SCIENTIFIC PAPERS

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MODELLING THE BENCHMARK SPOT CURVE FOR THE SERBIAN MARKET

ABSTRACT: *The objective of this paper is to estimate Serbian benchmark spot curves using the Svensson parametric model. The main challenges that we tackle are: sparse data, different currency denominations of short and longer term maturities, and infrequent transactions in the short-term market segment vs daily traded medium and long-term market segment. We find*

that the model is flexible enough to account for most of the data variability. The model parameters are interpreted in economic terms.

KEY WORDS: *fixed-income market, benchmark spot curves, yield curve modelling, Nelson-Siegel Model, Svensson model, fitting procedures*

JEL CLASSIFICATION: C02, C21, C61, G12

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1. INTRODUCTION

The notion of a spot curve (also known as zero-coupon curve or term structure of interest rates) is among the most fundamental concepts in modern finance.[1] It reflects market consensus on interest rates, that is, on lending and borrowing conditions in a country, at a particular point in time. It also provides the basis for the construction of discount factors, and thus for valuation of investment projects and financial securities. Last but not least, estimates of spot curves are fundamental ingredients of contemporary risk management models. The aim of the paper is to estimate benchmark spot curves for Serbia using a parsimonious approach. In doing so, we need to take into account issues specific to emerging markets in general and the Serbian market in particular.

Accurate estimation of spot curves is important for businesses and regulators alike. Yet such estimation is a rather challenging task. A successful model has to incorporate known empirical facts about interest rates: that interest rates are non-negative and show mean-reversion, that changes in interest rates are not perfectly correlated and also that the volatility of short-term rates is typically higher than the volatility of medium- and long-term rates. In order to find risk factors that cause changes in interest rates the Principal Component Analysis (PCA) method is frequently used. Its purpose is to explain variations of observed changes in interest rates with a small set of uncorrelated factors. Numerous studies across different fixed income markets show that the first three principal components explain most of the changes in interest rates. These factors are referred to, respectively, as level, slope and curvature factors.[2]

Apart from the three risk factors, actual bond prices are impacted by other considerations. For example, bonds that differ only by their liquidity may have different prices and, therefore, different yields to maturity. Consider, for example, long-term issues in the year of their maturity. They are, typically, much less liquid in comparison with a fungible money market instrument. For this reason such bonds are commonly sold at a discount. Different tax treatment may also induce price distortions. In Canada,[3] for example, purchasing a bond at a discount provides both a tax reduction and a tax deferral. In order to construct a benchmark

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[1] In principle, one can distinguish between the yield and spot curves. In case of zero-coupon bonds, bond yields coincide with the appropriate spot rates. Since the Serbian government bond market currently consists solely of zero-coupon bonds, no distinction can be made between the yield and spot rate curves.

[2] See Martellini, Priaulet and Priaulet (2003) for a summary of this strain of research.

[3] See Bolder and Stréliski (1999)

government bond spot curve that reflects only systematic characteristics of the market we need to eliminate issues that distort market prices.[4]

In this paper we use a parametric approach to modelling spot curves. In a parametric approach one fits market data to a pre-specified functional form. One way to do so is to use different curve parameters for different maturity segments of the spot curve. One practical way to accomplish that is to use Polynomial or Exponential splines.[5] Another approach is to find a single set of parameters that would fit the entire maturity horizon. This is commonly done using Nelson-Siegel with a Svensson extension.[6] The selection of a particular modelling approach depends, among other things, on whether we focus on fitting accuracy or the parsimony of the model.[7]

If a number of data points is sufficiently large, splines allow for high fitting accuracy. Fitting accuracy, however, comes at the expense of over-parameterization. For this reason, spline models often have poor out-of-sample performance, especially in case of curve extrapolation.[8] The problem is exacerbated in the case of a small number of data points (this is often the case in emerging markets such as Serbia). Another drawback of the spline models is that estimated parameters cannot be easily interpreted in economic terms.

For these reasons, we focus on the main practical alternative, namely the Svensson (1994) parametric model (we refer to it as the Svensson model). This allows modelling of the entire maturity horizon of the spot curve. It requires estimation of a significantly smaller number of parameters than a typical spline model. This makes the Svensson model more appropriate in markets with sparse data. Furthermore, the parameters of the model have a clear economic interpretation.

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4   See *http://www.ecb.int/stats/money/yc/html/technical_notes.pdf* for a detailed description of the eligibility requirements imposed by the European Central Bank for estimation of euro zone yield curves.

5   For more details about different spot curve estimation techniques see Martellini, Priaulet and Priaulet (2003) and Fabozzi (2000). Deacon and Derry (1994) compare performance of different fitting models on UK data. Bliss (1997) tests five different methods on U. S. bond market data.

6   See Nelson and Siegel (1987) and Svesson (1994).

7   Using either splines or the Nelson-Siegel-Svensson model one obtains a static spot curve, i.e. a spot curve calculated in a particular point in time. For spot and forward curve dynamics see, e.g. Martellini, Priaulet and Priaulet (2003).

8   See Shea (1984) for a critique of polynomial and exponential spline models in spot curve estimations.

Using the Svensson model allows the reconciliation of stylized empirical facts and accommodation of idiosyncrasies related to the bond market in focus.

In this paper we show how the Svensson model can be implemented in constructing benchmark spot curves for the Serbian government bond market, a small, underdeveloped bond market with euro-denominated medium and longer maturities and a short-maturity tail denominated in the local currency, the Serbian dinar. We show that the model is flexible enough to take into account several peculiarities of the Serbian market.

The remainder of the paper is organized as follows. Section 2 describes the model. Section 3 describes the Serbian bond market and draws the implications of its structure on spot curve modelling. Section 4 contains the description of the estimation procedures and presents the empirical results. Section 5 concludes.

2. THE MODEL

In order to price an arbitrary stream of cash flows, in principle one needs to know discount factors corresponding to every conceivable investment horizon. On the other hand, there is a finite number of bond maturities available in any given market. The purpose of spot curve modelling is to interpolate (and, when possible, extrapolate) the existing market data to model the missing maturities.

We define a bond price as the present value of cash flows promised to bond holders. We assume that this definition correctly captures important information related to bond prices and that the difference between the modelled and actual market prices is pure noise. Using continuously compounded interest rates the bond price is given by the following expression:

$$B = \sum_{i=1}^{n} CF_i e^{-R^c(0,\theta_i)\theta_i} \tag{1}$$

Here CF_i is i-th promised cash flow due to be received at time θ_i, $R^c(0, \theta_i)$ is the annual continuously compounded spot interest rate corresponding to maturity θ_i and, finally, n is the number of cash flows.

Nelson and Siegel (1987) propose a model that assumes more smoothness in the underlying relation than one observes in the actual market data. In their view, an over-parametrized model that follows all of the wiggles in the observed market data is less likely to have predictive power than a smoother model. Indeed, bond

trading is not always continuous and the available data may reflect transactions made at different points in time during the trading day. Bonds of specific maturities may sell at a discount or at a premium due to transaction cost differences, tax differential, etc. Smoothing the data, therefore, may help us capture systematic, essential characteristics of the data; departures from the fitted curve, from this standpoint, reflect idiosyncrasies of specific issues, i.e. pricing errors.

