STRESS CONCENTRATION DUE TO A SPHERICAL VOID UNDER HERTZIAN CONTACT

Assoc. Prof, Stelian **ALACI**, Universitatea Suceava, stalaci@yahoo.com
Lecturer Florina Carmen **CIORNEI**, Universitatea Suceava, florina@fim.usv.ro
Prof. Dumitru **AMARANDEI**, Universitatea Suceava, mitica@fim.usv.ro
Assoc Prof., Constantin **FILOTE**, Universitatea Suceava, filote@eed.usv.ro
Assoc. Prof., Delia-Aurora **CERLINCĂ**, Universitatea Suceava, delia@fim.usv.ro
Lecturer Luminița **IRIMESCU**, Universitatea Suceava, lumi@fim.usv.ro

Abstract: *The present paper presents the method of estimating the stress concentrator effect of a spherical void from an elastic half-space. An essential part consists in estimation of FEM error by finding the contact pressure from half-plane using an analytical method. Next, the stress concentrator effect of the same void, except for placed into elastic space, is found.*

Keywords: MATHCAD, hole, stretched.

1. Introduction

In mechanical structures, a large variety of parts create point Hertzian contacts which are characterized by occurrence of strong stress concentrators. In addition, the metallic materials from which these parts are made, may present, as a result of processing method, any kinds of heterogeneities such as voids or inclusions. These also lead to local stress concentrations. First, this aspect was pointed out by Kirsch, [1], who showed that on the contour of a circular hole from an elastic plane uniaxially stretched at infinity, the stress is three times greater than the traction stress of the plane. Miller and Keer, [2], found the effect of interaction between a void and an inclusion both situated into an elastic half-plane pressed by a cylinder.

For spatial problems, solving the problems involving voids is more complex due to a greater number of components of the stress tensor and displacement vector and to increased difficulty of the boundary conditions form. The simplest problem, namely the spherical cavity from an elastic space stretched at infinity, [3], requires elaborate mathematical relations, established by Lurie, [4].

Applying the finite element method allows finding the stress and strain state, in any case of intricate boundary conditions. Though, with reference to contact problems, additional concern is required for accurate results. This apprehension is raised by rapid spatial stress variation in contact regions and therefore a coarse meshing doesn't allow stress evaluation with required accuracy.

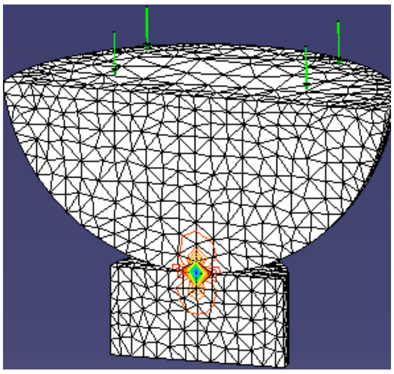
2. Figuring the meshing region dimension

In order to find the dimensions of the analyzed zone necessary to range the errors into an accepted accuracy domain, the problem of a sphere normally loaded on an elastic half-space was considered. For this problem, as a particular case of Hertzian problem, the analytical solutions are known, [5]. For a first phase, the dimensions of the studied zone were considered of the same magnitude order as the dimensions of the sphere. The meshing is presented in Fig. 1a, and the σ_z stress and the deformed shape of the netting is shown in Fig. 1b. From Fig. 1b it can be observed that, due to great value of mesh element size, the sphere-plane contact takes place only in one point, and actually the Boussinesq problem is solved, namely the normal concentrated force acting on a half-space.

In order to characterize the stress field, it is required a size of the meshing element smaller than the contact zone dimensions. This fact is illustrated in Fig. 2a, where the considered contact region dimensions are ten times the dimension of the contact radius $a = 0.296mm$. The Hertzian contact parameters were computed in MATHCAD and a sequence of the program is given in Fig. 2b. It can be seen that the theoretical value obtained for maximum contact pressure is $p_0 = 5.44 \cdot 10^8 Pa$ and the numerical value being $p_0 = 5.46 \cdot 10^8 Pa$, from FEA, the correct choice of meshing element size is validated. In Fig. 3, the position of a level curve for σ_z stress in an axial plane is presented comparatively. In Fig. 4 it can be observed the continuous curvature of the contacting surfaces after deformation.

