

## FRACTURE MECHANICS ANALYSIS OF DAMAGED TURBINE ROTOR DISCS USING FINITE ELEMENT METHOD

by

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*This paper presents evaluation fracture mechanics parameters in low pressure turbine components. Critical locations such as keyway and dovetail area are experiencing stress concentration leading to crack initiation. Stress intensity factors were evaluated using the J-Integral approach available within ANSYS software code. The finite element method allowed the prediction of the point of crack initiation and the crack propagation using the orientations of the maximum principal stresses. Special attention in this investigation is focused to develop analytic expressions for stress intensity factors at critical location of low pressure steam turbine disc.*

Key words: *turbine discs, stress intensity factor, J-integral, finite elements*

### Introduction

In low pressure (LP) steam turbine, rotating discs are simultaneously subjected to mechanical and moderate thermal load [1, 2]. A disc is loaded under internal pressure due to shrink fit on a shaft. Thus, additional blade effects may be taken into consideration and modeled by an external tensile load at the outer radius of the disc when the disc rotates. Since material behavior is temperature dependant, changes of material properties throughout the disc should be considered. In order to attain a certain and reliable analysis, solution should consider changes in material specification caused by temperature. Therefore, engineers have strong interests in monitoring and analyzing of rotating components in jet engines to improve safety and to reduce maintenance cost. To prevent catastrophic failure of the engine, they have developed different techniques to analyze structures. The engineering field of fracture mechanics was established to develop a basic understanding of such problems [3, 4]. Today, fracture mechanics is not only of academic interest, but plays an increasingly important role in structural design [5-8]. Recently, a more stringent safety criterion, assuming a pre-existing flaw in critical component, has been adopted to assess service life of aircraft [9-13]. This emphasizes the significance of fracture mechanics as a tool for analysis.

Mechanical loading is not the only factor considered in the design of structures or structural components. Other possible situations, such as temperature loading, have to be

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considered. In the operation of gas turbine engines, for example, thermal stresses can be as high as, or higher than, the centrifugal stresses. The combinations of thermal, centrifugal and gas bending stresses at elevated temperatures result in high local stresses which can lead to cracking of the turbine blades and rotor disks. Thus, thermal effects should not be ignored. Special attention in this paper is focused to develop analytic expressions for stress intensity factors at low pressure steam turbine disc. Finite Element Method (FEM) is used to determine critical location, zone of the stress concentration at low pressure disc, in which rotor blades are connected with rotating disc. At critical location of rotating disc various crack lengths are assumed. For various crack lengths at LP steam turbine disc stress intensity factors are determined under thermomechanical loads. Using these discrete values of SIF's analytic formulae for SIF at cracked LP disc are derived here. The stress intensity factor is valuable data in the prediction of service lives of turbine discs. Crack growth behavior is a major issue in a variety of rotating components for which analytic formula of SIF is necessary.

### Analysis with respects to fracture mechanics

Fracture mechanics provides the concepts and equations to determine how cracks grow and their effect on the strength of structure. The critical crack size  $a_c$  can be determined from the well known equation:

$$K_I = K_{IC} \quad (1)$$

where  $K_I$  is the mode I stress intensity factor and  $K_{IC}$  is the strain fracture toughness of material. Although several stress intensity factor handbook have been published, the available solutions are not always adequate for particular engineering applications. This is especially true for cracks subjected to thermal stresses.

Finite element method (FEM) and boundary element method (BEM) are the most widely used techniques for evaluating stress intensity factor (SIF). The most important region in modeling the fracture region is the region around the crack, where crack tip elements with singularity are used [14-18]. One of the most common methods to calculate SIF is to apply the FEM and J-integral method, by using the following relationship:

$$K = \sqrt{\frac{E J}{1 - \nu^2}} \quad (2)$$

where  $E$  is the Young's modulus and  $\nu$  the Poisson's ratio. Generally, the stress intensity factors are additive, provided different loading conditions induce the same mode of crack extension. Hence, the SIF for the interaction of mechanical and thermal loads can be expressed as:

$$K_I = (K_I)_M + (K_I)_T \quad (3)$$

where  $(K_I)_M$  is the stress intensity factor due to mechanical load and  $(K_I)_T$  is the stress intensity factor due to temperature effect.

