

**PROCESS PARAMETERS EFFECT ON  
MATERIAL REMOVAL MECHANISM AND CUT  
QUALITY OF ABRASIVE WATER JET  
MACHINING**

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## PROCESS PARAMETERS EFFECT ON MATERIAL REMOVAL MECHANISM AND CUT QUALITY OF ABRASIVE WATER JET MACHINING

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**Abstract.** *The process of the abrasive water jet cutting of materials, supported by the theories of fluid mechanics, abrasive wear and damage mechanics, is a high-tech technologies that provides unique capabilities compared to conventional machining processes. This paper, along the theoretical derivations, provides original contributions in the form of mathematical models of the quantity of the cut surface damage, expressed by the values of cut surface roughness. The particular part of this paper deal with the results of the original experimental research.*

*The research aim was connected with the demands of industry, i.e. the end user. Having in mind that the conventional machining processes are not only lagging behind in terms of quality of cut, or even some requests are not able to meet, but with the advent of composite materials were not able to machine them, because they occurred unacceptable damage (mechanical damage or delamination, fiber pull-out, burning, frayed edges).*

**Key words:** *Abrasive water jet cutting, Damage mechanics, aluminium specimens test, mathematical modeling*

### 1. INTRODUCTION

Abrasive water jet (AWJ) cutting is a non-conventional machining process that uses high velocity water with abrasives for cutting a variety of materials. Using the damages mechanics, as the basis of the machining process, the material damage is very small and can be controlled. It is a non-contact process which produces narrow kerf on the material, without heat affected zone. Abrasive water jet cutting has become a highly developed industry technology. It is most suitable process for very thick, highly reflective or highly thermal-conductive materials, as well as hard materials. Abrasive water jets can cut a wide range of thickness. Typical thickness are 100 mm for stainless steel, 120 mm for aluminium, 140 mm for stone, 100 mm for glass, but not limited. AWJ makes it possible to cut random contours, very fine tabs and filigree structures. Abrasive water jet cutting is capable of produce parts which do not require further processing with tolerances of  $\pm 0.1$  mm. Toxic fumes, recast layers, slag and thermal stress are totally eliminated.

Abrasive water jet cutting belongs among complicated dynamical and stochastic processes with incomplete information about mechanism and side effects character. In AWJ cutting, the final cut surface roughness and the dimensional accuracy depend on the process parameters including the water pressure, the abrasive mesh number, the abrasive mass flow rate, the feed rate, and the orifice and abrasive nozzle diameters [1], [2].

## 2. ABRASIVE WATER JET CUTTING

Abrasive water jet is the cutting tool. The cutting process is most similar to the grinding. The difference is that the abrasive particles are moved through the material by water rather than by a solid wheel. Abrasive water jet cutting process can be divided into subsequent steps:

- Transformation of the potential energy of water under high pressure into kinetic energy of a water jet.
- Transfer of a part of the kinetic energy of the high-speed water jet to abrasive particles by accelerating them and focusing the resulting abrasive water jet.
- Use of the kinetic energy of the abrasive particles to remove small chips of the work material.

In the process of abrasive water jet cutting the high pressure pump produces the required pressure up to 400 MPa. A high pressure supply line directs the pressurized water from the pump to the cutting head (Figure 1).

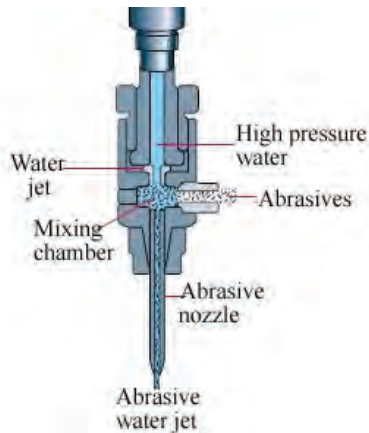


Fig. 1 Abrasive water jet cutting head

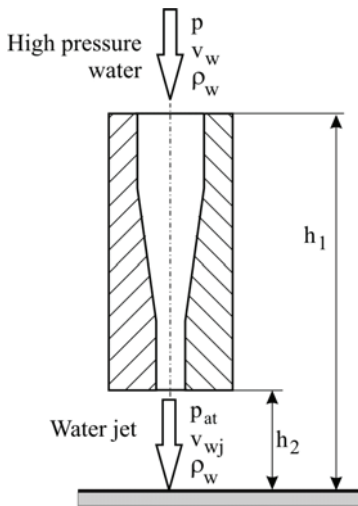
When the pressurized water comes out from the orifice, a water jet is created. The result is a very thin, extremely high velocity (approx. 900 m/s) water jet. Then, solid abrasive particles are added and mixed with the water jet. Resulting abrasive water jet is focused to the material through abrasive nozzle.

