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The Timoshenko Three-Beams Technique To Estimate The Main Elastic Moduli Of Orthotropic Homogeneous Materials

Dr. Kamal K.M. Saify * **Dr. Adnan N.J. Al-Temimy**** **Dr. Muhsin J.J. *****

[*] *The Technical College-Baghdad / Department of Dies & Tools.*

[**] *Mechanical Engineering Dept./College of Engineering./ University of Baghdad*

[***] *Mechanical Engineering Dept./ College of Engineering/ University of Al-Nahrain*

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Abstract:-

A New developed technique to estimate the necessary six elastic constants of homogeneous laminate of special orthotropic properties are presented in this paper for the first time. The new approach utilizes the elasto-static deflection behavior of composite cantilever beam employing the famous theory of *Timoshenko*. Three extracted strips of the composite plate are tested for measuring the bending deflection at two locations. Each strip is associated to a preferred principal axis and the deflection is measured in two orthogonal planes of the beam domain. A total of five trails of testing is accomplished and the numerical results of the stiffness coefficients are evaluated correctly under the contribution of the macromechanics and the approximate bending theory. To insure the validity of the new approach, separate individual tensile tests are performed, and the corresponding results are compared. Excellent agreements are obtained between the different approaches. The ease, simple and accurate predictions are well confident by the new technique.

Keywords: Timoshenko ,Beam**,** composite beam.

1. Introduction

The development of composite materials offers great potential in advanced civilian and non-civilian structural applications since the late thirties of the last century [1,2] and still now in rapid progress and evolution [3,4]. The recent century began with a new technological development of the "*smart*" composite structures [5,6,7] where a large strength-to-weight ratio is achieved, besides the ability to react actively to disturbance forces while maintaining structural integrity. The assignment of the mechanical engineering properties, of such materials, are strongly demanded for design and behavior analysis. The orthotropic elastic constants (total nine in number) represent an important set of those properties. Starting with the familiar *Young*, shear moduli and *Poisson*'s ratio, the traditional static tensile test satisfies, to some level, the mentioned objective but involves uncertainty of the results (due to the localized deformation near the end fixture of the tensile sample (see Ref.[8], *chap. Micromechanics*) as well as the weak and simple base theory it adopts where the transverse shear effects are ignored as usual [8,9]), and also the relative cost of the test requirements (the available of tensile M/C and minimum three samples to be distracted later). Moreover, the traditional test is not able to determine more than four elastic constants in its best conditions (refer to [8]). The theoretical and experimental attempts of *Tsai*[9] and nextly *Halpin and Tsai*[10] in the static micromechanics of composites, was found satisfactory if the pre-limitations in analysis were released.

Their formulations require that the physical and geometrical properties of the composite constituents as well as a suggestion of two new factors insurt in the formulae, all ought to be prepared in advance. Again only four elastic constants could be obtained by these approaches. Dynamic tests, were firstly conducted by *Goens*[11] to determine the shear modulus of an isotropic bar under torsion. Later on, the two elastic constants (*Young* and shear moduli) were obtained by *Pickett*[12], *Hasselmen* [13] and *Spinner & Teff*[14] using two independent tests, the bending vibration and torsional ones. *Rubben & Scharr*[15] applied excellently both the static tensile test and torsional vibration test to estimate the nine elastic constants of composite using three chosen samples for the two tests, one of which its fabrication procedure was seemed difficult to be achieved and required much care and accuracy. *Deobald & Gibson* [16] used the classic orthotropic plate theory of *Kerishoff* to compute the four elastic constants employing the new modal analysis technique (MAT) [17]. *Saify & Al-Temimi*[18] were the first who succeeded to obtain the two "*effective*" elastic moduli by one test of flexural vibrations of prismatic bar. Recently reference [19] presented a developed "three theories technique (TTT)" to determine all the elastic constants set of anistropic material employing the MAT and basing theoretically upon his "*exact*" orthotropic simply-supported plate theory from *Levinson*[20] first concept of the exact isotropic plate theory. The conditions, required to make this approach successful, are: a test rig for sample boundary supporting and the set-up instrumentation of the applied MAT. It seems, generally, that a chosen approach, to determine the orthotropic elastic constants of a composite material, is often incorporating some technical (and/or theoretical) limitations in the employment. The need of (i) inexpensive test, (ii) acceptable base theory, (iii) few tested samples and (v) many estimated elastic constants, is the most preferable thing to put forwards for achieving such aim. Too many demands against so humble abilities!.