Anyhow, the J-integral is path dependent for cases which include residual, inertial or thermal stress terms or loadings along the crack face, as well as for three-dimensional structures of non-homogeneous materials in the direction of crack advance [19]. Some efforts have been made to modify the J-integral to become valid parameter for two-dimensional thermal cases, *e. g.* Wilson and Yu. [20]:

$$J^* = \int_{\Gamma} (W^* dx_2 - \sigma_{ij} \frac{\partial u_i}{\partial x_1} n_j dS) + \frac{E\alpha}{1-2\nu} \int \varepsilon_{ii} \frac{\partial \theta}{\partial x_1} dA \quad (4)$$

where

$$W^* = W - \frac{E\alpha\theta}{2(1-2\nu)} \varepsilon_{ii} \quad (5)$$

$$\sigma_{ij} = \lambda \varepsilon_{ii} \delta_{ij} + 2\mu \varepsilon_{ij} - \frac{E\alpha}{1-2\nu} \theta \delta_{ij} \quad (6)$$

$$W = \frac{1}{2} \sigma_{ij} \varepsilon_{ij} \quad (7)$$

and  $\mu$  and  $\lambda$  are Lamé's constants,  $\theta$  is the temperature,  $\alpha$  is the coefficient of thermal expansion. Combining FEM with modified J\*-integral approach to analyze thermomechanical problems with respects to fracture mechanics is considered in [21-26].

### Numerical analysis and validation

Aim of this work is to determine fracture mechanical parameters in turbine stage of low pressure steam. To determine fracture mechanical parameters, the finite element method and J-integral approach was used. According to external load conditions, the temperature on upper surface of rotor disc is 193 °C and on lower surface is 180 °C. Temperature distributions at part of a low-pressure turbine disc is shown in fig. 1.

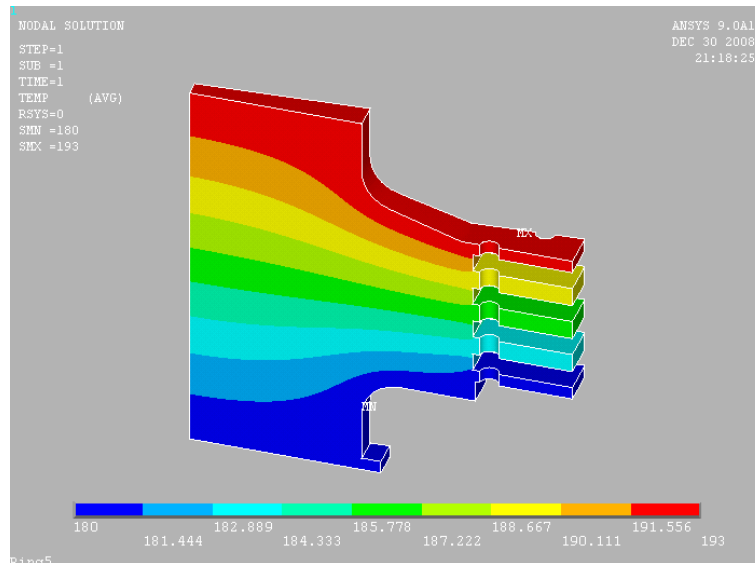
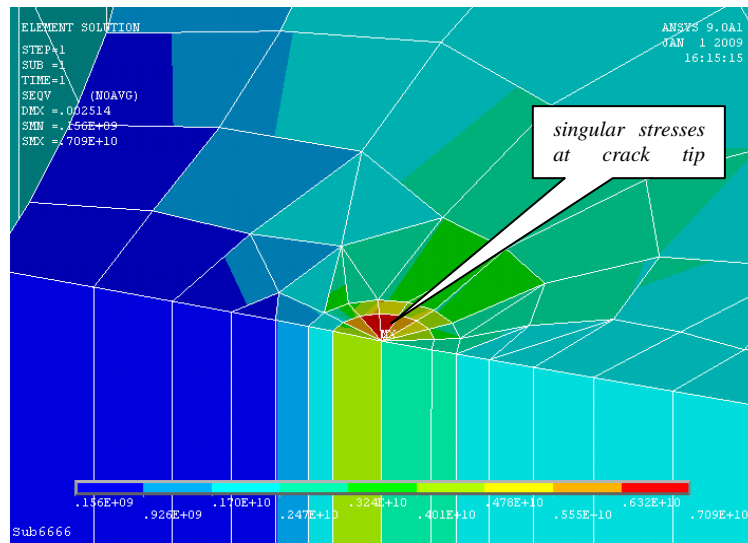


Figure 1. Temperature distributions at part of a low-pressure turbine disc

In stress analysis, disc was modeled with SOLID45 type elements consisting of eight nodes. Further analysis is needed for the parts (areas) that are subjected to extreme values stress and strain and are likely to develop cracks or presence of one. For submodeling, SOLID 95 type elements were used with 20 nodes. These elements are suitable for modeling area around crack tip and correctly describe singular stress and strain fields (Fig. 2).

