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Routes for exceptional loads: a new soil mechanics perspective

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Routes for exceptional loads: a new soil mechanics perspective

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Off-site prefabrication can bring cost, quality and programme benefits to construction projects but often requires the transportation of large, indivisible loads (in the order of 1000–10 000 t) on temporary routes that can cross soft soils. Through simple numerical modelling, this paper demonstrates that the fundamental behaviour of the ground supporting these large loads can differ significantly from that expected in conventional road design practice; the interaction between many closely spaced wheels means the vehicle's influence depth and failure mechanism are significantly deeper. Surface soils are less influential. Deeper soil was found to be more prone to local yield, developing large localised strains at low proportions (10–30%) of the ultimate capacity. Instead of designing temporary roads to avoid yield and degradation under cyclic loads, significant savings may be possible if limited degradation is permitted, with recovery through consolidation between loads. Investigation and monitoring of deep subsoils during operations is recommended for real-time evaluation of geotechnical risk.

Notation

B	loaded width (m)
D	pavement depth (m)
E	Young's modulus (MPa)
E_0	Young's modulus at top of layer (MPa)
K_0	coefficient of lateral earth pressure at rest (dimensionless)
$K_{0,NC}$	normally consolidated coefficient of lateral earth pressure at rest (dimensionless)
$K_{0,OC}$	overconsolidated coefficient of lateral earth pressure at rest (dimensionless)
m_1	increase in Young's modulus with depth (MPa/m)
N_{Su}	shear strength ratio of clay subgrade, $S_u/\gamma'_p D$ (dimensionless)
p'	mean normal effective stress (kPa)
q	deviator stress (kPa)
S_u	undrained shear strength (kPa)
s_E	settlement in linear-elastic model (m)
s_P	settlement in Mohr–Coulomb model (m)
x	lateral distance from vehicle or wheel centreline (m)
z_{NC}	depth to normal consolidation (m)
γ_p	total unit weight of pavement fill (kN/m^3)
γ'_p	effective unit weight of pavement fill (kN/m^3)
γ_s	unit weight of subgrade (kN/m^3)
$\Delta\sigma'_z$	increment in vertical effective stress from wheel load (kPa)
Λ	utilisation of ultimate capacity (i.e. ratio of applied load to failure load, ω/ω_{ult}) (dimensionless)
ν	Poisson's ratio (dimensionless)
σ'_z	vertical effective stress (kPa)
ϕ'	angle of internal shearing resistance (degrees)
ω	uniformly distributed wheel load stress on pavement (kPa)

ω_{ult} ultimate ω at failure applied on a single wheel (or for each separate wheel in the case of multi-wheel models) in Mohr–Coulomb model (kPa)

1. Introduction

Demand for off-site prefabrication in the construction, mining and oil and gas industries (see Cronin, 2015; Mammoet, 2017) can necessitate the transportation of large indivisible loads to remote locations on temporary roads. These loads may be of the order of 1000–10 000 t, transported by large, multi-wheeled mobile platforms (e.g. 6 m wide by 60 m long, supported on 88 wheels). Economic limitations due to the road's short design life, coupled with logistical constraints, may preclude the use of ground improvement where soft soils are crossed, and the risk of cyclic degradation must be managed in such extreme conditions. Subgrade soil behaviour under these unconventional conditions is poorly understood and requires novel research for efficient road design.

Conventional pavements are designed to allow only a small amount of local yielding under each passage, otherwise cumulative strains render the pavement unserviceable. Unbound temporary roads need not be so robust (i.e. cumulative strains could be tolerated), but they must avoid deep-seated slip failure. This can form under repeated loads as localised yielding in the subgrade spreads. Understanding local yielding behaviour of heavy-haul road subgrade soil is fundamental for successful design. In the work reported here, a parametric finite-element analysis study was conducted to investigate how it varies with

- vehicle dimensions
- the proportion of capacity mobilised

- differences in pavement–subgrade system properties
- assumptions made in modelling soil behaviour.

Simple analysis methods are purposefully used to allow yielding behaviour to be isolated and examined. It is demonstrated that the practice of minimising local yield, when applied to large vehicles, can be prohibitively expensive for temporary assets; a procedure that instead monitors real-time degradation may yield cost benefits. Implications for site investigation and monitoring of heavy-haul roads are also discussed.

2. Literature review

2.1 Transmission of loads through pavements

2.1.1 Domain of conventional road research

Conventional pavement design aims to maintain low subgrade stress, well below its shear strength, by spreading wheel loads through an engineered fill layer (Brown, 1996; Frost *et al.*, 2004). The resulting small strains are almost entirely recoverable, and irrecoverable strain is only measurable after multiple cycles (Brown, 1996). Within this small-strain regime (i.e. <0.5% strain), elastic behaviour and constant post-cyclic shear strength can be assumed with acceptable accuracy (Díaz-Rodríguez and López-Molina, 2008; Wang *et al.*, 2014). Tannant and Regensburg (2001) presented a resilient-modulus design method that prescribes mine-road fill depths specifically to limit strains to this regime (1500 to 2000 microstrain). Subjecting subgrades to larger strains causes irrecoverable strain and degradation of shear strength (Brown *et al.*, 1977; Wang *et al.*, 2014).

Conventional pavement design is typically based on the response to the passage of a single wheel (Boulbibane *et al.*, 2005; Brown, 1996; Sharp and Booker, 1984), as wheels on conventional vehicles are considered to be sufficiently separated to avoid interaction between wheel pressure bulbs. Individual pressure bulbs from closely spaced wheels can interact to form wider, deeper pressure bulbs (as encountered by Gräbe and Clayton (2009) when investigating deformation in railway foundations, where sleepers are more closely spaced than conventional road wheel loads). The interaction between closely spaced wheel loads (or railway sleepers) also potentially changes the failure mechanism from being localised under a wheel to acting over the whole vehicle (Lehtonen *et al.*, 2015). This interaction and the deeper stressed zone must be considered in heavy-haul road design.

Conventional pavement design generally presumes that the upper subgrade layers are the most influential on the development of progressive failure (Brown, 1996; Little, 1992). Even heavily loaded mine- and forestry-haul roads use this approach; pavement design thickness is often based on either the California bearing ratio (CBR) or the resilient modulus of the subgrade surface (FCE, 2011; Kaufman and Ault, 1977; Tannant and Regensburg, 2001). For interacting multi-wheeled

vehicles, the dominance of the upper, most overconsolidated layers will diminish and deeper, normally consolidated, saturated soils will become increasingly influential.

2.1.2 Composite fill–subgrade system behaviour

A pavement is often described as a ‘composite system’ (Brown, 1996; Frost, 2000; Hyde, 1974); strains are considered compatible at the interface and the stress distribution is influenced by the ratio of the pavement stiffness to the subgrade stiffness (relative stiffness). The role of engineered fill is primarily to spread the wheel load to keep subgrade stresses sufficiently low (Frost *et al.*, 2004). Load on the subgrade is more efficiently distributed if the pavement relative stiffness increases, but above an optimum relative stiffness the pavement shears locally, reducing the composite capacity (Sharp and Booker, 1984).

Deeper pavements spread loads on the subgrade more widely, but the load-spreading angle decreases until a limiting depth is reached where behaviour is completely controlled by the pavement (Boulbibane *et al.*, 2005; Burd and Frydman, 1997; Houlsby and Burd, 1999; Sharp and Booker, 1984). This effect has also been observed in scaled foundation tests (Ismail Ibrahim, 2016; Laman *et al.*, 2012).

2.1.3 Influence of principal stress rotation

A moving wheel load continuously rotates the principal stresses (Brown, 1996). Cyclic principal stress rotation softens soil, causing significantly larger cumulative strains (Arthur *et al.*, 1980; Jefferies *et al.*, 2015; Xiao *et al.*, 2014). Cyclic principal stress rotation has been found to result in accelerated failure (Gräbe and Clayton, 2009) when compared with a cyclic load that does not move (Brown and Chan, 1996).

