

## Effects of Site-Dependent Errors on the Accuracy of C/A Code DGPS Positioning

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### Abstract

Several differential GPS processing techniques can be used; for instance, single differencing and double differencing, which are popular in practice. Irrespective of the DGPS processing technique used, the ultimate accuracy of the user-location depends on the existence of non-common or site-dependent errors, which occur at the points of observation and the reference. Of these, the most common and dominant site-dependent error is the multipath. Therefore, this research evaluates the effects of site-dependent errors on C/A code differential GPS correction accuracies by providing special emphasis on the multipath error. For the analyses, four segments of about 24-hour continuous static C/A code based DGPS observations were conducted at three precisely known ground stations and four different multipath environments were introduced by placing three different types of artificial signal reflectors at one of the observation stations. By using the known GPS receiver-reflector configuration, pseudo-range multipath was precisely calculated for each observation segment. C/A code DGPS positioning accuracies before and after multipath mitigation were presented by evaluating the effect of the most dominant site-dependent error, i.e., multipath, on C/A code DGPS correction accuracies.

*Keywords:* Multipath Error; C/A Code DGPS Corrections; Positional Accuracies.

### 1. Introduction

Differential GPS (DGPS) can be considered as the most effective technique that minimizes or completely remove the influence of almost all common mode errors from GPS observations [1]. According to the concept of DGPS technique, the estimated total pseudo-range error at the reference station is specified as the correction for other GPS receiver observations performed over a limited geographical area. However, it is apparent that the estimated total error for each individual pseudo-range is the combination of common and non-common (site-dependent) errors. Therefore, the non-common errors (multipath, receive and measurement noises), which affect the observation of the reference station contributes as accuracy diminishing factors for DGPS correction data estimations [11]. Furthermore, amongst all non-common error sources, multipath is predominant; hence, mitigation is essential to improve the accuracy of DGPS corrections.

Based on the above facts, therefore, most permanent GPS reference stations are capable of calculating its correction data with particular reference to common mode errors by minimizing the effect of multipath through careful site selection and/or augmentation with additional hardware such as utilizing choke-ring antennas [10]. These approaches are only effective for reducing the effects of multipath and ensuring the quality of DGPS correction estimation. However, in most of the practical situations, multipath-free or comparatively low site selection is not an easy task to be accomplished. Therefore, some residual multipath error, receiver, and measurement noises always remain and demolish the quality of

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the correction data generated at the reference station [8]. However, by utilizing the double difference DGPS processing method, the non-common bias term of the receiver clock can be eliminated [3]. Yet, the error terms caused by multipath remain unchanged as in the case of single and double difference DGPS. However, most of the applications focus on handling the systematic errors that can be appropriately modeled and ignore the non-common bias that may actually exist. To further improve the accuracy and reliability of GNSS applications, such non-common bias must be handled especially when they are significant [12].

The combination of non-common mode errors both at the reference and the point of observations, and the remaining common mode errors characteristically form a noise-like variation in the time domain [5]. For some GPS applications, however, the estimation of precise carrier phase DGPS correction is crucial; for instance, in deformation or landslide monitoring and large structure dynamic monitoring, where most analyses and predictions are based on the noisy-like variation of the position of observation. In these cases, it is critical to correctly interpret the actual observations as closely as possible by eliminating the combined residual effects of multipath and other systematic errors. Most modern GPS receivers are employing multipath mitigation algorithms, however, none of presently available hardware and software methods are effective in complete elimination of multipath error at challenging multipath environments. Having realized the importance and undesirable consequence of multipath error, several researchers have proposed different techniques to eliminate it from GPS observations. Several investigations have been conducted to explore how the multipath error from carrier phase GPS observations could be eliminated (For example [7, 16, 17, 18] and so forth). In addition, Azarbad et al., 2014 tested the use of stationary wavelet transform (SWT) in GPS signal data processing to extract the a low-frequency multipath error term of double difference (DD) residuals and improved the ultimate differential accuracy. Father, the use of support vector machine (SVM) to separate the unimodal error terms of GNSS pseudorange measurement into three categories, clean, multipath and non-line-of-sight (NLOS) has shown significant classification accuracy of about 75% [9]. Signal-to-Noise Ratio (SNR) and equivalently  $C/N_0$  are well-known quality metrics investigated for the purpose of GNSS stochastic models and measurement weighting schemes to minimize the effect of multipath [14]. Hence, one of the common possibilities of improving the positional accuracy at challenging environments is to adopt advanced satellite selection methods based on signal-to-noise ratio (SNR) values to mask measurements having degraded quality due to the effect of multipath [17].

