

Numerical wind wave model with a dynamic boundary layer

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Received: 05 March 2001 – Revised: 23 July 2001 – Accepted: 22 January 2002

Abstract. A modern version of a numerical wind wave model of the fourth generation is constructed for a case of deep water. The following specific terms of the model source function are used: (a) a new analytic parameterization of the nonlinear evolution term proposed recently in Zakharov and Pushkarev (1999); (b) a traditional input term added by the routine for an atmospheric boundary layer fitting to a wind wave state according to Makin and Kudryavtsev (1999); (c) a dissipative term of the second power in a wind wave spectrum according to Polnikov (1991). The direct fetch testing results showed an adequate description of the main empirical wave evolution effects. Besides, the model gives a correct description of the boundary layer parameters' evolution, depending on a wind wave stage of development. This permits one to give a physical treatment of the dependence mentioned. These performances of the model allow one to use it both for application and for investigation aims in the task of the joint description of wind and wave fields.

1 Introduction

The problem of constructing an effective and physically well-grounded wind wave model is far from being solved. One reason for this circumstance dwells in the extreme complexity of the processes that take place at the air-sea interface and are responsible for the wind wave evolution. Therefore, despite progress in the scope gained over the last 10–15 years (see, for example, monographs Efimov and Polnikov, 1991; Komen et al., 1994; Lavrenov, 1998), some questions have been raised at present, which can be solved in order to construct a new generation model.

In reality, all known wind wave models incorporate, as a rule, empirical parameterization of the nonlinear evolution term with ambiguous fitting parameters (the SWAMP group, 1985; Komen et al., 1994). From the fundamental point of view, this fact is a weak link of any model. Additionally, all

known models are constructed without a dynamic fitting of an air boundary layer to a state of wind waves (a dynamic boundary layer regime). At the same time, recent advances in the wind wave theory permit one to remove the principal drawbacks mentioned above in a wind wave model and to construct more elaborate models with a dynamic boundary layer. According to the project SWAMP (the SWAMP group, 1985) classification, such models could be attributed to the next fourth generation.

As a prototype of a new generation model, one may point to the spectral wind wave model presented in Zakharov et al. (1999). But due to the use in this model of the so-called “narrow directional approximation”, the model is only applicable for narrow angular spread spectra. The use of this model for arbitrary, non-uniform wind fields needs a special basing.

The model proposed in this paper has significant differences from the model of Zakharov et al. (1999). First, this model is based on the diffusive approximation for the exact nonlinear term, derived recently in Zakharov and Pushkarev (1999). This is the more universal approximation with respect to the one used in Zakharov et al. (1999). As shown in a special investigation (Polnikov, 2002), this approximation is applicable even in the case of mixed sea. Second, in our case, the more advanced model of a dynamic boundary layer proposed in Makin and Kudravtsev (1999) is used. Third, the square power in a wind wave spectra dissipation term is used in the model. Effectiveness of such a dissipation term is shown in Polnikov (1985, 1991), and the foundation of it is given in Polnikov (1994, 1995). Some experimental evidence of such a dissipation is given in Donelan (1998). All these elements of the new model warrant a priori a wider domain of applicability with respect to all known models.

The most important new fields of applicability of the proposed numerical model are the following:

1. Investigation of the air-sea interaction physics;
2. Investigation of the boundary layer parameters' dependence on the sea state;

3. Combination of the wind wave model with an atmosphere circulation model to provide changeable boundary conditions at the air-sea interface.

All other traditional fields of application described in monographs mentioned above are available for the proposed wind wave model as well.

2 Main equations

The main evolution equation for the frequency-angle wind wave spectrum, $S(\sigma, \theta, \mathbf{x}, t)$, has the kind

$$\frac{\partial S}{\partial t} + C_{gx} \frac{\partial S}{\partial x} + C_{gy} \frac{\partial S}{\partial y} = F \equiv NL + In - Dis. \quad (1)$$

Here, σ and θ represent the frequency and the angle of the wave component, respectively; \mathbf{x} is the vector of the horizontal coordinates, t is the time variable, C_{gx} , C_{gy} are the group velocity components of the wave, F is the source function including the following evolution mechanisms: NL is the rate of conservative nonlinear energy transfer through the wave spectrum (“nonlinearity”); In is the rate of energy transfer from the wind to the waves (“input”); Dis is the rate of wave energy loss due to wave breaking and interaction with the upper layer turbulence (“dissipation”).