Self-weight stresses are low beneath a conventional road (Brown, 1996); therefore, lateral stresses arising in front of (and behind) a moving wheel will cause large rotation of the principal stresses. Overlapping adjacent stress bulbs will to some extent ‘cancel out’ the lateral stress pulse between adjacent axles, causing these lateral stresses to occur at greater depths where principal stress rotation from a given lateral stress increment is reduced as a result of the increased self-weight stress. It is possible for two ‘regimes’ to exist – a shallow zone where principal stresses are rotated significantly by the passage of traffic and a deeper one where stress rotation is small. This is a complex stress–strain environment: the shallower overconsolidated zone experiences rapidly changing stresses (both magnitudes and principal directions) while the deeper zone, more prone to compressive yield, experiences smaller stress changes but is still likely to degrade quickly.

2.2 Progressive failure and shakedown

A stable equilibrium (‘shakedown’), whereby only recoverable strain occurs under load, is reached if the pavement–subgrade

system can effectively become ‘prestressed’ by a residual stress field to remain within elastic limits both under load and at rest (Ponter *et al.*, 1985; Sharp and Booker, 1984; Zhao *et al.*, 2008). Residual stresses are assumed to be mobilised through limited local yield during initial load passages. This suggests an additional reserve of resistance to repeated load slightly above the load that causes first local yield, but significantly below the static load causing collapse (Pande, 1982; Sharp and Booker, 1984). Constant strength and stiffness parameters are typically assumed (Boulbibane *et al.*, 2005; Sharp and Booker, 1984; Zhao *et al.*, 2008), characteristic of small- to medium-strain soil behaviour (Díaz-Rodríguez and López-Molina, 2008), to model mathematically a limiting safe load or ‘shakedown limit’. It should be noted that the shakedown limit differs from the cyclic threshold stress observed experimentally (e.g. Heath *et al.*, 1972) – the former considers the whole pavement–subgrade system theoretically whereas the latter indicates loss of strength of a single soil element once sufficiently large strains occur (e.g. Wang *et al.*, 2014). Full-scale pavement tests (Sharp and Booker, 1984) and scaled laboratory experiments on a range of soils (Juspi, 2007) have been found to compare well with theoretical shakedown limits. This suggests that residual stresses are the dominant factor in achieving equilibrium, and material strengths only degrade for loads above the shakedown limit. In particular, overconsolidated surface soils are predominantly elastic when loaded well below their shear strength (Brown, 1996; Schofield and Wroth, 1968) and the conventional approach of limiting subgrade surface strains is clearly valid for roads resisting predominantly small, separate wheel loads.

If the residual stress field required to counteract heavier loads cannot be achieved without yielding at rest, strain accumulates with each load application and the road fails progressively, typically accompanied by rising subgrade pore water pressures that reduce the strength (e.g. Frost, 2000; Gräbe and Clayton, 2009). Avoiding such degradation by limiting strains for large multi-wheeled vehicles crossing soft ground, where stresses extend to great depth, is likely to require a much thicker pavement layer. Such an approach would likely be unjustifiably costly and impractical for a temporary road. A better understanding of strength degradation from load and recovery from consolidation in medium/large-strain regimes, as discussed by Krechowiecki-Shaw *et al.* (2016), could achieve a more economic and practical design.

2.3 Local yield

As a surface load increases from zero towards the failure load, the first local yield is reached and plastic deformation occurs at a single point. A further increase in load causes more widespread yielding, redistributing stresses until the shear strength is mobilised over the full slip circle at failure (Madabhushi and Haigh, 2015; Osman and Boulton, 2005). In studying local yield, it is convenient to consider utilisation (i.e. the proportion of capacity mobilised, Λ),

given by

$$1. \quad \Lambda = \omega/\omega_{ult}$$

Elastic–plastic finite-element modelling and corroborating field tests carried out by D’Appolonia *et al.* (1971) indicate that the first local yield depends heavily on the in situ stress state and shear strength. Both of these are determined by overconsolidation, which can be quantified by N_{Su} (see Equation 2, after Burd and Frydman (1997)). For normally consolidated soil, this may occur at $\Lambda = 12\text{--}25\%$, while local yield in heavily overconsolidated surface soils may only occur for $\Lambda > 50\%$ (D’Appolonia *et al.*, 1971). Berardi and Lancellotta (2002) made similar observations related to oil tank settlements.

$$2. \quad N_{Su} = S_u/\gamma'_p D$$

Burd and Frydman (1997) also found that N_{Su} influences the failure mechanism: normally consolidated subgrades mobilised yield over larger volumes of soil, particularly developing yield in the compression zone directly beneath the load.

At rest, a normally consolidated soil has a lower value of K_0 (coefficient of lateral earth pressure at rest) and thus requires a smaller deviator stress increment to reach compressive yield. While compressive yield in deep normally consolidated soil is not a common problem for conventional roads, below a heavy-haul road it is a concern and was therefore investigated in this study. This paper focuses on the development of local yielding in soft soil at depth below unbound heavy-haul roads, rather than attempting accurate stress or settlement predictions. A number of simplifying assumptions were made in the finite-element analyses to allow a ‘first-approximation’ of the problem to be developed.

3. Analysis method

3.1 Modelling philosophy

Simple linear-elastic and linear-elastic, perfectly plastic (‘Mohr–Coulomb’) models were used for the soil layers. While being unrealistic descriptors of element-level behaviour, over a soil mass these models can

- offer good agreement with yield development and failure mechanisms of site trials (D’Appolonia *et al.*, 1971; Ismail Ibrahim, 2016), and
- allow the study of local yielding effects in isolation (D’Appolonia *et al.*, 1971).

Single wheel loads were compared with multi-wheeled vehicles to identify changes in behaviour dependent on the interaction between wheels. The vehicle considered is long relative to its width and the wheels are closely spaced in the longitudinal direction (Figure 1). The three-dimensional layout was thus

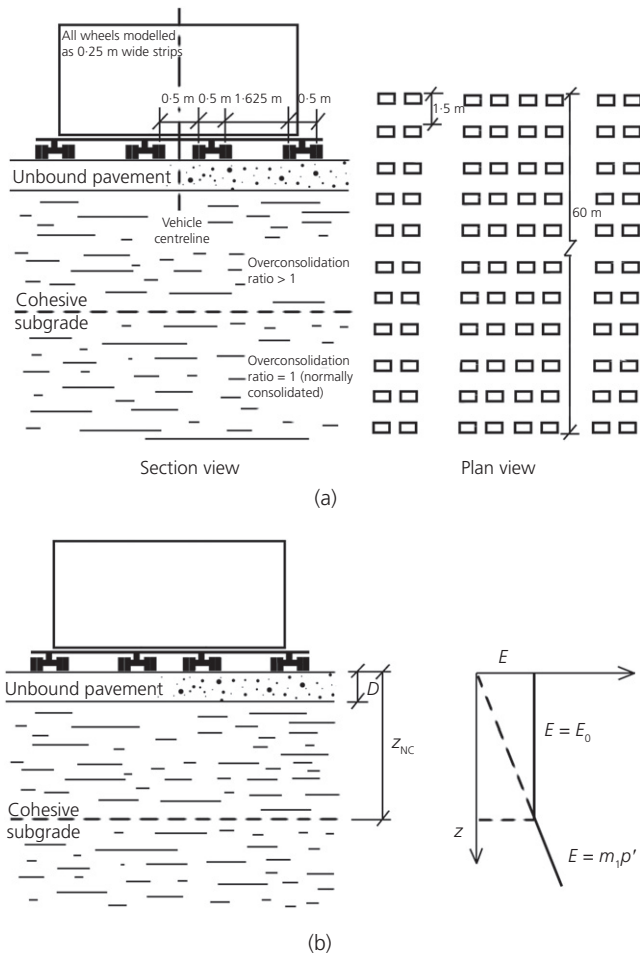


Figure 1. Problem definition: (a) vehicle geometry; (b) ground model for analysis. As a plane-strain analysis was used, the model geometry was based solely on the section view; the plan view is for information only

simplified to a two-dimensional, plane-strain analysis based on the section view. While a single wheel is more accurately represented by an axisymmetric (circular) load, conventional haulage vehicles often have multiple axles with small longitudinal spacing; some longitudinal interaction is likely to occur, making the response more similar to a strip load, hence single wheel loads are also simplified to a plane-strain strip load. Sharp and Booker (1984), Boulbibane *et al.* (2005) and Juspi (2007) similarly analysed moving single wheel loads as plane-strain strips.

