Theoret. Appl. Mech., Vol.32, No.3, pp. 223–234, Belgrade 2005

Radiation effects on flow past an impulsively started vertical plate with variable temperature and mass flux

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Abstract

An analysis is performed to study the thermal radiation effects on unsteady free convective flow over a moving vertical plate in the presence of variable temperature and uniform mass flux. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. The temperature is raised linearly with time and the concentration level near the plate are raised linearly with time. The dimensionless governing equations are solved using the Laplace transform technique. The velocity and skin-friction are studied for different parameters like the radiation parameter, Schmidt number, thermal Grashof number, mass Grashof number and time. It is observed that the velocity increases with decreasing radiation parameter.

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List of symbols

a^* absorption coefficient
A constant
C' species concentration in the fluid
C'_w concentration of the plate
C'_∞ concentration in the fluid far away from the plate
C dimensionless concentration
C_p specific heat at constant pressure
D mass diffusion coefficient
g acceleration due to gravity
Gr thermal Grashof number
Gc mass Grashof number
j'' mass flux per unit area at the plate
k thermal conductivity of the fluid
Pr Prandtl number
q_r radiative heat flux in the y-direction
R radiation parameter
Sc Schmidt number
T'_∞ temperature of the fluid far away from the plate
T'_w temperature of the plate
T' temperature of the fluid near the plate
t' time
t dimensionless time
u velocity of the fluid in the x-direction
u_0 velocity of the plate
U dimensionless velocity
y coordinate axis normal to the plate
Y dimensionless coordinate axis normal to the plate

Greek symbols

β volumetric coefficient of thermal expansion
β^* volumetric coefficient of expansion with concentration
μ coefficient of viscosity
ν kinematic viscosity
ρ density

τ dimensionless skin-friction
θ dimensionless temperature
η similarity parameter
$erfc$ complementary error function

1 Introduction

Radiative convective flows are encountered in countless industrial and environment processes e.g. heating and cooling chambers, fossil fuel combustion energy processes, evaporation from large open water reservoirs, astrophysical flows, solar power technology and space vehicle reentry. Radiative heat and mass transfer play an important role in manufacturing industries for the design of reliable equipment. Nuclear power plants, gas turbines and various propulsion device for aircraft, missiles, satellites and space vehicles are examples of such engineering applications. If the temperature of the surrounding fluid is rather high, radiation effects play an important role and this situation does exist in space technology. In such cases, one has to take into account the effect of thermal radiation and mass diffusion.

England and Emery[1] have studied the thermal radiation effects of a optically thin gray gas bounded by a stationary vertical plate. Soundalgekar and Takhar[2] have considered the radiative free convective flow of an optically thin gray-gas past a semi-infinite vertical plate. Radiation effect on mixed convection along a isothermal vertical plate were studied by Hossain and Takhar[3]. In all above studies, the stationary vertical plate is considered. Das *et al*[4] have analyzed radiation effects on flow past an impulsively started infinite isothermal vertical plate. The governing equations were solved by the Laplace transform technique. Raptis and Perdikis[4] studied the effects of thermal radiation and free convection flow past a moving vertical plate. The governing equations were solved analytically. Thermal radiation on unsteady free convection and mass transfer boundary layer over a vertical moving plate were analyzed by Raptis and perdikis[5].

However the thermal radiation effects on moving infinite vertical plate in the presence variable temperature and mass flux is not studied in the literature. It is proposed to study thermal radiation effects on flow past an impulsively started infinite vertical plate with variable temperature and mass flux. The dimensionless governing equations are solved using the Laplace transform technique.

2 Basic equations and analysis

Thermal radiation effects on unsteady flow of a viscous incompressible fluid past an impulsively started infinite vertical plate with variable temperature and mass flux is studied. The x-axis is taken along the plate in the vertically upward direction and the y-axis is taken normal to the plate. Initially, the plate and fluid are at the same temperature and concentration. At time $t' > 0$, the plate is given an impulsive motion in the vertical direction against gravitational field with constant velocity u_0 in a fluid, in the presence of thermal radiation. At the same time the plate temperature is raised linearly with time and the mass is diffused from the plate to the fluid at an uniform rate. The fluid considered here is a gray, absorbing-emitting radiation but a non-scattering medium. Under the above assumptions the flow is governed by the following equations:

$$\frac{\partial u}{\partial t'} = g\beta(T - T_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u}{\partial y^2} \tag{1}$$

$$\rho\, C_p \,\frac{\partial T}{\partial t'} = k\, \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_r}{\partial y} \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D\, \frac{\partial^2 C'}{\partial y^2} \tag{3}$$