Nelson and Siegel (1987) describe instantaneous forward rate $f(0, \theta)$, i.e. the marginal cost of borrowing or lending money for an infinitely short period of time at date θ as seen from today, in the following functional form:[9]

$$f(0,\theta) = \beta_0 + \beta_1 e^{-\frac{\theta}{\tau_1}} + \beta_2 \frac{\theta}{\tau_1} e^{-\frac{\theta}{\tau_1}} \tag{2}$$

The spot rate is related to instantaneous forward rates by the following relation:

$$R^c(0,\theta) = \frac{1}{\theta} \int_0^\theta f(0,s)\,ds \tag{3}$$

Performing the integral in (3) the authors arrive at the following functional form approximating the spot curve on a chosen date:

$$R^c(0,\theta) = \beta_0 + \beta_1 \left[\frac{1-e^{-\frac{\theta}{\tau_1}}}{\frac{\theta}{\tau_1}} \right] + \beta_2 \left[\frac{1-e^{-\frac{\theta}{\tau_1}}}{\frac{\theta}{\tau_1}} - e^{-\frac{\theta}{\tau_1}} \right] \tag{4}$$

Parameters of the model have an important economic interpretation.[10] Parameter $\beta_0 = \lim_{\theta \to +\infty} R^c(0,\theta)$ can be interpreted as the long-term level of interest rates. Parameter $\beta_1 = \lim_{\theta \to 0}\left(R^c(0,\theta) - \beta_0\right)$ is a spread between the short and long term interest rates. Quantity $\lim_{\theta \to 0} R^c(0,\theta) = \beta_0 + \beta_1$ represents the short (overnight) lending rate. Finally, β_2 is the curvature factor. It determines the magnitude and the direction of the spot curve hump.

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9 This functional form is derived as a solution to an ordinary differential equation. See Nelson-Siegel (1987).

10 See Diebold, Piazzesi and Rudebusch (2005) for the discussion on linkages between movements in the model parameters and macroeconomic variables

One can view the spot rates represented by the Nelson-Siegel model (4) as a superposition of short, medium and long-term components. Parameter τ_1 is the rate of decay. It determines how fast the values of the short and medium-term component decay to zero, i.e. how fast they lose their importance. The long-term rate β_0 is unaffected by the decay parameter. When the value of τ_1 is small, most of the impact comes from the short-maturity segment. In that case (4) provides a particularly good fit for short maturities. In contrast, (4) with large values of τ_1 provides a better fit for longer maturity ranges.

Graph 1 shows sensitivity curves with respect to the three β parameters. Formally:

$$S_i = \frac{\partial R^c(0,\theta)}{\partial \beta_i} \tag{5}$$

From (4) and (5) we obtain that for the long term component $S_0=1$, for the short-term component $S_1 = \dfrac{1-e^{-\frac{\theta}{\tau_1}}}{\dfrac{\theta}{\tau_1}}$ and for the medium-term component

These quantities are plotted on Graph 1 for the fixed value $\tau_1 = 2$. The shapes of the three lines imply that changes in beta parameters can be regarded as parallel shift, the change of slope and the change of curvature, respectively.

$$S_2 = \frac{1-e^{-\frac{\theta}{\tau_1}}}{\dfrac{\theta}{\tau_1}} - e^{-\frac{\theta}{\tau_1}}$$

The Nelson-Siegel model is able to generate different empirically observed shapes of spot curves including monotonic curves, humps and S shapes. On the other hand, it generates inappropriate curve shapes in cases when there are two or more humps.

With a single decay parameter there is always a trade-off between the fitting potential in the short and long maturity regions. In order to overcome this problem and to provide a better fit over all maturity horizons, Svensson (1994) proposes a functional form with an additional short-term component and with two, instead of one, decay parameters. We refer to this model as the Svensson model:

$$R^c\left(0,\theta\right) = \beta_0 + \beta_1\left[\frac{1-e^{-\frac{\theta}{\tau_1}}}{\frac{\theta}{\tau_1}}\right] + \beta_2\left[\frac{1-e^{-\frac{\theta}{\tau_1}}}{\frac{\theta}{\tau_1}} - e^{-\frac{\theta}{\tau_1}}\right] + \beta_3\left[\frac{1-e^{-\frac{\theta}{\tau_2}}}{\frac{\theta}{\tau_2}} - e^{-\frac{\theta}{\tau_2}}\right] \quad (6)$$

Graph 1. Sensitivities of Nelson-Siegel spot curve
with respect to parameter changes
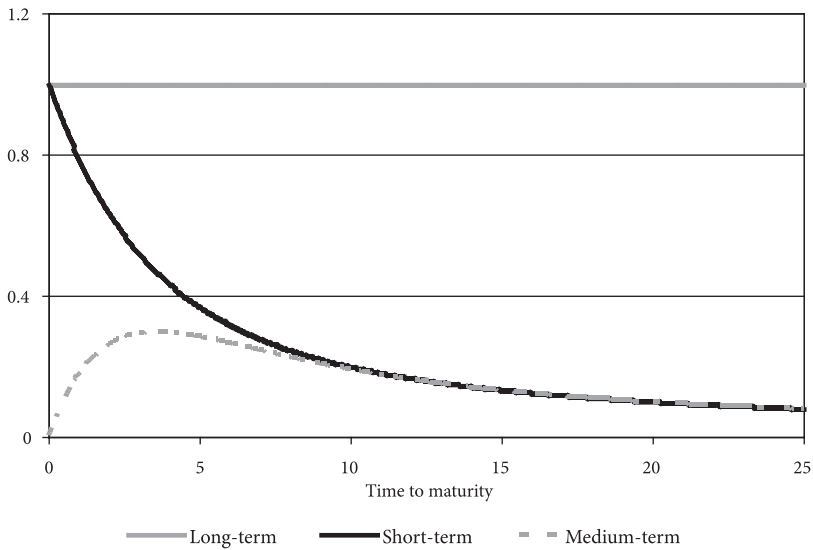
The Svensson model can successfully fit curves with an additional hump as compared to the simple Nelson-Siegel model. The empirical importance of the extension comes from the fact that money markets are usually more volatile compared to the rest of the maturity horizon. Thus, a model aiming to fit a spot curve over the entire maturity horizon should be flexible enough to capture volatile short-maturity as well as less volatile longer-maturity segments.

There are numerous studies confirming the empirical applicability of the Svensson model in developed markets. Svensson (1994) applies the model to Swedish market data and concludes that the Svensson model provides a very good fit even in cases when the original Nelson-Siegel version does not work sufficiently well. Utilizing the estimated spot curves and the implied forward curves, the author discusses the implied market consensus regarding the expected real rates, inflation rates and the inflation risk premium in the Swedish market. Bolder and Stréliski (1999) apply the Svensson model to Canadian market data. They discuss filtering criteria,

different optimisation algorithms and different goodness-of-fit tests. Yu and Fung (2002) apply both Nelson-Siegel and Svensson models to estimate spot curves based on exchange fund bills and notes issued by the government of Hong Kong. They find a satisfactory fit. However, parameter estimates are sometimes sensitive to the minimisation method being used. They employ both price errors and yield errors minimisation and find higher differences between the two methods under the Svensson model than under the Nelson-Siegel model. The European Central Bank (ECB) employs the Svensson model to calculate daily spot, forward, and par yield curves corresponding to benchmark sovereign securities. The ECB also provides daily time series of the estimated model parameters.[11]

3. SERBIAN FIXED INCOME MARKET

3.1. The Market Description

At present, the fixed income market in Serbia consists of euro-denominated and dinar-denominated T-bills.[12]

Euro-denominated bonds determine medium- and longer-term market segments and are comprised of bonds based on frozen foreign currency savings of Serbian citizens in failed government-owned banks. These bonds are Frozen Foreign Currency Savings Bonds (FFCBs).[13] These bonds were issued in 2002 in annual series from A2002 to A2016, to the total amount of EUR 4.2 billion. These securities have not been issued since. Starting March 24th 2003, they have been traded on the Belgrade Stock Exchange (BELEX) and OTC or interbank market[14]. Each individual series is due on May 31st of the corresponding maturity year. For modelling purposes it is important to note that FFCBs are zero-coupon bonds.