Bernoulli's equation is the law of conservation of energy applied to an ideal fluid as follows:

$$p + \frac{\rho_w v_w^2}{2} + \rho_w gh = const \tag{1}$$

where:  $p$  - water pressure  
 $v_w$  - velocity of water  
 $\rho_w$  - density of water  
 $g$  - acceleration due to gravity and  
 $h$  - height of the observed points above the reference plane

By observing the leakage of high pressure water jets in the air and using equation (1), one can determine the leakage velocity of water jet from a nozzle based on water pressure.



$$p + \frac{\rho_w v_w^2}{2} + \rho_w gh_1 = p_{at} + \frac{\rho_w v_{wj}^2}{2} + \rho_w gh_2 \text{ or}$$

$$p - p_{at} = \frac{1}{2} \rho_w (v_{wj}^2 - v_w^2) + \rho_w g (h_2 - h_1)$$

for  $p_{at} \ll p$ ;  $v_{wj} \gg v_w$ ;  $h_1 \approx h_2$

$$p = \frac{1}{2} \rho_w v_{wj}^2$$

Fig. 2. Bernoulli's equation applied to the leakage of water jets in the air

If we ignore the difference in altitude (several millimeters) and assuming that the speed of the water on nozzle entrance is negligible compared to the speed of the jet at the nozzle exit (several hundred times), and the atmospheric pressure (1 bar) is much smaller than the water pressure at the entrance to the nozzle (4000 bar), we get the equation for calculating the velocity of the water jet after exiting the water nozzle:

$$v_{wj} = \sqrt{\frac{2p}{\rho_w}} \tag{2}$$

A schematic diagram of a high-speed water jet in air is shown in Figure 3. The jet consists of three regions, namely, the initial region, the main region, and the final region.

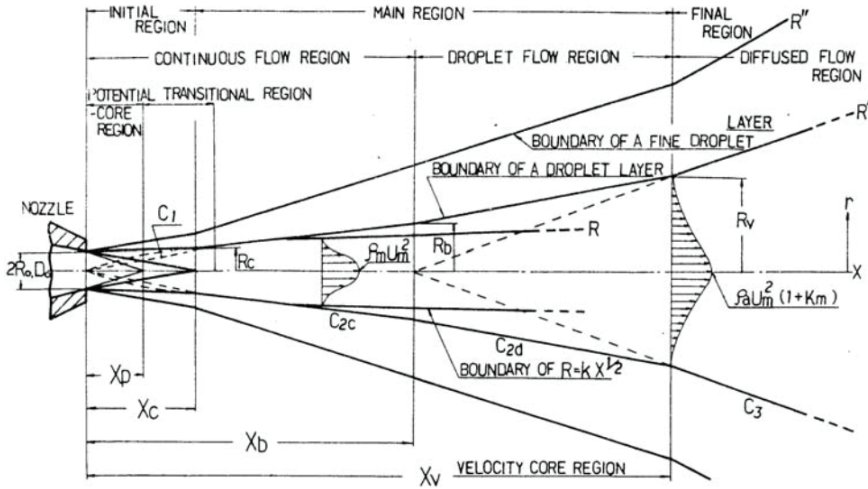


Fig. 3 Structure of a water jet in air [3]

Leakage velocity of water jet from a nozzle is crucial because its role is to accelerate the abrasive particles. Due to the extra weight, abrasive particles, however, cannot achieve velocity of water jet but only a part of that velocity.