A range of subgrade models was simulated to represent normally consolidated to lightly overconsolidated, soft to very soft deposits with a high water table (i.e. areas with low bearing capacity likely to present difficulties to temporary roads, such as alluvial deltas). Soil parameters were chosen to reflect a low-plasticity (plasticity index, PI=10–15%) silty to very silty clay. Overconsolidation was represented using the simplified model presented by Foye *et al.* (2008) in which the

overconsolidated layer is modelled with a constant stiffness and shear strength, while deeper normally consolidated soil increases in stiffness and strength with depth (Figure 1). The properties of the pavement layer were also varied to investigate its influence on single and multiple wheel loads.

While principal stress rotation is known to influence the soil stress–strain response (Jefferies *et al.*, 2015), such effects were not modelled in this work for the sake of simplicity and due to difficulties in obtaining realistic parameters. However, general trends in stress rotation were considered, as these can have a large influence on the subgrade degradation environment. The finite-element software Midas GTS NX (Midas, 2016) was used for all finite-element analyses.

3.2 Soil models

3.2.1 Linear-elastic models

Three separate linear-elastic clay subgrade models (representing varying degrees of overconsolidation) and a single granular pavement model, with properties as given in Table 1, were analysed. Below the depth to normal consolidation, z_{NC} (Figure 1(b)), the subgrade soil is assumed to be normally consolidated and hence Young’s modulus is described by

$$3. \quad E = m_1 p'$$

The case 1 subgrade model is such that z_{NC} coincides with the top of subgrade, to provide a theoretical minimum strength and stiffness.

The clay subgrade K_0 is assumed to take the normally consolidated value, following Brooker and Ireland (1965) (Equation 4). The granular pavement follows the work of Jaky (1948) (Equation 5).

$$4. \quad K_{0,NC} = 0.95 - \sin \phi'$$

$$5. \quad K_{0,NC} = 1 - \sin \phi'$$

For the purpose of determining $K_{0,NC}$, for the low-plasticity clay subgrade, $\phi' = 30^\circ$ (BSI, 2015).

Several road thicknesses were considered, overlying each subgrade model. Subgrade effective stresses varied as a result, thereby changing z_{NC} (Table 2).

3.2.2 Mohr–Coulomb models

The granular pavement was modelled as drained material with zero effective cohesion and $\phi' = 40^\circ$ (unless otherwise stated as 32°), roughly corresponding to typical values for a granular sub-base and general earthworks fill, respectively (BSI, 2015; Burd and Frydman, 1997; Sharp and Booker, 1984). Other

Table 1. Parameters assumed for linear-elastic and Mohr–Coulomb models. The subgrade models were based on normally consolidated or lightly overconsolidated, soft to very soft alluvial clay of low plasticity (i.e. $PI = 10\text{--}15\%$); $m_1 = 457 \text{ MPa/m}$ (i.e. $0.001m_1 = 0.457 \text{ kPa/m}$)

	Unit weight: kN/m^3	Undrained shear strength at top of layer, S_u : kPa	Young's modulus at top of layer, E_0 : MPa	Poisson's ratio, ν	Coefficient of lateral earth pressure at rest, K_0
Unbound pavement, γ_p Cohesive subgrade, γ_s	21	—	50	0.263	0.36
Case 1	17	$0.001m_1p'$	m_1p'	0.495	0.45
Case 2	17	5.91	5.91	0.495	0.45
Case 3	17	15.26	15.26	0.495	0.45

Table 2. Model parameters assumed for various pavement designs and corresponding depths to normal consolidation required to satisfy Equation 1

Pavement thickness: m	Depth to water table: m	z_{NC} for case 2: m	z_{NC} for case 3: m
0.25	0	2.700	7.192
0.50	0	2.561	7.053
1.00	0	2.283	6.775
1.50	0	2.005	6.497
0.25*	0.25	2.359	6.851
1.50*	1.50	1.500	4.450

*Groundwater level at base of pavement

parameters were unchanged from the linear-elastic models. The clay subgrade was modelled as undrained cohesive material; that is, with $\phi' = 0$ and undrained shear strength set such that $E/S_u = 1000$, in agreement with typical values for low-plasticity, normally consolidated or lightly overconsolidated clay (D'Appolonia *et al.*, 1971; Jamiolkowski *et al.*, 1979).

To calculate ω_{ult} , the strength reduction method (SRM) was used with a nominal surface load. The SRM reduces (or increases) strength parameters (i.e. S_u and $\tan \phi'$) by an SRM factor until equilibrium is met (Midas, 2016). By modifying the surface load accordingly, an SRM factor of 1.0 was achieved, which was taken to correspond to the ultimate pressure. The bearing capacities of single-material models (drained and undrained) obtained in this manner were compared with the classical closed-form equations proposed by Brinch Hansen (1970) and were found to differ by less than 4%, confirming the reasonable accuracy of the finite-element model.

To improve computational efficiency, artificial symmetry was imposed through the centreline of the wheel or vehicle. This approach is commonly used in modelling single loads (Burd and Frydman, 1997; Ismail Ibrahim, 2016) but is not necessarily applicable to a multi-wheeled vehicle as an asymmetric slip may arise between groups of wheels. To test the applicability of this simplification for closely spaced wheels, a Mohr–Coulomb model with artificial symmetry about the centreline

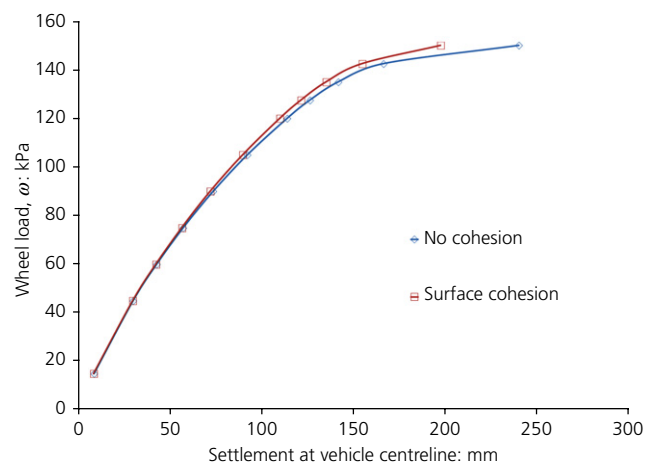


Figure 2. Load–settlement response at centreline of multi-wheel Mohr–Coulomb model for a 1.5 m total pavement depth, showing impact of adding cohesion to the uppermost 0.25 m of the pavement layer

was compared to a full-width model. The differences in bearing capacity and surface settlement profiles were found to be negligible (<1%). The failure mechanism was found to remain symmetrical up to 95% of bearing capacity. Above this load, the solution struggled to converge and the asymmetries noted were most likely as a result of convergence algorithms rather than an asymmetrical failure mode.

For multi-wheel models with a 1.5 m pavement depth, large localised movement of nodes at the pavement surface adjacent to wheels caused slow convergence at low utilisations. By including a small effective cohesion of 5 kPa over the uppermost 0.25 m of the 1.5 m thick pavement layers, convergence times were improved. A comparison of bearing capacity and settlements indicated a negligible effect on global behaviour (Figure 2).

3.2.3 Mohr–Coulomb model with overconsolidated earth pressures

The previous Mohr–Coulomb models only considered changes to strength and stiffness resulting from overconsolidation. The

further refinement of including overconsolidated lateral earth pressures was thus incorporated, allowing comparison of the relative effects of lateral earth pressures on the composite response. A constant K_0 was assumed from the base of the pavement to z_{NC} . While the actual K_0 for overconsolidated soil varies from a maximum near the surface to approach $K_{0,NC}$ at depth, using a mean K_0 over the layer produces reasonable correspondence with field responses (Levenburg and Garg, 2014). The value of K_0 at the midpoint of the layer was used, calculated following Ladd *et al.* (1977) and Mayne and Kulhawy (1982), and assuming an exponent of 0.8 as per the work of Burd and Frydman (1997) (Equations 6 and 7).

$$6. \quad \frac{(S_u/\sigma'_z)}{(S_u/\sigma'_z)_{NC}} = \text{OCR}^{0.8}$$

$$7. \quad \frac{K_{0,OC}}{K_{0,NC}} = \text{OCR}^{\sin \phi}$$

where OCR is the overconsolidation ratio. Compaction of the granular layer will 'lock in' lateral stresses (Brown, 1996). Accordingly, analyses with varying K_0 in the granular layers were undertaken (using $K_0 = 1.0$ and $K_0 = 3.0$).

