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STABILITY BOUND ANALYSIS OF SINGULARLY PERTURBED SYSTEMS WITH TIME-DELAY

This paper considers the stability bound problem of singularly perturbed systems with time-delay. Some stability criteria are derived by constructing appropriate Lyapunov-Krasovskii functionals. The proposed criteria are less conservative than the existing ones. Two numerical examples are given to illustrate the advantages and effectiveness of the proposed methods.

Keywords: singularly perturbed systems; stability bound; linear matrix inequality; Lyapunov-Krasovskii functional.

Multiple time-scale systems occur widely in chemical processes, robotic systems, aerospace engineering, power systems and magnetic-ball suspension systems due to the existence of small parameters such as the quality, inertia, conductivity, and capacitance. Multiple time-scale systems are usually modeled as singularly perturbed systems (SPSs), with a small singular perturbation parameter ε determining the degree of separation between the slow and fast modes of the systems. Stability is a fundamental problem in control theory and applications. The stability problem for SPSs is more complex than that for normal systems since their robustness with respect to the singular perturbation parameter should be considered. The stability problem for SPSs is known as the stability bound problem, which is referred to as the problem of determining the stability bound $\bar{\varepsilon}$ such that the system is stable for $\forall\varepsilon \in (0,\bar{\varepsilon}]$. Many frequency-domain and time-domain methods for stability bound problem of SPSs have been reported (see, for example [1]).

Time-delay is common in practical applications, which may result in undesirable system responses or even instability of the system. Research on stability analysis of time-delay systems is essential for practical applications. Stability criteria for time-delay systems can be classified into delay-independent and delay-dependent criteria [2-4]. For SPSs with time-delay, two cases have been studied, in which: 1) delays are proportional to perturbed parameter and 2) delays and perturbed parameter are independent. For the first case, some significant results have been reported for singularly perturbed systems by reduction technique [5-10]. For the more general case that time-delay and perturbed parameter are independent, there are mainly two classes of approaches: frequency domain techniques and Lyapunov-Krasovskii functional based approaches. Stability of SPSs with time-delay in slow state has been studied in [11-14] by reduction technique. However, when time-delay exists in both slow and fast states, the reduction technique does not work since the slow and fast states cannot be separated completely [15].

Lyapunov-Krasovskii functional based approaches, which are usually expressed in linear matrix inequalities (LMIs), are popular in recent years because LMI conditions can be easily verified by using convex optimization algorithms [1,16]. Most of the existing stability criteria for SPSs with time-delay

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require the singular parameter to be small enough and cannot produce an estimate of the stability bound of the systems (for example see [15,17-19]). Recently, the stability bound problem of SPSs with time-delay has attracted much attention. In [20], the exact stability bound for discrete multiple time-delay singularly perturbed systems was examined. In [21] and [22], the stability bound problem of continuous-time SPSs with time-delay was considered and some sufficient conditions were proposed by choosing appropriate Lyapunov-Krasovskii functionals. Since the existing methods are all sufficient conditions, it is significant to establish less conservative stability criteria.

In this paper, we construct a new Lyapunov--Krasovskii functionals for SPSs with time-delay, by which delay-dependent and independent stability criteria are proposed. Some examples are given to illustrate the obtained methods. The contributions of the paper are as follows: 1) the obtained methods are less conservative than the existing ones since they are derived by more general Lyapunov-Krasovskii functionals; 2) using these methods, stability bound of the SPS can be computed; 3) the proposed Lyapunov-Krasovskii functionals can be used to study other analysis and synthesis problems of SPSs with time-delay.

Preliminaries

Consider the following SPS with time-delay:

$$\begin{cases} \mathbf{E}(\varepsilon)\dot{x}(t) = Ax(t)+Dx(t-d), & t>0 \\ x(t) = \phi(t), & t\in[-d,0) \end{cases} \quad (1)$$

where

$$\mathbf{E}(\varepsilon)=\begin{bmatrix} I & 0 \\ 0 & \varepsilon I \end{bmatrix}$$

The following lemmas [23] will be used in the sequel.