The volume flow rate of water may be expressed as:

$$q_w = A_{orifice} v_{wj} \tag{3}$$

$$\text{for } v_{wj} = \sqrt{\frac{2p}{\rho_w}} \text{ and } A_{orifice} = \frac{\pi}{4} d_0^2$$

where  $d_0$  is water orifice diameter,

$$q_w = \frac{\pi}{4} d_0^2 \sqrt{\frac{2p}{\rho_w}} \tag{4}$$

Total power of the water jet can be given as:

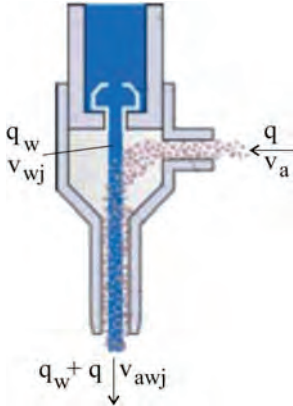
$$P_{wj} = p q_w \tag{5}$$

$$P_{wj} = p \frac{\pi}{4} d_0^2 \sqrt{\frac{2p}{\rho_w}}$$

$$P_{wj} = \frac{\pi}{4} d_0^2 \sqrt{\frac{2p^3}{\rho_w}} \tag{6}$$

As the high velocity water jet streams through orifice into the mixing chamber, low pressure (vacuum) is created within the mixing chamber. Metered abrasive particles are

introduced into the mixing chamber through a port (Figure 4). During mixing process, the abrasive particles are gradually accelerated due to transfer of momentum from the water phase to abrasive phase and when the jet finally leaves the abrasive nozzle, phases, water and abrasive, are assumed to be at same velocity.



where:

$q$  - abrasive particle flow rate

$v_a$  - initial velocity of abrasive particles

Fig. 4 Mixing process

The law of conservation of momentum says that the total momentum of any closed system, i.e., the vector sum of the momentum vectors of all the things in the system, is a constant. The momentum of air before and after mixing will be neglected due to very low density. Further, it is assumed that after mixing both water and abrasive phases attain the same velocity of abrasive water jet. Moreover, when the abrasive particles are fed into the water jet through the port of the mixing chamber, their velocity is also very low and their momentum can be neglected.

$$q_w v_{wj} + q v_a = (q_w + q) v_{awj} \tag{7}$$

$$v_{awj} = \frac{q_v}{q_w + q} v_{wj}$$

$$v_{awj} = \frac{v_{wj}}{1 + \frac{q}{q_w}} \tag{8}$$

As during mixing process momentum loss occurs as the abrasives collide with the water jet and at the inner wall of the abrasive nozzle multiple times before being entrained, velocity of abrasive water jet is given as,

$$v_{awj} = \eta \frac{v_{wj}}{1 + \frac{q}{q_w}} \tag{9}$$

where  $\eta$ - momentum loss factor, whose values lies arond 0.65-0.85 [4].

The abrasive flow rate determines the number of impacting abrasive particles as well as their kinetic energies. The energy of the abrasives can then be expressed as:

$$E_{awj} = \frac{1}{2} q v_{awj} = \frac{1}{2} q \eta^2 \frac{v_{wj}^2}{\left(1 + \frac{q}{q_{wj}}\right)^2} \quad (10)$$

Combination of equations (2) and (10) gives abrasive particles kinetic energy (power) needed to overcome the fracture energy of the material in order to damage (cut) workpiece material.

$$E_{awj} = \frac{1}{2} q \eta^2 \frac{2p}{\rho_v} \frac{q_{wj}^2}{(q_{wj} + q)^2} \quad (11)$$

### 3. MATERIAL REMOVAL MECHANISM OF ABRASIVE WATER JET MACHINING

High-velocity-water jets used in water jet processes can be categorized according to the fluid medium as either pure water jets or abrasive water jets.

The most common water jets used in water jetting processes are continuous pure water jets in air issued from a nozzle having a circular cross section. This type of water jet is widely used in water jetting industries for cleaning, surface preparation, and cutting of soft materials.

The material removal capability of abrasive water jets, in which abrasive particles are added to the water stream, is much larger than the material removal capability of the pure water jets. In an abrasive water jet, the stream of the water jet accelerates abrasive particles, which erode the workpiece material.

#### 3.1. Micro-mechanism of material removal in abrasive water jet cutting

Impact of solid particles is the main mechanism in the process of removing material by abrasive water jet [5]. Meng and Ludema [6] have defined a sub-mechanism for separating solid particles from the surface of the workpiece material, such as cutting and brittle fracture. These mechanisms do not operate separately but simultaneously. Presence of individual mechanisms of separation depends on many factors, such as stroke angle, the kinetic energy of abrasive grains, abrasive particle shape, material properties of the workpiece and ambient conditions.

