SELECTING THE WAY OF MEASURING THE PRICE EVOLUTION USING THE METHOD OF INDICES

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Abstract

The price indices *have a long history and a large variety of uses, from the adjustment of the wages, pensions and payments included in a long-term contract, the deflation of aggregates in National Accounts, to the elaboration of economic policies.*

Having identified the purpose of the index, we`ll have to choose the target index and the calculation formula, this operation being carried out based on the observed prices and on the quantity and quality weights.

In the statistical practice, the price index is often calculated by aggregating the elementary indices using the weighted arithmetic mean, using annual weights from a period that is previous to the reference period.

In this situation, the question about the possible impact of the weights update (by prices) on the interpretation of price indices becomes legitimate, also the question about the influence of using this approach on measuring the price change. We can get a possible answer to this question using the Lowe and Young indices introduced by the Consumer Price Index international manual.

Key words: inflation, price indices, consumer prices, weighting system, reference period, current period, update.

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The intent of aiming for a target index is important for at least two reasons. Aiming for an ideal target leads to a better reference for the calculation of the future indexes. It is necessary to set a measurable target that is able to quantify the value of any statistical distortion: the difference between what it is effectively measured and what it should be measured. Even the target index cannot be calculated in real time, it can be compiled afterwards, when the necessary data become available.

Possible targets for measuring the inflation

When the main purpose of the index is measuring "the inflation" or the "pure" price change, it can be argued that the ideal index could be a "basket-type" one of this type: $P_{Lo}\left(p^0, p^t, q^b\right)=\frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b}$.

The index of this type measures the ratio between the expenditures on the same "basket" (q_i^b) from two different time periods, namely 0 şi t. This respective index is known in the specialized literature as a Lowe-type index[1]. If b=0, the Lowe index becomes a Laspeyres-type index, $P_L\left(p^0, p^t, q^0\right)=\frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0}$ or if b=t then the index is a Paasche index:

$$P_P\left(p^0, p^t, q^t\right)=\frac{\sum_{i=1}^{n} p_i^t q_i^t}{\sum_{i=1}^{n} p_i^0 q_i^t} .$$

The Lowe index can be regarded as a general definition of the basket-type index, as long as it is not necessary to make reference to a specific period for the considered quantities in the selected basket.

An immediate question arises about how the quantities in the equation referring to the Lowe index are chosen. An obvious response should be that these quantities must be as representative as possible for the period that the index will be covering. It can be assumed that a good estimation of the representativity is given by some certain types of quantity averages from both the initial period and the final period. There are some types of indices that are using quantity averages for the aggregation on different levels. The best known are the Walsh şi Marshall-Edgeworth indexes[2].

The Walsh index.

1. In 1823 Joseph Lowe published a study on agriculture, commerce, finance – developing the concept of a price index as a monetary value change of a certain set, or basket of goods and services, this approach still being in use.

2. Alfred Marshal (1842-1924) and Francis Ysidro Edgeworth (1845 –1926) important economists.

$$P_W\left(p^0, p^1, q^0, q^1\right) = \frac{\sum_{i=1}^{n} p_i^1 \sqrt{q_i^0 q_i^1}}{\sum_{j=1}^{n} p_j^0 \sqrt{q_j^0 q_j^1}} = \frac{\sum_{i=1}^{n} \left(p_i^1 / \sqrt{p_i^0 p_i^1}\right)\sqrt{s_i^0 s_i^1}}{\sum_{j=1}^{n} \left(p_j^0 / \sqrt{p_j^0 p_j^1}\right)\sqrt{s_j^0 s_j^1}} =$$

$$= \frac{\sum_{i=1}^{n} \left(p_i^1 / p_i^0\right)\sqrt{\left(s_i^0 s_i^1\right) / \left(p_i^1 / p_i^0\right)}}{\sum_{j=1}^{n} \sqrt{\left(s_i^0 s_i^1\right) / \left(p_i^1 / p_i^0\right)}} = \sum_{i=1}^{n} \left(p_i^1 / p_i^0\right) s_i^w ,$$

where s_i^0 and s_i^1 represent the weights ($p_i q_i \Big/ \sum_{i=1}^{n} p_i q_i$) in the periods 0 and 1 respectively.