4. Modelling outcomes

4.1 Linear-elastic stress bulbs

A greater relative stiffness of the pavement (i.e. over a softer subgrade) distributed vertical stress more efficiently at a wider load-spreading angle (Figure 3 and Section 2.1.2), particularly for thin pavements. Subgrade stress bulbs were also deeper for thin pavements, particularly those with high relative stiffness.

Modelling of the large multi-wheeled vehicle confirmed that the closely spaced stress bulbs beneath individual wheels join to form a resultant stress bulb on the scale of the whole vehicle (Figure 4). This acts similarly to a single load of the same width as the vehicle, meaning that even a 1.5 m deep pavement behaves as 'thin' in relation to the combined stress bulb, as evidenced by the wide load-spreading angle in the pavement and the deep subgrade stress bulb (similar to a single wheel load applied to a thin pavement). Changes to relative stiffness and pavement depth did not significantly affect the multi-wheel stress distribution patterns (Figure 4), although deeper pavements were found to protect the subgrade from high localised stresses.

4.2 Local yield of Mohr–Coulomb models

The bearing capacity (ω_{ult}) was found to improve at diminishing rates with increasing fill depth, eventually becoming limited by the capacity of the fill itself (Figure 5), in agreement with the findings of Burd and Frydman (1997) and Ismail

Ibrahim (2016). Limited local yielding at the pavement surface was common to all the models but was relatively small except at high utilisations (Figure 6). Local yield in the subgrade was identified when the behaviour diverged from the 'pavement-only' results. Normally consolidated subgrades showed local yielding at lower utilisations, while overconsolidated subgrades showed little local yield until larger utilisations, followed by rapid plastic settlement; this is similar to the findings of D'Appolonia *et al.* (1971). Differences were more pronounced for thinner pavements, indicating the increased importance of the subgrade in these cases.

Single-wheel analyses with differing subgrade strengths, groundwater levels and pavement depths showed similar behaviour for similar N_{Su} ratios (Figure 7); a deeper pavement or lower water table increased the subgrade effective stresses, reducing the tendency for overconsolidated-like behaviour. The pavement depth itself was also found to influence local yield. The magnitude of plastic settlement for a constant N_{Su} ratio reduced for a deeper pavement (compare the subgrade case 1 results in Figure 6), but the tendency for local yielding to begin at low utilisation (10–30%) was similar.

Load–settlement behaviour of multi-wheel analyses could not be similarly normalised by N_{Su} at the base of the pavement (Figure 8). Following Foye *et al.* (2008), an influence depth for shear strength equal to one footing width (taken as the vehicle width, i.e. 6 m) was used to compute N_{Su} . The resultant values were similar to those of the normally consolidated single-wheel models (subgrade case 1), indicating that local yield behaviour is also determined by the size of the whole vehicle.

For the single-wheel analyses, increasing plastic settlement coincided with shear stresses exceeding the subgrade shear strength in the compression zone at the top of the subgrade (Figure 9). As the load increased, the yield extent spread laterally to the passive wedge zone. A similar tendency was apparent for the multi-wheel analyses, although yield occurred at low utilisations in the compression zone for all subgrade models, similarly to the normally consolidated single-wheel analyses (Figure 9).

4.3 Development of failure mechanisms

The form of the single-wheel failure mechanism was influenced by both the pavement thickness and N_{Su} . Thick pavements and overconsolidated subgrades developed extension strains in a passive wedge confined to the pavement layer, while thin pavements and soft subgrades tend to develop strain bulbs in the subgrade compression zone. Figure 10 shows the progression of failure mechanisms with increasing load by indicating shear strains in excess of 1%, representing significant post-yield straining. Failure mechanisms extending into overconsolidated subgrades (i.e. through thin pavements, not

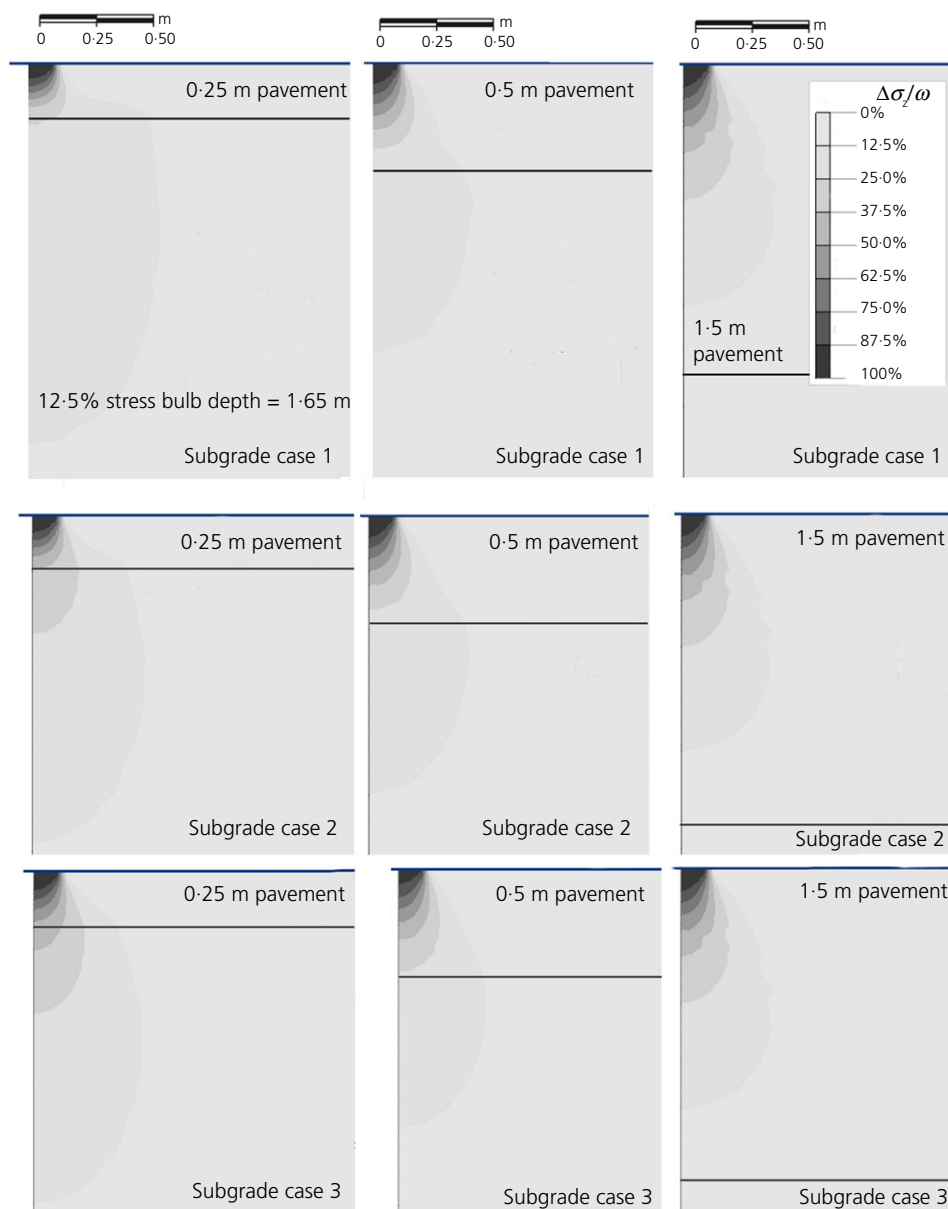


Figure 3. Increments in vertical stress, expressed as a percentage of wheel load pressure ω , from a 0.25 m wide surface strip load with Mohr–Coulomb models of varying subgrade stiffness and pavement depth

localised to the pavement) were found to be smaller, similar to the findings of Burd and Frydman (1997).