Considering the mechanical properties and behavior on impact, the material of the workpiece can be classified into two groups. Some belong to the group of ductile materials, which are characterized by deformation properties, while others are brittle.

For ductile (deformable) material, the process of separating the material is divided into two mechanisms: micro-cutting and separating by material plastic deformation (Figure 5) [7].

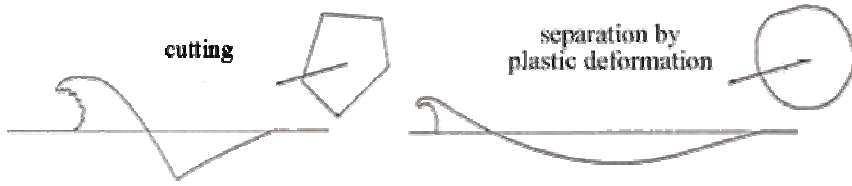


Fig. 5. The process of separating the ductile material

When observing the cutting process of brittle materials, a series of researches led to the identification of the mechanism of separation of materials, consisting of the phenomenon of brittle fracture (Figure 6) and plastic deformation. At low angles of attack are visible scratches, but it occurs to some extent and intercrystalline fracture [8], [9]. In contrast, intercrystalline fracture is the predominant method of removing material at the corner of the administrative impact. Traces of plastic deformation are present, but to a much lesser extent than at low angles of attack.

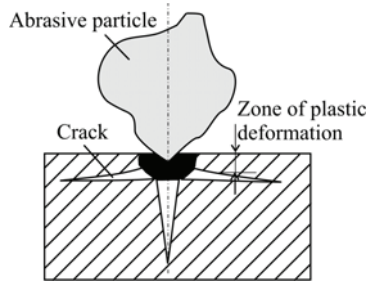


Fig. 6. Impact of abrasive particle in the surface of brittle materials

### 3.2. Macro-mechanism of material removal in abrasive water jet cutting

Hashish [10] proposed a general model, in which a stable cutting process takes place to a certain depth of penetration of abrasive water jet, followed by the formation of steps on the surface of the cut. Below the critical depth, the processing is unstable resulting in the creation of striated or wavy surface of the cut.

With increasing depth and creating steps, the removal mechanism is changing from cutting to the separating material by plastic deformation. The above-described mechanism, cyclic repeating, resulting in different types of material damage, which is the subject of study of damage mechanic.

The biggest problem with abrasive water jet machining, was reflected in disparity of the machined surface quality. This disparity is manifested by different parameters of cut quality as follow: surface roughness, machined surfaces deviation from the vertical plane-taper of the cut and the appearance of curved lines on machined surface-striate formation [11], as shown in Figure 7. All these phenomena significantly affect the restrictions of using abrasive water jet machining.





Fig. 7. Cut surface generated in abrasive water jet cutting of aluminium alloy

The cut surfaces produced by abrasive water jet cutting typically exhibit a smooth upper zone followed by a lower striated zone. These phenomena can be related to the jet loss of energy during the cutting process, e.g. deformation of the sharp edges of the abrasive particles as illustrated in Figure 8 [12].

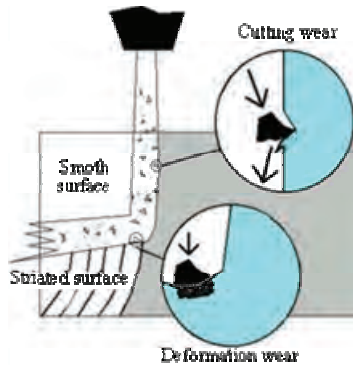


Fig. 8. Formation of different regions in abrasive water jet cutting

## 4. EXPERIMENTAL WORK

### 4.1. Cut quality

In the abrasive water jet cutting "cut quality" is a term that describes the combination of characteristics such as geometry of cut (kerf width -  $w$ , kerf taper -  $\alpha$ ) and cut surface quality (cut surface roughness -  $R_a$ ). Standards for describing the cut quality, resulting in abrasive water jet cutting, are not yet established [13]. Parameters that define the cut quality (geometric characteristics of cut quality and cut surface quality) in abrasive water jet cutting are shown in Figure 9.