$$\begin{bmatrix} \mathbf{Z}^T(0)A+A^T Z(0)+\mathbf{Q}+YE(0) \\ +E(0)Y^T+dX_1+dA^T MA & \mathbf{Z}^T(0)D - YE(0)+E(0)N^T+dX_2+dA^T MD \\ * & -\mathbf{Q} - NE(0) - E(0)N^T+dX_3+dD^T MD \end{bmatrix}<0 \quad (11)$$

$$\begin{bmatrix} \mathbf{Z}^T(\bar{\varepsilon})A+A^T Z(\bar{\varepsilon})+\mathbf{Q}+YE(\bar{\varepsilon}) \\ +E(\bar{\varepsilon})Y^T+dX_1+dA^T MA & \mathbf{Z}^T(\bar{\varepsilon})D - YE(\bar{\varepsilon})+E(\bar{\varepsilon})N^T+dX_2+dA^T MD \\ * & -\mathbf{Q} - NE(\bar{\varepsilon}) - E(\bar{\varepsilon})N^T+dX_3+dD^T MD \end{bmatrix}<0 \quad (12)$$

Lemma 1. Given $\varepsilon > 0$, symmetric matrix $\mathbf{S}_1$, $\mathbf{S}_2$ and $\mathbf{S}_3$, if:

$$\mathbf{S}_1 \geq 0 \quad (2)$$

$$\mathbf{S}_1+\bar{\varepsilon}\mathbf{S}_2>0 \quad (3)$$

$$\mathbf{S}_1+\bar{\varepsilon}\mathbf{S}_2+\bar{\varepsilon}^2\mathbf{S}_3>0 \quad (4)$$

then:

$$\mathbf{S}_1+\varepsilon\mathbf{S}_2+\varepsilon^2\mathbf{S}_3>0, \quad \forall\varepsilon\in(0,\bar{\varepsilon}] \quad (5)$$

Lemma 2. If there exists matrix $\mathbf{Z}_i (i=1,2,\ldots,5)$ with $\mathbf{Z}_i=\mathbf{Z}_i^T (i=1,2,3,4)$ satisfying the following LMIs:

$$\mathbf{Z}_1>0 \quad (6)$$

$$\begin{bmatrix} \mathbf{Z}_1+\bar{\varepsilon}\mathbf{Z}_3 & \bar{\varepsilon}\mathbf{Z}_5^T \\ \bar{\varepsilon}\mathbf{Z}_5 & \bar{\varepsilon}\mathbf{Z}_2 \end{bmatrix}>0 \quad (7)$$

$$\begin{bmatrix} \mathbf{Z}_1+\bar{\varepsilon}\mathbf{Z}_3 & \bar{\varepsilon}\mathbf{Z}_5^T \\ \bar{\varepsilon}\mathbf{Z}_5 & \bar{\varepsilon}\mathbf{Z}_2+\bar{\varepsilon}^2\mathbf{Z}_4 \end{bmatrix}>0 \quad (8)$$

then:

$$\mathbf{E}(\varepsilon)\mathbf{Z}(\varepsilon)=(\mathbf{E}(\varepsilon)\mathbf{Z}(\varepsilon))^T=\mathbf{Z}^T(\varepsilon)\mathbf{E}(\varepsilon)>0, \quad \forall\varepsilon\in(0,\bar{\varepsilon}] \quad (9)$$

where:

$$\mathbf{Z}(\varepsilon)=\begin{bmatrix} \mathbf{Z}_1+\varepsilon\mathbf{Z}_3 & \varepsilon\mathbf{Z}_5^T \\ \mathbf{Z}_5 & \mathbf{Z}_2+\varepsilon\mathbf{Z}_4 \end{bmatrix} \quad (10)$$

MAIN RESULTS

Delay-dependent stability criterion

Consider the SPS with time-delay given by Eq. (1).

Theorem 1. Given $\bar{\varepsilon}>0$, $d>0$, system (1) is asymptotically stable for $\forall\varepsilon\in(0,\bar{\varepsilon}]$, if there exist a symmetric positive-definite matrix $\mathbf{Q}>0, \mathbf{M}>0$, symmetric semi- positive matrix:

$$\mathbf{X}=\begin{bmatrix} X_1 & X_2 \\ * & X_3 \end{bmatrix}\geq 0$$

matrices $\mathbf{Y}$, $\mathbf{N}$ and and $\mathbf{Z}_i (i=1,2,\ldots,5)$ with $\mathbf{Z}_i=\mathbf{Z}_i^T$ $(i=1,2,3,4)$, satisfying the LMIs (6)-(8) and:

$$\mathbf{\Psi}=\left[\begin{array}{cc|c} X_1 & X_2 & Y \\ * & X_3 & N \\ \hline * & * & M \end{array}\right]\geq 0 \quad (13)$$