Values of stress intensity factors for different crack sizes and loads are shown in tab. 1 and fig. 3. Disks were made of 0.35C 0.65Mn 0.9Cr 0.3Mo steel grade steel (34HN3M steel according to GOST). Material properties for disk of turbine are as follow: density  $\rho = 7820 \text{ kg/m}^3$ , Young modulus  $E = 186 \text{ GPa}$ , Poisson coefficient  $\nu = 0.3$ , coefficient of thermal expansion  $\alpha = 13.32 \text{ } \mu\text{m/m}^\circ\text{C}$ , thermal conductivity  $\lambda = 36 \text{ W/m}^\circ\text{C}$  and  $K_{IC} = 120 \text{ MPa(m)}^{1/2}$ .



**Fig. 2.** von Mises stresses in submodel from thermal and mechanical loads ( $n = 3000 \text{ o/min}$ ) in elements, crack length  $a = 0.01 \text{ m}$

**Table 1.** Stress Intensity Factors  $K_I$  [ $\text{MPa} \sqrt{\text{m}}$ ] for different crack lengths and rotational speed

Crack lengths [m]	a=0,010m	Rotational speed [o/min]	n=3000	35,930	35,930	35,713	35,713	35,858	35,858
	a=0,010m		n=3300	36,368	36,368	36,248	36,248	36,295	36,295
	a=0,015m		n=3000	35,971	35,971	35,755	35,755	35,899	35,899
	a=0,015m		n=3300	36,415	36,415	36,297	36,297	36,342	36,342
	a=0,020m		n=3000	39,354	39,354	39,908	39,908	39,279	39,279
	a=0,020m		n=3300	39,827	39,827	40,397	40,397	39,750	39,750
	a=0,025m		n=3000	40,172	40,172	40,802	40,802	40,096	40,096
	a=0,025m		n=3300	40,653	40,653	41,299	41,299	40,574	40,574
	a=0,030m		n=3000	41,104	41,104	42,363	42,363	41,025	41,025
	a=0,030m		n=3300	41,596	41,596	42,875	42,875	41,515	41,515
Position of crack tip from the top surface [m]			0	0,002	0,004	0,006	0,008	0,010	

Previous finite element results can be used to determine analytic expressions for stress intensity factors. These analytic expressions for stress intensity factors can be used for crack growth analyses and residual life estimations. By using values of stress intensity factors for different crack lengths, as shown in Table 1, analytic expressions were established for different load conditions. The values of stress intensity factors can be found by solving Eq. (8) and (9) for different crack lengths,  $a$ , on the top surface of disc and rotational speed ( $n = 3000 \text{ o/min}$  and  $n = 3300 \text{ o/min}$ ), respectively:

$$K_I = 112.432 - 178009a + 1.43727 \times 10^6 a^2 - 4.7944 \times 10^7 a^3 + 5.724 \times 10^8 a^4 \quad (8)$$

$$K_I = 113.443 - 179366a + 1.44842 \times 10^6 a^2 - 4.832 \times 10^7 a^3 + 5.76933 \times 10^8 a^4 \quad (9)$$

Using previous derived analytic formulæ for the stress intensity factors, values of the stress intensity factors  $K_I$  can be controlled in the simple manner near to its critical value  $K_{IC}$ . In accordance to computation results, under thermo-mechanical loads, value of the SIF is  $K_I = 42,88 \text{ MPa(m)}^{1/2}$  and its fracture toughness is  $K_{IC} = 120.0 \text{ MPa(m)}^{1/2}$ .

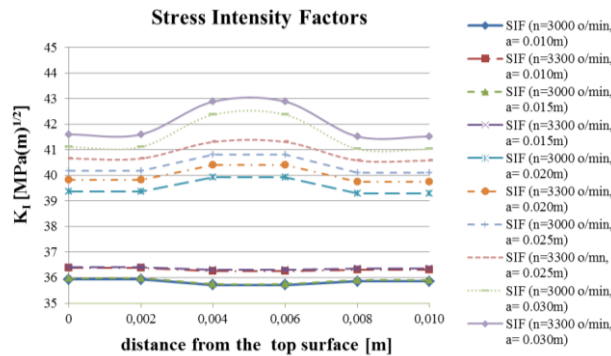


Fig. 3. Stress Intensity Factors for different crack lengths and rotational speed

## Conclusions

The method developed in this paper was applied to compute the stress intensity factors in linear elastic fracture mechanics (LEFM) crack growth in plain stress problem. The finite element method has been used to calculate thermal and inertia stresses in a low-pressure turbine disc. These results were used to determine stress intensity factors using J-integral approach. In many cases, the J-integral will be easiest means of calculating stress intensity factors. This method is easy to use when the software supports determination of the contour integral. The results are fairly accurate even for coarse meshes because the contours can be chosen remote from the near crack tip region. The calculated values of stress intensity factors are used for evaluation Eq. (8) and (9), which present analytical expression for stress intensity factors where  $a$  is crack length.

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