

# The crack kinking out of an interface

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## Abstract

Kinking of a plane strain crack out of an interface between the two dissimilar isotropic elastic materials is analyzed. Analysis is focused on the initiation of kinking and thus the segment of the crack leaving the interface is imagined to be short compared to the segment in the interface. The analysis provides the stress intensity factors and energy release rate of the kinked cracks in terms of the corresponding quantities for the interfacial crack. The energy release rate is enhanced if the crack heads into the more compliant material and is diminished if the crack kinks into the stiff material.

## 1 Introduction

Bounded interface between the two dissimilar materials often separates by cracking, because the toughness of the interface is low compared to that of the abutting materials. At certain instance, crack kinks out of the interface and will advance in one of the two materials. The crack can also be lying entirely within one of the two materials parallel to the interface - subinterfacial crack. Such questions, like whether the crack, lying on interface, would advance along the interface or would kink out of the interface, are important in the design of the interface between fiber and matrix, when a crack, propagating in the matrix, approaches

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a fiber is deflected to continue along the interface, thereby allowing the fiber to survive. Analysis of subinterfacial crack and problem of crack kinking out of the interface can be applied in coatings and glued joints.

There is a number of papers in which were analyzed problems of crack penetration or deflection at an interface. Cook and Erdogan (1972) and Erdogan and Biricikoglu (1973) investigated the behavior of a crack penetrating the interface at right angles. Gorre and Venezia (1977) analyzed several problems of penetration and deflection for a main crack impinging the interface at the right angle. Additional works in this direction were by Lu and Erdogan (1983). Behavior of crack approaching an interface at an oblique angle has been analyzed in studies by Erdogan and Arin (1975), Lardner et al. (1988) and He and Hutchinson (1988). From the same aspect He and Hutchinson (1988) considered a problem of a crack kinking out of an interface into one of the abutting materials. Hutchinson et al. (1987) investigated crack paralleling an interface between the two dissimilar materials.

## 2 Kinking of a crack out of an interface

In this paper an analysis of a crack kinking out of an interface is carried out with providing the condition needed to assess if an interfacial crack will tend to propagate in the interface or whether it will advance by kinking out of the interface. The analyzed geometry is shown in Figure 1.

The main interfacial crack lies on the interface between the two different semi - infinite isotropic elastic solids. A straight crack segment of length  $a$  and angle  $\omega$  kinks downward into material 2. The length  $a$  is assumed to be small compared to the length of the crack itself, and thus the asymptotic problem for the semi - infinite main crack is analyzed. The stress field prior to kinking ( $a \rightarrow 0$ ) is the singularity field of an interface crack characterized by a complex stress intensity factor,  $K = K_1 + iK_2$ . The crack tip field at the end of the kinked crack is characterized by combination of the standard Mode I and Mode II stress intensity factors,  $K_I$  and  $K_{II}$ . The analysis provides the relationships between  $K_I$  and  $K_{II}$  for the kinked crack and  $K_1$  and  $K_2$  for the interface crack as dependent on the kink angle  $\omega$  and the material

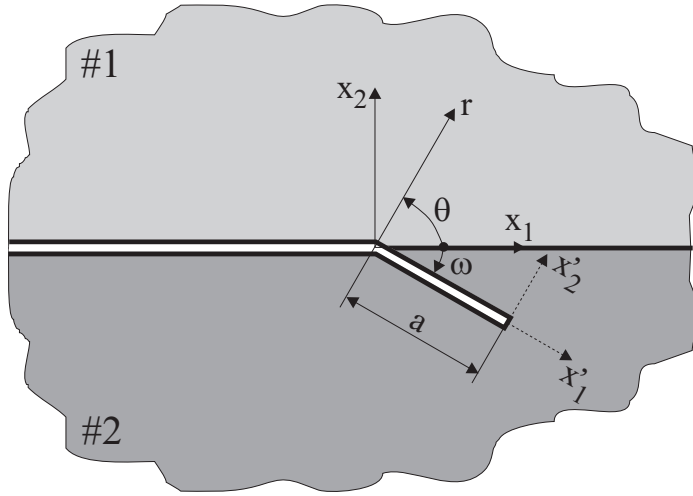


Figure 1: Geometry of a kinked crack

moduli. The energy release rate of the kinked crack is also related to the energy release rate of the interface crack.