Although the highest possible accuracy is obtained through carrier phase DGPS, the less accurate C/A code DGPS positioning has several significant merits in various types of positioning and navigation applications due to its applicability in a wide range of GPS receivers from low-cost to very expensive ones. In addition, the operational distance of C/A code-based DGPS is several hundreds of kilometers whilst the carrier phase DGPS operations are limited to only within several tens of kilometers [4]. Most geospatial professionals in Sri Lanka use single-frequency (L1) GPS receivers with or without applying real-time or post-processing C/A code differential techniques on their geospatial data collection and other location-based applications. GPS Aided Geo Augmented Navigation (GAGAN), the regional Space Based Augmentation Service (SBAS), is also available for national geospatial applications. Dammalage et al., 2017 tested the performance of GAGAN service and confirmed its accuracy is almost identical for the accuracy of C/A code DGPS. GAGAN service is freely accessible by any SBAS active single-frequency (L1) GPS receiver to minimize most of the dominant common errors caused by ionosphere, satellite clock and orbit. The GNSS market report published by the European GNSS Agency in 2015 illustrates that more than 91% of the GNSS devices are being used in location-based services (LBS) and vehicle navigation as of 2015. Majority of these LBS and road applications are based on CA code measurements and utilized mostly at challenging urban environments. However, a single-frequency (L1) GPS receiver operates in a built-up or multipath prone environment, the C/A code pseudorange measurements are highly degraded by the multipath error and none of the differential correction methods would accomplished of achieving expected accuracies. Moreover, the effect of multipath on CA code based measurements is almost ten times greater than that of the carrier phase measurements [13].

Considering the practical advantages and the significant effect of the multipath error on C/A code measurements, Dammalage et al., 2010, has proposed an effective technique to eliminate the effect of multipath from continuous static observation after double differencing the carrier phase GPS observations. By adopting the same technique, this study was formulated to investigate the C/A code DGPS positioning accuracies prior to and after multipath correction with evaluating the effect of the most dominant site-dependent error on C/A code DGPS positioning accuracies.

## 2. C/A Code-based DGPS Corrections

Propagating GPS signals through ~20,200 km from satellite to the receiver antenna are essentially affected by the medium. GPS signals transmit at significantly high frequencies L1 and L2, respectively 1.57524 GHz and 1.2276 GHz; therefore, the atmospheric effects are comparatively smaller but significant inaccurate measurements. In addition to atmospheric effects, there are several other sources of errors, which affect both the code and carrier phase-based range measurements. The magnitude of each potential error sources,  $\Delta t_{SV}$ ,  $\Delta t_R$ , I, T, E, MP,  $\epsilon$  on GPS pseudo-range observables are listed in Table 1 as standard deviations. The errors listed in common mode are standard for any GPS receiver operating in a limited geographic region without considering the receiver type. The magnitude of non-common

mode errors is dependent on the receiver type, local environment, and the technique used to mitigate the effect of those errors.

**Table 1. Error budget on pseudo-range measurements [13]**

	Error Source	Potential Error Size (rms*)
<b>Common mode</b>	Satellite Clock ( $\Delta t_{SV}$ )	2 m
	Ephemeris ( $E$ )	2 m
	Ionosphere ( $I$ )	2-10 m
	Troposphere ( $T$ )	~2.5 m (at sea level)
<b>Non-common mode</b>	Multipath** ( $MP$ )	Code: 0.5-1 m Carrier: 0.5-1 cm
	Receiver Clock ( $\Delta t_R$ ) and Noise ( $\epsilon_r$ )	Code: 0.25-0.5 m
		Carrier: 1-2 mm

\*Standard Deviations are added in the root-mean-square sense

\*\* In a clean environment

In single difference pseudo-range DGPS, the total pseudo-range error is estimated at the reference station and utilized as the respective pseudo-range correction for the user GPS receiver observations. The pseudo-range error at the reference station,  $e_R^i(t)$ , and the equivalent DGPS correction,  $c_R^i(t)$ , for a satellite ‘i’ at time ‘t’ thus takes the following form in Equation 1. Subscripts ‘R’, ‘U’ and ‘RU’ refer to the effect at reference, user, and both stations, respectively.

$$e_R^i(t) = -c_R^i(t) = -c \times [\Delta t_R - \Delta t_{SV}^i] - I_{RU}^i(t) - T_{RU}^i(t) - MP_R^i(t) - \epsilon_R(t) \tag{1}$$