The term $\frac{\partial q_r}{\partial y}$ represents the change in the radiative flux with distance normal to the plate. The initial and boundary conditions may be expressed as:

$$t' \leq 0: \quad u = 0, \qquad T = T_\infty, \qquad C' = C'_\infty \ \text{ for all } \ y$$

$$t' > 0: \quad u = u_0, \quad T = T_\infty + (T_w - T_\infty)\ A\ t', \quad \frac{\partial C'}{\partial y'} = -\frac{j''}{D} \ \text{at} \ \ y = 0$$

$$u = 0, \qquad T \to T_\infty, \qquad C' \to C'_\infty \ \text{ as } \ y \to \infty \tag{4}$$

where $A = \frac{u_0^2}{\nu}$.

The local radiant for the case of an optically thin gray gas is expressed by

$$\frac{\partial q_r}{\partial y} = -4a^*\ \sigma\left(T_\infty^4 - T^4\right) \tag{5}$$

It is assume that the temperature differences within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding T^4 in a Taylor series about T_∞ and neglecting higher-order terms, thus

$$T^4 \cong 4T_\infty^3\ T - 3T_\infty^4 \tag{6}$$

By using equations (5) and (6), equation (2) reduces to

$$\rho\ C_p\ \frac{\partial T}{\partial t'} = k\ \frac{\partial^2 T}{\partial y^2} + 16a^*\sigma T_\infty^3\ (T_\infty - T) \tag{7}$$

On introducing the following dimensionless quantities:

$$U = \frac{u}{u_0},\ t = \frac{t' u_0^2}{\nu},\ Y = \frac{y u_0}{\nu},\ \theta = \frac{T - T_\infty}{T_w - T_\infty},$$

$$Gr = \frac{g\beta\nu(T_w - T_\infty)}{u_0^3},\ C = \frac{C' - C'_\infty}{\left(\frac{j''\nu}{Du_0}\right)},\ \ Gm = \frac{\nu g \beta^* \left(\frac{j''\nu}{Du_0}\right)}{u_0^3}, \tag{8}$$

$$Pr = \frac{\mu C_p}{k},\ Sc = \frac{\nu}{D},\ R = \frac{16a^*\nu^2\sigma T_\infty^3}{ku_0^2},\ K = \frac{\nu K_l}{u_0^2}$$

in equations (1) to (4), leads to

$$\frac{\partial U}{\partial t} = Gr\,\theta + Gc\,C + \frac{\partial^2 U}{\partial Y^2} \tag{9}$$

$$\frac{\partial \theta}{\partial t} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial Y^2} - \frac{R}{Pr}\,\theta \tag{10}$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2} \tag{11}$$

The initial and boundary conditions in non-dimensional form are

$$\begin{array}{lllll} & U=0, \quad \theta=0, & C=0, & \text{for all} & Y, t \le 0 \\ t>0: & U=1, \quad \theta=t, & \dfrac{\partial C}{\partial y} = -1, & \text{at} & Y=0 \\ & U=0, \quad \theta\to 0, & C\to 0 & \text{as} & Y\to\infty \end{array} \tag{12}$$

All the physical variables are defined in the nomenclature. The solutions are obtained for hydrodynamic flow field in the presence of first order chemical reaction.

The equations (9) to (11), subject to the boundary conditions (12), are solved by the usual Laplace-transform technique and the solutions are derived as follows:

$$\begin{aligned} \theta = \frac{t}{2}\Big[&\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at}) \\ &+\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at})\Big] \\ &-\frac{\eta Pr\sqrt{t}}{2\sqrt{R}}\Big[\exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}-\sqrt{at}) \\ &-\exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr}+\sqrt{at})\Big] \end{aligned} \tag{13}$$

$$C = 2\sqrt{t}\left[\frac{exp(-\eta^2 Sc)}{\sqrt{\pi}\ \sqrt{Sc}} - \ \eta\ erfc(\eta\sqrt{Sc})\right] \tag{14}$$