When looking at changes in FFCB yields one observes three sub-periods: from March 2003 to the end of 2005; from January 2006 to the end of 2008; and from

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11   *http://www.ecb.int/stats/money/yc/html/index.en.html*

12   The official currency in Serbia is the Serbian dinar (RSD).

13   Source: Belgrade Stock Exchange (BELEX).

14   The Serbian financial market is plagued with low liquidity and the FFCB bond market is no exception. While, formally, trading of these securities is continuous, in practice the actual number of daily transactions can sometimes be quite small. There are, sometimes, days without open market transactions. Apart from the open market transactions there are also over-the-counter trades. In fact, there is anecdotal evidence that as much as 75% of all trades have been over the counter. Unfortunately, no data is available to us about the OTC trades.

January to December 2009. Two main characteristics of the first trading period are: a drop in yields across different maturities and a narrowing of the range of yields over different maturity horizons. At the very start of continuous trading the yields on FFCBs were ranging from 9.5% to as much as 16.3%.[15] Gradually, the market matured so that, by the end of 2005, the range of yields was between 4.5%-5.5%. Only yields related to series in a maturity year close to the maturity date do not manifest a general decreasing trend in that period. This phenomenon can be explained by high transaction costs. It became less pronounced as the market matured.

Initially, FFCBs were the main source of liquidity in the Serbian bond market. There are several reasons for this. These securities are euro-denominated securities, thus face no exchange rate risk in a dual currency (dolarized) economy where the euro is effectively a reserve currency. These bonds are also tax exempt. Finally, they can be used as a special case of convertible bonds – the face value could be used for purchasing shares of companies undergoing privatization in Serbia. In this way investors that did not have cash in hand could use the bonds purchased in the secondary market to purchase companies. This put most of the demand pressure on the most deeply discounted bonds and brought about inverted spot curves. In addition, the first year of continuous FFCB trading was often characterized by significant yield oscillations over the maturity horizon. Movements in the FFCB market are impacted by a limited set of determinants rather uniformly. This is reflected by high correlation coefficients among FFCB yield series. The exceptions were short maturity issues, due to the previously mentioned phenomenon of high yields near maturation. Table 1 shows the correlation matrix for the period March 24th 2003 - May 20th 2005.

Table 1. Correlation matrix for FFCB yields March 24th 2003 - May 20th 2005

|	A2005	A2006	A2007	A2008	A2009	A2010	A2011	A2012	A2013	A2014	A2015	A2016
A2005	1.00											
A2006	0.62	1.00										
A2007	0.59	1.00	1.00									
A2008	0.58	0.99	1.00	1.00								
A2009	0.57	0.99	0.99	1.00	1.00							
A2010	0.55	0.98	0.99	0.99	1.00	1.00						
A2011	0.54	0.97	0.98	0.99	0.99	1.00	1.00					
A2012	0.53	0.97	0.98	0.99	0.99	0.99	1.00	1.00				
A2013	0.52	0.96	0.97	0.98	0.99	0.99	1.00	1.00	1.00			
A2014	0.50	0.95	0.96	0.97	0.98	0.98	0.99	0.99	0.99	1.00		
A2015	0.46	0.92	0.94	0.95	0.96	0.97	0.97	0.97	0.98	0.99	1.00	
A2016	0.43	0.90	0.91	0.92	0.94	0.94	0.95	0.95	0.95	0.96	0.99	1.00

15   All yield values presented in the paper are continuously compounded annual yields calculated by using the actual/365 day-count notation.

From the beginning of 2006 until the end of 2008, FFCB yields gradually increased, although not uniformly across all maturities. The increase in yields or, equivalently, a decrease in prices is related to a reduction of demand for FFCBs in that period. The reduction is the result of two fundamental factors. The main one is that in that period the Serbian dinar (RSD) was quite stable and even appreciated vis-à-vis the euro. This stability reduced the attractiveness of euro-denominated FFCB bonds. In addition, many of the most attractive companies undergoing privatization had already been privatized. This reduced the demand from this source for FFCBs even further. In 2008, spot curves were predominantly flat with pronounced parallel shifts. In the fourth quarter of 2008 yields ranged from 7%-8%. The increase in average yields at that time reflects lower investor interest for this paper and coincides with the beginning of the global financial crisis (most sub-investment grade paper had at that time a sharp increase in spread with respect to risk-less assets).

The year 2009 brought once again an increase in demand for FFCBs across all outstanding maturities, mostly because the RSD started to weaken vis-à-vis euros and investment in FFCBs is the principal hedge of Serbian institutional investors against the currency risk. In particular, the demand for such securities from institutional investors has risen again. In the fourth quarter of 2009 yield ranges were similar to those at the end of 2005, roughly 4.5%-5.5%.

Graph 2 summarizes the evolution of average FFCB yields in the period between March 2003 and Dec 2009.

Graph 2. Monthly FFCB yields for the period
March 24th 2003 – December 29th 2009
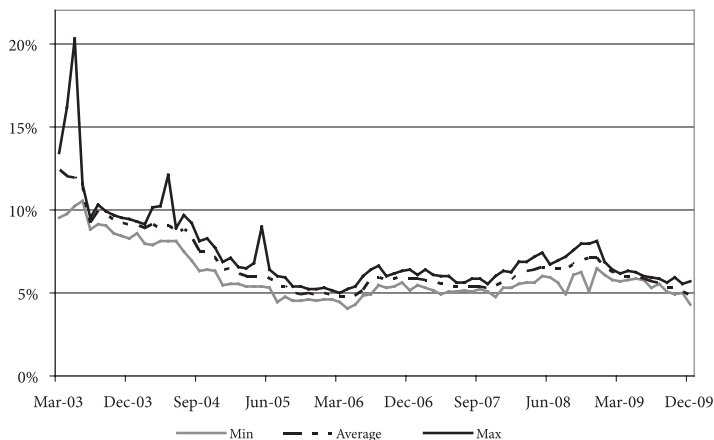
Note that there is no primary market for euro-denominated bonds in Serbia: as mentioned before, FFCBs were issued only once and never again. Because of this, observable maturities are not fixed - the longest maturity and the number of outstanding maturities decline over time. As a particular bond series matures, the overall number of securities outstanding (and maturities available) is reduced by one.

In addition to euro-denominated FFCBs, there are RSD-denominated government securities (T-bills) in Serbia. They comprise the money market in the country and have maturities of up to 12 months.

T-bills are auctioned by the Republic of Serbia for the purpose of financing the short-term budget deficit.[16] They have been issued in maturities of three and six months since 2003 and 12 months since 2009.[17] Graph 3 illustrates realized yields on T-bills for the April 2003-Dec 2005 period.

Graph 3. Realized yields on T-bills April 15th 2003-December 27th 2005
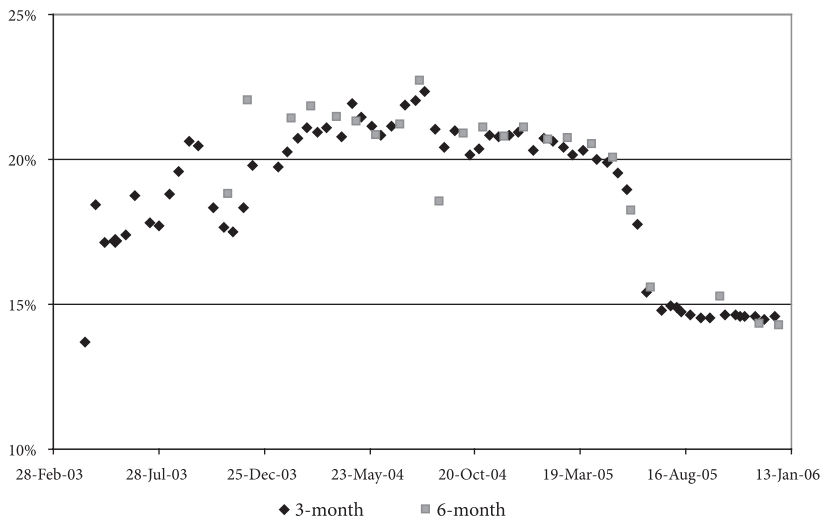
Up to the beginning of 2009 and an increased need of the government for fresh loans, T-bill auctions were not conducted on a regular basis and yields were not always synchronized with the repo transactions. The situation changed in 2009.

