Research on the properties of a hydrostatic transmission for different efficiency models of its elements

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The article presents some considerations on the effect of the assumed mathematical models of the pump efficiency and the hydraulic engine, exerted upon the static and dynamic properties of a hydrostatic transmission. For this purpose some simulation tests of the transmission described have been carried out with two models: one - simplified, containing efficiency constants, and the other - an extended one with various efficiency values.

Key words: hydrostatic transmission, efficiency, mathematical model, simulation tests.

Introduction

Modern design works entail still better and better knowledge of the phenomena that may occur in real systems. In the case of hydrostatic transmissions, an essential problem may be the variability of efficiency coefficients of both the pump and the hydraulic engine. Usually, while modelling a transmission it is proposed that the indicated efficiency values are constant, assuming average levels, in most cases as high as in the catalogues of typical parts. It seems that such an assumption is not justified and the efficiency values should be treated as variable. Preliminary simulation tests (Jedrzykiewicz et al., 1995) indicate that such an approach is not without foundations. This article is aimed at showing some simulation tests of a transmission described with two models - one is simplified and with constant efficiency values; the other is extended and with variable efficiency values. In the simulation tests a modern MATLAB-SIMULINK packet (Matlab, 1992), (Simulink, 1992) has been applied. This packet is commonly used in research and technical calculations.

Assumptions

Mathematical modelling and simulation tests of real systems are usually carried out after having assumed a set of simplifying proposals. Such assumptions should arise out of the construction of the systems and regard the degree of being familiarised with the phenomena under modelling. It could seem that the more complex a mathematical model is, the better it is - as it describes the object under investigation in a more exact and precise way; thus it is more reliable. On the other hand, the more complex a model is, the more difficult are its formulation, modification and analysis. All these factors lead to two diametrically opposed postulates:

- models should be little complicated, thus soluble in an easy and fast way,
- models should exactly description work of real systems.

In the case of a hydrostatic transmission, usually the following set of assumptions is taken up:

1. The transmission works in an open circuit of fluid.
2. The properties of the transmission do not depend upon the direction of revolutions, therefore a one-way operation of the pump and the hydraulic engine can be assumed.
3. The rotational velocity of the engine driving the pump is constant.
4. The transmission is at the thermal equilibrium, therefore changes in the operational parameters do not appear as a temperature function.
5. Fluid stream pulsation's, arising out of the construction of the pump and the hydraulic engine, are negligibly small.
6. The transmission is compact, thus energy losses in the duct connecting the pump and the hydraulic engine are negligibly small.

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(Received September 29, 1997)

7. Natural volumetric and mechanical losses in both the pump and the hydraulic engine are described with either constant or variable efficiency coefficients.
8. The high pressure fluid volume consists of the volume of the fluid in the pump, in the hydraulic engine and in the duct connecting the pump with the engine.
9. Fluid bulk module is constant.
10. The safety valve opening pressure exceeds the operational pressures of the transmission.

A mathematical transmission model

Basing upon the said simplifying assumptions, the calculating diagram for the transmission, as presented in Fig. 1, has been accepted.