Solution for these problems depends only on two special parameters, Dundurs (1969):

$$\alpha = \frac{\mu_2(\kappa_1 + 1) - \mu_1(\kappa_2 + 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \quad , \quad \beta = \frac{\mu_2(\kappa_1 - 1) - \mu_1(\kappa_2 - 1)}{\mu_2(\kappa_1 + 1) + \mu_1(\kappa_2 + 1)} \quad (1)$$

where:  $\kappa_i = 3 - 4\nu_i$  for plane strain and  $\kappa_i = (3 - \nu_i)/(1 + \nu_i)$  for plane stress. In expressions for parameters  $\alpha$  and  $\beta$ ,  $\mu_i$  and  $\nu_i$  are shear modulus and Poisson's ratio, respectively, and the subscript  $i=1, 2$  identifies the material as indicated in Figure 1. Both  $\alpha$  and  $\beta$  vanish when the materials are identical.

The stress field for the semi - infinite interface crack ( $a=0$ ) has the form:

$$\sigma_{\alpha\beta} = Re \left[ K(2\pi r)^{-\frac{1}{2}} r^{i\epsilon} \Sigma_{\alpha\beta}(\theta) \right] \quad , \quad (2)$$

where  $i = \sqrt{-1}$ ,  $r$  and  $\theta$  are polar coordinates at the origin,  $K = K_1 + iK_2$  is the complex stress intensity factor, and

$$\varepsilon = \frac{1}{2\pi} \ln \left( \frac{1 - \beta}{1 + \beta} \right) \quad (3)$$

is the bielastic constant or oscillatory index. The angular dependence  $\Sigma_{\alpha\beta}(\theta)$  is complex in general but universal for a given material pair. On the interface ahead of the tip the tractions are:

$$\sigma_{22} + i\sigma_{12} = K(2\pi r)^{-\frac{1}{2}} r^{i\varepsilon}. \quad (4)$$

The notation used here follow those introduced by Rice (1987) and Hutchinson, Mear and Rice (1987), which in turn are based on the early papers on the subject by England (1965), Erdogan (1965) and Rice and Shih (1965). The interface stress intensity factors are defined so that  $K_1 \rightarrow K_I$  and  $K_2 \rightarrow K_{II}$  when the dissimilarity between the two materials vanishes. Note also that, when  $\beta = 0$  and  $\varepsilon = 0$ ,  $K_1$  measures the normal component of the traction singularity acting on the interface, while  $K_2$  measures the shear component with standard definitions for a stress intensity factor.

The complex stress intensity  $K = K_1 + iK_2$  is taken as the prescribed loading parameter in the present study. For a specific interface crack problem  $K$  will necessarily have the dimensional form:

$$K = K_1 + iK_2 = (\text{applied load})\sqrt{L} L^{-i\varepsilon} f, \quad (5)$$

which follows from equation (4), where  $L$  is a characteristic length such as crack length or ligament length and  $f$  is a dimensionless function of the geometry and material moduli.

The stress field at the tip of the kinked crack in material 2 is the classical field with conventional stress intensity factors  $K_I$  and  $K_{II}$  such that:

$$\sigma_{2'2'} + i\sigma_{1'2'} = (K_I + iK_{II})(2\pi r)^{-\frac{1}{2}}, \quad (6)$$

on the line ahead of the tip ( $x'_1 > 0$ ,  $x'_2 = 0$ ), where  $x'_1 x'_2$  is the coordinate system centered at the tip of the kinked crack.

As already stated, the problem considered is the asymptotic one where  $a$  is small compared to the all other relevant length quantities so that the interface is taken as semi - infinite. The relationship between the

stress intensity factors of the kinked crack and the prescribed complex stress intensity factor,  $K$ , can be written as, He and Hutchinson (1988):

$$K_I + iK_{II} = cK a^{i\varepsilon} + \bar{d} \bar{K} a^{-i\varepsilon}, \tag{7}$$

where  $(\cdot)$  denotes complex conjugation and  $c$  and  $d$  are complex functions of  $\omega$ ,  $\alpha$  and  $\beta$ . The factors  $K_I$  and  $K_{II}$  have dimensions of stress  $\cdot (\text{length})^{1/2}$  while  $K$  has the form (5). By dimensional analysis,  $a$  must be combined with  $K$  as  $K a^{i\varepsilon}$  or its conjugate since in the asymptotic problem  $a$  is the only length quantity other than the length quantities implicit in  $K$  in (5). Equation (7) is a general representation of  $K_I + iK_{II}$  consistent with this observation and with linearity. Use of  $\bar{d}$  in (7) is for convenience. When  $\varepsilon = 0$ , i.e., when materials across the interface are identical, then the material dissimilarity vanishes, or when  $\beta = 0$ , real and imaginary parts of (7) become:

$$\begin{aligned} K_I &= (c_R + d_R)K_1 - (c_I + d_I)K_2 \\ K_{II} &= (c_I - d_I)K_1 + (c_R + d_R)K_2 \end{aligned}, \tag{8}$$

where:  $c = c_R + ic_I$  and  $d = d_R + id_I$ . Coefficients  $c(\omega)$  and  $d(\omega)$  in original form (tabulated) as obtained by He and Hutchinson (1988) are given in Appendix.