The Marshall-Edgeworth index.

$$P_{ME}\left(p^0, p^1, q^0, q^1\right) = \frac{\sum_{i=1}^{n} p_i^1 \left\{\left(q_i^0 + q_i^1\right)/2\right\}}{\sum_{j=1}^{n} p_j^0 \left\{\left(q_j^0 + q_j^1\right)/2\right\}} = \frac{\sum_{i=1}^{n} p_i^1 \left\{\left(\frac{v_i^0}{p_i^0} + \frac{v_i^1}{p_i^1}\right) \Big/ 2\right\}}{\sum_{j=1}^{n} p_j^0 \left\{\left(\frac{v_i^0}{p_i^0} + \frac{v_i^1}{p_i^1}\right) \Big/ 2\right\}} =$$

$$= \frac{\sum_{i=1}^{n} \left(p_i^1 / p_i^0\right) p_i^0 \left\{\left(\frac{v_i^0}{p_i^0} + \frac{v_i^1}{p_i^1}\right) \Big/ 2\right\}}{\sum_{j=1}^{n} p_j^0 \left\{\left(\frac{v_i^0}{p_i^0} + \frac{v_i^1}{p_i^1}\right) \Big/ 2\right\}}$$

$$= \frac{\sum_{i=1}^{n} \left(p_i^1 / p_i^0\right) \left\{\left[v_i^0 + \left(v_i^1 / \left(p_i^1 / p_i^0\right)\right)\right] / 2\right\}}{\sum_{j=1}^{n} \left\{\left[v_i^0 + \left(v_i^1 / \left(p_i^1 / p_i^0\right)\right)\right] / 2\right\}} = \sum_{i=1}^{n} \left(p_i^1 / p_i^0\right) s_i^{ME} ,$$

where $v_i^t = p_i^t q_i^t$ represent the value in the both periods 0 and 1.

Consequently, both Walsh index and the Marshall-Edgeworth index are special cases of the Lowe index. If $q_i^b = \left(q_i^0 q_i^1\right)^{1/2}$, then the Lowe becomes a Walsh index and in the case when $q_i^b = (q_i^0 + q_i^1)/2$, then the Lowe index becomes a Marshall-Edgeworth index.

The Walsh index is known in the statistical literature as a superlative index, and so are the Fisher and the Tornqvist indexes. The notion of a superlative index can be defined using the axiomatic approach[1], which implies the respective indexes are put to specific mathematical tests analyzing their properties.

The Walsh index, compared to the Fisher[2] and Tornqvist[3] indexes, has a feature that is important from a practical point of view: it can be calculated as a weighted arithmetic mean of the price ratios. The Walsh index can be calculated by the aggregation of the elementary indexes – using their weight in the total expenditure.

Other options for defining the target index may exist; the Lowe index calculation formula that use the quantities from the reference period being an obvious solution. It`s estimated that, in practice, the Lowe index should be expressed in value weights terms – by directly using the quantities:

$$P_{Lo}\left(p^0, p^t, q^b\right) = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} \left(p_i^t / p_i^0\right) p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^{0b},$$

unde $s_i^{0b} = \frac{p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\left(p_i^0 / p_i^b\right) p_i^b q_i^b}{\sum_{i=1}^{n} \left(p_i^0 / p_i^b\right) p_i^b q_i^b} = \frac{\left(p_i^0 / p_i^b\right) s_i^b}{\sum_{i=1}^{n} \left(p_i^0 / p_i^b\right) s_i^b}, \; s_i^b = \frac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}.$

The price ratios at an elementary level are aggregated by using a hybrid of the value weights, s_i^{0b} (the quantities from the b time period expressed by the prices in the 0 period). The hybrid weights can be calculated by using the method of updating (through prices) of the value in the period b to the value in the 0 period.