2.1 Nonlinear evolution mechanism

To describe the nonlinear evolution term of the model, we use the formula derived in Zakharov and Pushkarev (1999) in the diffusion approximation for the exact nonlinear kinetic integral. In the frequency-angle representation, this formula has the kind

$$NL(\sigma, \theta) = \left. \frac{\partial S(\sigma, \theta)}{\partial t} \right|_{NL} = cg^{-4} \sigma L \left[\sigma^{12} S^3(\sigma, \theta) \right], \quad (2)$$

where L is the differential operator of the second order

$$L = \frac{1}{2} \frac{\partial^2}{\partial \sigma^2} + \frac{1}{\sigma^2} \frac{\partial^2}{\partial \theta^2}. \quad (3)$$

The difference from the original paper by Zakharov and Pushkarev (1999) is that the gravity acceleration constant, g , and the only fitting coefficient, c , are introduced here in the explicit form. As it was shown in a special study (Polnikov, 2002), the value of c is of the order of 0.1.

2.2 Input mechanism

In the energy containing frequency domain ($0.5\sigma_p \leq \sigma \leq 2.5\sigma_p$, where σ_p is the peak frequency), the input term has the traditional kind

$$In = \beta(\sigma, \theta, U) \sigma S(\sigma, \theta), \quad (4)$$

and the growing increment, β , is given by the empirical formula (Snyder et al., 1981)

$$\beta(\sigma, \theta, U) = \max \left(0, a \frac{\rho_a}{\rho_b} \cdot \left(\frac{28U_* \sigma}{g} \cos(\theta - \theta_u) - b \right) \right) \quad (5)$$

Here, the following notions are used: ρ_a and ρ_b represent the air and water density, respectively, U_* is the friction velocity, and θ_u is the local wind direction. Parameters a and b are varying in the following intervals: $a \cong 0.2 - 0.3$ and $b \cong 0.9 - 1$.

In the higher frequency domain $\sigma > 2.5\sigma_p$, the Plant’s parameterization for β is used (Plant, 1982)

$$\beta = (0.04 \pm 0.02) \left(\frac{U_* \sigma}{g} \right)^2 \cos(\theta - \theta_u). \quad (6)$$

2.2.1 Dynamic boundary layer

A fitting of the air boundary layer to the sea state (a dynamic boundary layer regime) is realized by means of using a special procedure for calculating the friction velocity, U_* . For this aim the new model of the boundary layer evolution over the sea surface is used (Makin and Kudryavtsev, 1999). The essence of this model is the following.

As usual, it is proposed that the total momentum flux to waves, τ , can be represented as a sum of the turbulent flux, τ^t , and the wave-induced flux, τ^w :

$$\tau \equiv U_*^2 = \tau^t + \tau^w. \quad (7)$$

Hereafter, the flux τ is normalized to the air density for simplicity. Further, the turbulent flux is calculated by the use of the following equation for a turbulent shear layer

$$\tau^t(z) = K \frac{\partial U(z)}{\partial z}. \quad (8)$$

A use of the turbulent energy balance equation with some simplifications permitted Makin and Kudryavtsev to determine an expression for the vertical eddy viscosity coefficient, K . The latter allows one to solve Eq. (8) analytically and to write an explicit expression for the vertical wind velocity profile of the kind

$$U(z) = U_*^2 \int_{z_0^v}^z \left[1 - \frac{\tau^w(z)}{U_*^2} \right] K^{-1} dz = \frac{U_*}{\kappa} \int_{z_0^v}^z \left[1 - \frac{\tau^w(z)}{U_*^2} \right]^{\frac{3}{4}} d(\ln z). \quad (9)$$

Here, z_0^v is the viscous sub-layer width the order of which is defined by the formula $z_0^v \cong 0.1 \frac{v}{U_*^t}$, where v is the kinematic viscosity of the air, and U_*^t is the turbulent part of the friction velocity.