Relationship between the energy release rate  $G_0$  of the interface crack advancing in the interface and complex interface stress intensity factor  $K$  is, (Malyshev and Salganik, 1965);

$$G_0 = \left( \frac{\kappa_1 + 1}{\mu_1} + \frac{\kappa_2 + 1}{\mu_2} \right) \frac{K \bar{K}}{16ch^2(\varepsilon\pi)}. \tag{9}$$

The energy release rate  $G$  of the kinked crack ( $a_i0$ ) is given by:

$$G = \frac{\kappa_2 + 1}{8\mu_2} (K_I^2 + K_{II}^2). \tag{10}$$

Using equation (7), one obtains:

$$G = \frac{\kappa_2 + 1}{8\mu_2} [ (|c|^2 + |d|^2) K \bar{K} + 2Re(c d K^2 a^{2i\varepsilon}) ]. \tag{11}$$

To reduce expression (11),  $K$  should be written as:

$$K \equiv K_1 + iK_2 = |K| e^{i\gamma} L^{-i\varepsilon}, \quad (12)$$

where, according to (6)  $L$  is the length quantity characterizing the specific interface crack problem when  $a=0$ . The real angular quantity  $\psi$  will be used as the measure of the loading combination. Then by (9), (11) and (12):

$$G = q^{-2}G_0 \left[ (|c|^2 + |d|^2) + 2\text{Re}(cd e^{2i\bar{\psi}}) \right], \quad (13)$$

where:  $q = \sqrt{\frac{1-\beta^2}{1+\alpha}}$  and  $\bar{\psi} = \psi + \varepsilon \ln\left(\frac{a}{L}\right)$ .

When  $\varepsilon = 0$ , the stress intensity factors,  $K_I$  and  $K_{II}$  and  $G$  are independent of  $a$ . This is the case for similar moduli across the interface ( $\alpha = \beta = 0$ ). By (3)  $\varepsilon = 0$  whenever  $\beta = 0$  regardless of the value of  $\alpha$ . The oscillatory behavior of the interface crack fields and  $a$ -dependence of  $G$  only appear when  $\beta \neq 0$ .

In the normal loading case, which is considered here, the contact between the crack faces is possible. The contact zone is very small and in that case and it can be neglected. For the shear loading case, however, contact zone can be as large as  $1/3$  of the crack length. This means that the kink length should be longer than the contact zone. Again, here is considered the case of normal loading, so this does not apply.

Since the crack has been taken to kink downward, the loading combinations which result in  $K_I > 0$  (i.e. an opening at the tip) and an opening at the kink will generally require  $K_1 > 0$  and  $\psi \geq 0$ .

When  $\beta \neq 0$ , and therefore  $\varepsilon \neq 0$ , the interface crack with  $a=0$  suffers contact between the crack faces within some distance from the tip. Contact between crack faces is less likely for the kinked crack ( $a > 0$ ,  $\omega > 0$ ) loaded such that  $K_I$  and  $K_{II}$  are positive, since this will open up the crack at the kink. However, contact will occur for  $\varepsilon \neq 0$ , when  $a$  is sufficiently small compared to  $L$ .

When  $a/L$  becomes sufficiently small,  $G$  oscillates between a maximum  $G_+$  and minimum  $G_-$  that are:

$$\begin{aligned} G_+ &= q^{-2}G_0(|c| + |d|)^2 \\ G_- &= q^{-2}G_0(|c| - |d|)^2 \end{aligned}, \quad (14)$$

and which depend on  $K_1$  and  $K_2$  only through  $G_0$ . For values of  $a/L$

outside the oscillatory range,  $G$  approaches  $G^*$  given by (13) with  $\bar{\psi} = \psi$ , i.e.,

$$G^* = q^{-2}G_0 [ (|c|^2 + |d|^2) + 2\text{Re}(cd e^{2i\psi}) ] \quad (15)$$

Note that  $G^*$  coincides with  $G$  when  $\varepsilon=0$ . Contact between the crack faces is not valid for the predictions of  $G$  from (13), when  $a/L$  is in range where oscillatory behavior occurs.