3.Finding the stress field from a half-space with void

A spherical void of radius equal to the contact area radius was considered into the half-space. The stress field was found for the position of the spherical cavity centre at a distance $h = 1;2;3mm$ with respect to the half-space plane boundary. In Fig. 5 the analytical results are presented for the contact parameters of the compact half-space compared to σ_z stress value found for "with void" case, when the spherical void is situated at a depth $h = 1mm$. One can notice that the maximum contact pressure is not altered by the void presence.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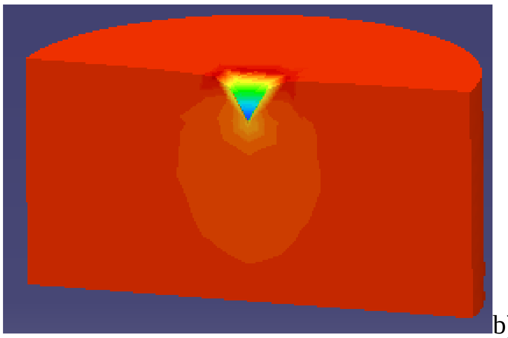
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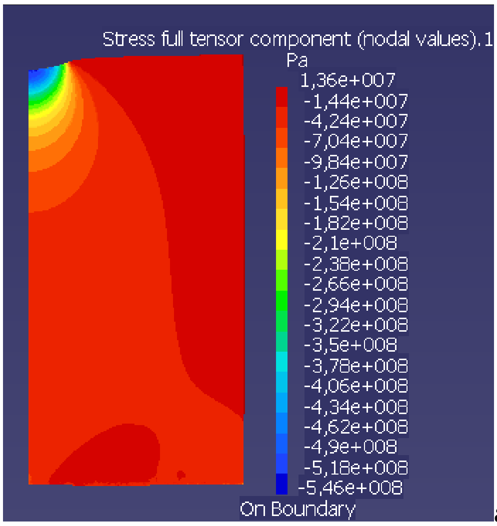
Fig.1. Meshing of the contacting elements (a), and contact pressure on deformed mesh, (b)



a)

$Q := 100$

$R := 40 \cdot 10^{-3}$

$E_1 := 2.1 \cdot 10^{11} \quad \nu_1 := 0.3 \quad E_2 := E_1 \quad \nu_2 := \nu_1$

$p_0 := \frac{1}{\pi} \cdot \sqrt[3]{\frac{3}{2} \cdot \frac{k^2}{\eta^2} \cdot Q} \qquad p_0 = 5.44 \cdot 10^8$

$\delta := \frac{1}{2} \cdot \sqrt[3]{\frac{3}{2} \cdot \eta^2 \cdot k \cdot Q^2} \qquad \delta = 1.917 \cdot 10^{-6}$

$a := \sqrt[3]{\frac{3}{2} \cdot \frac{\eta}{k} \cdot Q} \qquad a \cdot 10^3 = 0.296$

b)

Fig.2. Finding numerical (a) and analytical (b) maximum contact pressure

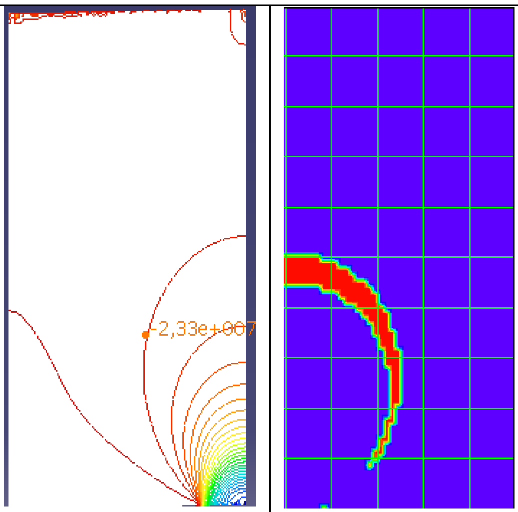


Fig.3. Comparison of FEA and theoretical level curve for $\sigma_z = 2.33 \cdot 10Pa^7$