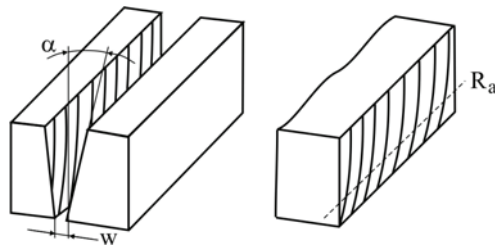


Fig. 9. Characteristics of cut quality in AWJ

The surface roughness is used to describe the cutting surface and gives an indication of whether the subsequent machining required. It is defined using the value of roughness average  $R_a$ . Cut surface can never be ideally smooth. It consists of small, finely spaced surface irregularities (micro irregularities - roughness) formed in the course of treatment. Additionally, there are surface irregularities of greater spacing (macro irregularities - waviness), which may be periodically repeatable.

#### 4.2. Experimental set-up

A series of water jet cutting experiments were conducted using a Byjet 4022 abrasive water jet cutting machine (Bystronic AG, Switzerland). As workpiece material, aluminium alloy AA-ASTM 6060 (EN: AW-6060; ISO: Al MgSi) was used. Alloy 6060 is one of the most popular of the 6XXX series alloys. Typical uses include architectural sections, sections fit for forming processes and automotive parts. The aluminium alloy was chosen as a workpiece material because the material is very attractive, possess resistance to corrosion and can provide significant value for the end user. Also, aluminum and its alloys are characterized by high reflectivity and thermal conductivity. This makes them relatively difficult to cut with lasers. Abrasive water jet cutting, which does not create an observable heat affected zone, is much more useful for cutting aluminum for modern applications.

Although AWJ cutting involves a large number of variables and virtually all these variables affect the cutting results (kerf width, taper and surface roughness), only few major and easy-to-adjust dynamic variables were considered in the present study. Those are: feed rate (the speed at which the cutting head moves along workpiece during cutting operation), material thickness and abrasive flow rate. The other process parameters were kept constant using the standard machine configuration ( $d_0 = 0.3$  mm;  $d_A = 1.02$  mm;  $p = 400$  MPa).

#### 4.3. Results and discussion

In the present study, surface roughness as assessed by the centre-line average roughness  $R_a$  (according to standard ISO 4287:1997) was used in evaluating the cut quality. Surface roughness was measured at upper and lower region of the cut surface, and at the middle of the cut. These measurements were taken for each cut away from the ends of the slots to eliminate any effect of the cutting process at the jet entry and exit. The surface roughness was measured perpendicularly to the jet penetration axis, and parallel to the cutting head feed direction.

The cut surface has better quality at upper region (entrance area) of the jet. From the middle of the thickness downwards, the surface quality deterioration is observed. As the penetration depth of abrasive water jet increases, the jet loses its energy due to the jet–material interaction, mutual particle impacts, etc. This situation results in rougher surface characteristics at the lower region of the cut surface. Figure 10 shows dependence of roughness average (Ra) at upper, middle and lower region of the cut surface of different feed rate values for material thickness of 10 mm.

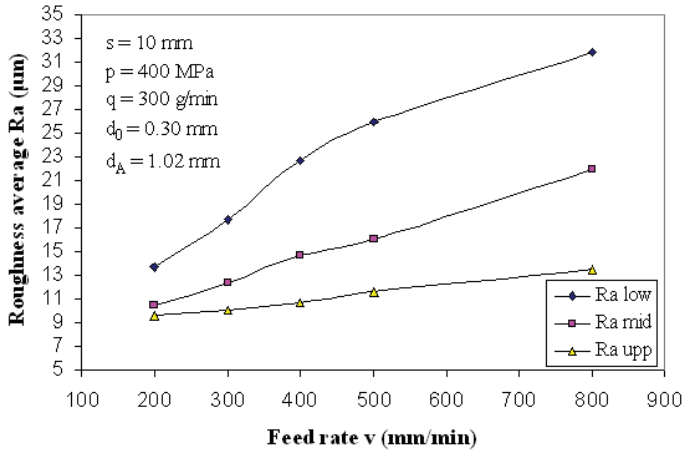


Fig. 10. Roughness average Ra in dependence of feed rate when material thickness is 10 mm at upper, middle and lower zone of the cut

The results of determining surface roughness at lower region of the cut surface with respect to the material thickness, feed rate and abrasive flow rate are graphically represented on Figure 11.