In presenting results for the energy release rate when  $\varepsilon \neq 0$  a feature  $G^*$  appears. From a physical standpoint  $G^*$  should be relevant if there exists a crack emanating from the interface whose length is greater than the zone of contact. That is,  $G^*$  should be relevant for testing for kinking if the fracture process on the interface is large compared to the contact zone predicted in the idealized elasticity solution. If it is not, then more attention must be paid to the  $a$ -dependence of  $G$  and to consideration of contact. In any case,  $G^*$  should play a prominent role in necessary conditions for a crack kinking out of an interface because once nucleated the kinked crack has an energy release rate which rapidly approaches  $G^*$  as it's length increases.

### 3 Results and discussion

Plots of  $G/G_0$  versus  $\omega$  for various  $\psi$  are shown in Figure 2 for  $\alpha = 0.75, 0.5, 0, -0.5, -0.75$ , by using equation (13) and symbolic programming package *Mathematica*.

All information for crack kinking out of an interface can be derived from  $c(\omega)$  and  $d(\omega)$  from equations (8-10). These coefficients are available in tabulated form in He and Hutchinson (1988). In this paper their results are approximated in present notation:

$$\begin{aligned} c(\omega) &= \frac{1}{2} \sqrt{\frac{1-\beta}{1+\alpha}} \left( e^{-\frac{i\omega}{2}} + e^{-\frac{3i\omega}{2}} \right) \\ d(\omega) &= \frac{1}{4} \sqrt{\frac{1-\beta}{1-\alpha}} \left( e^{-\frac{i\omega}{2}} - e^{-\frac{3i\omega}{2}} \right) \end{aligned} \quad (16)$$

These coefficients, given in tabulated form, were processed by *Mathematica* programming routine. The values from tables were entered into the program and the graphs for  $c(\omega)$  and  $d(\omega)$  were obtained. The curves for coefficients are best fitted by expressions given in (16).

This enables obtaining diagrams  $G/G_0$  and  $G^*/G_0$  versus  $\omega$  that are more accurate than for tabulated case of  $c(\omega)$  and  $d(\omega)$  by He and Hutchinson (1988).

This approximation is correct for  $\omega < 45^\circ$ , error is about 1%, and for  $\omega > 45^\circ$ , error is between 5% and 10%, as shown in Appendix.

The case with  $\beta = 0$  and  $\alpha \neq 0$  represents an insight into interface problems without the added complication of oscillations, or contact associated with nonzero  $\varepsilon$ . Roughly speaking,  $\alpha > 0$  implies that material 1 is stiffer than material 2, and vice versa. In this paper the crack is always taken to kink downward into material 2 so that the relevant range of loading is restricted to  $K_1 > 0$  and  $\psi > 0$ .

The qualitative features which emerge from the directional dependence of the energy release rate in Figure 2 are the following: The more compliant is the material into which the crack kinks, the larger is the energy release rate, all other factors being equal. Conversely, if the lower material, into which the crack kinks is relatively stiffer ( $\alpha < 0$ ) then the energy release rate is reduced. These features are consistent with the role of moduli differences across an interface when a crack approaches the interface from within one of the two materials. When the differences are relatively large the energy release rate for a crack kinking into the stiff material can be less than the interface release rate  $G_0$  for all combinations of loading, as can be seen in Figure 2(e) for  $\alpha = -0.75$ . This suggests that under combinations when the compliant material is tough and the stiff material and the interface are each relatively brittle with comparable toughness, the crack will tend to be trapped in the interface for all loading combinations. If the stiff material is even less tough than the interface the crack may leave the interface but not necessarily by kinking. For example, when  $\alpha = -0.75$  in Figure 2(e), the largest energy release rate occur when  $\omega$  is small approaching zero, suggesting that the crack may smoothly curve out of the interface. such a path would not necessarily satisfy  $K_{II} \cong 0$ .

As discussed in connection with equation (13),  $G$  is not independent of  $a$  when  $\varepsilon \neq 0$ , but  $G$  approaches  $G^*$  for all but very small  $a$ . Plots of  $G^*/G_0$  as a function of  $\omega$  are shown in Figure 3 for  $\alpha = 0.5$ ,  $\beta = 0.25$  and  $\alpha = -0.5$ ,  $\beta = -0.25$ .