The present paper looked for accomplishing most of these demands through the adoption of *Timoshenko* beam theory [21] which is still found as an acceptable engineering theory. A non-destructive static deflection test (instead of the destructive tensile test) of a composite cantilever strip may be sufficed to obtain the two elastic constants associated with the principal axes of the testing sample. Utilizing the familiar configuration of the samples, originally used in the tensile test, the present approach would be able to determine a maximum of six independable elastic constants from three samples preserving the same simple test set-up. The new approach had been examined, for validity assessment, by a resonant frequency test and the comparison of experimental results were made among all mentioned approaches. The present T3BT reflected very obviously its reliability and success in the achievement comparing with other techniques in literature till the time of submitting this report.

2. Theoretical Analysis

Referring to **Fig.1**, the composite sample, under consideration, is modeled as a rectangular beam (strip) with its length, thickness and breadth are denoted by L_1,L_2 and L_3 respectively. The beam is supposed to be bent statically in the plane (1-2) due to an arbitrary distributing load P_{12} (per unit breadth) on the beam upper surface. Generally, the load may be a function of location η along the major axis-1. In accordance to *Timoshenko* beam theory the constitutive-displacement relationships and the force-moment equilibrium conditions state that:

$$Q = \frac{5}{6} G_{12} A_1 \left(y + \frac{dd}{dh} \right), \mathrm{M} = \mathrm{E}_1 I_3 \frac{dy}{dh}$$

......(1)

$$\frac{dQ}{dh} = P_{12}(h) \quad , \quad \frac{dM}{dh} - \mathrm{Q} = 0$$

......(2)

where:

$$\mathrm{A}_1 = L_2 . L_3 \quad , \quad \mathrm{I}_3 = \frac{1}{12} L_3 .(L_2)^3$$

......(3)

The quantity ($5/6G_{12}$), in Q-expression, is the "*effective*" shear modulus of the strip material [20], in which the factor (5/6) refers to *Reissner*'s shear coefficient, while G_{12} is simply the actual shear modulus associated with existed plane of deformation (1-2). The item E_1, in M-expression, is commonly

the *Young* modulus of the cantilever material in the major direction-1.

Solving of the ordinary differential eq.(2) for the loading condition of concentrated force P_0 at the beam tip (η=L_1), and using the results into eq.(1) yields to the general expressions of the displacement components δ (the local deflection) and ψ (the section rotation) as followings:

$$y = \frac{6P_0}{5G_{12}A_1} - \frac{P_0}{E_1 I_3} \left(L_1 h - \frac{1}{2} h^2 \right) + C_1$$

$$d = \frac{P_0}{E_1 I_3} \left(\frac{1}{2} L_1 h^2 - \frac{1}{6} h^3 \right) + C_1 h + C_0$$

......(4)

Applying the beam condition at the clamped end $(h = 0 \Rightarrow y = d = 0)$, gives the exciplict formula for δ as varied with η, in the form:

$$d = \frac{p_0}{6E_1 I_3} \left(3L_1 h^2 - h^3 \right) + \frac{p_0}{5G_{12} A_1} h \qquad \text{....(5)}$$

The two elastic constants (E_1 and G_{12}), appeared in above equation, can be calculated whenever the deflection δ is precisely measured at two locations, say the strip tip (η=L1) and the mid-length (η=L_1/2), resulting in:

$$d_T = \frac{P_0 L_1^{\ 3}}{3E_1 I_3} + \frac{6P_0 L_1}{5G_{12} A_1}$$

......(6)

$$d_M = \frac{5P_0 L_1^{\ 3}}{48 E_1 I_3} + \frac{3P_0 L_1}{5G_{12} A_1}$$

where δ_T and δ_M represent the localized deflections at the beam tip and mid-length positions respectively. Eq.(6) suffices now to compute the two elastic constants of the strip from:

$$E_1 = \frac{P_0}{\left(\frac{2}{3} \right) L_3 \left(\frac{L_2}{L_1} \right)^3 \left(2d_T - d_M \right)}$$

...(7)

$$G_{12} = \frac{P_0}{\left(\frac{5}{18} \right) L_3 \left(\frac{L_2}{L_1} \right) \left(16 d_M - 5 d_T \right)}$$