With respect to the nature of each pseudo-range error component at the reference station, two error classes as common ( $C_{RU}^i(t)$ ) and non-common ( $nC_R^i(t)$ ) mode errors for user GPS receiver could be grouped and presented in Equation 2 and 3 respectively. The former group of errors affects almost equally at the reference and the user observations, whilst the latter group is specific to the reference station only. Given that, the pseudo-range correction ( $c_R^i$ ) components are grouped as Equation 4.

$$C_{RU}^i(t) = I_{RU}^i(t) + T_{RU}^i(t) + c \times \Delta t_{SV}^i \tag{2}$$

$$nC_R^i(t) = MP_R^i(t) + c \times \Delta t_R + \epsilon_R(t) \tag{3}$$

$$c_R^i(t) = C_{RU}^i(t) + nC_R^i(t) \tag{4}$$

In theory, the true pseudo-range DGPS correction for a user GPS receiver is the common mode error affected at the reference GPS station ( $e_R^i$ ). Yet, the presence of non-common errors on Equation 4 calculation diminishes the accuracy of DGPS corrections ( $c_R^i$ ). Therefore, an experiment was designed to investigate the effect of most dominant non-common or site-dependent error, multipath, on C/A code DGPS corrections.

### 3. Research Methodology and Data Acquisition

In this experiment, C/A code DGPS observations were made at three closely located, precisely known ground controls, adopting two receivers as permanent reference stations and the third one as a user GPS, as illustrated in Figure 1. Four segments of about 24 hours continuous static observations were conducted by introducing four different multipath environments to one selected GPS reference station, and the observations were made by placing three types of artificial signal reflectors to result in three different observation segments with multipath error and the fourth segment to be observed without a signal reflector, to have the minimum effect of multipath error on observations. Figure 1(a) and 1(b) presents the receiver configuration with and without signal reflector, respectively. The base-line distances D1, D2, and D3 were measured precisely by an electronic distance measurement (EDM) instrument. In addition, accurate carrier phase observations were used for precise estimation of planimetric coordinates of each observation station. Both reference stations, GPS 01 and 02, in Figure 1 were installed with a similar type of GPS receivers to avoid the receiver-specific bias or limitations. Therefore, it could be perceived that the pseudo-range corrections generated by these two reference stations, in the absence of artificial signal reflectors, should be similar (in nature) as that of generated during Day 4 observations by GPS 01 and 02 reference stations.

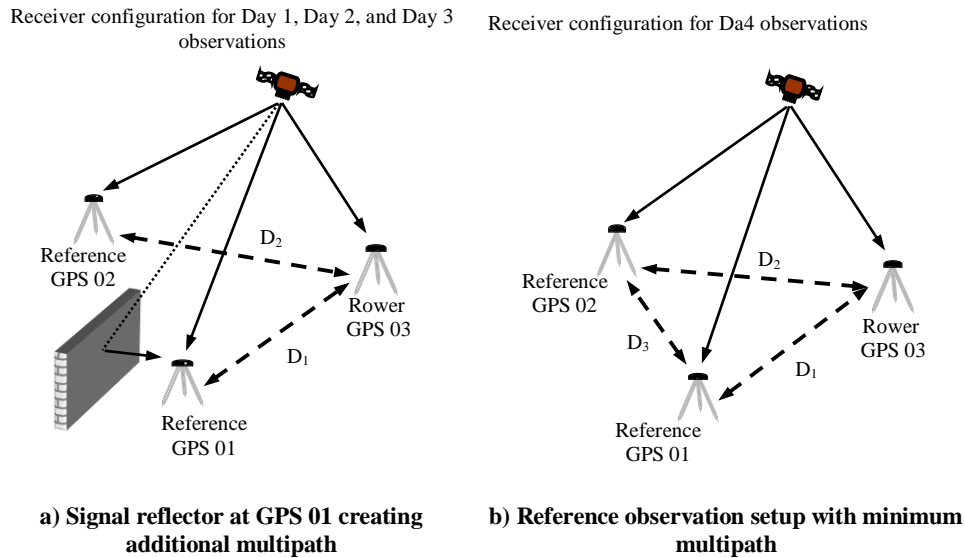


Figure 1. Field experiment setup

It is apparent that the residual errors affected by the atmosphere to the final position estimation are getting larger with the increase of the baseline distances. Therefore, this experiment was specially designed to have very short baseline observations, aiming to minimize all common mode errors to investigate the effects caused only by site dependent, especially the multipath error. Considering the given facts and making use of the known GPS receiver and reflector configuration, the pseudo-range multipath (excluding measurement and receiver noise) was calculated precisely for Day 1, 2, and 3 observations at reference GPS 01.