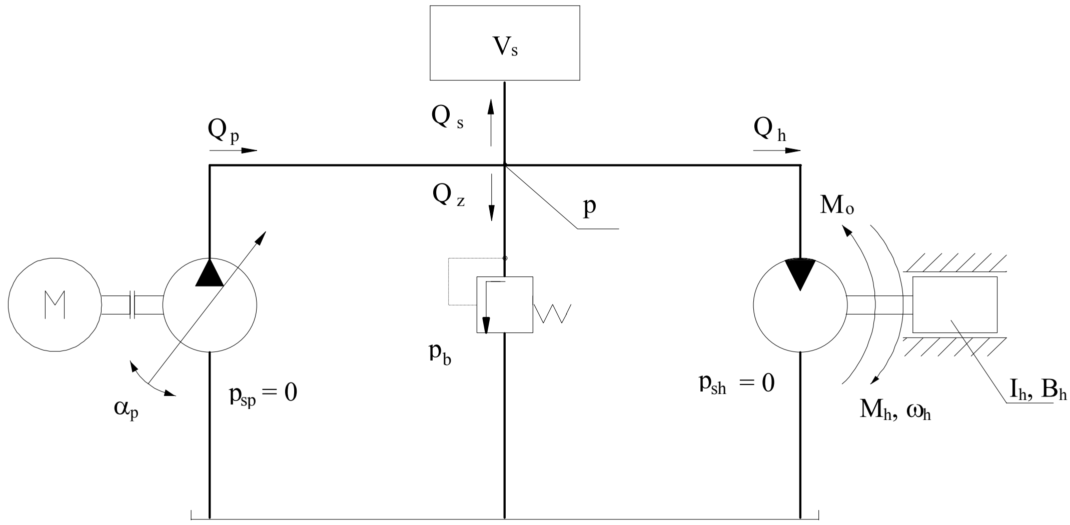

Fig. 1

The mathematical model of the transmission presented in Fig. 1 consists of three equations: flow-rates balance, equilibrium of loads and efficiency ones. Their general form is given below:

$$Q_p = Q_h + Q_s + Q_z \tag{1}$$

$$M_h = M_I + M_B + M_o \tag{2}$$

$$\eta_o = \eta_{vp}\eta_{mp}\eta_{vh}\eta_{mh} \tag{3}$$

where: Q_p is pump delivery, Q_h is hydraulic engine absorptivity, Q_s is flow rate resulting from fluid compressibility, Q_z is flow rate through the safety valve, M_h is torque generated by the hydraulic engine, M_I is torque resulting from the inertia of moving parts, M_B is torque resulting from viscous friction in moving parts, M_o is torque resulting from technological load, η_o is general efficiency of the transmission, η_{vp} is pump's volumetric efficiency, η_{mp} is pump's mechanico-hydraulic efficiency, η_{vh} is engine's volumetric efficiency and η_{mh} is engine's mechanico-hydraulic efficiency.

Respective terms of the equations (1) - (3), basing upon (Jedrzykiewicz, 1981), (Jedrzykiewicz, 1996), (Stryczek, 1995) are shown below:

$$Q_p = \alpha_p K_{qp} \eta_{vp} \tag{4}$$

where: Q_p - pump delivery [m^3/s], α_p - deflection angle of either a pump disk or casing [^{o}], K_{qp} - pump's efficiency coefficient [$m^3/(^o \cdot s)$], η_{vp} - pump's volumetric efficiency coefficient [-].

$$Q_h = \frac{K_{qh}\omega_h}{\eta_{vh}} \tag{5}$$

where: Q_h - engine absorptivity [m^3/s], K_{qh} - absorptivity coefficient [m^3], ω_h - angular velocity of the engine shaft [1/s], η_{vh} - volumetric efficiency coefficient [-].

$$Q_s = \frac{V_s}{E_s} Dp \tag{6}$$

where: Q_s - flow rate related to compressibility [m^3/s], Vs - fluid volume, subject to pressure effect [m^3], E_s - fluid bulk module [Pa], p - fluid pressure [Pa], D - differentiating operator.

$$Q_z = K_{zb}(p - p_b) \qquad \text{for} \quad p > p_b \tag{7}$$

$$Q_z = 0 \qquad \text{for} \quad p \le p_b \tag{8}$$

where: Q_z - flow rate through a valve [m^3/s], K_{zb} - average slope of the valve static characteristics [m^5/(N·s)], p - system operational pressure [Pa], p_b - valve opening pressure - set while in operation [Pa].

$$M_h = K_{mh}\Delta p_h \eta_{mh} \tag{9}$$

where: M_h - rotational moment generated by the engine [N·m], K_{mh} - moment coefficient [m^3], Δp_h pressure fall in the engine [Pa], the value being $\Delta p_h = p - p_{sh} = p$, η_{mh} - coefficient of engine's mechanico-hydraulic efficiency [-].

$$M_I = I_h D\omega_h \tag{10}$$

$$M_B = B_h\omega_h \tag{11}$$

where: I_h - moment of inertia of the engine and machine parts, reduced upon the engine shaft [N·m·s ^{2}], B_h - resistance coefficient of viscous friction in the engine and machine parts, reduced upon the engine shaft [N·m·s].