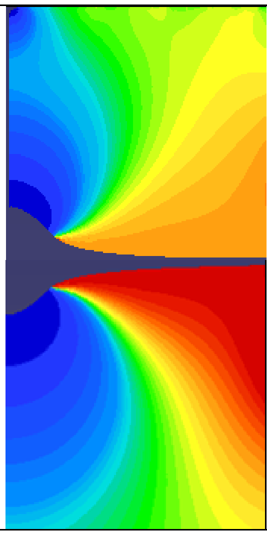


Fig.4. FEA illustration of deformed surfaces of the two bodies

$Q := 100$

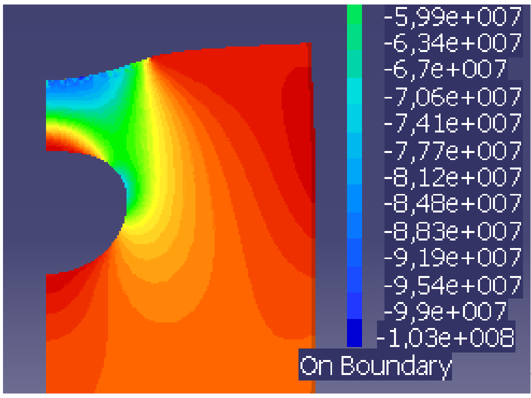
$R := 500 \cdot 10^{-3}$

$E_1 := 2.1 \cdot 10^{11} \quad \nu_1 := 0.3 \quad E_2 := E_1 \quad \nu_2 := \nu_1$

$$p_0 := \frac{1}{\pi} \cdot \sqrt[3]{\frac{3}{2} \cdot \frac{k^2}{\eta^2} \cdot Q} \qquad p_0 = 1.01 \cdot 10^8$$

$$a := \sqrt[3]{\frac{3}{2} \cdot \frac{\eta}{k} \cdot Q} \qquad a \cdot 10^3 = 0.688$$

a)



b)

Fig.5. (a) Analytical calculus for contact parameters in the case of compact half-space, and: (b) FEA values obtained for σ_z in the presence of spherical void

In order to characterize the stresses, spherical coordinates were used, since the border conditions are expressed versus these coordinates, with the centre of coordinate system in the centre of the void and oriented as seen in Fig. 6.

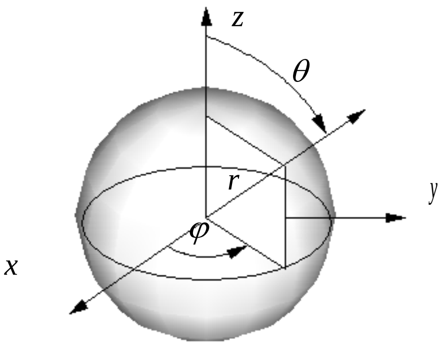


Fig. 6. Spherical coordinate system

Both the normal stresses in spherical coordinates for the précised values of position depth and the equivalent von Misses stress are arranged in Table 1. Due to axial symmetry of the problem, the displacements of the points from any axial plane are zero and for this reason, aiming a superior computer resource capacity exploit, the study was made for a narrow slice, explicitly having an angular width of 3^{o}. From Table 1 the conclusion that all the stresses decrease as the depth of the void centre increases, can be traced. Regarding stresses, among these, the stress reaching the maximum value is σ_{θ}, which attains its maximum in a point moving towards the equator of the void as the depth of the void centre increases. Moreover, it is noticed an asymptotical increase of all stresses values and this aspect leads to the conclusion that at enough great depths, the stresses do not depend on the void centre depth.