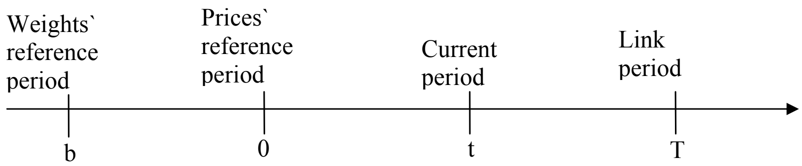
1. The Walsh index meets 16 of the 20 tests of the axiomatic approach. The only index meeting all the 20 tests is the Fisher index. The Laspeyres and Paasche indices are on the second place with 17 tests met, but the fact that they don`t meet one of the most important tests – the one referring to switching the period – make these two indices to be subject to severe limitations.

2 Irving Fisher (1867-1947) was an American economist from the neoclassical school.

3 The Törnqvist index theory has been attributed to Leo Törnqvist (1936).

Calculating formulas used on a regular basis

The most frequent situation is that, when the monthly index has to be calculated - for example - for the months following the initial period 0, until the moment T. The period between 0 and T can be one year or longer, depending on how often the weights are updated. The diagram below shows the situation when the information about the value indicator considered for building the weights, is available in a certain period b, which is prior to the period 0 of reference for the index.



In this case, the situation is complicated by the fact that – most often - the weight`s reference period precedes and is longer than the prices` reference period. Usually, the weights refer to a period of an year, while the compared prices are collected monthly over an year, which is next to the year of reference for the weights.

Most statistical offices in the world, if not all of them, **calculate the consumer prices indices** by using **the weighted arithmetic mean** of the elementary indices. Starting from this, it`s estimated that every national statistical office is going to decide whether the weights have to be updated from the period when the households` expenditures are available to the prices` reference period or they should be used without any adjustment. The decision depends on both the target index and on practical and empirical reasons.

Updating the weights – the Lowe index

By the updating approach, the weights are brought to the same period as the reference prices. If a statistical office decides to update the weights, then the index they get is Lowe-type one, which is defined as an index with fixed quantities that measures the value of the same (annual) “basket” of goods and services from one period to another.

The updated weights are set by multiplying the expenditures available at elementary level in the period b by the index of the period 0 as against the period b, at the same level. A price index, calculated by using the updated annual weights, measures the change from a month to another of the total cost of an annual “basket” of goods and services. Usually, the reference year of the weights precedes the prices` reference period by at least two years.

The Lowe index is by no means a Laspeyres index, taking into consideration that the reference periods of the weights and of the prices – respectively – are not the same.

When the weights are attached to one year and the prices are monthly ones, it`s impossible to calculate – even retrospectively – a Laspeyres price index. The Lowe index can be expressed as a ratio of two Laspeyres indices, one of them being calculated for the period from b to 0, while the other one is calculated for the period from b to t.

$$P_{Lo}\left(p^0, p^t, q^b\right) = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} \Bigg/ \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b} = P_L\left(p^b, p^t, q^b\right) / P_L\left(p^b, p^0, q^b\right)$$

It results from this equation that the Lowe index calculated for the period between 0 and t will provide the same procentual change as the one measured by the ratio of the two Laspeyres indices. The price change from one month to another can be measured directly by the Lowe index by updating the weights of the annual period b as against to the reference month taken into consideration for the prices.