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16   Source: Serbian Ministry of Finance
17   From the second half of 2009, the Serbian Ministry of Finance started to auction 1-year T-bills, but they are not included in the analysis.

Around that time, an auction platform for T-bills was introduced. In addition to three and six-month bills, a one-year maturity bill was introduced. A trend of reduction in dinar-denominated yields is observed in 2009. This trend parallels that of major base rates (EUROBOR, LIBOR), as well as that of the reduction of yields of euro-denominated Serbian government bonds.

For the past several years, the National Bank of Serbia (NBS) has been entering into repurchase agreements (REPO). NBS frequently issues foundation (underlying) securities for these transactions commonly known as NBS bills. It should be noted that NBS bills are securities that are issued exclusively for this purpose (in most other countries, the role of foundation security for repo transactions is typically played by T-bills, notes and bonds). Repo transactions have been conducted on a relatively regular basis since January 2005. Repo agreements enable the NBS to make interventions in liquidity and money creation. There are two main objectives in issuing repos: (1) to inject or absorb liquidity in the banking system through open market operations (the so-called fine-tuning of the monetary policy) and (2) to generate signals about risk-free interest rates. In 2005, two-week, 1-month and 2-month repos were auctioned; in 2006, the NBS issued two-week and 2-month repos. As of 2007 the NBS has auctioned only two-week repos. Graph 4 illustrates realized repo yields in 2005.[18]

Graph 4. Realized yields on repo transactions
January 31st 2005-December 30th 2005

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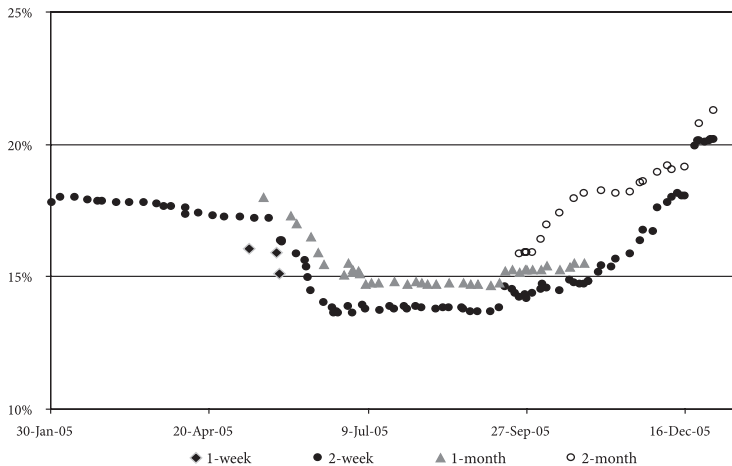
18   Source: National Bank of Serbia

In 2009 repo rates decreased along with T-bill rates. As a result, T-bill and repo rates are now much closer together than they were in previous years.

In summary, the Serbian bond market consists exclusively of zero-coupon government bonds. For short maturities there are, from before 2009, irregularly-issued securities denominated in dinars. For medium and longer-term maturities, there are euro-denominated bonds. These bonds were issued only once and are gradually maturing. In contrast to the money market instruments, euro-denominated bonds are traded continuously, i.e. there are prices available on a daily basis.

### 3.2 Implications of the Data Features on Spot Curve Modelling

The data features of the Serbian fixed income market that affect spot curve modelling can be divided into advantages and disadvantages. The main advantage is that all of the fixed-income securities currently available are zero-coupon bonds. This makes spot rate calculations easy. Also, none of the securities have embedded options. Finally, FFCBs are traded on the Belgrade Stock Exchange (BELEX) on a regular basis. The list of disadvantages is much longer: data sparseness; securities denominated in different currencies (euros and RSD); a gradual decrease in available maturities; irregular auction schedules for dinar-denominated securities; transaction cost effects in FFCB yields for near-maturity bonds.

The stated characteristics of the market indicate the existence of two distinct segments of the spot curve: a dinar-denominated short maturity segment and a euro-denominated medium and longer maturity segment. Obviously it would be easier to deal with the two segments separately.[19] On the other hand, fitting the curve over the entire horizon of the observed maturities makes it easier to find the economic interpretation of the model parameters. The problem of reconciling money market and FFCB yields boils down to reconciling dinar-denominated short-term securities with yields under control of the state authorities and euro-denominated yields of FFCBs that are the reflection of the market consensus. One of the important issues is to choose the base currency for the entire spot curve. Given that most of the maturities are covered only by euro-denominated bonds,

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19 Such a separate approach can be found for more developed debt markets as well, since liquidity and volatility of yields in the money market segment is typically higher compared to the rest of the maturities (see Nelson and Siegel (1987)). For U.S. government securities, Bliss (1997) fits separately two spot curves, one consisting of issues with maturities less than 5 years and another with maturities over 5 years.

the euro is the natural base currency. Indeed, if we were to choose the dinar as the base currency, we would need to forecast spot exchange rates up to the longest outstanding maturity (i.e., up to May 2016). No such procedure is currently in existence, especially for emerging markets. With the euro as the base currency, implied euro yields can be calculated for any historical spot curve. One should bear in mind, however, that implied euro yields of dinar-denominated securities are highly sensitive to exchange rate fluctuations (this sensitivity is higher the lower the maturity). Graph 5 illustrates implied euro repo yields in 2005.

Graph 5. Implied euro yields on repo transactions
January 31st 2005-December 30th 2005
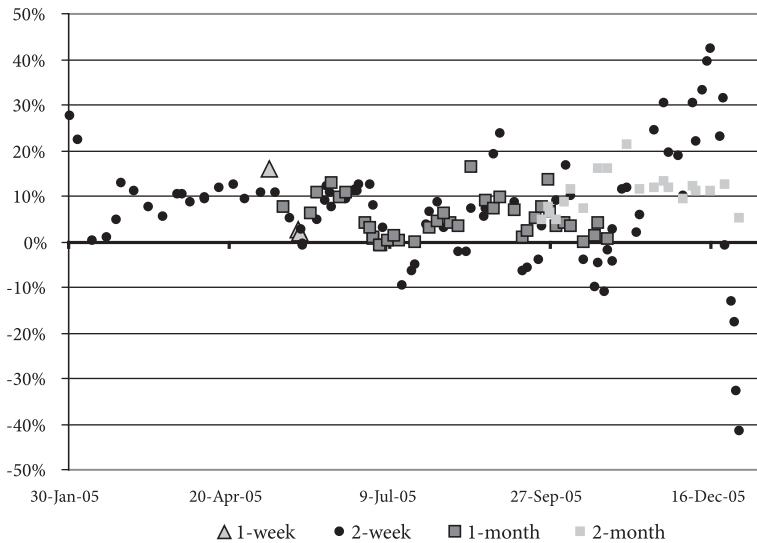
4. THE EMPIRICAL ANALYSIS

Our main aim in this section is to test the applicability of the Svensson model in the Serbian market and to provide a procedure for parsimonious spot curve estimation over the entire horizon of the available maturities. Before presenting the results, issues related to data filtering, estimation methods, fitting techniques, parameter selection, and Serbian data idiosyncrasies are considered.