To calculate the pseudo-range multipath error caused by the reflector at the observation site, Equation 4 is reformulated as Equations 5 and 6 for GPS 01 pseudo-range corrections for the receiver configurations (a) and (b) illustrated in Figure 1 (a) and (b) respectively.

$$c_R^{R1,a}(t_a) = C_{RU}^{R1,a}(t_a) + nC_R^{R1}(t_a) + MP^{R1,a} \tag{5}$$

$$c_R^{R1,b}(t_b) = C_{RU}^{R1,b}(t_b) + nC_R^{R1}(t_b) \tag{6}$$

Where,  $MP^{R1,a}$  represents the multipath effects at reference GPS 01 due to the reflector. ‘R1’, ‘R2’, and ‘a’, ‘b’ are introduced to illustrate reference GPS 01 and 02, and receiver configurations in Figures 1(a) and (b) respectively. Further, ‘U’, ‘R’, and ‘RU’ refer to the effect at the user, reference, and both GPS receiver stations. Notations ‘ $t_a$ ’ and ‘ $t_b$ ’ represent the local time with the respective sidereal shift of 236 seconds per each successive day. Therefore, the difference of the pseudo-range correction at reference GPS 01 ( $dc_R^{R1}$ ) is obtained by subtracting Equation 6 from 5.

$$dc_R^{R1} = [C_{RU}^{R1,a}(t_a) - C_{RU}^{R1,b}(t_b)] + [nC_R^{R1}(t_a) - nC_R^{R1}(t_b)] + MP^{R1,a} \tag{7}$$

In this experiment, two geodetic grade GPS receivers of the same make were utilized at reference GPS 01 and 02 for both configurations (a) and (b). Hence, the receiver noise components are approximately identical ( $nC_R^{R1}(t_a) \approx nC_R^{R1}(t_b)$ ). Accordingly, Equation 7 reduces to the form as given by Equation 8.

$$dc_R^{R1} = [C_{RU}^{R1,a}(t_a) - C_{RU}^{R1,b}(t_b)] + MP^{R1,a} \tag{8}$$

Similarly, the difference of the pseudo-range correction at reference GPS 02 for both configurations (a) and (b) can be written as Equation 9, where there is no additional multipath effect.

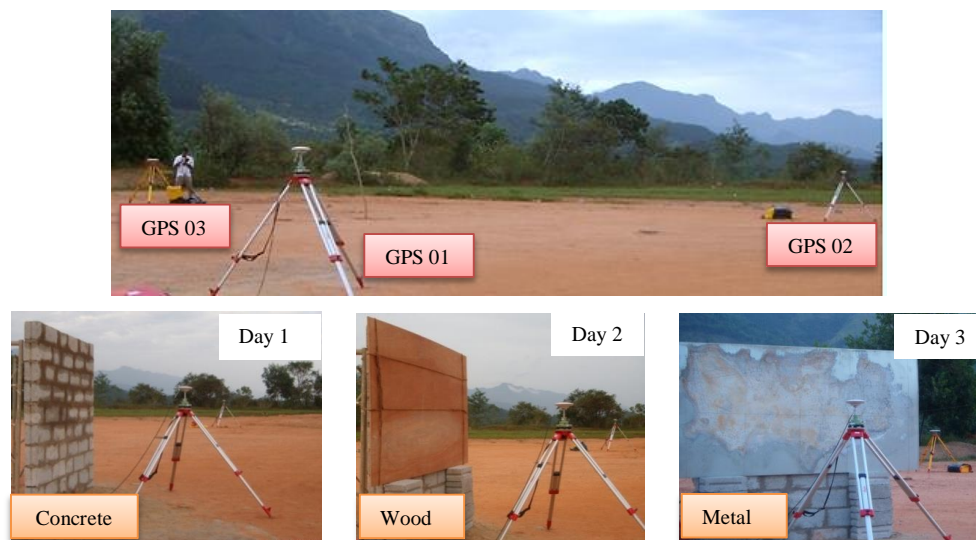
$$dc_R^{R2} = [C_{RU}^{R2,a}(t_a) - C_{RU}^{R2,b}(t_b)] \tag{9}$$