Proof. Define a quadratic Lyapunov-Krasovskii functional as follows:

$$V(x_t)=x^T(t)E(\varepsilon)Z(\varepsilon)x(t)+\int_{t-d}^{t}x^T(s)Qx(s)ds+$$
$$+\int_{-d}^{0}\int_{t+\theta}^{t}(E(\varepsilon)\dot{x}(s))^T ME(\varepsilon)\dot{x}(s)dsd\theta$$

It can be seen that:

$$\mathrm{E}(\varepsilon)\mathrm{Z}(\varepsilon)=\begin{bmatrix} I & 0\\ 0 & \varepsilon I\end{bmatrix}\begin{bmatrix} Z_1+\varepsilon Z_3 & \varepsilon Z_5^T\\ Z_5 & Z_2+\varepsilon Z_4\end{bmatrix}=$$
$$=\begin{bmatrix} Z_1+\varepsilon Z_3 & \varepsilon Z_5^T\\ \varepsilon Z_5 & \varepsilon Z_2+\varepsilon^2 Z_4\end{bmatrix}=$$
$$=(\mathrm{E}(\varepsilon)\mathrm{Z}(\varepsilon))^T=\mathrm{Z}^T(\varepsilon)\mathrm{E}(\varepsilon)$$

By Lemma 2 and LMIs (3)-(5), we have:

$$\mathrm{E}(\varepsilon)\mathrm{Z}(\varepsilon)=\mathrm{Z}^T(\varepsilon)\mathrm{E}(\varepsilon)>0,\quad \forall\varepsilon\in(0,\bar{\varepsilon}]$$

Thus $V(x_t)$ is a positive Lyapunov-Krasovskii functional.

Taking the derivative of $V(x_t)$ along the trajectories of system (1), we have:

$$\dot{V}(x_t)\Big|_{(1)}=\frac{\mathrm{d}}{\mathrm{d}t}\left(x^T(t)\mathrm{E}(\varepsilon)\mathrm{Z}(\varepsilon)x(t)\right)+$$
$$+\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{t-d}^{t}x^T(s)Qx(s)\mathrm{d}s\right)+$$
$$+\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{-d}^{0}\int_{t+\theta}^{t}(E(\varepsilon)\dot{x}(s))^T ME(\varepsilon)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right)$$

where:

$$\frac{d}{dt}\left(x^T(t)E(\varepsilon)Z(\varepsilon)x(t)\right)=\frac{d}{dt}\left(x^T(t)E(\varepsilon)\right)Z(\varepsilon)x(t)+$$
$$+x^T(t)E(\varepsilon)\frac{d}{dt}\left(Z(\varepsilon)x(t)\right)=\dot{x}^T(t)E(\varepsilon)Z(\varepsilon)x(t)+$$
$$+x^T(t)E(\varepsilon)Z(\varepsilon)\dot{x}(t)=\left(E(\varepsilon)\dot{x}(t)\right)^T Z(\varepsilon)x(t)+$$
$$+x^T(t)\left(E(\varepsilon)Z(\varepsilon)\right)^T\dot{x}(t)=\left(E(\varepsilon)\dot{x}(t)\right)^T Z(\varepsilon)x(t)+$$
$$x^T(t)Z^T(\varepsilon)\left(E(\varepsilon)\dot{x}(t)\right)=\left(Ax(t)+Dx(t-d)\right)^T Z(\varepsilon)x(t)+$$
$$+x^T(t)Z^T(\varepsilon)(Ax(t)+Dx(t-d))=x^T(t)(A^TZ(\varepsilon)x(t)+$$
$$+Z^T(\varepsilon)Ax(t))+\left(Dx(t-d)\right)^T Z(\varepsilon)x(t)+$$
$$+\left(Z(\varepsilon)x(t)\right)^T Dx(t-d)=2x^T(t)Z^T(\varepsilon)Ax(t)+$$
$$+2x^T(t)Z^T(\varepsilon)Dx(t-d)=2x^T(t)Z^T(\varepsilon)(Ax(t)+$$
$$+Dx(t-d))$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{t-d}^{t}x^T(s)Qx(s)\mathrm{d}s\right)=x^T(t)Qx(t)-$$
$$-x^T(t-d)Qx(t-d)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{-d}^{0}\int_{t+\theta}^{t}(E(\varepsilon)\dot{x}(s))^T ME(\varepsilon)\dot{x}(s)\mathrm{d}s\mathrm{d}\theta\right)=$$
$$=\int_{-d}^{0}\frac{\mathrm{d}}{\mathrm{d}t}\left(\int_{t+\theta}^{t}(E(\varepsilon)\dot{x}(s))^T ME(\varepsilon)\dot{x}(s)\mathrm{d}s\right)\mathrm{d}\theta=$$
$$=\int_{-d}^{0}[(E(\varepsilon)\dot{x}(t))^T M(E(\varepsilon)\dot{x}(t))-$$
$$-(E(\varepsilon)\dot{x}(t+\theta))^T M(E(\varepsilon)\dot{x}(t+\theta))]\mathrm{d}\theta=$$
$$=\int_{-d}^{0}(E(\varepsilon)\dot{x}(t))^T M(E(\varepsilon)\dot{x}(t))\mathrm{d}\theta- \tag{14}$$
$$-\int_{-d}^{0}(E(\varepsilon)\dot{x}(t+\theta))^T M(E(\varepsilon)\dot{x}(t+\theta))\mathrm{d}\theta=$$
$$=d(E(\varepsilon)\dot{x}(t))^T M(E(\varepsilon)\dot{x}(t))-$$
$$-\int_{t-d}^{t}(E(\varepsilon)\dot{x}(\omega))^T M(E(\varepsilon)\dot{x}(\omega))\mathrm{d}\omega$$