While the results are obtained for the Serbian market, similar techniques can be used in other markets with similar characteristics (multiple currencies, sparse data, etc).

4.1 Data Filtering

We first need to construct the dataset that is used to perform the analysis. In order to fit the entire spot curve to the entire maturity horizon we analyse the data from the period from March 2003 to December 2005. This period captures most of the shapes that the empirical yield curves have produced to date. The dates are not always equidistant in time.

In order to derive representative spot curves, data filtering deserves considerable attention. One filtering goal is to help us capture systematic features. It is important to identify and exclude securities that bear significantly lower or higher yields than other, otherwise identical, securities due to some idiosyncratic features, and assess potential impact on the results if such data are not excluded. By keeping distorting data in our dataset the spot curve could be pulled up or down, thus causing under-pricing or over-pricing of securities. This phenomenon is usually referred to as downward (upward) bias of the spot (yield) curve. The set of filtering rules depends on the objectives of the analysis as well as specific features of the market.

We have already mentioned that implied euro yields for the shortest maturities are extremely sensitive to the euro/dinar exchange rate and time to maturity. E.g. implied repo yields in 2005 ranged between -41.59% and +42.42%. The first, naturally imposed filter is to exclude the data leading to negative implied yields. Additionally, we need to exclude unrealistically high yields. While the exclusion of the negative yields has no alternative, the boundary of acceptably high yields is somewhat arbitrary and depends on the objectives of the analysis as well as on modelling priorities. There is also the question of unacceptably low yields. We decided not to exclude issues with low yields (as long as they are not negative) but rather to test their influence on the spot curve estimation process. The upper boundary in implied yields is set at 22%. This boundary provides a good balance between keeping the largest number of data in an already sparse data set and obtaining a sample that results in acceptable fitting values. While the cut-off yield value is rather high, it is not unrealistic. By using this we have in mind high starting FFCB yields and allow for an exchange rate risk premium for dinar-denominated securities. Table 2 shows the filtering results regarding dinar-denominated securities. The first column corresponds to the type of security in question. The second column denotes the total number of bonds in the stated period. The third column shows the number of realized euro yields that are greater than zero (the first filtering criterion). The last column shows the

total number of realised euro yields that are less than 22%, among those that are greater than zero (the second filtering criterion).

Of the entire FFCB yield set for the stated period we excluded only yields of the series in the maturity year for being greater than 22%. Regarding series A2003, A2004 and A2005 we have excluded 23, 4 and 1 yield respectively.

Table 2. Filtering results for dinar-denominated securities
March 24th 2003 - December 30th 2005

Type of securities	Total number of occurances	>0	<22%
NBS-bills	449	232	210
1-week	104	47	32
2-week	131	59	55
1-month	126	69	67
2-month	88	57	56
REPO	153	126	115
2-week	94	72	61
1-month	39	35	35
2-month	19	19	19
T-bills	90	84	84
3-month	67	62	62
6-month	23	22	22

The maximum number of securities in the sample on a single date is 17 while the maximum number of dinar-denominated securites is 4.

4.2. Estimation Procedures

To fit the data to the Svensson model we employ the method of ordinary least squares (OLS method). The OLS method is based upon the minimisation of the sum of squared differences (errors) between the modelled and actual market values (either prices or yield values, depending on the approach). If the market data contains coupon bonds one has to employ price OLS minimisation. It is well known that unweighted price OLS minimisation may result in poor estimates of short-term yields. In particular, price OLS minimisation procedure tends to overfit the long-term segment of the spot curve at the expense of the short-term

segment. This phenomenon is referred to as heteroskedasticity of yield errors. In order to correct for the bias one may impose, for each bond, weights as follows:

$$w_i = \frac{\dfrac{1}{D_i}}{\displaystyle\sum_{i=1}^{n} \dfrac{1}{D_i}} \tag{7}$$

Here, D_i is Macaulay duration of the i-th bond in the estimation sample and n is the number of bonds in the estimation sample. Such a weighting scheme assigns higher weights to shorter maturities, thus potentially mitigating the bias. Bliss (1997) finds that inverse duration weighting produces the highest proportion of correctly priced out-of-sample bonds. Other weighting schemes are noticeably less successful.

Unlike most of the fixed-income markets, the Serbian government bond market lacks coupon-bearing bonds. In a zero-coupon bond environment spot rates coincide with yields to maturity. Thus spot rates can be extracted directly from the observed bond prices. Therefore, direct yield OLS minimisation can be employed in addition to the price OLS minimisation. This gives us an excellent opportunity to compare and contrast the results obtained from the price and yield-based OLS minimisation. In the yield OLS minimisation approach, all of the sample yields are of the same importance, i.e. there is no need for a weighting scheme. Yield OLS minimisation should provide the best fit of yields across the entire maturity horizon. From here on, we refer to the unweighted price OLS, weighted price OLS, and yield OLS minimisation procedures as UNWE, WE and YIELD procedures, respectively.

The Svensson model is sensitive to the choice of the estimation procedure and to the starting values of the parameters used in the minimisation procedure. The robustness of the solution depends on whether or not the obtained solution is, indeed, a global minimum. In order to deal with this issue Nelson and Siegel (1987) search for the optimal solution over a grid of τ parameter in increments of 10. Bolder and Stréliski (1999) first fix the τ parameters and estimate parameters, β and then fix parameters β and estimate parameters τ. We estimate the set of parameters for different levels of τ parameters. The grid of τ parameters we select is 1 through 50 in increments of 5, as well as a set of high values of τ (100, 200, and 500). For each date, as an initial guess we use the set of estimated parameters obtained for the preceding date in the sample. This approach leads to no more

than 3 different local minima for the chosen grid of τ parameters. It also leads to different solutions for higher τ values. This is mostly the consequence of a small number of observed maturities relative to similar studies from Western countries. Note that we must impose additional constraints that prevent the occurrence of economically unjustifiable solutions. This helps us narrow down the set of feasible minimisation solutions. In particular, the lower boundary for the overnight rate $\beta_0 + \beta_1$ is set at 4% (in general), and at 1% (for high values of τ_2). Also, the lower boundary for the long-term limit rate β_0 is set to 10% (for the spot curves in 2003) and to 5% (for the spot curves in 2004 and 2005).

Goodness-of-fit is arguably the most important criterion of an appropriate curve fitting procedure. It measures how well the model fits the underlying data. When using UNWE or WE procedures one minimises errors in terms of bond prices while targeting, in fact, yield errors. On the other hand, with the YIELD method we minimise yield errors directly. In order to compare the three methods we may use goodness-of-fit both in terms of yields and prices. In terms of yields, the goodness-of-fit is most frequently defined as

$$\text{Root mean squared error } (RMSE) = \sqrt{\sum_{i=1}^{n} \frac{\left(Error_i - \overline{Error}\right)^2}{n}} \tag{8}$$

as well as

$$\text{Average absolute yield error } (AABSYE) = \frac{\sum_{i=1}^{n} \left|Error_i - \overline{Error}\right|}{n} \tag{9}$$

Here, $Error_i = Yield_{imodel} - Yield_{imarket}$ is yield error referring to the i-th yield to maturity of the estimation sample, $\overline{Error}$ is the average yield error and n is the number of bonds in the estimation sample. RMSE can actually be considered as standard deviation of errors. As a consequence it is rather sensitive to outliers. Thus we also calculate AABSYE values as an alternative measure that is not so sensitive to extreme points.

While the goodness-of-fit in terms of yield errors is of primary importance, since our aim is to accurately fit the spot curve, one may also want to check the goodness-of-fit in terms of price errors. A natural measure to consider is the sum of squared price differences or SSPD. By construction, UNWE should lead to the smallest value of SSPD while YIELD should lead to the lowest value of RMSE.