$$\eta_{vp} = \varepsilon_v \left[1 - A_{vp\mu} \frac{\Delta p_p}{(\varepsilon_p + \varepsilon_{ep})(n_p + \varepsilon_{np})} - A_{vp\rho} \frac{\sqrt{\Delta p_p}}{(\varepsilon_p + \varepsilon_{ep})(n_p + \varepsilon_{np})} \right] \tag{12}$$

where: Δp_p - pressure fall in the pump [Pa], its value being $\Delta p_p = p - p_{sp} = p$, n_p - rotational speed of the pump driving shaft [rev/s], $A_{vp\mu}$ - fluid loss coefficient during laminar flow [m^2·rev /(N·s)], determined from experimental data, $A_{vp\rho}$ - fluid loss coefficient in turbulent flow [m·rev/($N^{0.5}$·s)], determined as well from experimental data, ε_p - pump delivery setting coefficient [-], ($\varepsilon_p = 1$ for a constant delivery pump and $0 \le \varepsilon_p \le 1$ for a variable delivery pump), ε_{ep} - a very small positive number e.g. 10e-10 [-] enabling calculations to be continued when $\varepsilon_p = 0$, ε_{np} - a very small positive number, e.g. 10e-10 [rev/s] that enables to keep on calculating when $n_p = 0$, ε_v - corrective factor depending upon the expression in square brackets (ε = 1 where the expression is not negative, $\varepsilon_v = 0$ where the expression is negative - it results from the fact that the efficiency cannot be negative while considering energy losses).

$$\eta_{mp} = \frac{1}{1 + A_{mpf} + A_{mp\mu}\dfrac{n_p}{\Delta p_p + \varepsilon_{pp}} + A_{mp\rho}\dfrac{n_p^2\sqrt[3]{\varepsilon_p^2}}{\Delta p_p + \varepsilon_{pp}}} \tag{13}$$

where A_{mpf} - loss coefficient of mechanical friction [-] determined from experimental data, $A_{mp\mu}$ - loss coefficient of viscous friction [N·s/(rev·m^2)] determined from experimental data, $A_{mp\rho}$ - hydraulic loss coefficient in turbulent flow [N·s^2/(rev^2·m^2)] determined also from experimental data, ε_{pp} a very small positive number e.g. 10e-10 [Pa] enabling the calculations to be carried on when $\Delta p_p = 0$.

$$\eta_{vh} = \frac{1}{1 + B_{vh\mu}\dfrac{\Delta p_h}{(\varepsilon_h + \varepsilon_{eh})(n_h + \varepsilon_{nh})} + B_{vh\rho}\dfrac{\sqrt{\Delta p_h}}{(\varepsilon_h + \varepsilon_{eh})(n_h + \varepsilon_{nh})}} \tag{14}$$

where: n_h - rotational speed of the hydraulic engine shaft [rev/s], $B_{vh\mu}$ - fluid loss coefficient in laminar flow [m^2·rev/(N·s)] determined experimentally, $B_{vh\rho}$ - fluid loss coefficient in turbulent flow [m·rev/(N$^{0.5}$·s)], also determined experimentally, ε_h - engine absorptivity setting factor [-], ($\varepsilon_h = 1$ for constant absorptivity engines, $0 \le \varepsilon_h \le 1$ for variable absorptivity engines), ε_{eh} and ε_{nh} - very small positive numbers per analogiam to the abovesaid.

$$\eta_{mh} = \varepsilon_m \left[1 - B_{mhf} - B_{mh\mu}\frac{n_h}{\Delta p_h + \varepsilon_{ph}} - B_{mh\rho}\frac{n_h^2\sqrt[3]{\varepsilon_h^2}}{\Delta p_h + \varepsilon_{ph}} \right] \tag{15}$$

where: B_{mhf} - loss coefficient of mechanical friction [-] determined from experimental data, $B_{mh\mu}$ - loss coefficient of viscous friction [N·s/(m^2·rev)] determined from experimental data, $B_{mh\rho}$ - hydraulic loss coefficient in turbulent flow [N·s^2/(m^2·rev^2)] determined as well from experimental data, ε_{ph} - a very little positive number analogous to the abovesaid, ε_m - corrective factor of the same meaning as for the pump.

Under the simulation research programme of the transmission, two versions of its mathematical model have been considered:

Model 1. Efficiency values of the pump and hydraulic engine in formulae (3), (4), (5) and (9) assume constant values, as in catalogue specifications.

Model 2. Efficiency values of the pump and hydraulic engine in formulae (3), (4), (5) and (9) are variable, according to formulae (12) - (15).