Then:

$$\dot{V}(x_t)\Big|_{(1)}=2x^T(t)Z^T(\varepsilon)\left(Ax(t)+Dx(t-d)\right)+$$
$$x^T(t)Qx(t)-x^T(t-d)Qx(t-d)+d(E(\varepsilon)\dot{x}(t))^T\times$$
$$\times M(E(\varepsilon)\dot{x}(t))-\int_{t-d}^{t}(E(\varepsilon)\dot{x}(\omega))^T M(E(\varepsilon)\dot{x}(\omega))\mathrm{d}\omega\le$$
$$\le 2x^T(t)Z^T(\varepsilon)\left(Ax(t)+Dx(t-d)\right)+x^T(t)Qx(t)-$$
$$-x^T(t-d)Qx(t-d)+d(E(\varepsilon)\dot{x}(t))^T M(E(\varepsilon)\dot{x}(t))-$$
$$-\int_{t-d}^{t}(E(\varepsilon)\dot{x}(\omega))^T M(E(\varepsilon)\dot{x}(\omega))\mathrm{d}\omega++2x^T(t)Y\times \tag{15}$$
$$\times\left(E(\varepsilon)x(t)-\int_{t-d}^{t}E(\varepsilon)\dot{x}(s)\mathrm{d}s-E(\varepsilon)x(t-d)\right)+$$
$$+2x^T(t-d)N(E(\varepsilon)x(t)-\int_{t-d}^{t}E(\varepsilon)\dot{x}(s)\mathrm{d}s-$$
$$-E(\varepsilon)x(t-d))++d\xi^T(t)X\xi(t)-$$
$$-\int_{t-d}^{t}\xi^T(t)X\xi(t)\mathrm{d}s\triangleq\xi^T(t)\hat{\Phi}(\varepsilon)\xi(t)-$$
$$-\int_{t-d}^{t}\eta^T(t,s)\psi\eta(t,s)\mathrm{d}s$$

where:

$$\xi(t)=\begin{bmatrix}x^T(t) & x^T(t-d)\end{bmatrix}^T$$

$$\hat{\Phi}(\varepsilon)=\begin{bmatrix}\Phi_{11}(\varepsilon)+dA^TMA & \Phi_{12}(\varepsilon)+dA^TMD\\ * & \Phi_{22}(\varepsilon)+dD^TMD\end{bmatrix}$$

$$\eta(t,s)=\begin{bmatrix}x^T(t) & x^T(t-d) & \left(E(\varepsilon)\dot{x}(s)\right)^T\end{bmatrix}^T$$

$$\Phi_{11}(\varepsilon)=\mathrm{Z}^T(\varepsilon)A+\mathrm{A}^TZ(\varepsilon)+\mathrm{Q}+YE(\varepsilon)+\mathrm{E}(\varepsilon)Y^T+dX_1$$

$$\Phi_{12}(\varepsilon)=\mathrm{Z}^T(\varepsilon)D-\mathrm{Y}E(\varepsilon)+\mathrm{E}(\varepsilon)N^T+dX_2$$

$$\Phi_{22}(\varepsilon)=-\mathrm{Q}-N\mathrm{E}(\varepsilon)-\mathrm{E}(\varepsilon)N^T+dX_3$$