The benefit of above formulae, comes from that its mathematical scheme can be held correctly for general beam rotation of the coordinate axes system. It does not enforce any preferable choice of the directions (1,2,3) to be adjusted for any sides of the strip (length, thickness or breadth), i.e. eq.(7) can be utilized for any axis rotation through (90^0) about its plane. This will serve to compute another set of two elastic constants (*Young* and shear moduli) corresponding to the new axes system. In order to estimate a maximum number of these sets of moduli values, the job was devoted to conduct the deflection tests, firstly on a beam-A whose major axis coinciding with the fiber axis (i.e. parallel), secondly on a beam-B whose major axis is perpendicular to the fiber axis (i.e. normal) and thirdly on a beam-C whose major axis is at 45^0 with the fiber axis (i.e. inclined). These three beams are actually cut from the composite laminate as illustrated by Fig.(2). Each beam is then tested independently one or two times.

In each time the deflection axis is altered by (90^0) rotation about the major axis. Denoting the Cartesian plane (xy) as the mid-plane of the composite laminate, where the fibers are along x-axis, and choosing z-axis to be orthogonal with (xy) through out the laminate thickness, then the complete deflection tests may be organized as followings:

(i) Beam-A (parallel):

(1)Test-1: The major axis-1 is x-axis and the deflection axis-2 is z- axis, from which E_x and G_{xz} can be estimated.

(2)Test-2: The major axis-1 is x-axis and the deflection axis-2 is y-axis, from which E_x and G_{xy} would be then computed.

(ii) Beam-B (normal):

(1)Test-1: The major axis-1 is y-axis and the deflection axis-2 is z- axis, from which E_y and G_{yz} can be estimated.

(2)Test-2: The major axis-1 is y-axis and the deflection axis-2 is x- axis, from which E_y and G_{xy} would be then computed.

(iii) Beam-C (inclined):

(1) Test-1: The major axis is 1-axis and the deflection axis is 2-axis, from which E_1 and G_{12} can be evaluated. Transforming the results to the actual laminate axes (xyz), then the two elastic constants (G_{xy} and ν_{xy}) can be estimated, from this test, using (see Ref.[8]):

$$\frac{1}{G_{xy}} = \left(\frac{4}{E_1} + \frac{1}{4G_{12}}\right) - 2\left(\frac{1}{E_x} + \frac{1}{E_y}\right)$$
$$\frac{2\nu_{xy}}{E_x} = \left(\frac{1}{4G_{12}}\right) - \left(\frac{1}{E_x} + \frac{1}{E_y}\right) \qquad \ldots(8)$$

By now, the present 3-beam samples are non-destructively tested by simple deflection tests to estimate the six elastic constants (E_x, E_y, G_{xy}, ν_{xy}, G_{xz} and G_{yz}) of the given orthotropic material. **Table(1)** summarizes the total five tests procedure, previously explained . Note that the constants E_x and E_y would be averaged from the test results of beam-A and B respectively. The same thing might be done for G_{xy} from all the three beam tests, whereas no averaging is there for G_{xz}, G_{yz} and ν_{xy} since they are computed one time only.

3. Numerical results, discussions and comparison

The ever best method to check for the validity of the present and relevant techniques to estimate the different elastic constants of an orthotropic material, is the adoption of a reference sample whose material elastic moduli had been precisely obtained and verified frequently by some reliable technique, other than these mentioned here, and see whether the present approaches retain the same elastic constants values. Unfortunately, this trail failed due to the absence of such material. However, the present aim can be achieved alternatively by adoption of the same experimental and theoretical results of the different approaches. The "*best*" approach is that which maintaining the minimum deviations of the results throughout all cycles of the tests. It is a simple sort of "*optimization*" of the different four approaches: The classical tensile test, The strength of material approach, The elasticity approach and finally the present T3BT.

The present manufactured composite plate was firstly well prepared and the three strips(A,B,C) were perfectly cut along the corresponding directions as clarified by **Fig.2**. Appendices(A,B) display the main formulations to calculate the corresponding four elastic constants (E_x, E_y, G_{xy} and ν_{xy}) in the light of the approaches respectively.