Block diagrams of the transmission

By using the graphical editor of the MATLAB-SIMULINK packet, basing upon equations (1) - (15 detailed block diagrams and a general), diagram have been made (shown in Figs 2, 3 and 4).

Simulation tests

Basing upon the catalogue and experimental data-sheets, the following coefficients of the transmission models have been found: A_α=16 [^{o}], A_m=250 [N·m], $A_{vp\mu}$=9.4860e-8 [m^2·rev/(N·s)], $A_{vp\rho}$=-7.3539e-5 [m·rev/(N$^{0.5}$·s)], A_{mpf}=-1.4151e-3 [-], $A_{mp\mu}$=-9.1763e+6 [N·s/(m^2·rev)], $A_{mp\rho}$=3.7036e+5 [N·s^2/(m^2·rev^2)], B_h=12 [N·s·m], $B_{vh\mu}$=4.8705e-9 [m^2·rev/(N·s)], $B_{vh\rho}$=1.4612e-5 [m·rev/(N$^{0.5}$·s)],

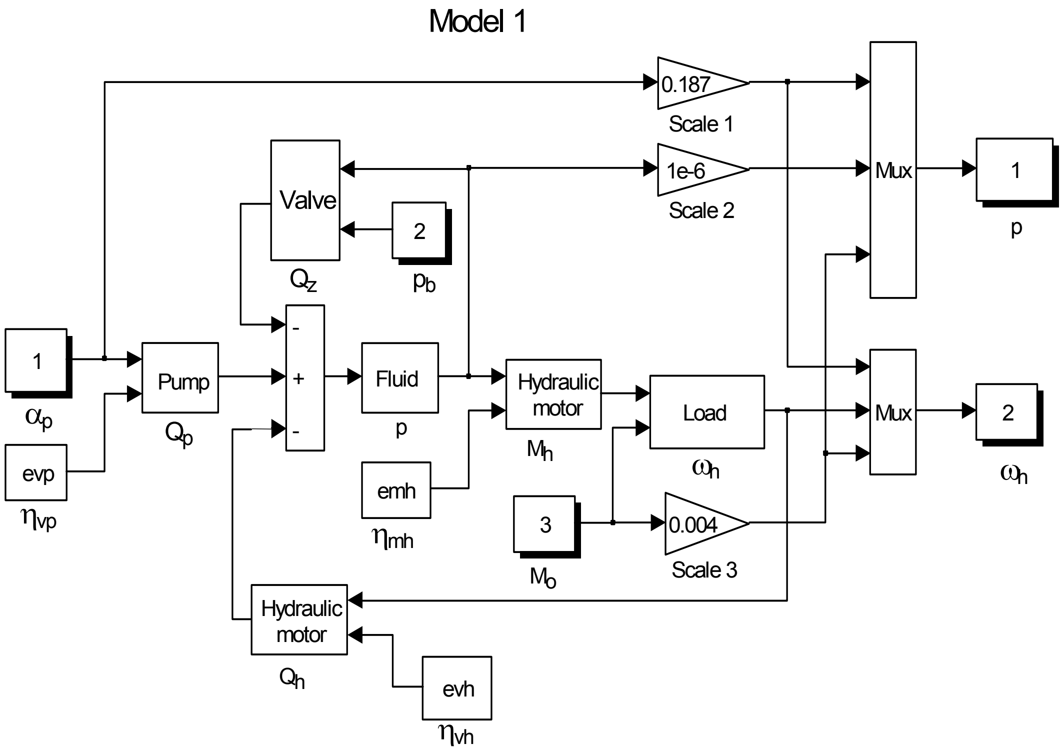
Model 1
0.187
Scale 1
Valve
1e-6
Scale 2
Mux
1
p
2
Qz
pb
1
Pump
-
+
-
Fluid
Hydraulic motor
Load
Mux
2
αp
Qp
p
Mh
ωh
ωh
evp
emh
ηvp
ηmh
3
0.004
Mo
Scale 3
Hydraulic motor
Qh
evh
ηvh