The multi-wheel failure mechanism was governed by wheel load interactions: a single wheel load applied to a 1.5 m deep pavement caused failure entirely within the pavement layer (Figure 10), but the multi-wheel failure was a deep global slip over the vehicle's full width (Figure 11). The large yielded volume of soil in the compression zone was also similar to the normally consolidated single-wheel response, validating

the assertion that a whole-vehicle-scale response is dominated by soil at depth and not the pavement layers (see Tables 3 and 4).

4.4 Influence of in situ stress state

Burd and Frydman (1997) suggested that lateral earth pressure coefficients have little effect on the ultimate bearing capacity, and the analysis from this study agrees with that suggestion. However, higher subgrade lateral earth pressures were found to reduce local yielding, in agreement with the work of

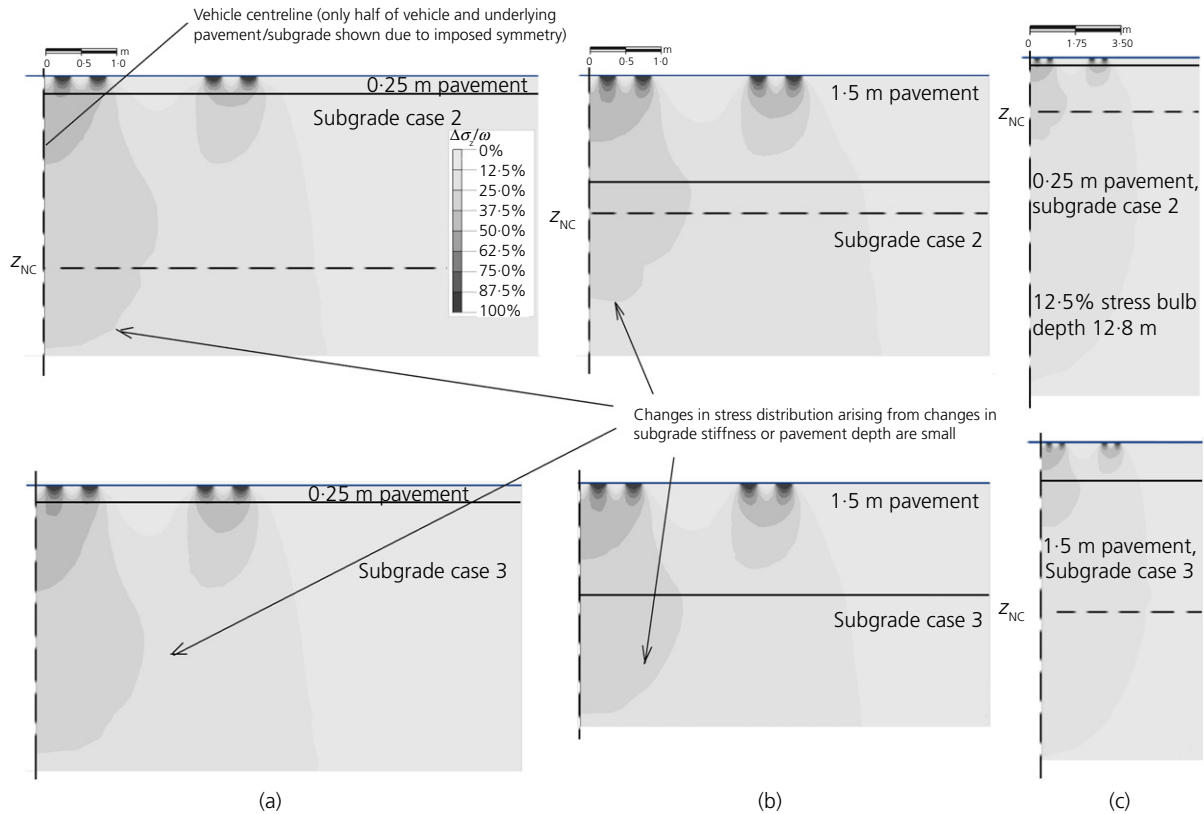


Figure 4. Increments in vertical stress, expressed as a percentage of wheel load pressure ω , from a multi-wheeled vehicle with Mohr–Coulomb models of varying subgrade stiffness and pavement depth: (a) and (b) interaction between individual wheels; (c) large-scale stress bulbs over the whole-vehicle width, extending to great depth, similar to the 0.25 m pavement depth, subgrade case 1 stress bulbs in Figure 3

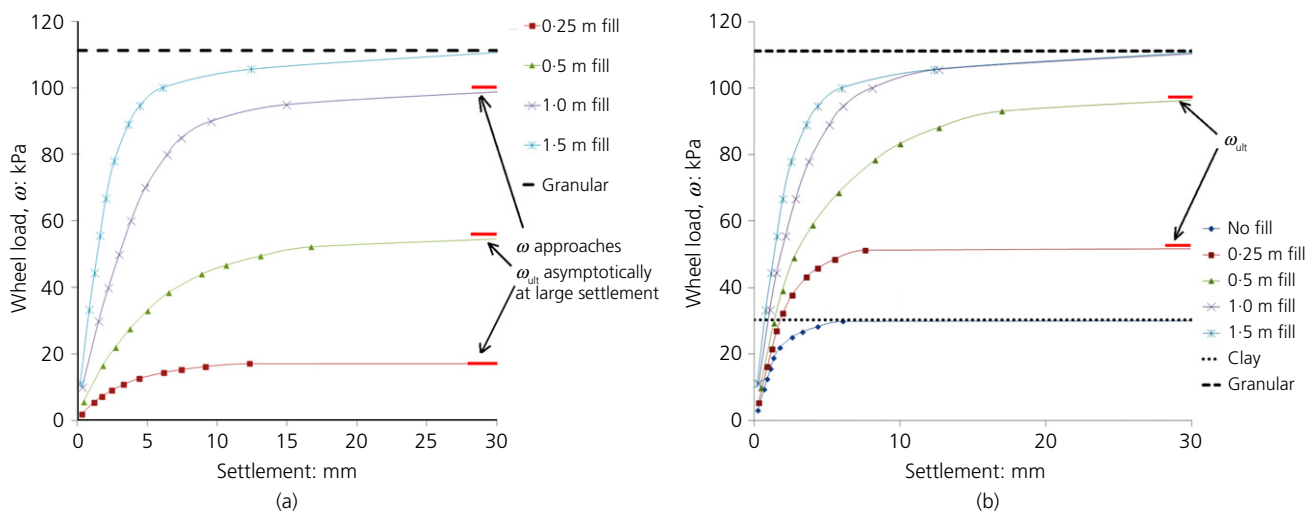


Figure 5. Load–settlement response for single-wheel models with varying pavement depths. Dashed lines indicate the closed-form capacity of the subgrade and pavement fill, calculated using the equations of Brinch Hansen (1970). (a) Subgrade case 1. (b) Subgrade case 2

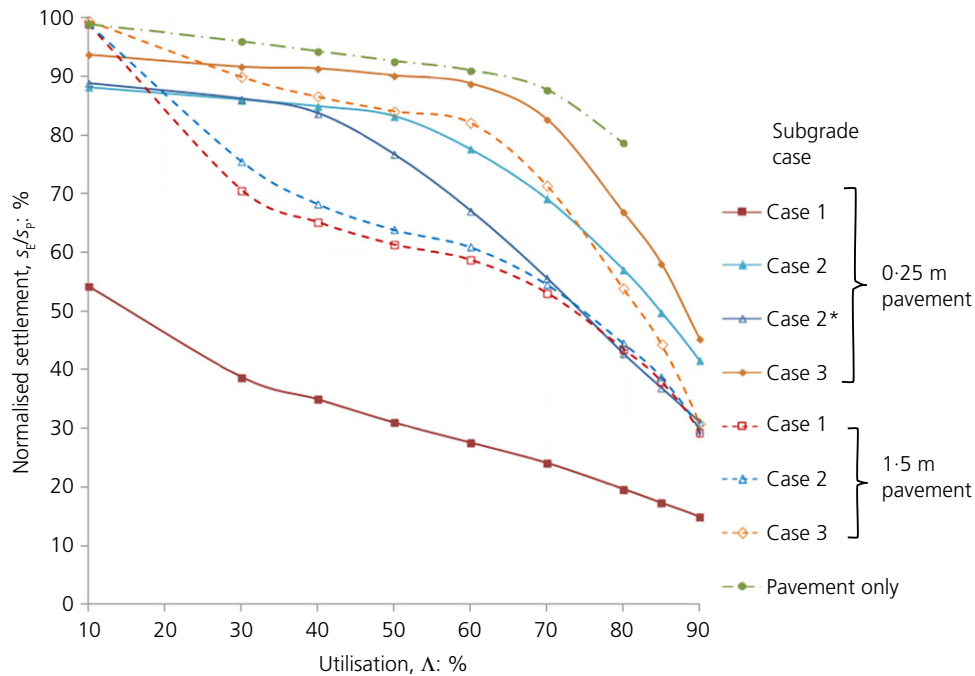


Figure 6. Normalised settlement plotted against utilisation for a range of single-wheel Mohr–Coulomb models. A normalised settlement of 100% indicates no yield (fully elastic); decreasing normalised settlement indicates increasing yield; * indicates groundwater level at base of pavement