Comparing the Lowe and Laspeyres indices

The usual Laspeyres price index, between the months 0 and t, can be defined by using the prices and quantities of both the reference month 0 and the current month:

$$P_L\left(p^0, p^t, q^0\right) = \frac{\sum_{i=1}^{n} p_i^t q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{\sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} \left(\frac{p_i^t}{p_i^0}\right) s_i^0 = \sum_{i=1}^{n} s_i^0 r_i \equiv r^*,$$

where $r_i = p_i^t / p_i^0$ and $s_i^0 \equiv \frac{p_i^0 q_i^0}{\sum_{j=1}^{n} p_j^0 q_j^0}$; i=1,….,n

The quantity index Laspeyres $Q_L\left(q^0, q^b, p^0,\right)$, which compares the quantities q^b from the year b with the corresponding quantities q^0 from the month 0 - by the prices p^0 in the month 0, is shown below:

$$Q_L\left(q^0, q^b, p^0\right) = \frac{\sum_{i=1}^{n} p_i^0 q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^0} = \frac{\sum_{i=1}^{n} \left(\frac{q_i^b}{q_i^0}\right) p_i^0 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} = \sum_{i=1}^{n} \left(\frac{q_i^b}{q_i^0}\right) s_i^0 = \sum_{i=1}^{n} s_i^0 t_i \equiv t^*,$$

where $t_i = q_i^b / q_i^0$; i=1,....,n

The relation between the Lowe index $P_{Lo}\left(p^0, p^t, q^b\right)$, which uses the quantities in the year b as weights for the comparison of the prices in the month t with those in the month 0, and the corresponding Laspeyres index $P_L\left(p^0, p^t, q^0\right)$, which uses the quantities in the month 0 as weights for the comparison of the prices in the month t with those in the month 0 is as follows:

$$P_{Lo}\left(p^0, p^t, q^b\right) = \frac{\sum_{i=1}^{n} p_i^t q_i^b}{\sum_{i=1}^{n} p_i^0 q_i^b} = P_L\left(p^0, p^t, q^0\right) + \frac{\sum_{i=1}^{n}\left(r_i - r^*\right)\left(t_i - t^*\right)}{Q_L\left(q^0, q^b, p^0\right)}.$$

It follows that the Lowe price index that uses the quantities in the year b as weights, $P_{Lo}\left(p^0, p^t, q^b\right)$, equals the Laspeyres index that uses the quantities in the month 0 as weights, $P_L\left(p^0, p^t, q^0\right)$, plus the ratio between the covariance $\sum_{i=1}^{n}\left(r_i - r^*\right)\left(t_i - t^*\right)s_i^0$ and the quantity index Laspeyres of the month 0 compared to the year $Q_L\left(q^0, q^b, p^0,\right)$.

Although the sign and the value of the covariance term $\sum_{i=1}^{n}\left(r_i - r^*\right)\left(t_i - t^*\right)s_i^0$ are actually an empirical question, we`ll show here a few reasonable enunciations on the connection between the two indices:

- ✓ If the covariance term is zero, the Lowe price index coincides with the Laspeyres price index
- ✓ If this covariance is negative, the Lowe index will be lower than the Laspeyres index

✓ If the covariance is positive, the Lowe index will be higher than the Laspeyres index.

From a conceptual point of view, the Lowe index is clear enough and it assumes that the quantities are constant over the entire period from the year b up to the moment when their update takes place (period T). If the consumer doesn`t respond to the price changes by the substitution of goods and services, the Lowe index can be regarded as a superlative index. From an economical point of view this index is quite improbable – and the Lowe index, calculated for the period from 0 to t, will most probable be higher than the Laspeyres index for the same period, calculated by using the weights from the period 0.

Using the original weights – the Young index

If the annual weights aren`t updated, we consider a Young index: $P_Y\left(p^0, p^t, s^b\right) = \sum_{i=1}^{n} s_i^b \left(p_i^t / p_i^0\right)$, where $s_i^b = \dfrac{p_i^b q_i^b}{\sum_{i=1}^{n} p_i^b q_i^b}$.

The calculation of the Young index can be justified in the situation when the expenditures measured in the period b, which is the same for the weights, is a good estimation for the average of the expenditures incurred during the period from 0 to t. If the consumer has a normal substitution response, but the impact of the response doesn`t affect the weight of the expenditures at an elementary level, the Young index can be regarded as a superlative index.