Fig. 2

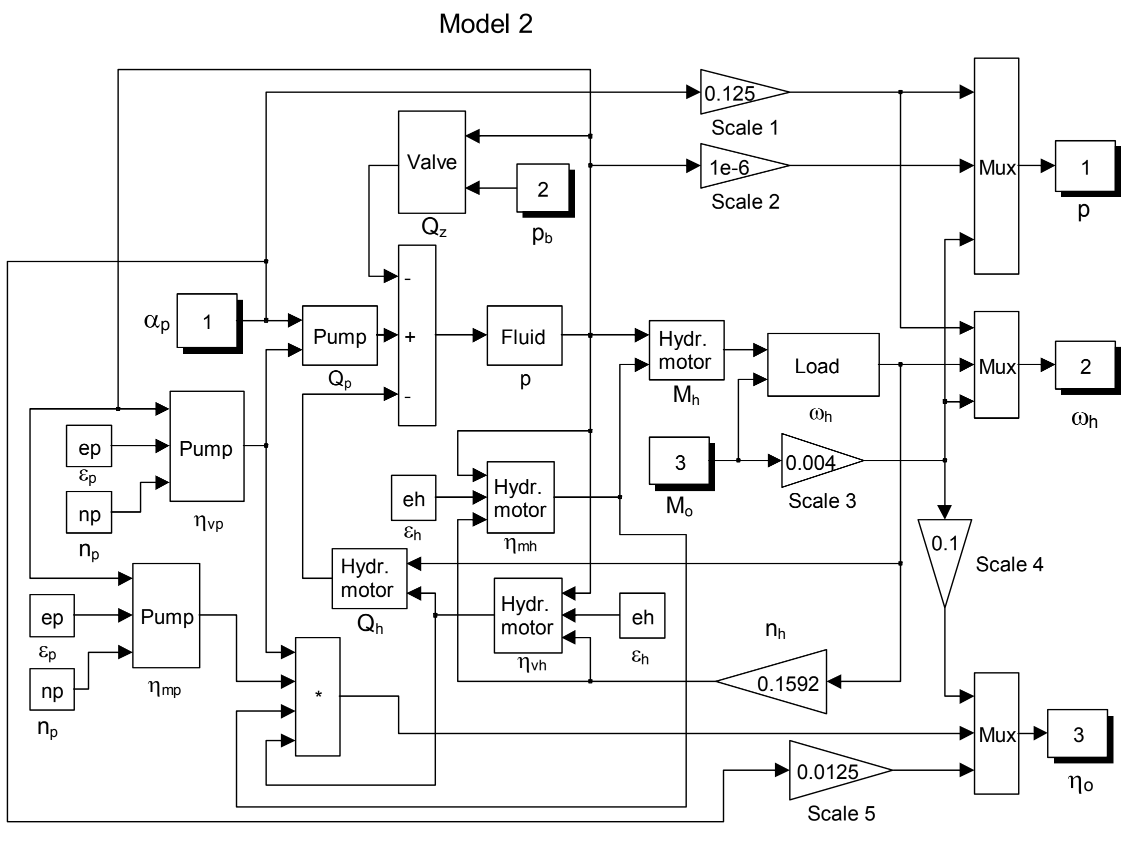
Model 2
0.125
Scale 1
Valve
1e-6
Scale 2
2
Mux
1
p
Qz
pb
αp
1
Pump
+
Fluid
Hydr. motor
Load
Mux
2
Qp
p
Mh
ωh
ωh
ep
Pump
εp
np
ηvp
np
eh
Hydr. motor
εh
ηmh
3
0.004
Mo
Scale 3
0.1
Scale 4
Hydr. motor
Qh
ep
Pump
εp
np
ηmp
np
Hydr. motor
ηvh
eh
εh
nh
0.1592
*
Mux
3
ηo
0.0125
Scale 5