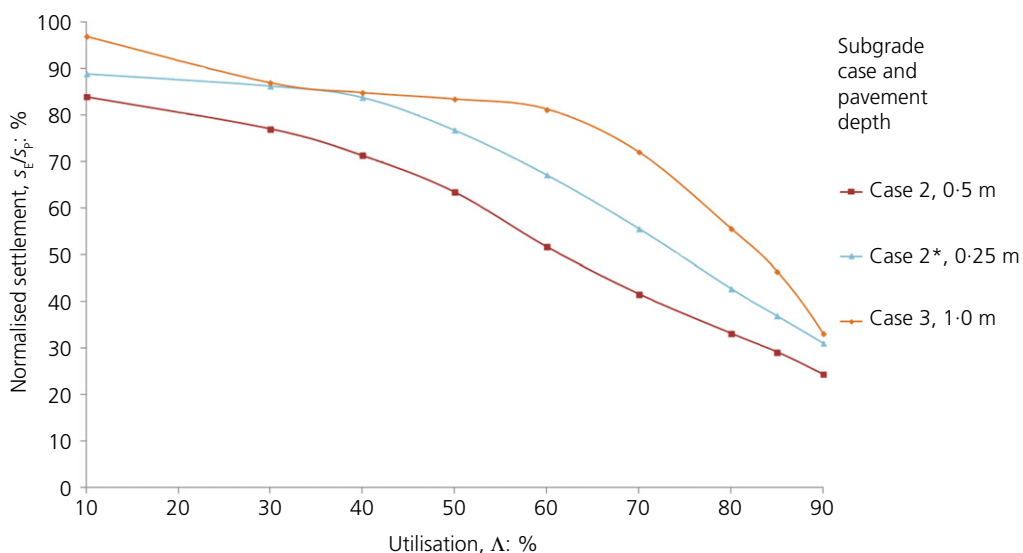


Figure 7. Normalised settlement plotted against utilisation for single-wheel Mohr–Coulomb models with similar N_{Su} ratios (the N_{Su} values were 1.06, 1.13 and 1.36); * indicates groundwater level at base of pavement

D’Appolonia *et al.* (1971). This was most pronounced for high overconsolidation and shallow influence depths (Figure 12), although the effect on load–settlement response was small

when compared with that arising from changes in shear strength (Figure 6). For the multi-wheeled models, the behaviour was less influenced by the upper strata and hence the

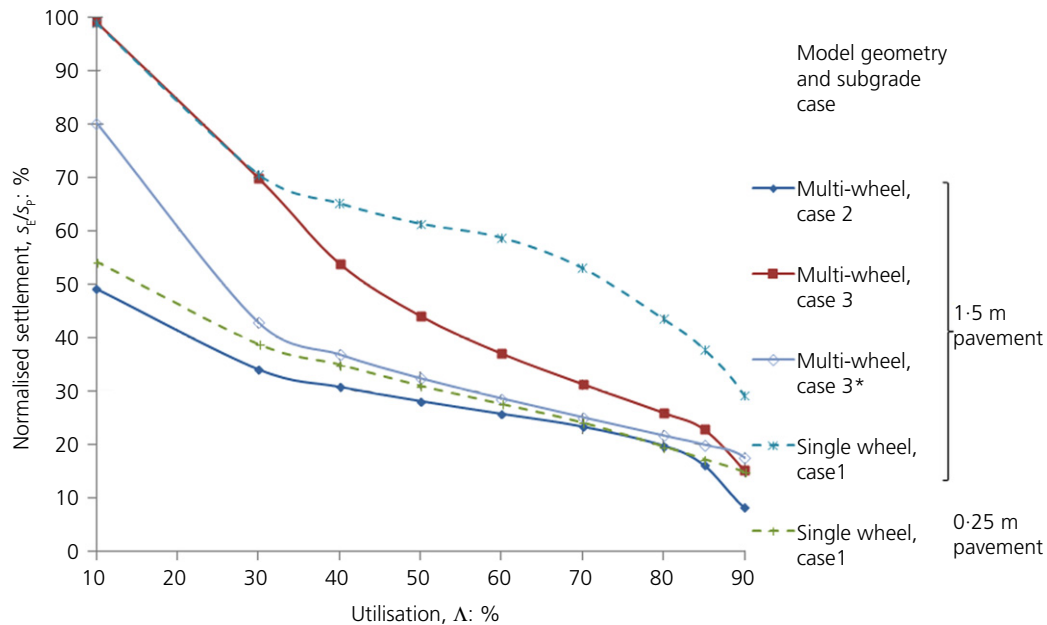


Figure 8. Normalised settlement plotted against utilisation for multi-wheel Mohr–Coulomb models with varying subgrade models. For comparison, normally consolidated (case 1) subgrade single-wheel models are also shown. Note that the shear strength and stiffness at the influence depth of 6 m were the same for subgrade cases 3* and 2 (* indicates groundwater level at base of pavement)

impact of the upper overconsolidated layer was even less significant.

Increasing K_0 in the granular fill had a negligible influence on settlement. This may be due to the simplistic choice of the material model, which assumed constant stiffness. More sophisticated models (e.g. Duncan and Chang, 1970; Wolff and Visser, 1994) that account for stiffening under increasing mean normal effective stress and strain-hardening under increasing deviator stress may indicate a greater influence of K_0 . This is important in understanding the response of an unbound pavement to a single wheel load (as suggested by Brown (1996)). It is likely that such sophisticated modelling of the pavement is not as necessary for large multi-wheeled vehicles because the influence of the pavement layers on the composite response is greatly reduced (Table 4).

4.5 Comparison of principal stress rotation

Significant rotation of the principal axes occurred within the pavement layers in front of or behind a single wheel, even at relatively low utilisations (Figure 13). Below the pavement, only small principal stress rotation occurred at low utilisation. This was more pronounced for the weaker subgrade models and can be attributed to two factors. Firstly, a lower bearing capacity (and therefore lower wheel pressures for the same degree of utilisation) means smaller relative changes to the in situ stress state. Secondly, a higher relative pavement stiffness causes greater load spreading (Figure 3), reducing the principal

stress rotation in the subgrade (Figure 13). The inclination of principal stresses reduced with depth, as self-weight stresses become dominant.

For the multi-wheel model at low utilisation, the subgrade principal stresses showed little inclination, reducing further with depth and increasing with increasing utilisation. At high utilisation (80%), a zone of principal stress reversal was apparent directly beneath the pavement (Figure 14) as the passive part of the failure mechanism was mobilised. This coincided with the development of large plastic strains throughout the entire compression zone and yield being initiated in parts of the extension zone (Figure 11).

5. Practical considerations for heavy-haul roads

The simplified modelling indicated that the response of an unbound pavement–subgrade system is fundamentally different when subjected to loading from a large vehicle with many closely spaced wheels rather than a single wheel. The influence of the pavement layer and the subgrade surface (dominant for the single-wheel case) is reduced and behaviour is more strongly influenced by the soil at depth (see Figures 4 and 8 and Table 4). Designing such roads by only investigating the subgrade surface and specifying a granular layer thickness to minimise irrecoverable subgrade strain from single, separate wheel loads (as is done for conventional roads) is thus inappropriate.