It follows from (12) and (13) that $\hat{\Phi}(0)<0$ and $\hat{\Phi}(\bar{\varepsilon})<0$. Then, by using Lemma 1, we have $\hat{\Phi}(\varepsilon)<0$, $\forall\varepsilon\in(0,\bar{\varepsilon}]$, which yields:

$$\xi^T(t)\hat{\Phi}(\varepsilon)\xi(t)<0 \tag{16}$$

In addition, inequality (13) implies that:

$$\int_{t-d}^{t} \eta^T(t,s)\psi\eta(t,s)\mathrm{d}s \geq 0 \tag{17}$$

Then, from (16) and (17) we obtain $\dot{V}(x_t)\big|_{(1)} < 0$. Therefore, system (1) is asymptotically stable for $\forall\varepsilon \in (0,\bar{\varepsilon}]$.

Remark 1. Delay-dependent stability conditions for SPSs were proposed by using the Lyapunov-Krasovskii functional and linear matrix inequality (LMI) technique [15,18]. However, the main results [15,18] can only judge the stability of SPSs for specified values of time-delay and singular perturbation parameter. The advantage of Theorem 1 is that an estimate of the stability bound can be attained.

Remark 2. Stability of SPSs with time-delay that are proportional to perturbed parameter was considered and an LMI-based sufficient condition was proposed by using Lyapunov-Krasovskii functional [10]. The Lyapunov-Krasovskii functional is as follows:

$$\begin{aligned} V(z_t,\dot{z}_t,\varepsilon) &= z^T(t)E_\varepsilon P_\varepsilon z(t) + \\ &+\varepsilon h_0\int_{-\varepsilon h_0}^{0}\int_{t+\theta}^{t}\exp(2v(s-t))\dot{z}^T(s)R_h\dot{z}(s)\mathrm{d}s\mathrm{d}\theta + \\ &+\int_{-\varepsilon r_0}^{0}\int_{t+\theta}^{t}\exp(2v(s-t))z^T(s)R_r z(s)\mathrm{d}s\mathrm{d}\theta \end{aligned}$$

The matrix P_ε is in the form:

$$\mathrm{P}_\varepsilon = \begin{bmatrix} P_1 & \varepsilon P_2^T \\ P_2 & P_3 \end{bmatrix}$$

Similarly to the proof of Theorem 1, the Lyapunov-Krasovskii functional, $V(z_t,\dot{z}_t,\varepsilon)$, can be generalized by setting:

$$\mathrm{P}_\varepsilon = \begin{bmatrix} Z_1+\varepsilon Z_3 & \varepsilon Z_5^T \\ Z_5 & Z_2+\varepsilon Z_4 \end{bmatrix}$$

and thus Theorem 7.1 in [10] can be further improved to obtain a less conservative stability criterion.

Delay-independent case

Consider system (1).

Theorem 2. Given $\bar{\varepsilon} > 0$, system (1) is asymptotically stable for $\forall\varepsilon \in (0,\bar{\varepsilon}]$ if there exist symmetric positive-definite matrix Q and matrices $\mathrm{Z}_i (i = 1,2,\ldots,5)$ with $\mathrm{Z}_i = \mathrm{Z}_i^T (i = 1,2,3,4)$, such that the LMIs (6)-(8), (18) and (19) are feasible.

$$\begin{bmatrix} \mathrm{Z}^T(0)A + A^T Z(0) + \mathrm{Q} & Z^T(0)D \\ D^T Z(0) & \mathrm{-Q} \end{bmatrix} < 0 \tag{18}$$

$$\begin{bmatrix} \mathrm{Z}^T(\bar{\varepsilon})A + A^T Z(\bar{\varepsilon}) + \mathrm{Q} & Z^T(\bar{\varepsilon})D \\ D^T Z(\bar{\varepsilon}) & \mathrm{-Q} \end{bmatrix} < 0 \tag{19}$$

Proof. Define a quadratic Lyapunov-Krasovskii functional as:

$$V(x_t) = x^T(t)E(\varepsilon)Z(\varepsilon)x(t) + \int_{t-d}^{t} x^T(s)Qx(s)\mathrm{d}s$$

where $x_t = x(t+\theta)$, $\theta \in [-d,0]$.