The common mode errors always make equal effects on each GPS receiver, which operates within a limited local geometric area. Therefore,  $C_{RU}^{R1,a}(t_a) = C_{RU}^{R2,a}(t_a)$  and  $C_{RU}^{R1,b}(t_b) = C_{RU}^{R2,b}(t_b)$ . The multipath error ( $MP^{R1,a}$ ) is thus calculated by subtracting Equation (9) from (8).

$$MP^{R1,a} = dc_R^{R1} - dc_R^{R2} \tag{10}$$

Hence, the multipath error ( $MP^{R1,a}$ ), caused by the signal reflector at GPS 01 reference station generated pseudo-range DGPS corrections derived by Equation 10, for Day 1, 2, and 3.

The observations used for the analyses were collected by employing three dual-frequency geodetic type GPS receivers. Two LEICA® – System 500 receivers were utilized at the two reference stations, GPS 01 and 02, and the observations at the user station GPS 03, were performed using a Trimble® 5700 instrument. GPS receivers were mounted over three precisely fixed ground control points to accomplish the configuration illustrated in Figure 1. Baseline lengths between each control points were predetermined by Electronic Distance Measurement (EDM) instruments. The distances were observed to be as,  $D1 = 20.0295 \pm 0.001$  m,  $D2 = 19.8930 \pm 0.001$  m, and  $D3 = 19.9590 \pm 0.001$  m. These precisely known measurements were used as ground truth to validate the accuracy of the results obtained throughout this experiment. The experiment site selected was such that, with minimum obstructions for satellite signals and even the satellites are at very low elevations ( $\sim 50^\circ$ ). The site was selected with 360-degree undisturbed open sky view, situated on a higher elevated flat ground as shown in Figure 2. At the site proximity, there were no buildings, electric cables, water bodies, and concrete, metal, or wood surfaces. Hence, this site configuration was assumed to be a minimum multipath environment for the respective observations.



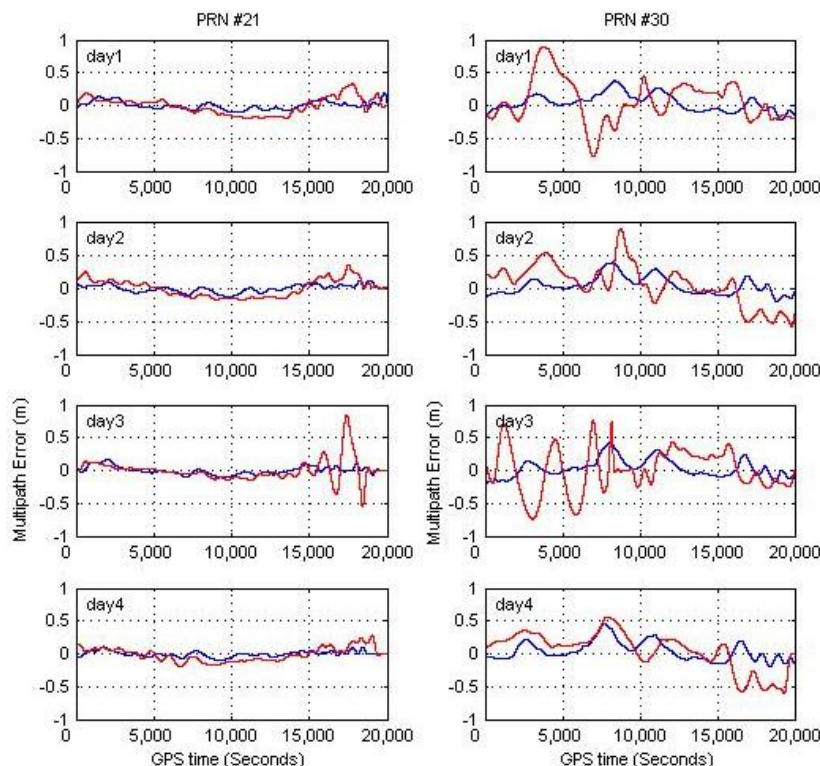
**Figure 2. Experiment site with GPS receivers at observation stations GPS 01, 02, and 03**

For the analysis, four days of 24-hour observations with 1-second interval were recorded according to the illustrations in Figures 1 and 2. Consequently, three days of observations, day 1, 2, and 3, were carried out with introducing additional multipath for the reference station, GPS 01, by concrete, wood, and metal reflectors respectively. The location, size, and the orientation of the reflector were maintained unchanged throughout day 1, 2, and 3 observations to minimize the bias due to the reflector location, size, and orientation from the multipath analyses. The observations of day 4 were performed without a reflector at GPS 01; however, the GPS receiver antenna height and location were maintained unchanged as day 1, 2, and 3 observations. Therefore, during the analysis, day 4 observations were chosen as the reference observations to evaluate the nature and the magnitude of multipath error introduced by the reflectors for day 1, 2, and 3 GPS measurements.