4.Finding stress concentrators around a spherical void from an elastic half-space with uniaxially stretched at infinity

The aspects revealed above suggests the necessity of evaluation the stress concentrator factor of a spherical void from an elastic half-space with uniaxial tension at infinity, fig. 7. In order to solve the problem numerically, the void is considered placed into a cylinder with much larger dimensions compared to the cavity radius. With the purpose of comparing the present problem to the precedent Hertzian type one, the cylinder is axially loaded with a surface force having the resultant equal to the normal force from previous Hertzian case. In Fig. 8, the equivalent von Mises stress is shown and it can be seen that the maximum is reached on the equator from the plane normal to the traction direction.

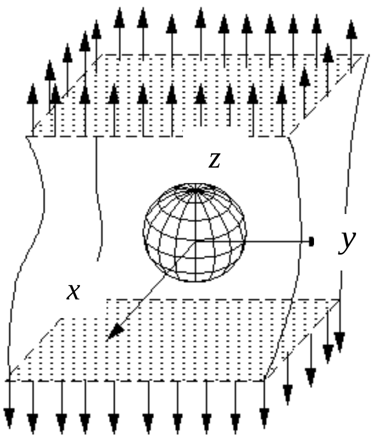


Fig. 7. Spherical cavity from a monoaxially stretched space

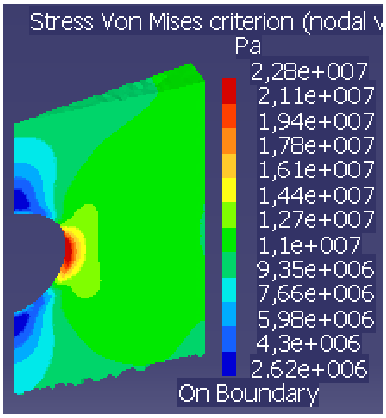
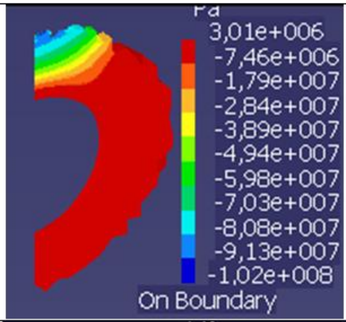


Fig. 8. Equivalent von Mises stress around the void

Solomon, [3], gives an analytical solution of the problem using the method of Lurie's operators and concludes that the maximum stress appears on the void equator from the plane normal to the stretching direction. The stress reaching the maximum value is the σ_{θ} stress and it takes a double value compared to its value at infinity. In Fig. 9 there are presented the normal spherical stresses and in Fig. 10 the principal stresses around the cavity are shown. It can be seen that the stress maximum value is $2.35 \cdot 10^{7}\,Pa$ while for the Hertzian case, the maximum pressure was $2.47 \cdot 10^{7}\,Pa$. The fact that it was not obtained a double value for maximum stress is a consequence of the assumption that the half-space can be modelled as a finite dimensions cylinder.