The Young index is a fixed weights index, and not a fixed quantities index, this meaning that it doesn`t measure the buying cost changes of a fixed basket of goods and services – like in the case of the Lowe index.

From an economical point of view, the use of the Young index requires the existence of a substitution elasticity at an elementary level from the period b to the period 0, but not from the period 0 to the period t.

The difference between the Lowe and Young indices:

$$P_{Lo}\left(p^0, p^t, q^b\right) - P_Y\left(p^0, p^t, s^b\right) = \sum_{i=1}^{n} s_i^{0b}\left(p_i^t / p_i^0\right) - \sum_{i=1}^{n} s_i^b\left(p_i^t / p_i^0\right) = \sum_{i=1}^{n}\left(s_i^{0b} - s_i^b\right)\left(p_i^t / p_i^0\right).$$

It follows that the Lowe index allows a bigger weight to those elementary indices whose prices have been raised above the average of the period from b to 0 and conversely, smaller weights to the elementary indices whose prices have been raised below the average of the same time period. In the situation when no long-term exists for the prices, the Lowe index is higher than the Young index.

Comparing the Young and Laspeyres indices

There are cases when the statistical offices and the users of the statistical data regard the Young index defined earlier here as being an approximation of the Laspeyres index. That is why, it is interesting to compare the two indices:

$$P_Y\left(p^0, p^t, s^b\right) - P_L\left(p^0, p^t, q^0\right) = \sum_{i=1}^{n} s_i^b\left(\frac{p_i^t}{p_i^0}\right) - \sum_{i=1}^{n} s_i^0\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left(\frac{p_i^t}{p_i^0}\right) = \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]r_i$$

$$= \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left[r_i - r^*\right] + r^*\sum_{i=1}^{n}\left[s_i^b - s_i^0\right], \text{ deoarece } \sum_{i=1}^{n} s_i^b = \sum_{i=1}^{n} s_i^0 = 1 \text{ rezultă}$$

$$P_Y\left(p^0, p^t, s^b\right) - P_L\left(p^0, p^t, q^0\right) = \sum_{i=1}^{n}\left[s_i^b - s_i^0\right]\left[r_i - r^*\right]$$

The Young index $P_Y\left(p^0, p^t, s^b\right)$ equals the Laspeyres index $P_L\left(p^0, p^t, q^0\right)$, plus the covariance between the difference of the weights from the year b and the month 0 and the relative prices deviation as against their average value.

A legitimate question arises: What happens to the expenditures` weight in the case of a product whose price is increased? The answer to this question depends on the elasticity of the demand observed in the case of the respective product.

Conclusion

Choosing the way of setting the weighting system influences both the price indices interpretation, and the measuring of the procentual changes of the prices.

If the statistical offices use the price updating method for the weights in order to measure the price change in two months, it can be said that the index we get is a Lowe-type one. The result is similar to the ratio of two monthly Laspeyres-type indices.

If the relative quantities tend to remain constant, and if the consumer doesn`t respond to the relative price change by replacing products, the Lowe index can be considered a superlative index.

If the weights aren`t price updated, and the monthly price comparison is carried out by using the original weights, the index is a Young-type one. In this case, if the consumer replaces the products without changing the expenditures weight, then the Young index is a good estimation of the superlative index.

In the case of a long-term price increasing tendency, the Lowe index will be higher than the Young index. Because the Young index approves to some extent the substitution effect, while the Lowe index doesn`t, we can say that the traditional tendency of overestimation in the case of the Laspeyres index is diminished by using the Young index.

For either long time periods or for periods when important structural changes happen, or for periods with unusual price movement it is recommendable that the weights should not be automatically price updated. The potential errors of the weighting system generated by the price updating method can be avoided by the frequent update of the weights and by reducing the time gap between the reference period of the weights and the reference period of the prices.

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