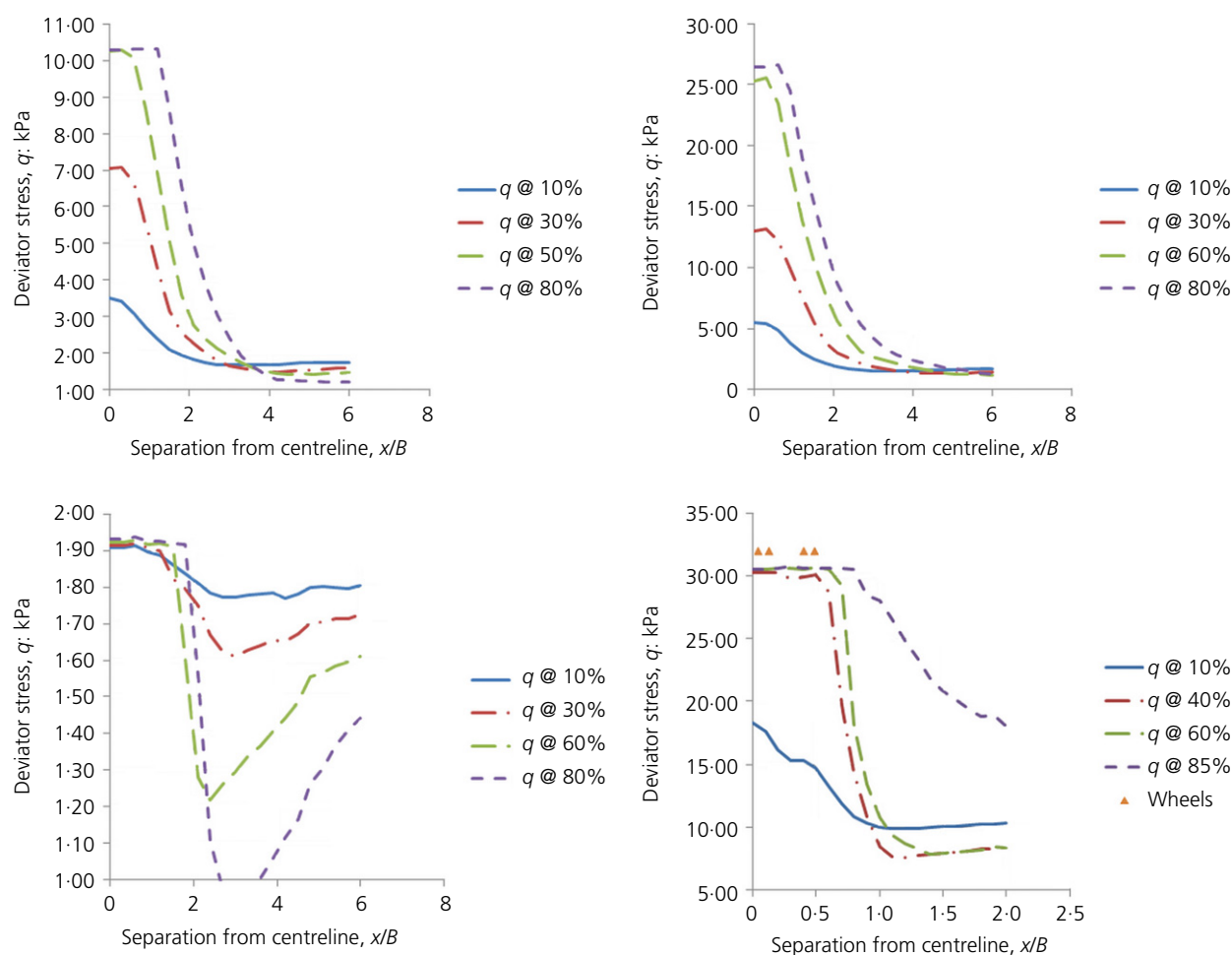


Figure 9. Deviator stress q (equal to shear stress $\times 2$) at base of pavement layer for various degrees of utilisation (Δ): (a) single wheel, subgrade case 2; (b) single wheel, subgrade case 3; (c) single wheel, subgrade case 1; (d) multiple wheels, subgrade case 3. (a)–(c) Pavement depth = 0.25 m, $B = 0.25$ m. (d) Pavement depth = 1.5 m, B assumed to be 6 m (whole-vehicle width)

A different site investigation and design approach is therefore necessary for roads carrying large multi-wheeled vehicles. Surface tests, such as the commonly used CBR test, dynamic probing or in situ surface stiffness tests (Frost, 2000), will still be useful in understanding resistance to rutting on the scale of a single wheel but less useful in understanding the behaviour of deeper soils when exposed to repetitions of multi-wheeled vehicles. The risk of degradation due to a whole-vehicle-scale mechanism will require investigation to greater depths using investigation techniques more common to the design of large foundations, such as percussive drilling or cone penetration tests. Furthermore, monitoring of pavement surfaces for rutting may not give an indication of deeper-seated strain development, which is likely to manifest over a larger area. Under large strains close to or exceeding yield, excess pore water pressures are expected to accumulate in the subgrade; monitoring pore water pressures at depth by way of piezometers in boreholes may therefore be more effective.

The finite-element results suggest that a large vehicle will generate local yielding at much lower utilisations (Figures 8 and 11). As shakedown theory indicates that cyclic failure is expected at loads slightly in excess of those causing the first local yield, a conventional design that aims to avoid cyclic degradation would need to limit the extents of yielding, either through a very deep pavement or high earthworks to spread transient stresses sufficiently, or by strengthening the ground with large-scale ground improvement. Both of these options are unlikely to be economically viable for a temporary road.

The excess pore water pressures generated by a single vehicle passage may initially reduce the strength of the subgrade, potentially making it unsafe for another vehicle to pass. However, consolidation of the subgrade is expected to increase its strength over time. By taking this into account, an economic observational design, similar in philosophy to that