Although the  $\beta$ -values in these examples are quite large the curves are quite similar to the curves in Figure 3 with the same values of  $\alpha$  and



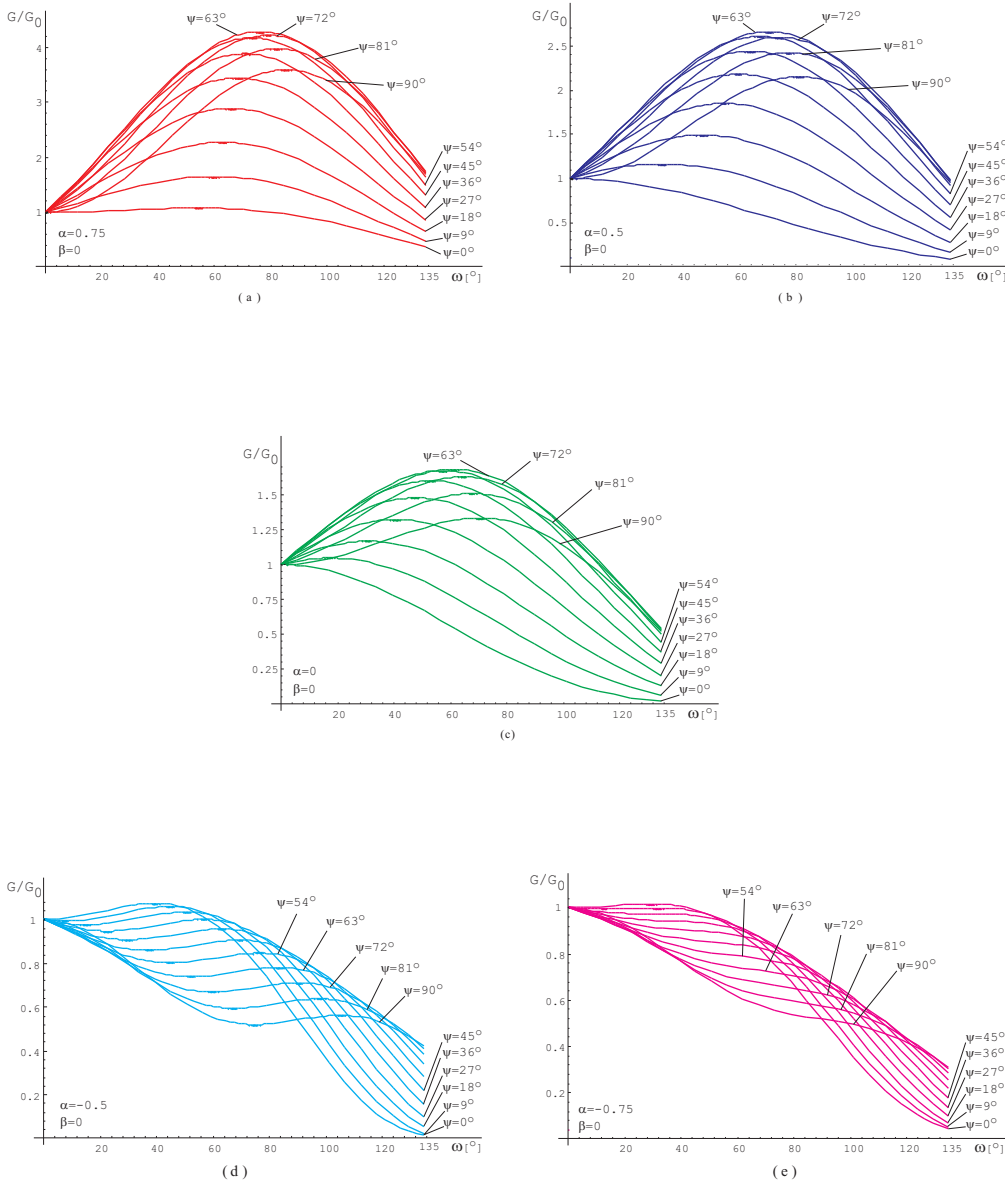


Figure 2: Variation of  $G/G_0$  with kink angle for loading combination specified by  $\psi = \arctg(K_2/K_1)$  for various values of  $\alpha$  and  $\beta = 0$ . The homogeneous case is in 2(c).