$$\begin{aligned}
U = & \left(1 + \frac{Gr}{b^2(1-Pr)}\right)\ erfc(\eta) \\
& + \frac{Gr\ t}{b(1-Pr)}\ \left[(1+2\eta^2)\ erfc(\eta) - \frac{2\eta}{\sqrt{\pi}}\ exp(-\eta^2)\right] \\
& - \frac{Gr\ exp(bt)}{2b^2(1-Pr)}\left[exp(2\eta\sqrt{bt})\ erfc(\eta + \sqrt{bt}) \right. \\
& \left. + exp(-2\eta\sqrt{bt})\ erfc(\eta - \sqrt{bt})\right] \\
& - \frac{Gc\ t\ \sqrt{t}}{3(1-Sc)\sqrt{Sc}}\ \eta\ (6+4\eta^2)\ erfc(\eta) \\
& + \eta\ \sqrt{Sc}\ (6 + 4\ \eta^2\ Sc)\ erfc(\eta\ \sqrt{Sc}) \\
& - \frac{Gr(1+bt)}{2b^2(1-Pr)}\left[exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} + \sqrt{at}) \right. \\
& \left. + exp(-2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} - \sqrt{at})\right] \\
& + \frac{\eta\ Gr Pr\sqrt{t}}{2b(1-Pr)\sqrt{R}}\left[exp(-2\eta\sqrt{Rt}) erfc(\eta\sqrt{Pr} - \sqrt{at}) \right. \\
& \left. - exp(2\eta\sqrt{Rt})\ erfc(\eta\sqrt{Pr} + \sqrt{at})\right] \\
& + \frac{Gr\ exp(bt)}{2b^2(1-Pr)}\left[exp(2\eta\sqrt{Pr(a+b)t}) erfc(\eta\ \sqrt{Pr} + \sqrt{(a+b)t}) \right. \\
& \left. + exp(-2\eta\sqrt{Pr(a+b)t})\ erfc(\eta\ \sqrt{Pr} - \sqrt{(a+b)t})\right]
\end{aligned} \tag{15}$$

where, $\eta = Y/2\sqrt{t},\ a = \frac{R}{Pr}$ and $b = \frac{R}{1-Pr}$.

3 Discussion of results

In order to get a physical view of the problem numerical calculations are carried out for different values of the radiation parameter, Schmidt number, thermal Grashof number and mass Grashof number and time. The purpose of the calculations given here is to assess the effects of the parameters R, Gr, Gc, Sc and t upon the nature of the flow and transport. The Laplace transform solutions are in terms of exponential and complementary error function.

The velocity profiles for different values of the radiation parameter($R = 0.2, 2, 5, 10$), $Gr = Gc = 5, Sc = 0.6, Pr = 0.71$ and $t = 0.2$ are shown in table 1. It is observed that the velocity increases with decreasing radiation parameter. This shows that velocity decreases in the presence of high thermal radiation.

The velocity profiles for different values of thermal Grashof number($Gr = 2, 5$), mass Grashof number($Gc = 2, 5$), $R = 5, Sc = 0.6, Pr = 0.71$ and time $t = 0.2$ are shown in table 2. It is clear that the velocity increases with increasing thermal Grashof number or mass Grashof number.

In table 3, the effect of velocity for different values of the time ($t = 0.2, 0.4, 0.6$), $Sc = 0.6, R = 0.2, Gr = Gc = 5$ and $Pr = 0.71$ are presented. It is observed that the velocity increases with increasing t.

From the velocity field, the effect of mass transfer on the skin-friction is studied and is given in dimensionless form as

$$\tau = -\left(\frac{dU}{dY}\right)_{Y=0} = -\frac{1}{2\sqrt{t}}\left(\frac{dU}{d\eta}\right)_{\eta=0} \tag{16}$$

Hence, from equation (15) and (16),

$$\begin{aligned}\tau &= \frac{1}{\sqrt{\pi\, t}} - \frac{Gr(1+bt)}{b^2(1-Pr)}\left(\sqrt{Pr} + \sqrt{\pi R t}erf(\sqrt{at})\right) \\ &+ \frac{Gr exp(bt)}{b^2(1-Pr)}\left(\sqrt{Pr} + \sqrt{Pr(a+b)\pi t}erf(\sqrt{(a+b)t})\right) \\ &- \frac{Gr Pr\sqrt{\pi t}}{2b(1-Pr)\sqrt{R}}erf(\sqrt{at}) - \frac{Gct\sqrt{\pi t}}{\sqrt{Sc}(1+\sqrt{Sc})}\end{aligned} \tag{17}$$

The numerical values of τ are presented in table 4. It is observed from this table, that an increase in the radiation parameter leads to fall in the value of the skin-friction. This trend is just reversed with respect to Schmidt number. As time advances the value of the skin-friction decreases. Moreover the value of the skin-friction decreases with increasing thermal Grashof number or mass Grashof number.