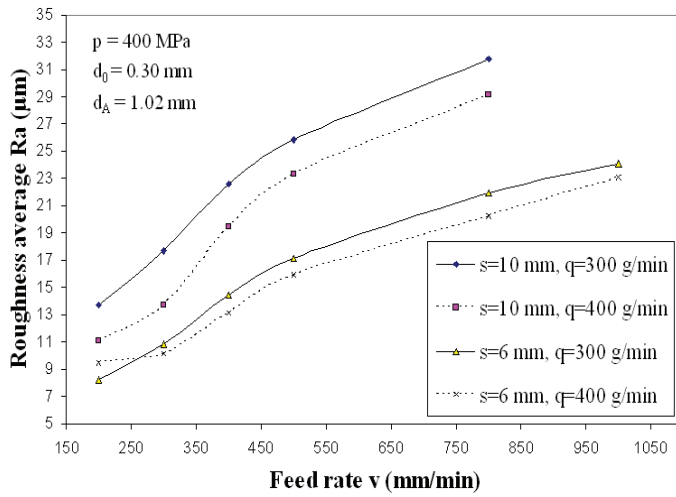


Fig. 11. Roughness average Ra in dependence of material thickness, feed rate and abrasive flow rate

It can be noticed that the surface roughness significantly increases as the feed rate increase. This may be anticipated as increasing the feed rate allows less overlap machining action and fewer abrasive particles to impinge the surface, deteriorating surface quality [14].

The influence of abrasive flow rate is found to be less significant on surface roughness. The increase in the number of impacting particles contributes to the improved surface finish. A high number of abrasive particles involved in mixing increases the probability of particle collision that decreases the average diameter of the impacting particles, so the roughness decreases with an increase of the abrasive flow rate. These results are in accordance with the literature [15].

The quantitative description of the process parameters effect on cut surface roughness was performed. Full factorial design for input factors (material thickness, feed rate and abrasive flow rate) at two levels, and output factor (roughness average Ra) with four centre point replications was adopted. Among the many process parameters that influence the cutting quality, three are selected and considered as factors in the experimental phase (Tab. 1).

Tab. 1. Process parameters and their levels

Factors	Factor level		
	-1	0	+1
Material thickness $s$ (mm)	6	8	10
Feed rate $v$ (mm/min)	200	400	800
Abrasive flow rate $q$ (g/min)	300	350	400

To identify the process parameters that are statistically significant in the process, the analysis of variance is performed. The significance of independent variables is interpreted in the Pareto chart. Pareto chart (Fig. 12) shows that feed rate, material thickness and abrasive flow rate have found to be the most sufficient factors that affects the cut surface roughness at abrasive water jet cutting in the experiment.

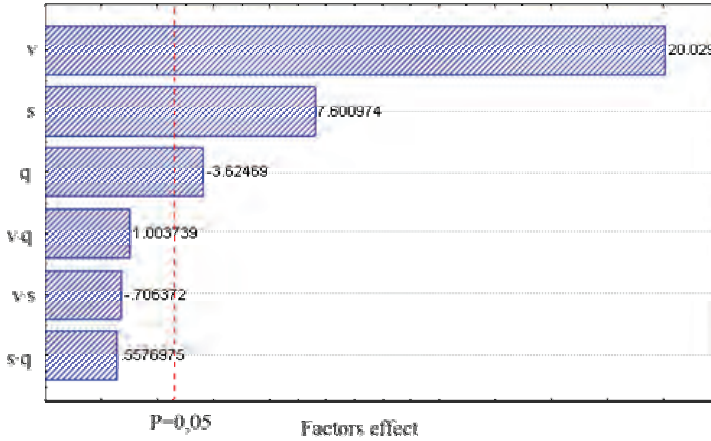


Figure 12. Pareto chart of level of significance for independent factors and their interactions at a 95% confidence interval

For the purposes of regression analysis were selected dimensions chosen factors (variables) as: s (mm), v (mm/min), q (g/min). To get the solution that best fits the experimental results, for the mathematical model of roughness average a power function is chosen (Eq.12):

$$R_a = C s^{p_1} v^{p_2} q^{p_3} \tag{12}$$

where  $R_a$  is an output variable, s, v and q are input (independent) variables, and C,  $p_1$ ,  $p_2$  and  $p_3$  are regression coefficients.