#### 4. Analysis of Results

Static continuous observations were performed according to the reflector-receiver configuration; based on which, pseudo-range multipath residuals were calculated by utilizing Equation 10 for both GPS 01 and 02 observations. The resulted multipath residuals were comparatively analyzed to investigate their magnitude and the nature of the effected multipath error for each satellite observables. Figure 3 illustrates the comparison for GPS 01 (red) and 02 (blue) pseudo-range residual multipath errors for PRN 21 and 30.





**Figure 3. Pseudo-range multipath residuals comparison for GPS 01 (red) and 02 (blue) of PRN 21 and 30**

According to Figure 3, there was no significant multipath error difference on PRN 21 for GPS 01 and 02 observations, which implies that the effect of multipath on PRN 21 at both observation stations was almost similar. Moreover, the magnitudes of multipath residuals were comparatively less for even day 1, 2, and 3 observations, where artificial signal reflectors were held at GPS 01 station. The only exception was observed with about +0.5 m magnitude of error at the latter part of the day 3 observations when the satellite was at very low in elevation. Hence, it is possible to conclude that the effect of multipath due to signal reflectors at GPS 01 on PRN 21 is very low. However, the same comparison for PRN 30 shows significant deviations for GPS 01 and 02 observations for all four days. The only difference for the observations at GPS 01 and 02 was the artificial signal reflector, and thus, it is apparent that the addition error observed, is only introduced by the reflector at GPS 01 observation site.

To evaluate the positional accuracy, C/A code-based single difference DGPS positioning was performed prior to and after multipath mitigation from the GPS 01 pseudo-range observations. For this analysis, DGPS processing was conducted by assuming GPS 01 as the reference station and GPS 02 as the rover. Accordingly, results for the base-line, GPS 01 – GPS 02, with 8 hours of 1-second static observation epochs (DGPS processed) were illustrated in Table 2. The effect of multipath on C/A code DGPS positioning accuracy is represented as a percentage of 1-second observation epochs within the specified three error limits for each day of DGPS observations post-processed before and after multipath mitigation.

**Table 2. Percentage of observation epochs, before and after multipath mitigation, with respect to the baseline distance accuracies**

Error	+5 cm	+10 cm	+20 cm
Day1	11.4 %	29.8 %	52.8 %
	70.1 %	82.9 %	94.2 %
Effect*	58.7 %	53.1 %	41.4 %
Day2	4.9 %	19.0 %	49.2 %
	69.9 %	85.1 %	99.5 %
Effect*	65 %	66.1 %	50.3 %
Day3	7.7 %	21.8 %	50.1 %
	64.7 %	74.8 %	91.5 %
Effect*	57 %	53 %	41.4 %
Day4	14.4 %	22.7 %	54.9 %
	74.3 %	86.7 %	100 %
Effect*	59.9 %	64 %	45.1 %

Before Multipath Mitigation  
 After Multipath Mitigation

\* Percentage of pseudo-range observations effected with multipath

The results justify that the number of observations affected by multipath and which introduces + 5 cm error limit for day 1 DGPS observation was 58.7% (70.1%-11.4%). By analyzing the same effect for each day and error limits, it is confirmed that about 40 to 65% of observations are always effected with multipath, even without a close reflector at the observation site.

According to some specific requirements of GPS positioning applications, both the accuracy and preciseness of DGPS observations are significant. The preciseness is utilized to emphasize the exact repeatability and reliability of achieving the same measurement, or as close as possible, comparatively for an extensive period of observation. Therefore, to illustrate the effect of multipath on both the accuracy and the precision, the two-dimensional (2D) DGPS processed positional errors were presented as scatter-plots concerning the error in the north (dN) and east (dE) directions for 20 hours static observations with 1-second intervals (72,000 observation epochs). The said scatter plots for each day of observations are comparatively examined before and after multipath mitigation and illustrated in Figure 4. The comparison for each day with concrete, wood, and metal reflectors and without a reflector is represented in Figure 4(a), (b), (c), and (d) respectively. Figure 4 illustrates the 2D positional error in centimeters (cm) and the north and east directional errors in meters (m). Accordingly, it is apparent that after multipath mitigation for each day of observations, the 2D error distribution is displaying more symmetrical and descending towards zero error with densifying the number of occurrences as close as possible to the zero value. That implies the effect of multipath diminishes not only the accuracy of GPS positioning but also its precision.