Note that RMSE is not the same as AABSYE. Thus, a procedure that minimises RMSE does not necessarily minimise AABSYE.

4.3. Fitting the Spot Curves Using the Three Procedures

In our analysis there is always a trade-off between the quality of fit of the short-term segment and the quality of fit over the rest of the maturity horizon. The question is: how do UNWE, WE, and YIELD methods fare when applied to the entire maturity horizon or to its separate segments? The reality in Serbia is that for the majority of dates the short maturity yields are quite turbulent. In such cases the WE procedure has to put considerable weight on the short maturity region at the expense of the longer maturities. In particular, the WE procedure does not generate the same results in cases where there are one, two, three, or more short-term securities for a given date. We find that with more than two short-term securities on a single date WE is biased towards the short-term end. On the other hand, in a number of cases the WE procedure results in rather high values of SSPD but the corresponding model curve more uniformly fits the yields over the entire maturity horizon than UNWE. Also, while the UNWE procedure does poorly in the short-term segment, the WE procedure, by construction, typically improves upon it.

While good yield difference optimization results always correspond to good yield fit along the entire maturity horizon, good unweighted price difference optimization results imply good medium and long-term segment fit but may actually produce poor short-term fit. Finally, good weighted price difference optimization results may produce poor long end fit.

Table 3 presents the comparison of the goodness-of-fit for the three methods on September 12th 2003. The chosen date features the maximum number of available securities (17 securities) and the maximum number of money market securities that passed the filtering criteria (4 securities). We report differences between the model and market values of yields as well as prices. We observe that the best results are obtained with the UNWE procedure (this is to be expected), which, in turn, did quite poorly for the short-term securities, i.e. for the NBS bills. Indeed, the model underestimated the yields of 1-Week and 2-Week NBS bills by 12.10% and 9.05% respectively. On the other hand, if the short end is excluded, the UNWE strategy provides excellent results.

Table 3. Comparative analysis of different fitting methods
for September 12th 2003[20]

Security	Yield difference			Price difference			Macaulay weights
	UNWE	WE	YIELD	UNWE	WE	YIELD	
1-week NBS-bills	(12.10)	0.75	0.77	0.23	(0.01)	(0.01)	52.95%
2-week NBS-bills	(9.05)	(0.64)	(0.66)	0.37	0.03	0.03	24.71%
1-month NBS-bills	(3.39)	(0.75)	(0.77)	0.28	0.06	0.06	12.35%
2-month NBS-bills	3.62	0.96	1.01	(0.59)	(0.16)	(0.17)	6.18%
A2004	(0.15)	(0.75)	(0.75)	0.10	0.50	0.51	1.41%
A2005	0.05	0.56	0.46	(0.07)	(0.82)	(0.68)	0.59%
A2006	(0.07)	0.16	0.18	0.14	(0.34)	(0.38)	0.37%
A2007	0.00	(0.04)	0.06	(0.00)	0.12	(0.14)	0.27%
A2008	0.06	(0.14)	(0.03)	(0.19)	0.42	0.08	0.22%
A2009	(0.03)	(0.28)	(0.20)	0.10	0.94	0.66	0.18%
A2010	0.03	(0.20)	(0.17)	(0.11)	0.71	0.59	0.15%
A2011	0.02	(0.14)	(0.16)	(0.08)	0.51	0.60	0.13%
A2012	0.00	(0.05)	(0.13)	(0.01)	0.18	0.49	0.12%
A2013	(0.05)	0.03	(0.11)	0.19	(0.10)	0.40	0.10%
A2014	0.01	0.24	0.06	(0.05)	(0.90)	(0.22)	0.09%
A2015	(0.22)	0.16	(0.06)	0.84	(0.58)	0.23	0.09%
A2016	0.21	0.75	0.50	(0.80)	(2.81)	(1.88)	0.08%

Table 4 presents SSPD, AABSYE and RMSE results for the chosen date regarding the three fitting methods. We observe that the best SSPD results are obtained with the UNWE procedure, while the best RMSE results we obtained by the YIELD procedure. Both of these results are, of course, expected. In addition, we find that YIELD results give second best SSPD. In terms of RMSE, WE gives results which are quite close to the YIELD and much better than UNWE. A similar conclusion is reached for the AABSYE criterion.

Table 4. SSPD, ABSYE and RMSE results, expressed in percentages,
for the three fitting methods on September 12th 2003

SSPD			AABSYE			RMSE		
UNWE	WE	YIELD	UNWE	WE	YIELD	UNWE	WE	YIELD
0.021	0.12	0.061	1.71	0.39	0.36	3.86	0.50	0.48

Combining the results presented in Tables 3 and 4 we reach the following conclusions (for that particular date). Namely, YIELD strategy is superior in

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[20]   Errors are calculated as a difference between the corresponding model and market values and are given in percentages.

terms of AABSYE if applied for all securities including the NBS bills.[21] Note that the WE method leads to only slightly worse AABSYE and RMSE results than the YIELD method. This demonstrates that Macaulay weighting does improve the results.

The UNWE strategy shows superior performance when NBS bills are excluded from the sample but over the entire maturity horizon leads to inferior results compared with the other two methods in terms of RMSE and AABSYE fit (see Table 4). The YIELD procedure, as expected, provides a balanced fit across different maturities. In particular, it provides slightly worse performance in the medium range than the UNWE procedure, but clearly outperforms it in the short maturities range.

Last but not least, note that for that particular date, in the case of the WE procedure, the sum of weights for NBS bills is 0.962 while the weight of a 1-week NBS bill alone is 0.53. Imposing such weights is the only way to change described unfavourable price error sensitivity when yields are the priority, but the overall performance of the WE procedure for this date is unsatisfactory. This is an exception rather than the rule. As we shall see below, the three procedures have a similar goodness-of-fit. This date was chosen to explain the comparative strengths and weakness of each of the estimation procedures.

4.4 Serbian idiosyncrasies

The longest outstanding maturity in our analysis ranges between 10.4 and 13.2 years.[22] The Serbian yield structure implies decreasing and concave spot curves in the long-term segment for most of the dates that we consider in our analysis. Series A2016, as a rule, has lower yields relative to the yields of other long-term bonds. Furthermore, yields of the A2015 series usually follow the same pattern when compared to its maturity predecessors. This has a considerable impact on the curve fitting procedure. In such a yield environment the Nelson-Siegel class of models tends to consistently overestimate the longest maturity yields.

There is a remedy for this problem, but it comes at a cost. Recall that the parameter τ_2 appears in the Svensson formula in the term attributed to the short-term segment flexibility. By selecting a high value of τ_2 (say, 100, 500, or 1000), one could obtain a much better fit in the longer maturity region. Indeed, in a

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21   Note that, while expected, this result is not guaranteed by construction. Namely, as stated above, a procedure that minimises RMSE does not necessarily minimise AABSYE.
22   Since FFCBs were issued only once, the longest available maturity decreases over time.

number of cases selecting high values of τ_2 improves the longer-maturity fit and leads to lower SSPD values. However, this is far from an ideal solution. These are the main shortcomings of such an approach: (1) Corresponding spot curves may become totally insensitive in the short-maturity end. This would be particularly problematic in the case of the WE approach which is supposed to improve the short-maturity fit; (2) Estimated overnight rates may become unrealistic; (3) Estimated long-term limit rate β_0 may be unrealistic as well (e.g. we obtained the value of β_0=75.6% for the June 20th 2005 estimate); and (4) This may lead to an increase in the estimated volatility of yields with maturities longer than the longest quoted maturity.