A logarithmic transformation of the (Eq. 12) in form of power function into a linear function

$$\ln R_a = \ln C + p_1 \ln s + p_2 \ln v + p_3 \ln q \tag{13}$$

allows us to perform linear regression technique.

The STATISTICA software package is used to determine regression equation coefficients, which give the level of roughness average  $R_a$  as a function of independent variables. The fit of the model is expressed by the coefficient of determination  $R^2 = 0.9862$ .

Within the regression analysis, the empirical model for roughness average could be expressed as:

$$R_a = 0.2913 \frac{s^{0.694} v^{0.642}}{q^{0.212}} \tag{14}$$

for the chosen dimensions:  $R_a$  ( $\mu\text{m}$ ) - roughness average,  $v$  (mm/min) – feed rate,  $s$  (mm) – material thickness,  $q$  (g/min) – abrasive flow rate

The regression analysis is applied in order to develop the response surfaces. Roughness average as a function of feed rate, and material thickness, for constant value of abrasive flow rate of 300 g/min is given in Fig. 13. These three-dimensional surface plot show predicted cut surface roughness as a function of independent variable - factors.

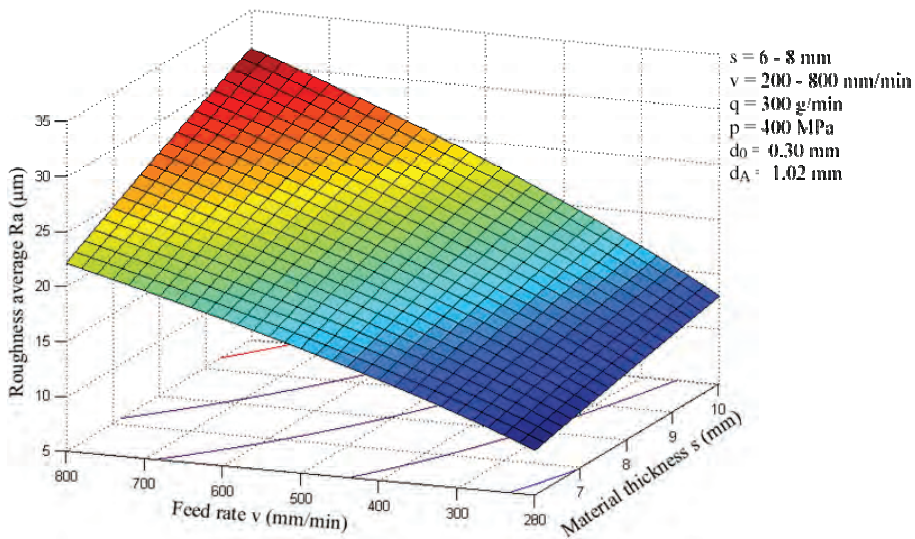


Fig. 13. Predicted average roughness as a function of feed rate  $v$  (mm/min) and material thickness  $s$  (mm) under given conditions

Proposed mathematical model allows us to choose a quantitative level of quality, which has not been the case in the theoretical and practical solution to this problem.

### 5. CONCLUSION

The flexibility and cool cutting characteristics of the abrasive water jet technique make it an important tool for cutting applications of new materials such as composites and sandwiched materials that are difficult to machine with traditional machining processes.

In abrasive water jet cutting the final cut surface roughness and the dimensional accuracy depend on the many process parameters. Summarizing the main features of the experimental results, the following conclusions may be drawn:

- As the feed rate increases, the AWJ cuts narrower kerf. This is because the feed rate of abrasive water jet allows fewer abrasives to strike on the jet target and hence generates a narrower slot.

- Higher abrasive flow rate produce greater kerf width, especially lower kerf width because the larger number of abrasive particles share in machining process which has positive effect on kerf geometry.
- The surface has better characteristics in the region that starts from the upper point where abrasive water jet begins to cut to the middle of the thickness. From the middle of the thickness downwards, the surface quality deterioration is observed.
- With an increase in the abrasive flow rate, the roughness is reduced. For high abrasive mass flow rates, the roughness is less sensitive to changes in the feed rate.

Experimental study shows that, among others, the most important factors influencing the cut surface roughness of aluminium alloy are nozzle feed rate and abrasive mass flow rate.