Thus, all boundary layer parameters, namely the friction velocity, U_* , the wind profile, $U(z)$, and the drag coefficient, C_d , could be found from Eq. (9) for the known wind at the standard horizon (for example, $U(10) = U_{10}$) and profile of the wave-induced momentum flux, $\tau^w(z)$. To determine the

latter, the following equation was derived

$$\frac{\tau^w(z)}{U_*^2} = \int_{\sigma_{min}}^{\sigma_{max}} d\sigma \oint_{\theta} d\theta \left[1 - \frac{\tau^w(0)}{U_*^2} \right] \cdot \left[\exp(-10zk) \cos(5\pi zk) R\left(\frac{c}{U_{10}}\right) \cdot k^2 S(\sigma, \theta) \cos(\theta) |\cos(\theta)| \right]. \quad (10)$$

Here, $k = \frac{\sigma^2}{g}$ is the wave number, $R\left(\frac{c}{U_{10}}\right)$ is the empirical long-wave cutting factor for the Plant's input term, a similar type to that given in Makin and Kudryavtsev (1999), $c = g/\sigma$ is the phase velocity of the wave component, and U_{10} is the known value of the wind velocity at the standard horizon, $z = 10$ m. The limits of integration, σ_{min} and σ_{max} , are defined by the lower edge of the numerical frequency interval and by a specially chosen upper limit for the high frequency spectrum tail, respectively.

Thus, the problem of estimation of the friction velocity, U_* , the drag coefficient, C_{10} , and the effective value of the roughness parameter, z_{0ef} , becomes closed for the given value U_{10} and the known sea state. Note that in order to find the effective roughness, z_{0ef} , one should take a hypotheses of the profile $U(z)$. In our estimations, we would use the standard logarithm wind profile

$$U(z) = \frac{U_*}{\kappa} \ln\left(\frac{z}{z_{0ef}}\right), \quad (11)$$

as it is usually done in practice (see, for example, Donelan, 1998).

It is important to mention that in this model all boundary layer magnitudes are functions of the sea state, for instance, of the wave age, A , defined by the ratio $A = c_p/U_{10}$ (where c_p is the phase velocity of the wave component corresponding to the peak of spectrum).

A difference of this model from the one in Zakharov et al. (1999) consists in the more general approach to estimation of the turbulent momentum flux, τ^t . This approach is based on Eq. (8) and on the use of the turbulent energy balance equations for determination of the vertical eddy viscous coefficient, K , (for details, see Makin and Kudryavtsev, 1999). Note that in this case (opposite to the model in Zakharov et al., 1999), the value of τ^t becomes dependent on the sea state, as well as on the value of τ^w .

2.3 Dissipation mechanism

The dissipation term is parameterized by the formula proposed in Polnikov (1991)

$$\text{Dis} = \max \left\{ 0, \frac{0.3 (\sigma - a\sigma_p) (\sigma - b\tilde{\sigma}_u) \sigma^6 S^2(\sigma, \theta) T(\theta)}{g^2 \sigma \tilde{\sigma}_u} \right\}, \quad (12)$$

where $a = 0.9$, $b = 0.9$;

$$\tilde{\sigma}_u = \frac{\sigma}{\min\left(\sigma_p, \frac{g}{U_{10}}\right)} \quad (13)$$

is the frequency normalized by an effective peak frequency, and

$$T(\theta) = \begin{cases} 1 & n\pi u|\theta - \theta_u| \leq \pi/2 \\ 1 - \cos(\theta - \theta_u) & \text{otherwise} \end{cases}. \quad (14)$$

Functions (13) and (14) were introduced into Eq. (12) in order to take into account a dissipation term dependence on the wind velocity, U_{10} , its direction, θ_u , and the peak frequency, σ_p . In the course of the model fitting, these functions may be changed, as well as the first three factors in Eq. (12). Therefore, some variations of the dissipation term are possible in a further elaboration of the model. But the general kind of term, which is of the second power in the spectrum, is assumed to be unchangeable, as it is theoretically grounded in Polnikov (1995). We should note here that the high power in the spectrum for the dissipation term was stated in Donelan (1998) on the basis of the experimental data analysis.