Another issue that requires attention is how to handle the overnight rate. In Serbia, the overnight interbank rate BEONIA (the Serbian counterpart of EONIA) was introduced in August 2005.[23] In the first months of its existence it exhibited high volatility. Including BEONIA in the estimation procedure has a considerable impact on the results. Recall that the sum $\beta_0+\beta_1$ is interpreted as an overnight rate. Imposing the constraint that $\beta_0+\beta_1$ equals the BEONIA rate, we would tie the left end of the curve to a known market value. This would reduce by one the available degrees of freedom, therefore increasing the model volatility of the short-term segment. If, on the other hand, the overnight rate is not fixed, different fitting procedures would lead to different estimates for the overnight rate $\beta_0+\beta_1$. This would affect, in turn, the long-term limit β_0 as well. We decided to include BEONIA as the overnight rate (from August 2005, when it became available).

4.5 Fitting Accuracy and Parameter Evolution

In this subsection we consider in more detail fitting accuracy and parameter evolution for the year 2005, for the entire maturity horizon. In that year, yields of the Serbian bond market and dinar/euro exchange rates show considerable stability. Largely because of the exchange rate stability, money market yields show relatively low volatility. In addition, FFCB yield ranges are tighter in 2005 than in any other year.

The sub-sample for 2005 consists of 35 different dates. Graph 6 depicts SSPD values obtained by applying WE and YIELD methods relative to the values obtained by the UNWE method (the latter method is, by definition, expected to provide the most accurate SSPD fit). One can see that the values corresponding to the WE

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[23] Interbank borrowing in Serbia is denominated in Serbian dinars.

method are always larger than the values obtained by the YIELD method, i.e. that for the sample at hand the SSPD values for the YIELD method lie between the other two methods.

By comparing the results obtained by applying the two price-fitting methods with the results obtained by the yield-fitting techniques we are in a position to determine the extent of accuracy loss when one cannot apply yield-based optimisation.

Graph 6. Comparison of YIELD and WE SSPD value obtained relative to
UNWE SSPDs January 14th 2005– December 27th 2005
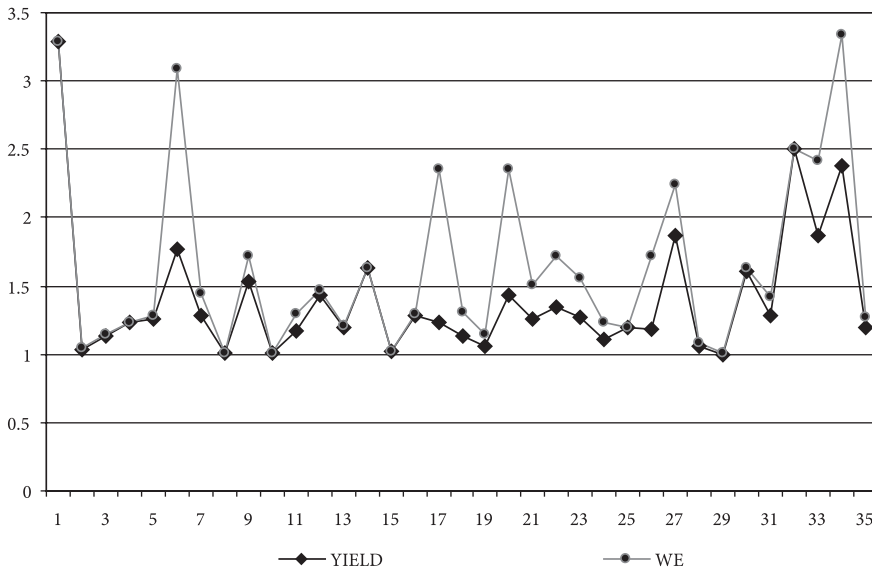
Graph 7 depicts RMSE values obtained by applying the WE method relative to the YIELD method, which is, by definition, expected to provide the most accurate RMSE fit. The values obtained by the UNWE method are not presented in the graph since they are uniformly much higher (sometimes as much as ten times) than corresponding WE values. The results presented in Graphs 6 and 7 are important because they show that by using WE and YIELD procedures in most cases one obtains a similarly good fit both in terms of price errors as well as, even more importantly, in terms of RMSE.

Graph 7. WE RMSEs obtained relative to YIELD RMSEs
January 14th 2005– December 27th 2005
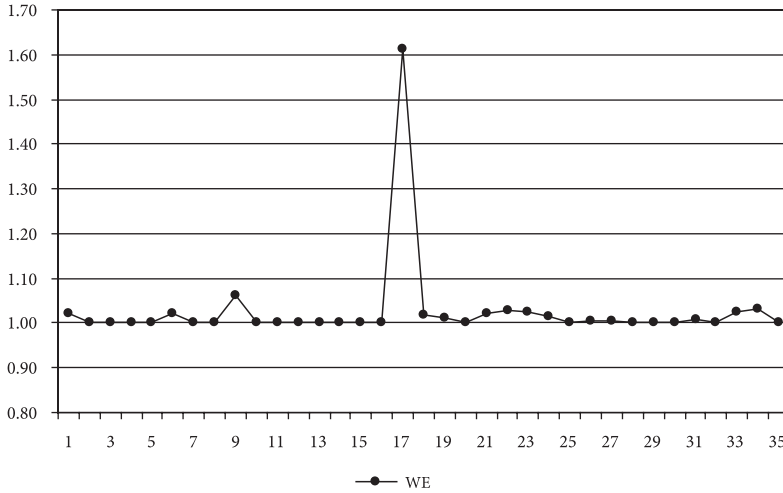
Consider now the time series of the model parameters. It turns out that, when the same set of parameters is supposed to fit the entire maturity horizon, parameters of the model change significantly over time. Graph 8 depicts the evolution of the parameter β_0 in 2005.[24] We observe that the WE and YIELD values of the parameter are quite volatile and similar to each other (the parameter estimates become more stable over time). On the other hand, the parameter estimated by the UNWE procedure shows a very different pattern. Generally speaking, the parameter estimated via the UNWE method shows much greater stability and, most of the time, smaller values of the parameters than the other two methods. We see that, at least where the value of the parameter β_0 is concerned, WE gives the procedure closest to the "ideal" procedure YIELD.

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[24]   In constructing this graph we exclude unrealistic vales obtained by high-τ_2 procedures.

Graph 8. Evolution of β_0 estimates obtained by the three procedures
April 26th 2005- December 27th, 2005
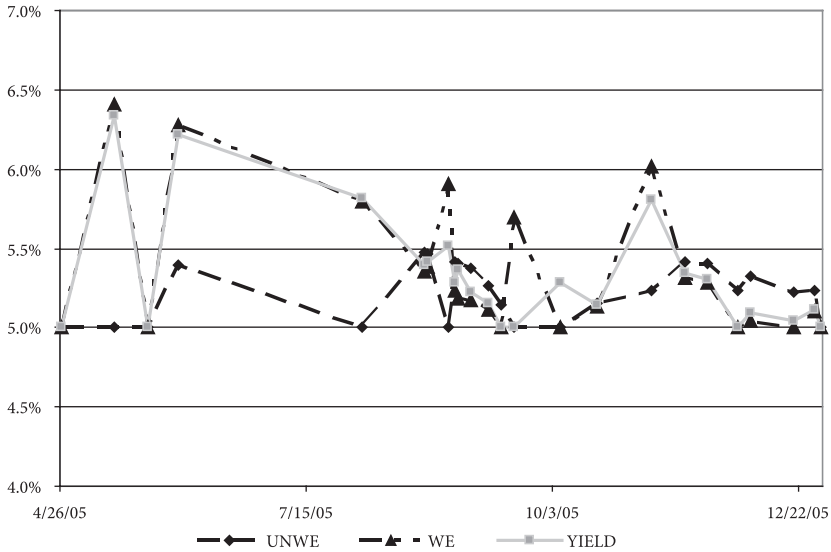

4.6 Volatility Comparison

In this subsection we test the consistency of market and model volatilities across different maturities. Graph 9 compares observed market volatility and model volatility based on the YIELD estimation procedure. For this purpose we plot volatilities based on the actual market data as well as data obtained by the YIELD estimation procedure. We do so for FFCB series A2006-A2016 since these securities were continuously traded in the course of 2005. We find that except for the A2006 series, the modelled volatility nicely approximates realized market volatility. The reason for the discrepancy for the A2006 series is quite straightforward. As of May 2005, the A2006 series represents the first maturity greater than those of the observed bills, which encountered volatility that is highly overestimated by the model.