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#### REFERENCES

55. Hloch S. and Fabian S., (2006), Qualitative analysis of AWJ factors affecting the surface roughness, *Wissenschaftliche Beitrage*, TFH Wildau, Germany, 113-119.
56. Lebar A. and Junkar M., (2004), Simulation of abrasive water jet cutting process, *Modeling Simul. Mater. Sci. Eng.* 12, 1159-1170.
57. Yanaida K. and Ohashi A., (1980), Flow Characteristics of Water Jets in Air, *Proceedings of 5th International Symposium on Jet Cutting Technology*, Paper A3, Hanover, 33-44.
58. Zhang S. and Nambiath P., (2005), Accurate hole drilling using an abrasive water jet in titanium, *American Waterjet conference*, Houston.
59. Guo Z., Ramulu M. Jenkins M.G., (2000), Analysis of the waterjet contact/impact on target material, *Optics and Laser in Engineering*, No. 33, 121-139.
60. Meng H.C. and Ludema K.C., (1995), Wear models and prediction equations, *Wear*, No. 181-183, 443-457.
61. Hutchings I.M., (1979), Mechanical and metallurgical aspects of the erosion of metals, *Proc. Corrosion/Erosion of Coal Converters*, Houston, pp. 393-428.
62. Lemma E., Deam R., Chen L., (2005), Maximum depth of cut and mechanics of erosion in AWJ oscillation cutting of ductile materials, *Journal of Materials Processing Technology*, No. 160, 188-197.
63. Zeng J. and Kim T.J., (1992), Development of an abrasive waterjet kerf cutting model for brittle materials, *Jet Cutting Technology*, Kluwer Academic Press, Dordrecht, 483-501.
64. Hashish M., (1987), Wear in abrasive-waterjet cutting systems, *Wear of Materials*, Vol. 2, New York, 769-776.
65. Chen F.L., Wang J., Lemma E., Siores E., (2003), Striation formation mechanisms on the jet cutting surface, *Journal of Material Processing Technology*, No. 141, 213-218.
66. Akkurt A., (2009), Cut front geometry characterization in cutting applications of brass with abrasive water jet, *Journal of Materials Engineering and Performance*, Volume 19, Issue 4, 599-606.

67. Janković P. and Radovanović M., (2008), Characteristics of Part Accuracy and Errors by Abrasive Water Jet cutting, *8th International Conference "Research and Development in Mechanical Industry"* RaDMI 2008, Užice, Serbia, 215-222.
68. Jankovic P.: Modeling of abrasive water jet cutting process and forming of technological processor, Doctoral dissertation, University of Nis (in Serbian), 2009.
69. Momber A., Kovacevic R., Schüneman R., et al, (1996), The influence of abrasive grain size distribution parameters on the abrasive water jet machining process, In: Rajukar K P (Eds) *Proc. 25 th North American Manuf. Engrs*, Dearborn, 21-26.

## UTICAJ PARAMETARA PROCESA SEČENJA ABRAZIVNIM VODENIM MLAZOM NA MEHANIZAM ODOŠENJA MATERIJALA I KVALIET REZA

**P. Janković, T. Igić, D. Nikodijević**

*Proces sečenja materijala abrazivnim vodenim mlazom, podržan teorijom mehanike fluida, abrazivnog habanja i mehanikom oštećenja, je visoko tehnološki postupak koji nudi jedinstvene mogućnosti u poređenju sa konvencionalnim postupcima obrade. U radu se, poret teorijskih izvođenja, daje originalni doprinos u obliku matematičkog modela veličine oštećenja površine reza, izražene vrednošću hrapavosti.*

*Cilj istraživanja, prikazanih u radu, povezan je sa zahtevima industrije, odnosno krajnjeg korisnika. Imajući u vidu da konvencionalni postupci obrade ne samo da su u zaostatku u što se tiče kvaliteta reza, već i da izvesne zahteve nisu mogli da ispune, sa pojavom kompozitnih materijala nisu uopšte bili u stanju da ih obrađuju, jer su se javljala nedozvoljena oštećenja, kao što su reslojavanje, izvlačenje vlakana, sagorevanje krajeva).*

**Ključne reči:** *Sečenje abrazivim vodenim mlazom, mehanika oštećenja, eksperimentalno ispitivanje uzoraka od aluminijuma, matematičko modeliranje*

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