Table(2) show the entire collection of the experimental readings of the strips static deflections (using electrical resistance strain gauges) corresponded to given concentrated load at the tip end and for all strips configurations and test trails as proposed by the T3BT. **Table(3)** presents the experimental acquired readings of the classical tensile test procedure made on the three strips and for all test trails as familiarly performed by this treatment. From these tests readings, the orthotropic elastic constants were computed and organized as shown by **Tables(4,5).** The T3BT gives the results of six elastic modulii, whereas the classical tensile test gives the results of four elastic constants. In closing, **Table(5)** illustrates the overall final values of the material elastic moduli as obtained by the current four approaches, mentioned before. In this table the results of the T3BT and the tensile tests are commonly averaged, from which the final standard deviations are computed easily.

A little consideration into the last argument of the standard deviations in **Table(5)** gives definitely that present T3BT estimates the accurate results in respect to the familiar tensile test approach, in addition to its ability of obtaining two further constants upon the common four ones. The mean value of these deviations, among the total six values from the T3BT, is no more than (0.053), while from the tensile test approach (with total four values) reaches to (0.108). It is very obvious that the present T3BT estimates the results two times accurate than the classical approach. Henceforth, the *Tsai* approach is more reliable in results than the strength of material approach which seems to be the worse one. The most beneficial thing regarding the present T3BT is its success in estimating the orthotropic shear moduli G_{xz} and G_{yz} that no other technique had achieved in similar proposition of the present work.

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APPENDIXES

Appendix(A): Strength of material approach.

Given the physical & mechanical properties of the fiber (E-glass) and matrix (epoxy) constituents of the composite material (the present fabricated laminate) as listed below:

Specification	Fiber (E-glass), V_f=45%	Matrix (epoxy) V_m=55%
Young modulus	72.40Gpa	3.40Gpa
Shear modulus	29.67Gpa	1.27Gpa
Poisson's ratio	0.220	0.34
Mass density	2.54×10^{-6} kg/mm3	1.22×10^{-6}kg/mm3

The apparent elastic constants and mass density of the orthotropic laminate may be computed as followings (see Ref.[8,9] where deep details on the chemical compositions are discussed):

$$E_x = V_f E_f + V_m E_m$$

$$\frac{1}{E_y} = \frac{V_f}{E_f} + \frac{V_m}{E_m}$$

$$\frac{1}{G_{xy}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \qquad \text{……(A -1)}$$

$$n_{xy} = V_f n_f + V_m n_m$$

$$r = V_f r_f + V_m r_m$$

with the notations (f, m) refer to the fiber and matrix constituents respectively

Appendix(B): The elasticity approach.

Referring to the theoretical concepts of *Tsai & Halpin* in the micromechanics of composite material of two constituents, discussed earliarly, the four apparent elastic constants were driven in the form of:

$$E_x = \bar{k}\,(E_m V_m + E_f V_f)\ , \quad \mathrm{E_y} = K_0\{(1-\bar{c})K_1 + \bar{c}K_2\}$$

$$n_{xy} = \{(1-\bar{c})K_3 + \bar{c}K_4\}\ , \quad \mathrm{G_{xy}} = \{(1-\bar{c})K_5 + \bar{c}K_6\} \qquad \text{……(B-1)}$$

where $\bar{k}$ and $\bar{c}$ are the effective "fudge" factor and the misalignment factor respectively. Their magnitudes are actually taken to be in the range (0.85-1.00) for the first factor and (0.0-0.4) for the second one, as proposed by the authors above. The six K's coefficients in eq.(B-1) are computed from:

$$K_0 = 2(1 - n_f V_f - n_m V_m)$$

$$K_1 = \frac{K_f (2K_m + G_m) - G_m (K_f + K_m) V_m}{(2K_m + G_m) + 2(K_f - K_m) V_m}$$

$$K_2 = \frac{K_f (2K_m + G_f) - G_m (K_f - K_m) V_m}{(2K_m + G_f) - 2(K_m - K_f) V_m}$$

$$K_3 = \frac{K_f n_f (2K_m + G_m) V_f - K_m n_m (2K_f + G_m) V_m}{K_f (2K_m + G_m) - G_m (K_f - K_m) V_m} \qquad \text{……(B-2)}$$

$$K_4 = \frac{K_f n_m (2K_f + G_f) V_m + K_m n_f (2K_m + G_m) V_f}{K_f (2K_m + G_f) - G_f (K_m - K_f) V_m}$$

$$K_5 = G_m \frac{2G_f - (G_f - G_m) V_m}{2G_m + (G_f + G_m) V_m}$$

$$K_6 = G_f \frac{(G_f + G_m) - (G_f - G_m) V_m}{(G_f + G_m) + (G_f - G_m) V_m}$$

with all other notations are as being defined in Appendix(A)

Table(2). The present T3BT readings of the composite cantilever strips under the proposed static deflection tests.(refer to Fig.(2)).

