + Q_1\frac{d}{2} - Q_2\frac{d}{2} \tag{14}$$

Fig. 8. Constructed cam mechanism.

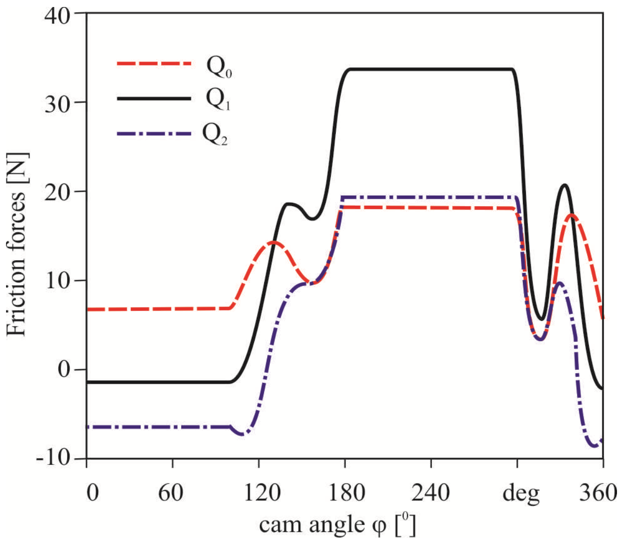

Fig. 9. Friction forces of cam mechanism with translating flat-face follower.

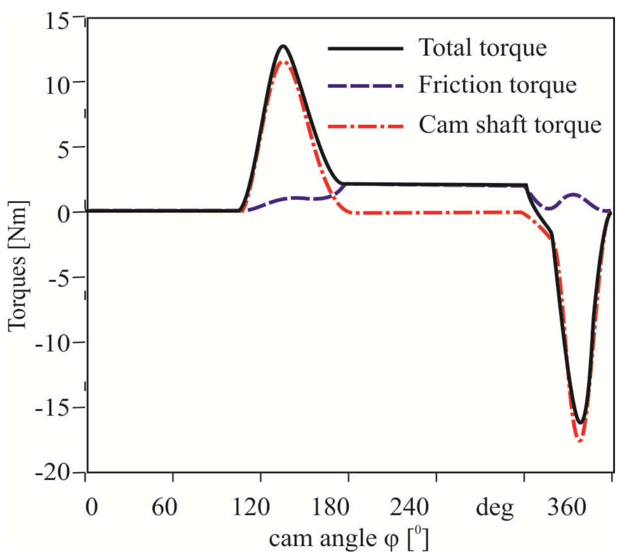

Fig. 10. Input torque with and without friction.

In the designed and constructed mechanism (Fig. 8) the data used in dynamic analysis are: μ=0.78, μ_0=0.15, m=1 kg, k=3004 N/m, s_0=13 mm, d= 50 mm and b=50 mm.

The damping coefficient is $c = 2\zeta \frac{\sqrt{1000\ k\ m}}{1000}$ with ζ=0.1. The spring constant k is chosen considering the spring force greater than inertia force corresponding to maximum deceleration, in order to avoid the jump phenomenon.

The parameter ξ is determined with the relation: ξ= (15-s)/b.

The diagram of friction forces versus cam angle is illustrated in Fig. 9.

In Fig. 10 is inserted the diagram of the input torque with and without friction.

6. CONCLUSION

In this paper the optimization of the design parameters of a cam mechanism with a flat-faced follower is approached. For this task the multi-objective optimization with genetic algorithm is applied using the high level programming language of MATLAB. The optimization satisfies constraints which are made in order to operate a cam mechanism properly. This procedure is automatic, gives results fast and it appears to be reliable. The final results provide useful information for a cam mechanism synthesis and can be used as a basis of final preference depending on the objectives that have to be succeeded.

Subsequently, after the cam mechanism synthesis, the applied friction forces are calculated. The most important conclusion is the fact that the friction forces are analogous with the action of the follower movement. This means that in the areas of dwell the friction forces are steady, whereas in the areas of rise or return the friction forces alter in an almost similar way.

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