By Lemma 2 and LMIs (6)-(9), we have Eq. (9).

Thus, $V(x_t)$ is a positive-definite Lyapunov-Krasovskii functional.

Taking the derivative of $V(x_t)$ along the trajectories of the system (22), we have:

$$\begin{aligned} \dot{V}(x_t) &= 2x^T(t)Z^T(\varepsilon)\left(Ax(t)+Dx(t-d)\right) + \\ &+x^T(t)Qx(t) - x^T(t-d)Qx(t-d) = \\ &= \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix}^T \times \\ &\begin{bmatrix} \mathrm{Z}^T(\varepsilon)A + A^T Z(\varepsilon) + \mathrm{Q} & Z^T(\varepsilon)D \\ D^T Z(\varepsilon) & \mathrm{-Q} \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-d) \end{bmatrix} = \\ &= \xi^T(t)\mathrm{W}(\varepsilon)\xi(t) \end{aligned} \tag{20}$$

where

$$\mathrm{W}(\varepsilon) = \begin{bmatrix} Z^T(\varepsilon)A + A^T Z(\varepsilon) + \mathrm{Q} & \mathrm{Z}^T(\varepsilon)D \\ D^T Z(\varepsilon) & \mathrm{-Q} \end{bmatrix}$$

and

$$\xi(t) = (x^T(t)x^T(t-d))^T$$

It follows from (18) and (19) that $\mathrm{W}(0) < 0$ and $\mathrm{W}(\bar{\varepsilon}) < 0$. Then, by using Lemma 1, we have $\mathrm{W}(\varepsilon) < 0$ for $\forall\varepsilon \in (0,\bar{\varepsilon}]$.

As a result, it follows from (20) that $\xi^T(t)\mathrm{W}(\varepsilon)\xi(t) < 0$. Then, we have $\dot{V}(x_t)\big|_{(1)} < 0$.

Hence, system (1) is asymptotically stable for $\forall\varepsilon \in (0,\bar{\varepsilon}]$.

Remark 3. Delay-independent stability conditions were proposed by using Lyapunov-Krasovskii functional and linear matrix inequality (LMI) technique and explicit stability bounds are attained by solving the convex optimization problem [17,21,22]. Since the Lyapunov-Krasovskii functional used in Theorem 2 is more general than that used in literature [17,21,22], Theorem 2 is expected to produce tighter stability bound than the method applied in the literature [17,21,22].

Remark 4. It is known that the LMI approaches in Theorems 1 and 2 also allow solutions for SPSs with uncertainties in the system matrices because the LMI conditions are affine in the system matrices. For norm-bounded uncertainties, using the routine method [24], we can obtain the corresponding result. For the polytopic uncertainties, LMIs should hold for all the vertices.

Remark 5. The first key point for deriving stability criterion of systems with time-delays is the choice of an appropriate Lyapunov-Krasovskii functional, which is usually composed of a quadratic term and several integral terms. In this paper, the quadratic term of the constructed Lyapunov-Krasovskii functional is:

$$x^T \begin{bmatrix} Z_1+\varepsilon Z_3 & \varepsilon Z_5^T \\ \varepsilon Z_5 & \varepsilon Z_2+\varepsilon^2 Z_4 \end{bmatrix} x$$

which is more general than the existing one:

$$x^T \begin{bmatrix} Z_1 & \varepsilon Z_5^T \\ \varepsilon Z_5 & \varepsilon Z_2 \end{bmatrix} x$$

Hence, the basic idea can be used to generalize the existing results [2-4,15,17-19,21,22] to deal with stability of singularly perturbed systems with time-delay.

EXAMPLES AND DISCUSSION

Example 1

We illustrate the advantage of Theorem 2 over the existing methods [17,21] by the following singularly perturbed system with time-delay:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + x_1(t-d) \\ \varepsilon \dot{x}_2(t) = -2x_1(t) - x_2(t) + 0.5x_2(t-d) \end{cases}$$

This system has been considered already [17,21].