3 Results

We do not dwell here on the methodological and technical details of calculations, which are typical for such types of problems (see, for instance, The SWAMP group, 1985). Thus, we address directly the results of the calculations for the direct fetch tests.

First, the key important results of the model testing are shown in Figs. 1–4. They give evidence of an adequate description of both the main empirical effects of wave evolution and the boundary layer parameters' variability. In particular, the following evolution effects are well reproduced: (a) the wave energy growing law (Fig. 1); (b) the spectrum evolution history and the "overshoot" effect (Fig. 2); (c) a decrease in the roughness parameter, z_{0ef} , with an increase in the wave age, A , (Fig. 3); (d) a decrease in the drag coefficient, C_{10} , with an increase in the wave age, A , (Fig. 4).

Note that the latter two effects may be reproduced only in models with a dynamic boundary layer. In our case, such explicit reproduction of them is presented in literature for the first time. It gives evidence of the rather good quality of the model. A comparison of the numerical dependence $z_{0ef}(A)$ with the empirical one taken from Drennan et al. (1999) (Fig. 5) shows a good quantitative correspondence between the theory and the experiment.

The verisimilitude of the mentioned effects' reproduction by the wind wave numerical model permits one to state a correctness of the ideas used for the boundary layer dynamics' description. Herewith, details of the model presented above give the possibility of treating the effects of the boundary layer dynamics. According to Sect. 2.2.1, it follows that the main reason for the effects mentioned above is connected to a falling of a spectrum tail level at high frequencies, accompanying a wave age increase. It leads to a decrease in the

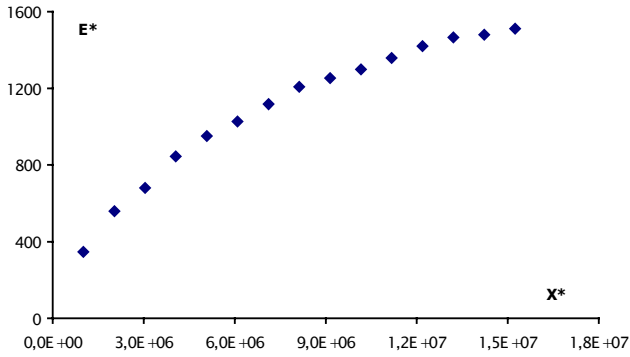


Fig. 1. Dependence of the nondimensional energy, $E^* = \frac{Eg^2}{U_*^4}$, on the nondimensional fetch, $X^* = \frac{Xg}{U_*^2}$, for a developed sea.

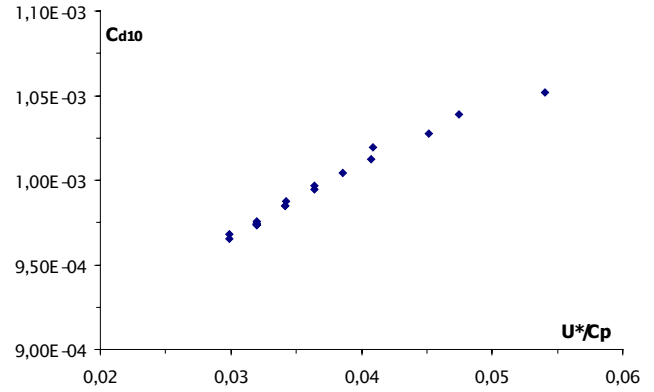


Fig. 4. Dependence of the drag, C_{d10} on the inverse wave age, U_*/C_p .