Graph 9. Comparison of market and model volatilities for FFCB yields for the period from January 14th 2005 to December 27th 2005.
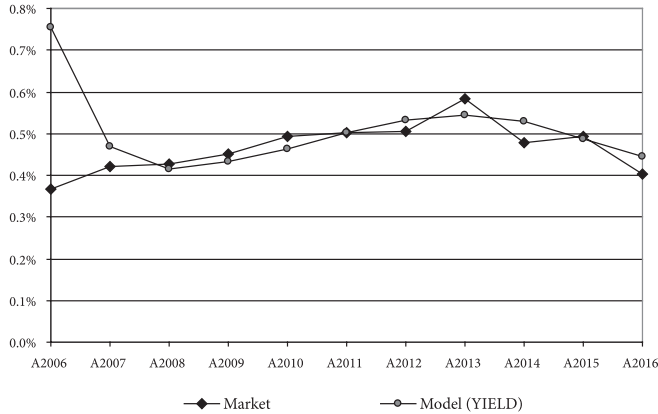
The term structure of volatility is a set of volatilities for government bonds of different maturities at a given point in time. Graph 10 illustrates the model volatility term structure for each of the three optimization procedures in the range from 2 weeks to 15 years. Apart from maturities that exist in the market the graph also contains extrapolated maturities for which there is no data available. As expected, the greatest differences in volatility are found for maturities of up to six months. Note also that modelled volatility for long maturities is significantly smaller than modelled volatility for short maturities. In addition, for long maturities there is practically no difference in modelled volatility between the three models.

Graph 10. Estimated volatility term structure January 14th– December 27th 2005
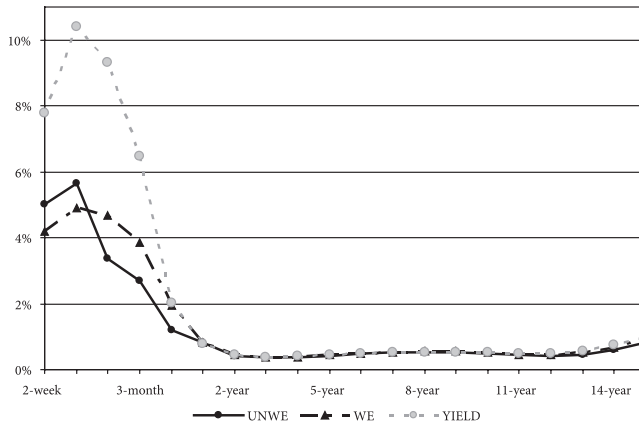
4.7 FFCB Spot Curves

Up to now, our estimation priorities have been to include the maximum number of signals from different market segments and to apply different fitting techniques in order to obtain the best in-sample fit. An alternative is to discard money market securities, the segment that introduces considerable instability in the estimation procedure. To that end, in this subsection we focus on the FFCB market segment only. We refer to corresponding spot curves as FFCB curves. We construct the FFCB curves for the period between September 2007 and December 2008. In that period the FFCB curves were practically flat or slightly decreasing. In particular, we are interested in the stability of the model parameters. Graph 11 shows the evolution of the parameter estimates using the YIELD method for the period under consideration. The lower boundary for the overnight rate is set to 3%. The lower boundary for the long-term limit rate is set at 4%.

Graph 11. Evolution of β_0 and β_1 estimates for FFCB yield curves obtained by the YIELD procedure for the period from September 4th 2007 to December 11th 2008
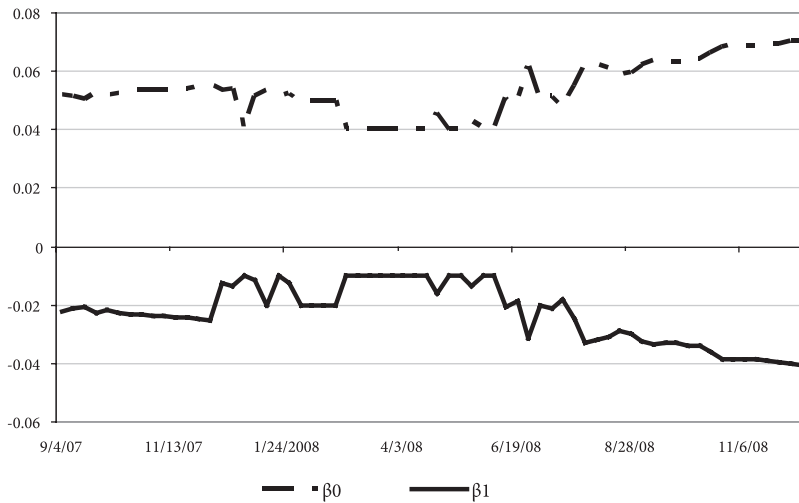
Graph 11 shows that when we consider FFCB spot curves only, rather than the spot curves that also include short maturities, the model parameters become quite stable and show clear trends. In the future, it will be important to study how these trends are impacted by underlying macroeconomic causes. Of course for such an analysis to be econometrically sound we would probably need a much better developed bond market.

5. CONCLUSIONS

Benchmark spot curves provide a market consensus on current borrowing and lending conditions in a country. This information is essential for policy makers, businesses and investors alike. An important challenge for policy makers is to determine in which way, with what intensity and at what time lag various policy measures impact the spot curve and, through it, the exchange rate, the real sector, and the economy as a whole. For serious policy use it is important to collect daily time series of benchmark government bond curves as well as their corresponding parameters using a consistent, well-defined estimation method. For this to be possible, in turn, there has to exist a liquid government bond market with bonds of representative maturities traded on a daily basis, as well as a comprehensive database that would store the corresponding historical data.

Businesses and investors use information provided by the benchmark spot curve in a multitude of ways. In determining the expected rate of return on an asset or an investment project modern financial theory suggests that one starts with a risk-less rate of return and adds to this rate an appropriate risk premium. In this context, the benchmark spot curve provides the natural choice of "risk-less" rates for various maturities.[25] Thus, knowing benchmark spot curves allows one, in principle, to value any asset or project. This is a precondition for the existence of a vibrant financial market and a vibrant economy. In addition, no risk management or asset/liability model, the cornerstones of the contemporary financial industry, can be implemented without regular use of benchmark spot curve estimates.

The question remains: which of the available spot curve estimation techniques would, overall, best suite policy makers, businesses and investors alike? The results of this paper demonstrate that the Svensson model may be a good candidate for such a model for the Serbian market. Together with the further development of both the primary and the secondary government bond market, a consistent use of this model would allow Serbian policy makers to capture important market signals from a time series of estimated model parameters. Such signals would be comparable with similar signals obtained from other markets that use the same methodology, including the European Central Bank estimates.

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[25] In practice, except for the most developed markets such as the American bond market, the government bond benchmark curve reflects important country risks. In the case of Serbia, a euro-based curve would contain a credit and liquidity risk premium, while a dinar-based curve would contain an exchange risk and liquidity risk premium. For a discussion of the interaction between exchange and credit risk in the Serbian economy see Božović et al (2009).

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