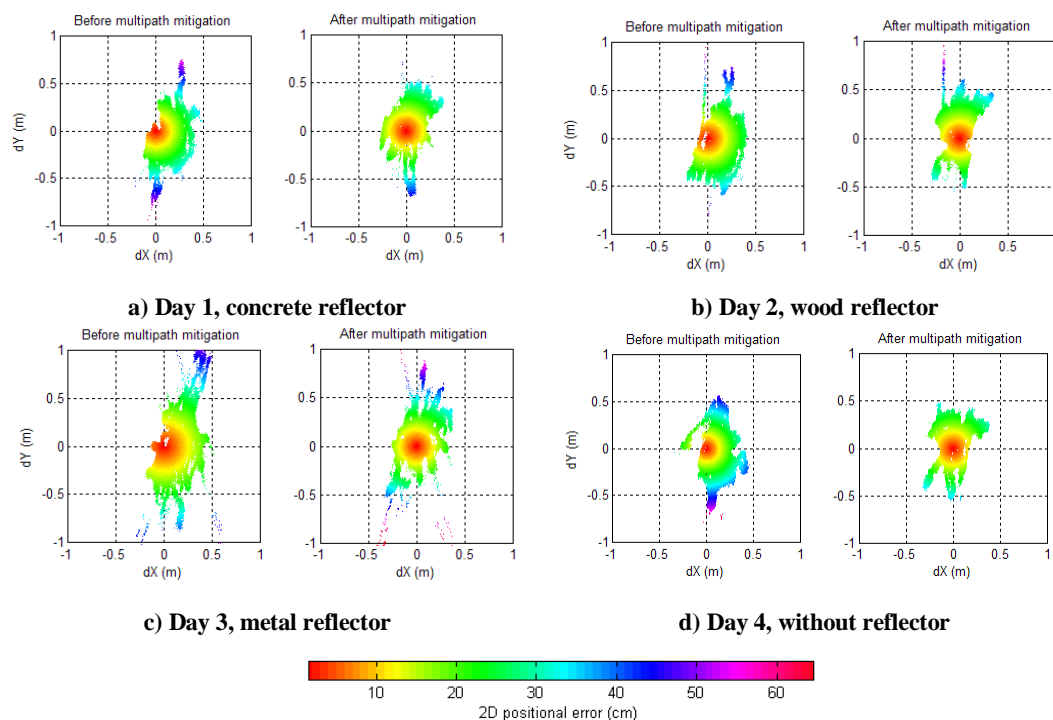


Figure 4. Scatter plots of 2D positional error (before and after multipath mitigation)

In order to have an explicit numerical illustration for the multipath effects presented graphically in Figure 4, the frequency of occurrence as a percentage for the same 72,000 observation epochs used for the above scatter plots are presented in Table 3. The percentages of epochs with and without multipath error for five different 2D positional error limits are illustrated.

Table 3. Frequency of occurrence as a percentage for 72,000 observation epochs: comparison for with and without multipath mitigation

		2D positional error of C/A code DGPS post-processed measurement									
		< 10 cm		< 20 cm		< 30 cm		< 40 cm		< 50 cm	
		With MP*	Without MP*	With MP*	Without MP*	With MP*	Without MP*	With MP*	Without MP*	With MP*	Without MP*
Day 1		32.2	40.8	66.7	71.1	87.4	89.1	95.3	96.6	96.8	99.6
Day 2		53.9	62.9	89.2	90.1	97.5	99.0	97.8	99.9	98.3	99.9
Day 3		54.0	80.5	87.3	94.4	94.4	98.7	96.8	99.3	97.4	99.9
Day 4		22.8	40.3	47.5	62.3	69.7	81.7	87.5	94.3	95.8	99.2

Unit of values are percentage (%)

\*MP: Multipath error

As expected, the percentage values emphasize the improvement caused by the multipath mitigation process. Most importantly, after multipath mitigation, almost 100% of observations show equal or less than 50 cm 2D positional error for each day of observations. Moreover, in theory, a 50 cm error is accepted as the inherent accuracy limitation of C/A code-based GPS positioning [13].