Table 1

<table>
<tr><th></th><th>σ_r</th><th>σ_φ</th><th>σ_θ</th><th>$\sigma_{vonMises}$</th></tr>
<tr><td>$h = 1$</td>
<td>
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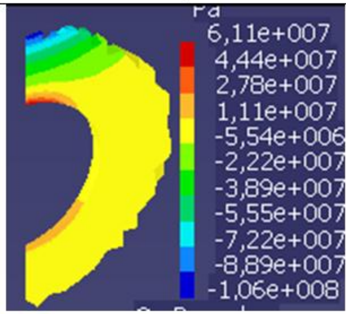
<td>
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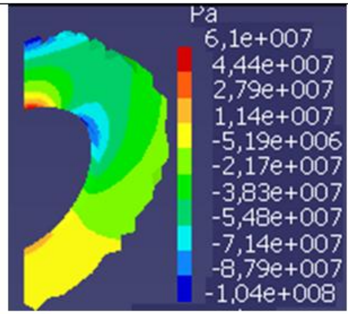
<td>
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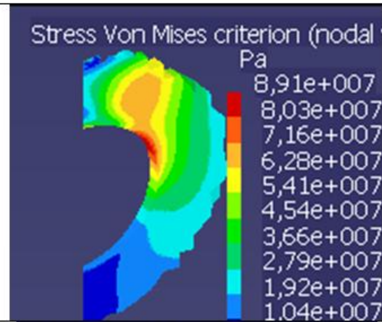
<td>
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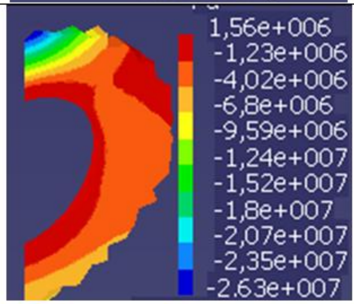
<td>
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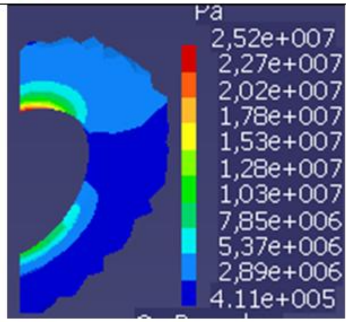
<td>
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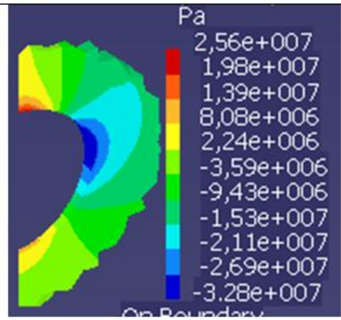
<td>
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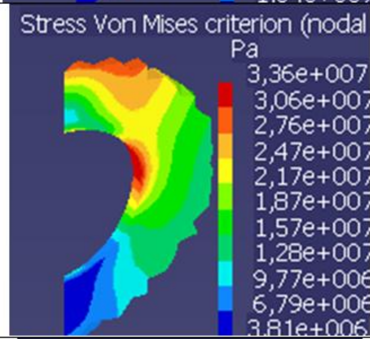
<td>
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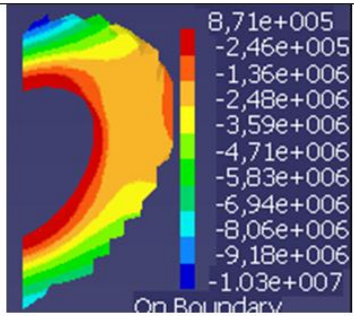
<td>
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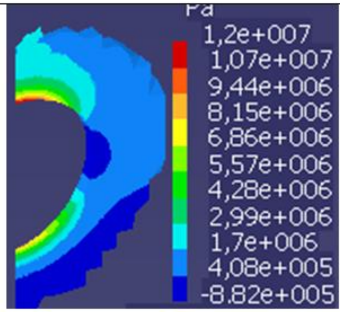
<td>
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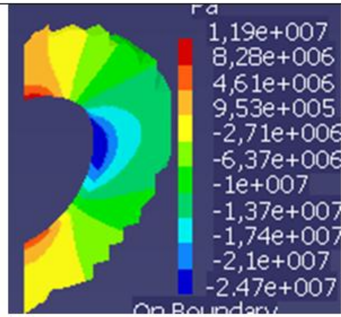
<td>
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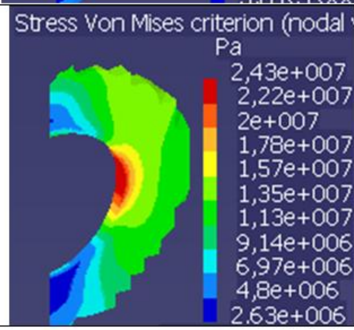
<td>
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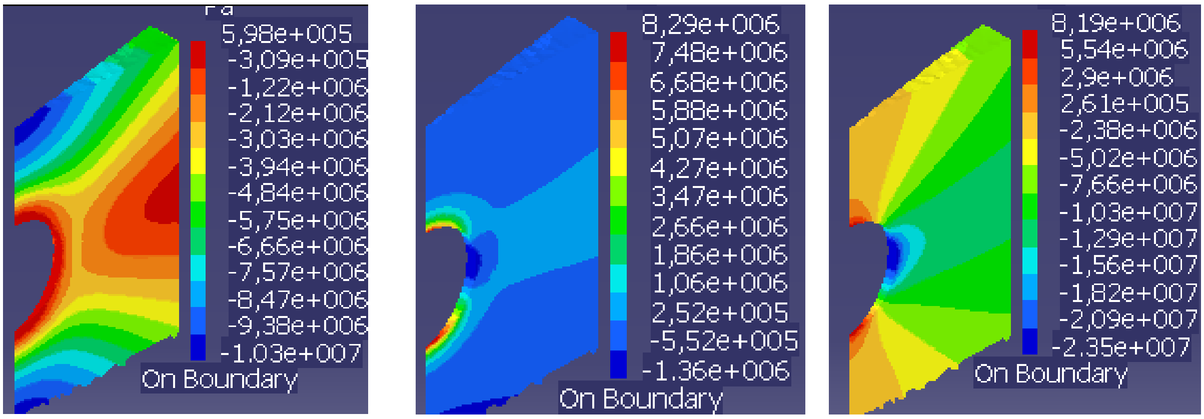