Scheme of test		Test-1			Test-2		
Reading Items		P_0 (kg)	δ_M (mm)	δ_T (mm)	P_0 (kg)	δ_M (mm)	δ_T (mm)
Strip-A	Trial(1)	1.00	1.5875	5.0763	50.00	0.4571	1.2955
	Trial(2)	1.50	2.3526	7.5231	75.00	0.6950	1.9544
	Trial(3)	2.00	3.1274	10.0007	90.00	0.8338	2.3428
Strip-B	Trial(1)	0.50	2.1473	6.8676	50.00	0.9374	2.8374
	Trial(2)	0.75	3.0527	9.7637	75.00	1.3575	4.0835
	Trial(3)	1.00	4.1033	13.1237	90.00	1.6415	4.9385
Strip-C	Trial(1)	25.0	0.8230	2.1363	-	-	-
	Trial(2)	35.0	1.1516	2.9673	-	-	-
	Trial(3)	45.0	1.4879	3.8349	-	-	-

Table(3). The present simple tensile test readings of the composite cantilever strips.

Reading Items		P_0 (kg)	ΔL* (mm)	Δb** (mm)
Strip-A	Trial(1)	300.0	0.800	0.098
	Trial(2)	350.0	0.964	0.078
	Trial(3)	400.0	1.096	0.072
Strip-B	Trial(1)	300.0	1.943	0.089
	Trial(2)	350.0	2.094	0.078
	Trial(3)	400.0	2.430	0.072
Strip-C	Trial(1)	300.0	0.710	-
	Trial(2)	350.0	1.312	-
	Trial(3)	400.0	1.571	-

(*) Longitudinal elongation of the tested strip.
(**) Lateral contraction of the strip.

Table(4). Computations of the elastic constants of the composite strip from two present theoretical/experimental approaches.

Approach		The estimated elastic moduli of the present orthotropic material					
		E_x (Gpa)	E_y (Gpa)	G_{xy} (Gpa)	ν_{xy}	G_{xz} (Gpa)	G_{yz} (Gpa)
T3BT (*)	Trial(1)	6.613	2.361	1.985	0.221	1.779	0.879
	Trial(2)	6.693	2.491	1.865	0.371	1.869	0.949
	Trial(3)	6.713	2.471	1.855	0.371	1.899	0.929
Tensile test (**)	Trial(1)	7.380	3.030	2.141	0.444	-	-
	Trial(2)	7.120	3.280	2.441	0.324	-	-
	Trial(3)	7.160	3.230	2.351	0.264	-	-

(*) refer to eqs.(5,6,7).
(**) $E_i=P_0.L_i/(L_j.L_k).\Delta L_i$, $\nu_{ij}=\Delta L_j.L_i/L_j$. ΔL_i (i,j,k=x,y,z or 1,2,3) with the help of eq.(7).

Table(5). Comparison of the estimated results of the orthotropic elastic moduli of the present composite material according to variety of current approaches.

Approach		The main elastic constants					
		E_x (Gpa)	E_y (Gpa)	G_{xy} (Gpa)	ν_{xy}	G_{xz} (Gpa)	G_{yz} (Gpa)
T3BT	Average	6.673	2.441	1.905	0.321	1.849	0.919
	σ(*)	0.043	0.057	0.064	0.071	0.051	0.029
Tensile test	Average	7.220	3.180	2.311	0.344	-	-
	σ(*)	0.123	0.108	0.126	0.075	-	-
Strength of material($)		7.081	4.003	1.502	0.294	-	-
Elasticity($$)		6.727	2.340	1.822	0.302	-	-

(*) Standard deviation $= \dfrac{\sqrt{\sum_{i=1}^{3}(value - average)_i^2}}{3}$

($) refer to Appendix(A).
($$) refer to Appendix(B).