Set $\bar{\varepsilon} = 0.4999$, solving the LMIs of Theorem 2, we have:

$$\mathrm{Q} = \begin{bmatrix} 119.4115 & 29.8952 \\ 29.8952 & 29.8836 \end{bmatrix},\ Z_1 = 80.0001,$$

Z_2 = 41.2452, Z_3 = 79.0514, Z_4 = 37.0304, Z_5 = = 59.7620.

From Theorem 2, this system is asymptotically stable for $\forall \varepsilon \in (0,\ 0.4999]$.

As shown in Table 1, the stability bound computed by Theorem 2 is larger than those given in literature [17,21]. Thus the newly developed methods are less conservative than the existing methods. In addition, the result in [15] presents a sufficient condition for the existence of stability bound, but cannot propose an estimate of the stability bound.

Table 1. Comparisons of stability bound of Example 1 for $0 < d < +\infty$

Method	Theorem 2.2 in [21]	Corollary 1in [17]	Theorem 2
$\bar{\varepsilon}$	0.3	0.4638	0.4999

Example 2

To illustrate the advantage of Theorem 1 over the existing methods [22], we consider a system in the form of (1) with:

$$\mathrm{A} = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix},\ \mathrm{D} = \begin{bmatrix} -1 & 1 \\ 0.6 & -0.5 \end{bmatrix}$$

Solving the LMIs in Theorem 1 with d = 6 and $\bar{\varepsilon}$ = 3, we have:

Z_1 = 1223.3708, Z_2 = 3292.7665, Z_3 = 665.8371, Z_4 = -174.2364, Z_5 = -1.275.8187,

$$\mathrm{Q} = \begin{bmatrix} 5007.4552 & -7455.4829 \\ -7455.4829 & 10838.0133 \end{bmatrix},$$

$$\mathrm{Y} = \begin{bmatrix} -490.3638 & 0.0214 \\ -56.2369 & -47.5006 \end{bmatrix},$$

$$\mathrm{N} = \begin{bmatrix} 366.5165 & 309.2107 \\ 291.2258 & 245.9806 \end{bmatrix},$$

$$\mathrm{M} = \begin{bmatrix} 2910.1337 & 2455.8222 \\ 2455.8222 & 2074.2968 \end{bmatrix},$$

$$\mathrm{X}_1 = \begin{bmatrix} 84.5662 & 5.4055 \\ 5.4055 & 11.0456 \end{bmatrix},$$

$$\mathrm{X}_2 = \begin{bmatrix} -60.5696 & -49.7500 \\ -9.2643 & -5.7005 \end{bmatrix},$$

$$\mathrm{X}_3 = \begin{bmatrix} 47.4802 & 35.0046 \\ 35.0046 & 33.3055 \end{bmatrix}.$$

By Theorem 1, the system is asymptotically stable for any $\varepsilon \in (0,3]$ and $d \in [0,6]$.

However, by Theorem 3 of [22], we found that for given $\bar{\varepsilon}$ = 3, the maximum allowable delay is 4.0438 and for given d = 6, the maximum stability bound is 1.0221.

This example shows that Theorem 1 of this paper is less conservative than Theorem 3 of [22].

Example 3

This example will show the advantage of Theorem 1 over the results of [12].

Consider the following system [12]:

$$\begin{cases} \dot{x}(t) = -x(t) - z(t) + u(t) \\ \varepsilon \dot{z}(t) = -z(t) + Kx(t-T) \end{cases}$$

where $0 \le K \le 1$.

In this example, K is constant but uncertain and satisfies $0 \le K \le 1$. As shown in Remark 4, LMI conditions in Theorem 1 are affine in the system matrices and thus, for the polytopic uncertainties, LMIs should be checked for all the vertices.

Solving the LMIs in Theorem 1 with T = 0.2, K = 0, $\bar{\varepsilon}$ = 10, we have:
Z_1 = 0.7627, Z_2 = 0.5450, Z_3 = 0.0024, Z_4 = -0.0264, Z_5 = 0.0345,