Fig.9. Normal spherical stresses

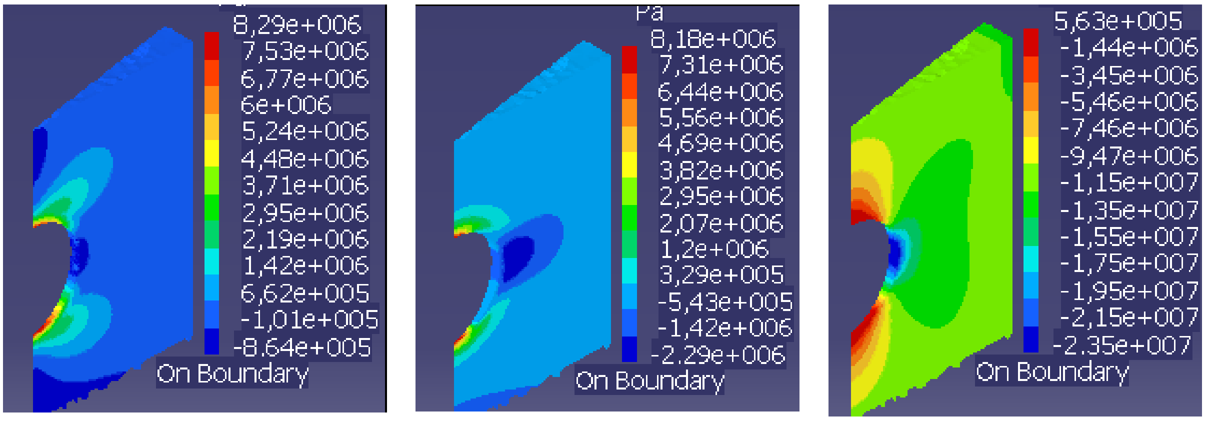

Fig.10. Principal stresses

5. Conclusions

The paper studies the stress concentrator effect of a spherical void from an elastic half-space in Hertzian contact with a normally loaded sphere. In a first stage, the dimensions of the study zone and the mesh element size in order to describe with a superior accuracy the stress field behaviour. To this end, the Hertz problem of a sphere pressed onto an elastic half-plane is analytically and numerically solved.

Subsequently the stress state around the void for different depth positions for the centre of the sphere is found. The maximum stress from the void surface tends asymptotically to a value that is, in conformity to Saint Venant principle, the same as the maximum stress from the surface of a void from an elastic space monoaxially stretched with a force equal to the load from Hertz problem. This limit value is matching the values given by the analytical solutions from literature.

References

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