Fig. 3

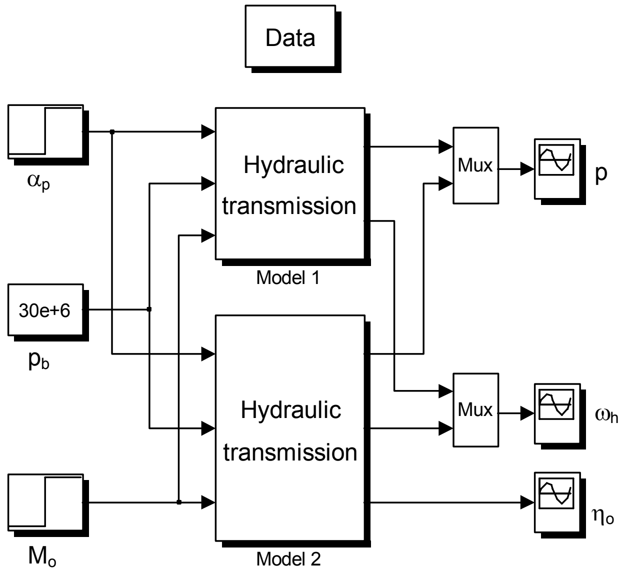

Fig. 4

B_{mhf}=1.0817e-2 [-], $B_{mh\mu}$=2.7200e+5 [$N\cdot s/(m^2\cdot rev)$], $B_{mh\rho}$=-1.1345e+4 [$N\cdot s^2/(m^2\cdot rev^2)$], E_s=1.65e+9 [Pa], I_h=0.04 [$N\cdot m\cdot s^2$], K_{qp}=2.6882e-5 [$m^3/(^o\cdot s)$], K_{qh}=3.979e-5 [m^3], K_{mh}=3.979e-5 [m^3], K_{zb}=0.20e-9 [$m^5/(N\cdot s)$], p_b=30.0e+6 [Pa], t_α=0.01 [s], t_m=0.05 [s], V_s=1.4145e-4 [m^3], η_{vp}=0.97 [-], η_{mp}=0.80 [-], η_{vh}=0.93 [-], η_{mh}=0.95 [-].

The tests were performed for a jump-like change in the pump delivery $\alpha_p=A_\alpha\cdot 1(t-t_\alpha)$ and a jump-like change in the motor load $M_o=A_m\cdot 1(t-t_m)$. The tests results are shown in Figs 5, 6 and 7. Fig. 5 shows changes in the angular velocity of a hydraulic engine, Fig. 6 - pressure variations in the press duct and Fig. 7 - changes in the general efficiency of the transmission - model 2.