$$\mathrm{Q}=\begin{bmatrix}0.6116 & 0.3384\\ 0.3384 & 0.4526\end{bmatrix},\quad \mathrm{Y}=\begin{bmatrix}-0.1401 & 0.0214\\ 0.1619 & -0.0158\end{bmatrix},$$
$$\mathrm{N}=\begin{bmatrix}0.1712 & -0.0042\\ -0.1034 & 0.0227\end{bmatrix},$$
$$\mathrm{M}=\begin{bmatrix}0.9383 & -0.2279\\ -0.2279 & 0.5266\end{bmatrix},\quad \mathrm{X}_1=\begin{bmatrix}0.9230 & 0.1728\\ 0.1728 & 0.6300\end{bmatrix},$$
$$\mathrm{X}_2=\begin{bmatrix}-0.2307 & 0.1892\\ 0.1890 & -0.1185\end{bmatrix},\ \mathrm{X}_3=\begin{bmatrix}0.9245 & 0.1775\\ 0.1775 & 0.6175\end{bmatrix}.$$

Solving the LMIs in Theorem 1 with T = 0.2, K = 1, $\bar{\varepsilon}$ = 10, we have:
Z_1 = 0.3516, Z_2 = 0.1238, Z_3 = 0.0004, Z_4 = -0.0045, Z_5 = 0.0108,

$$\mathrm{Q}=\begin{bmatrix}0.3074 & 0.1196\\ 0.1196 & 0.0972\end{bmatrix},\quad \mathrm{Y}=\begin{bmatrix}-0.0346 & 0.0065\\ 0.1339 & -0.0056\end{bmatrix},$$
$$\mathrm{N}=\begin{bmatrix}0.0509 & -0.0007\\ -0.0427 & 0.0112\end{bmatrix},\quad \mathrm{M}=\begin{bmatrix}0.3472 & -0.0935\\ -0.0935 & 0.1269\end{bmatrix},$$
$$\mathrm{X}_1=\begin{bmatrix}0.3710 & 0.0522\\ 0.0522 & 0.2349\end{bmatrix},\mathrm{X}_2=\begin{bmatrix}-0.0691 & 0.0582\\ 0.0469 & -0.0459\end{bmatrix},$$
$$\mathrm{X}_3=\begin{bmatrix}0.3696 & 0.0582\\ 0.0582 & 0.2042\end{bmatrix}.$$

By Theorem 1, the system is stable for T = 0.2, $0 \le K \le 1$ and $\forall \varepsilon \in (0,10]$. However, the result of [12] shows that the system is stable for T = 0.2 and $\forall \varepsilon \in (0,0.85)$. Thus, the obtained stability bound of this paper is much larger than that given by [12].

Given $\bar{\varepsilon}$ = 10, the maximal allowable time-delay for different K can be computed by Theorem 1. From Table 2, it can be seen that Theorem 1 is less conservative than Theorem 3 of [22].

Table 2. Comparisons of maximum allowable time-delay of Example 3 for $\bar{\varepsilon} = 10$

Theorem	K = 2	K = 3	K = 4
Theorem 3 of [22]	0.7813	0.4606	0.3171
Theorem 1	0.9666	0.5401	0.3728

In addition, when the singular perturbation parameter is known, we applied the proposed methods of this paper and [22] to Examples 2 and 3. We found that for a given singular perturbation parameter, these methods gave the same upper bound of time-delay for the system to be stable.

Thus, by Examples 2 and 3, the proposed methods in this paper have some the advantages over those of [22] when the singular perturbation parameter is not known and its upper bound is concerned.

CONCLUSION

This paper proposed two novel Lyapunov-Krasovskii functionals for SPSs with time-delay, by which delay-dependent and independent stability criteria were proposed. The proposed methods were shown to be less conservative than the existing ones because the adopted Lyapunov-Krasovskii functionals are more general. Furthermore, the proposed Lyapunov-Krasovskii functionals are expected to be used to study other analysis and synthesis problems of SPSs with time-delay.

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NAUČNI RAD

ANALIZA GRANICE STABILNOSTI SINGULARNO PERTUBOVANIH SISTEMA SA VREMENSKIM KAŠNJENJEM

U radu se razmatra problem određivanja granice stabilnosti singularno pertubovanih sistema sa vremenskim kašnjenjem. Izborom pogodnih funkcionala Lyapunov-Krasovskii izvedeni su određeni kriterijumi stabilnosti. Predloženi kriterijumi su manje konzervativni od postojećih. Data su dva numerička primera kojima se ilustruju prednosti i efektivnost predloženih metoda.

Ključne reči: singularno pertubovani sistemi, granica stabilnosti, linearne matrične nejednakosti, Lyapunov-Krasovskii funkcional