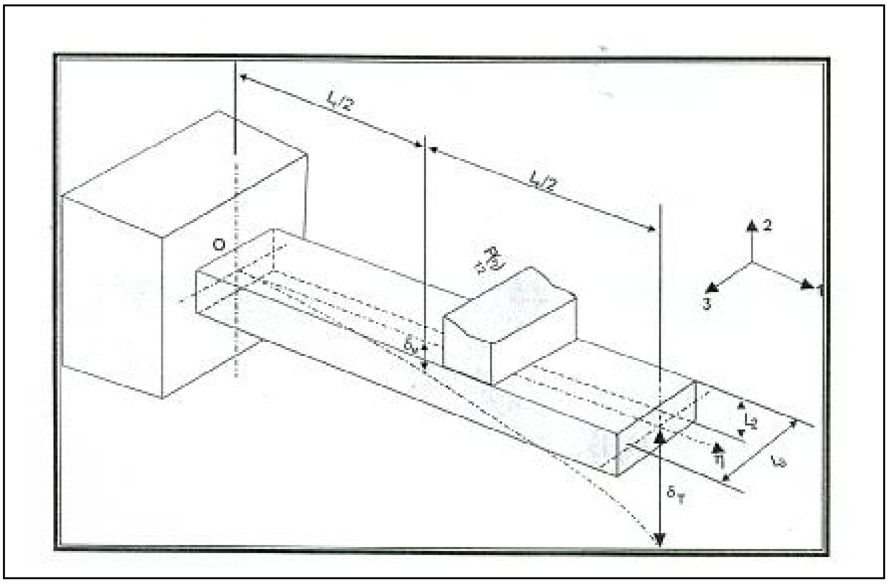

Figure (1)

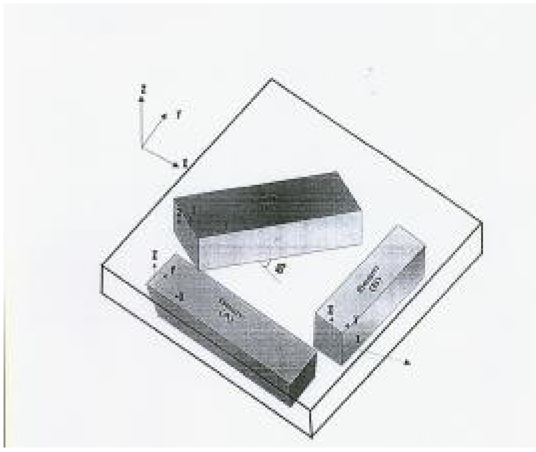

Figure (2)

تقنية تيموشينكو ثلاثية العتبة لتقدير معامل المرونة الرئيسي للمواد (المتجانسة ثلاثية البعد)

د.كمال مصطفى كمال محمود سيفي **د.عدنان ناجي جميل التميمي** **د.محسن جبر جويج**

قسم القوالب والعدد/الكلية التقنية-بغداد قسم الميكانيك/ كلية الهندسة قسم الميكانيك/ كلية الهندسة

جامعة بغداد جامعة النهرين

الخلاصة:

طريقة مطورة جديدة ، لحساب ثوابت المرونة الستة والضرورية لتحليل التصرف الميكانيكي والداينماكي للشرائح المركبة المتجانسة متعامدة الصفات الهندسية، قد قدمت في هذه الورقة للمرة الأولى من نوعها في الأساس النظري وإجراءات العمل. اعتمدت الطريقة على نظرية "تيموشنكو" للقضبان المركبة الناتئة والمنحنية سكونيًا. يتطرق الجانب العملي الى استخدام ثلاث شرائح مركبة من المادة على طول المحاور الأساسية الثلاث وإيجاد الازاحات السكونية المناظرة لكل شريحة وبمستويين متعامدين من منظومة المحاور الأساسية للتركيب. تم إجراء خمسة محاولات تجريبية بواقع اختبارين لكل محاولة وحساب معاملات الصلابة وفق معادلات "تيموشنكو" ونظرية الميكانيك الدقيق للمواد المركبة. لغرض إقرار الموثوقية للنتائج المختلفة، تم إجراء ثلاث اختبارات كلاسيكية للشد لجميع نماذج التجربة ومقارنة القراءات النهائية للصلابة في الاتجاهات الرئيسية. لقد أثبتت الطريقة المقدمة كفاءتها وصحة نتائجها بإعطائها أقل الانحرافات العددية مقارنةً بالطرق السابقة، كما امتازت ببساطة ودقة الشكل الرياضي للحل.