4 Conclusions

An analysis is performed to study flow past an impulsively started infinite vertical plate with variable temperature and uniform mass flux, in the presence of thermal radiation. The dimensionless governing equations are solved by the usual Laplace-transform technique. The conclusions of the study are as follows:

(i) The velocity increases with decreasing radiation parameter.

(ii) The velocity increases with increasing thermal Grashof number or mass Grashof number.

(iii) The skin-friction increases with decreasing radiation parameter.

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Table 1 Velocity for different R

η	$R = 0.2$	$R = 2$	$R = 5$	$R = 10$
0	1	1	1	1
0.25	0.8410	0.8395	0.8375	0.8350
0.5	0.6077	0.6058	06033	0.6001
0.75	0.3880	0.3864	0.3843	0.3818
1	0.2216	0.2205	0.2191	0.2175
1.25	0.1138	0.1132	0.1124	0.1116
1.5	0.0528	0.0525	0.0521	0.0517
1.75	0.0223	0.0221	0.0219	0.0218
2	0.0086	0.0085	0.0084	0.0084
2.25	0.0030	0.0030	0.0030	0.0030

Table 2 Velocity for different Gr and Gc

η	$Gr = 2, Gc = 2$	$Gr = 2, Gc = 5$	$Gr = 2, Gc = 5$
0	1	1	1
0.25	0.7692	0.8268	0.8375
0.5	0.5290	0.5932	0.6033
0.75	0.3270	0.3777	0.3843
1	0.1820	0.2154	0.2191
1.25	0.0912	0.1107	0.1124
1.5	0.0412	0.0514	0.0521
1.75	0.0168	0.0216	0.0219
2	0.0062	0.0083	0.0084
2.25	0.0021	0.0029	0.0030

Table 3 Velocity for different t

η	$t = 0.2$	$t = 0.4$	$t = 0.6$
0	1	1	1
0.25	0.8268	0.8568	0.9716
0.5	0.5932	0.6223	0.7429
0.75	0.3777	0.3980	0.4891
1	0.2154	0.2275	0.2857
1.25	0.1107	0.1171	0.1500
1.5	0.0514	0.0545	0.0714
1.75	0.0216	0.0230	0.0310
2	0.0083	0.0089	0.0124
2.25	0.0029	0.0032	0.0046

Table 4 skin-friction τ

R	t	Gr	Gc	Sc	τ
0.2	0.2	2	5	0.16	- 0.1391
0.2	0.2	2	5	0.6	-0.0808
0.2	0.2	2	5	2.01	0.3549
2	0.2	2	5	0.6	0.2874
5	0.2	2	5	0.6	0.2644
10	0.2	2	5	0.6	- 0.7029
5	0.2	2	2	0.6	0.7009
5	0.2	5	5	0.6	- 0.1400
5	0.4	2	2	0.6	- 3.1888
5	0.6	2	2	0.6	- 83.2727

Submitted on August 2005.

Uticaji radijacije na tečenje preko impulsivno pokrenute vertikalne ploče sa promenljivom temperaturom i fluksom mase

UDK 536.7

Izvedena je analiza u cilju studije efekta termičke radijacije na nestacionarno tečenje sa slobodnom konvekcijom preko pokretne vertikalne ploče u prisustvu promenljive temperature i uniformnog fluksa mase. Fluid se smatra sivom, apsorbujućom i emitujućom radijaciju sredinom ali bez rasejavanja. Temperatura se linearno podiže sa vremenom a nivo koncentracije blizu ploče se podiže linearno sa vremenom. Bezdimenzione jednačine problema se rešavaju korišćenjem Laplasove transformacije. Brzina i skin-trenje se proučavaju pri različitim vrednostima parametara kao što su: radijacioni parametar, Schmidt-ov broj, termički Grashofov broj, maseni Grashofov broj i vreme. Primećuje se da brzina raste sa opadanjem radijacionog parametra.