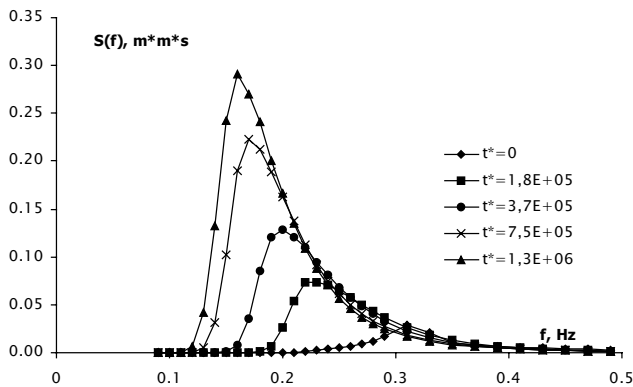


Fig. 2. Time history for $S(f)$ at some moments of the nondimensional time, $t^* = \frac{tg}{U_*}$.

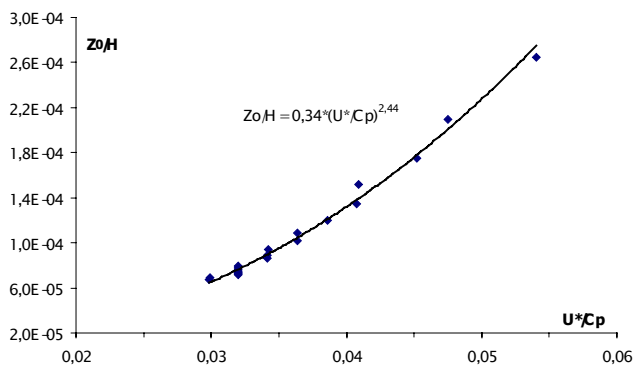


Fig. 3. Dependence of the normalized roughness, Z_0/\sqrt{E} , on the inverse wave age, U_*/C_p .

wave-induced momentum flux to waves, which, in turn, defines the proper behaviour of the boundary layer parameters, $z_{0ef}(A)$ and $U_*(A)$. The innermost reason for the spectrum tail behaviour dwells in an increase in the relative part of dissipation in the source function at each fixed frequency σ ,

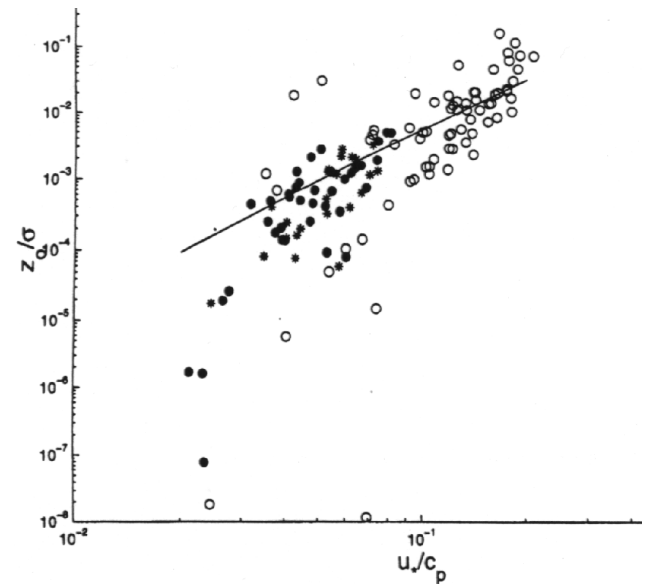


Fig. 5. Experimental dependence of normalized roughness, Z_0/\sqrt{E} , on the inverse wave age, U_*/C_p , following the paper Drennan et al. (1999).

during the wave development. Such an increase is enclosed in the model due to the parameterization (12) of the dissipation term. This is the essence of a physical treatment of the boundary layer parameters' dependence on the sea state.

Further elaboration of the model is connected to a more detailed comparison of numerical and experimental dependencies, both for the effects named above and for the other, more fine effects, describing the boundary layer parameters' dependence on the sea state. A total study of the model properties will be done on the basis of test calculations for the most informative test problems presented, for example, in Efimov and Polnikov (1991) or The SWAMP group (1985).

Acknowledgements. The work was supported by the Russian Foundation for Basic Research, projekt # 01-05-64580.

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