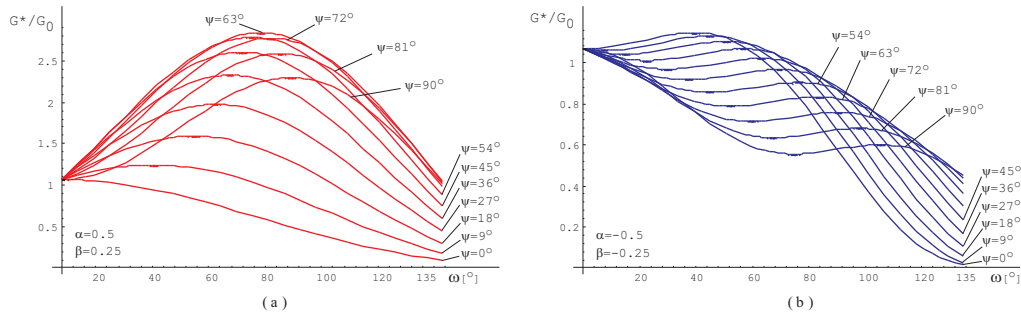


Figure 3:  $G^*/G_0$  versus  $\omega$  for two cases in which  $\beta \neq 0$

with  $\beta = 0$ .

## 4 Conclusion

The results for the kinked crack can be used to assess whether an interface crack will propagate in the interface or whether it will kink out of the interface. The simplest approach is to assume that the condition for propagation in an interface is  $G_0 = G_{0C}$  and that for propagation in material 2 is  $G_0 = G_{2C}$ . If  $G_{2C}$  is sufficiently large compared to  $G_{0C}$  the crack will never kink into material 2.

When material 2 is the more compliant material  $G_{2C}$  must be greater than the interface toughness  $G_{0C}$  by as much as 2.5 (for  $\alpha = 0.75$ ) if the crack is to stay in the interface for all  $\psi$ . On the other hand, when material 2 is relatively stiff (for  $\alpha = -0.65$ ) the crack will stay in the interface as long as  $G_{2C} \cong G_{0C}$ .

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## Appendix

Coefficients  $c(\omega)$  and  $d(\omega)$ , He and Hutchinson (1988).

$\alpha = 0, \beta = 0$			$\alpha = 0.56, \beta = 0.12$		
$\omega$	$c(\omega)$	$d(\omega)$	$\omega$	$c(\omega)$	$d(\omega)$
4	0.04820	0.02046	4	0.04279	0.01838
8	0.04981	0.02112	8	0.04216	0.01878
10	0.04988	0.02112	12	0.04208	0.01883
12	0.04992	0.02111	20	0.04204	0.01885
16	0.04996	0.02109	40	0.04202	0.01886
20	0.04998	0.02107	90	0.04201	0.01887
40	0.05001	0.02103			

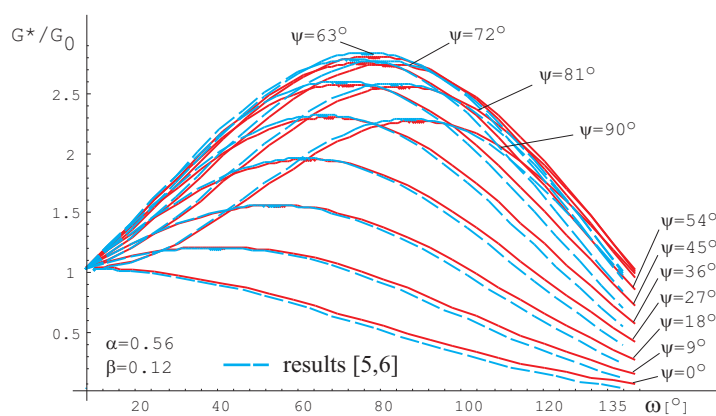


Figure 4: Comparative diagrams of  $G^*/G_0$  as function of  $\omega$  for  $\alpha = 0.56$ ,  $\beta = 0.12$ , obtained by the *Mathematica* programming routine (red lines) and results of He and Hutchinson (1988), for the same values of  $\omega$  (blue broken lines).

Submitted on June 2005.

## **Skretanje prsline sa interfejsa**

UDK 539.42, 539.421

Analizira se skretanje prsline sa interfejsa izmedju dva različita izotropna elastično - plastična materijala u uslovima ravanskog stanja deformacije. Analiza je fokusirana na inicijaciju skretanja pa se smatra da je segment prsline koji je van interfejsa mali u poredjenju sa segmentom na interfejsu. Analiza daje faktore intenziteta napona i brzinu oslobadjanja energije skrenute prsline u funkciji odgovarajućih veličina prsline na interfejsu. Brzina oslobadjanja energije je povećana ako prslina skreće u materijal veće popustljivosti, a smanjena je ako prslina skreće u krut materijal.