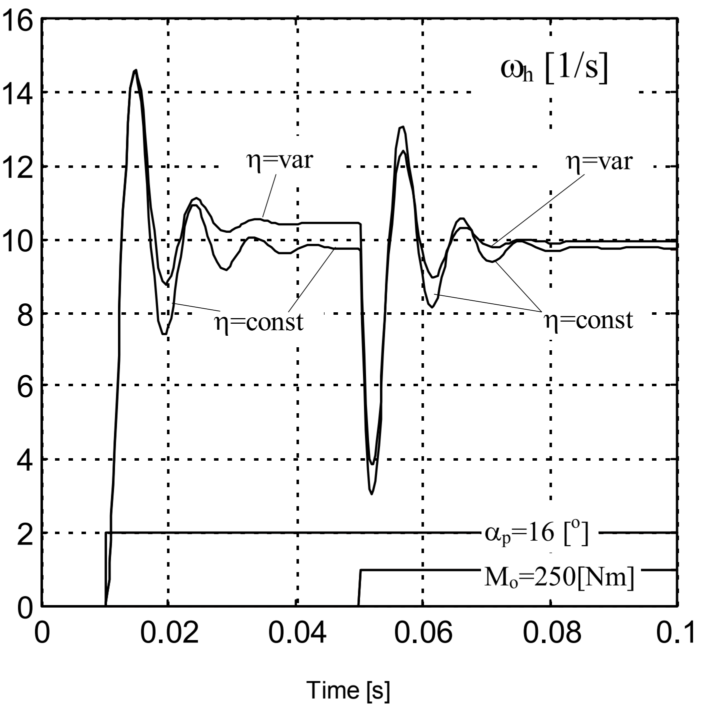

Fig. 5

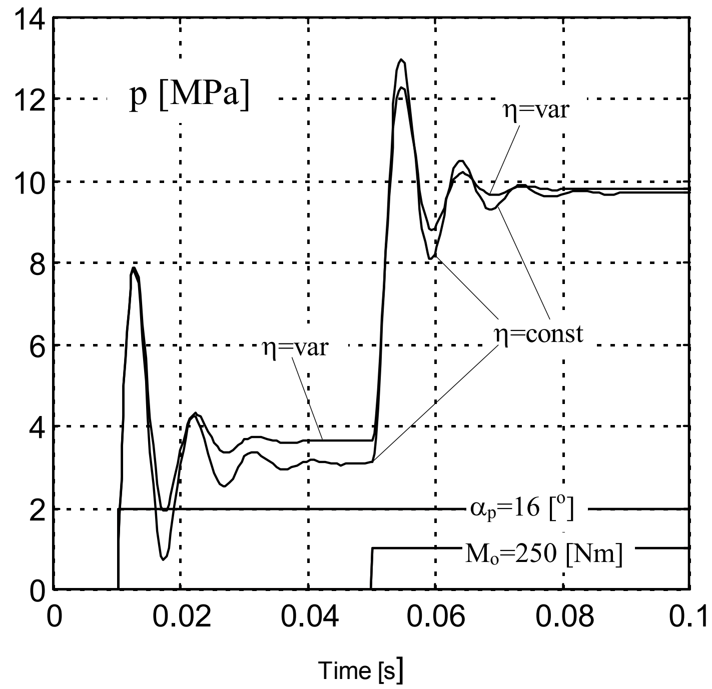

Fig. 6

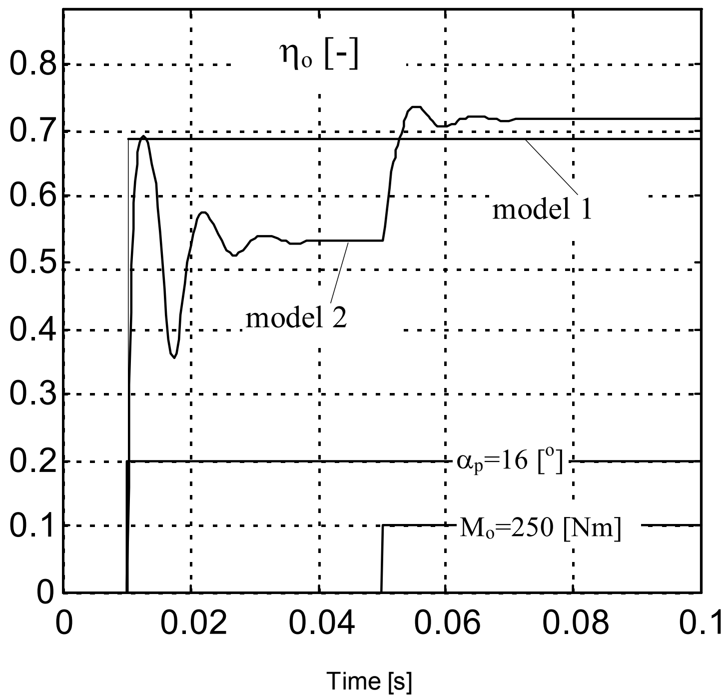

Fig. 7

Comparison of the results

It can be deduced form the presented simulation tests that the application of model 2 instead of model 1 can lead to essential quantitative differences in the rotational speed, pressure and general efficiency. Time courses of the first two speeds indicate lower over-controlling, smaller control times and higher stationary values. For a numerical presentation of the differences abovementioned, the following relative error has been defined:

$$\delta = \frac{W_{model2} - W_{model1}}{W_{model2}} 100[\%] \tag{16}$$

where: W_{model2}, W_{model1} - physical properties obtained, respectively, from model 2 and 1. Table 1 shows relative errors for the stationary operational states of the transmission, whereas Table 2 presents the maximum values of relative errors for non-stationary states.

Table 1. Stationary states

Physical quantity	Operation of unloaded transmission δ [%]	Operation of loaded transmission δ [%]
ω_h	6.8	1.9
p	13.6	1.1
η_o	-28.9	4.5

Table 2. Non-stationary states

Physical quantity	Operation of unloaded transmission δ_{max} [%]	Operation of loaded transmission δ_{max} [%]
ω_h	16.2	9.4
p	61.8	8.3
η_o	-93.0	6.8

Summary

Simulation tests, made on hydrostatic transmissions, indicate the necessity of having a mathematical model adapted to concrete design circumstances:

- Model 1 can be used exclusively in the situations during an operation under a load, close to the nominal one, only stationary states are essential; i.e., when the device operates for long time intervals under a constant load and unchanged control,
- Model 2 - can supply significant design tips when:
  - during a larger part of an operational cycle the operation of a transmission occurs under a slight load against the nominal one,
  - while operating under a load close to the nominal one, non-stationary states are significant; they occur relatively often under variable loads and controlling courses.

  As it can be seen, model 2 is exceptionally useful in case of analysing the operation of transmissions designed for automated devices.

The presented research was carried out within the framework of the Department of Process Control of, Contract no 11.130.174.

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