To emphasize the previously discussed performance of the multipath mitigation process, the comparison of 2D root mean square error (2D rms) values, before and after multipath mitigation, are also presented in Table 4. The 2D rms values are utilized to accentuate the improvement of the preciseness and the averaged positional error in East and North directions represent the accuracy of multipath mitigated observations.

**Table 4. Comparison of C/A code DGPS processed positional errors with and without multipath mitigation at the reference station**

	Root Mean Square Error (rms) for 2D positional error (2D rms)		Average Error for East & North directional errors (dE Avg. error, dN Avg. error)	
	With Multipath	Without Multipath	With Multipath	Without Multipath
Day 1	13.424	11.687	(16.41, -9.53)	(0.78, 4.65)
Day 2	12.926	11.851	(15.87, -8.14)	(-0.21, 5.62)
Day 3	18.882	13.728	(18.93, -3.47)	(3.52, 0.34)
Day 4	12.296	11.275	(14.42, -6.51)	(4.53, 0.18)

\*\*Unit of values: centimeter (cm)

The improvement of averaged East and North directional errors (dE Avg. error, dN Avg. error) are evidently observed to be approaching towards the absolute value of (0,0) after multipath mitigation. Moreover, the 2D rms value calculated for 72,000 observation epochs (20 hours observations with 1-second interval), before and after multipath mitigation, emphasize improvement of 1.737, 1.075, 5.154, and 1.021 cm for day 1, 2, 3, and 4 observations respectively. Based on which, the improvement of both accuracy and the precision is observed to be significant; with a slight difference in observations of each day. Hence, it reveals that both accuracy and precision of any DGPS observation depends on the magnitude and the nature of the effected multipath error.

The independent samples test (t-test) for the mean values of 2D positional errors of the same 20-hour observation with and without multipath error were tested to statistically verify the previously presented accuracy improvements after multipath mitigation. The T hypothesis test is defined as  $H_0 (X_w = X_{wo} : \text{Null hypothesis})$  and  $H_A (X_w \neq X_{wo} : \text{Alternative hypothesis})$ , where,  $X_w$  denotes the mean value calculated from the conventional DGPS technique, which includes the multipath error, while  $X_{wo}$  is the mean value calculated from the DGPS-corrected 2D positional errors based on reference station multipath mitigation. A 99% confidence level was used for hypothesis testing. Accordingly, Table 5 illustrates the summary of the results obtained from the hypothesis testing. It is evident from the obtained p-value (0.000) that the alternative hypothesis is achieved with a 99% confidence interval for each day of observations. Moreover, the standard deviations are also observed to be improved after multipath mitigation and based on which, the significant improvement is statistically confirmed.

**Table 5. Independent Sample Test (t-test) for mean values of 2D positional errors, calculated with and without multipath error**

Day	Multipath	N (sample size)	Mean (cm)	Standard Deviation (cm)	t - test	p - value
1	With	72001	25.045	13.424	35.952**	0.000
	Without	72001	22.807	11.687		
2	With	71919	25.242	12.926	52.689**	0.000
	Without	71919	21.950	11.851		
3	With	72001	28.540	18.882	114.197**	0.000
	Without	72001	19.147	13.728		
4	With	71999	25.710	12.296	87.738**	0.000
	Without	71999	20.351	11.275		

\*\* Significant at 99% confidence interval



## 5. Conclusion

Preliminary experiments and analyses were performed to investigate the characteristics of common and non-common mode errors before the compilation of research methodology, which is designed to achieve the goal of the study. Based on the analyses, the high- and low-frequency characteristics of common and non-common mode errors were identified respectively. Moreover, an artificial signal reflector was successfully experimented to generate a controllable additional multipath error on pseudo-range observations for further investigation of the said error. The analyses have stressed that the accuracy of DGPS-corrected observations depends on the amount of the remaining common mode errors (after DGPS), and the combined effect of non-common mode errors at the reference point and the point of observation. The effect of multipath on C/A code DGPS positioning accuracy was represented as a percentage of 1-second observation epochs within the specified three error limits, and it confirmed that about 40 to 65% of observations were always effected with multipath even without a close reflector at the observation site. Most importantly, after multipath mitigation, almost 100% of observations shows equal or less than 50 cm 2D positional error, for each day of observations. In theory, 50 cm error is accepted as the inherent accuracy limitation of C/A code-based GPS positioning. Further, the analyses disclosed that both accuracy and precision of any DGPS observation depend on the magnitude and the nature of the effected multipath error.

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