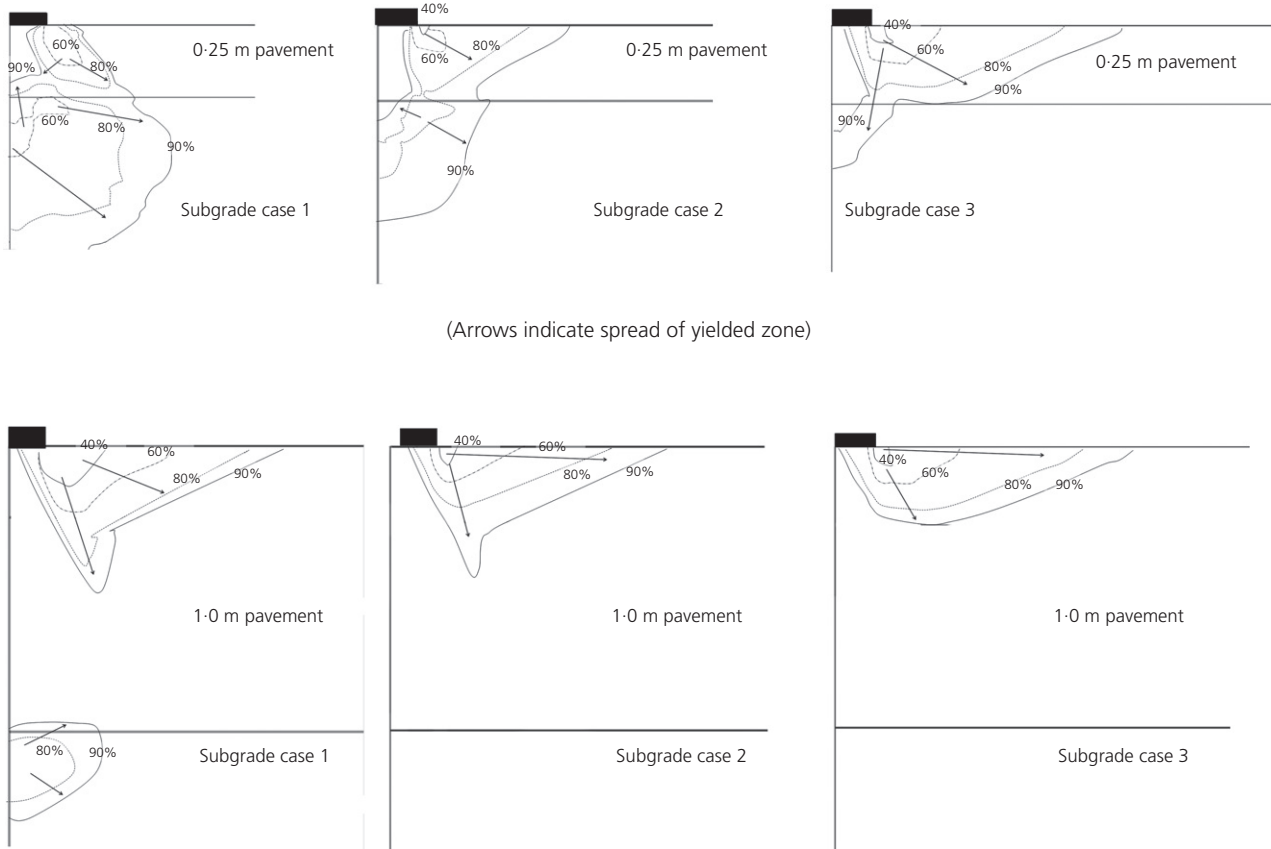


Figure 10. Progression of failure mechanisms indicated by 1% shear strain (i.e. large post-yield strain) contours for various degrees of utilisation (shown as percentage values on the plots) for single-wheel Mohr–Coulomb models

of the Cape Kennedy causeway to transport Apollo mission rockets (Peck, 1969), could be achieved: vehicles only traverse once the subgrade strength has sufficiently recovered. For this approach, the following aspects of subgrade behaviour, discussed further by Krechowiecki-Shaw *et al.* (2016), need to be understood.

- The rate at which consolidation occurs: monitoring in situ pore water pressures during operations would be more effective than estimation from laboratory tests on recovered samples and would provide a real-time indication of risk.
- Strength recovery as a function of excess pore water pressure dissipation: advanced laboratory testing will be required to determine the degree of consolidation necessary to recover the soil’s initial strength fully. This can then be used as a trigger level to allow passage of the next heavy vehicle.

If this approach is used, large strains are likely to develop (at least initially). Topping up of the pavement surface may be necessary to maintain design alignment. An additional complication arises if this design approach is used to mobilise high proportions of capacity and the compression zone of the

failure mechanism fully yields (Figure 11). Resistance to a failure mechanism is then provided by the ‘passive’ extension zone away from the loaded area. Not only is stability wholly dependent upon the strength of the extension zone being maintained, but the greater rotation of principal stresses (Figure 13) will also accelerate cyclic degradation. The response of soil to such an extreme combination of actions is not well understood and further research into this is merited.

6. Conclusions

The simple material models used in the finite-element analyses presented in this paper are easy to understand and apply, and the effects of yield can be isolated and investigated. It is recognised that these models are not representative of real soil behaviour, which often exhibits features such as strain- and pressure-dependent stiffness. Further investigations, isolating the effects of these properties, will be useful in developing a more realistic picture of a soil’s response. However, the modelling undertaken in this study clearly indicates that the composite pavement–subgrade response for a multi-wheeled large vehicle is fundamentally different to that of a conventional vehicle. Road design must therefore consider the following issues.

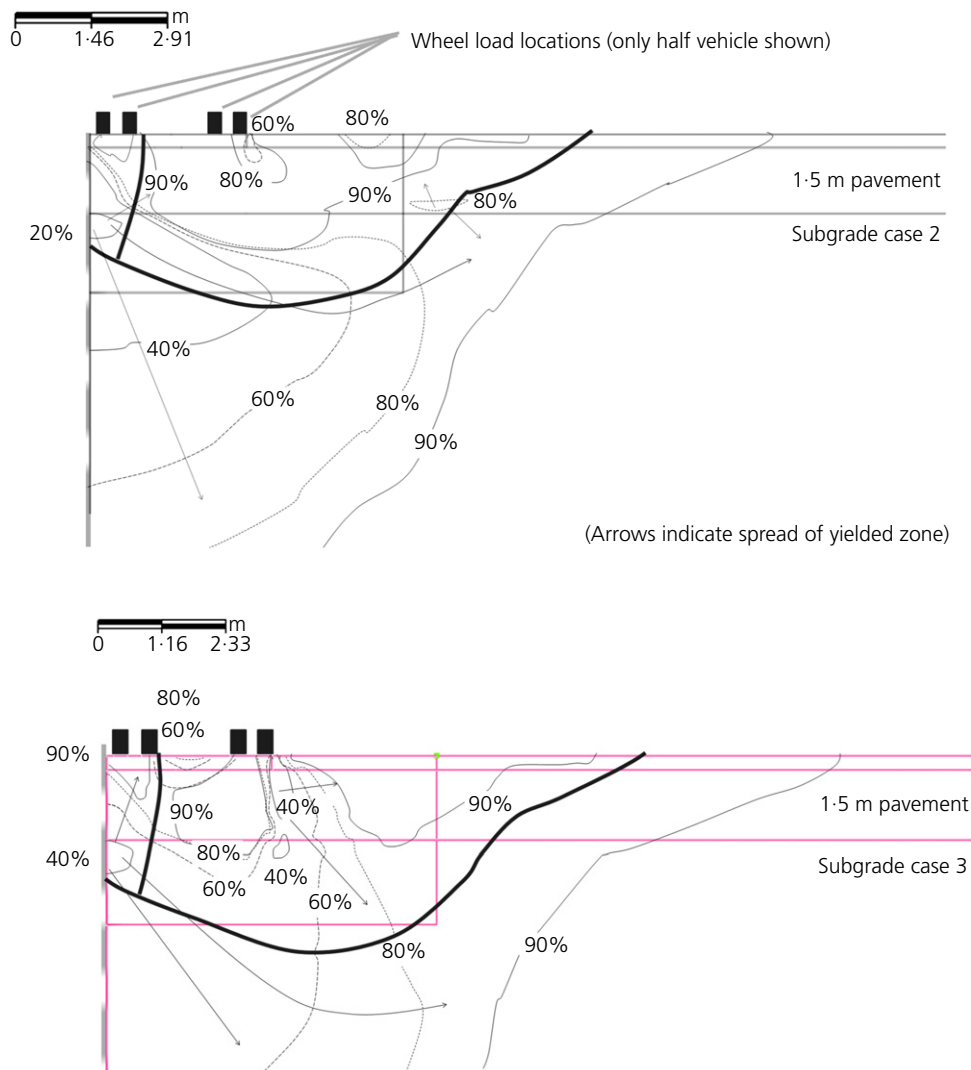


Figure 11. Progression of failure mechanisms indicated by 1% shear strain contours for various degrees of utilisation (shown as percentage values on the plots) for multi-wheel Mohr–Coulomb models. The thick lines follow the maximum shear strains developed at 95% utilisation (i.e. approximate to the final failure mechanism)

Table 3. Ultimate wheel pressures from finite-element single- and multi-wheel models. The bearing capacity of the pavement-only single-wheel model is 111 kPa

	Pavement depth: m	Ultimate wheel pressure, ω_{ult} : kPa		Ratio of ultimate pressures (case 3/case 2)
		Subgrade case 2	Subgrade case 3	
Single wheel	0.00	32	80	2.54
	0.25	54	105	1.94
	0.50	98	111	1.13
	1.00	111	111	1.00
	1.50	111	111	1.00
Multi-wheel	1.50	197	320	1.62

■ Interactions between wheels result in deeper soil (which is closer to a normally consolidated state and more prone to local compressive yielding) being mobilised and is thus far

more influential. The composite system responds on a whole-vehicle scale, similar to the response of a single wheel load applied to a thin pavement. Local yielding

Table 4. Ultimate wheel pressures for single- and multi-wheel models with varying pavement strength parameters

Subgrade model	Pavement depth: m	ω_{ult} : kPa	
		Pavement $\phi' = 40^\circ$	Pavement $\phi' = 32^\circ$
Single wheel, subgrade case 1	0.25	18	13
Single wheel, subgrade case 2	0.25	54	29
Single wheel, subgrade case 2*	0.25	63	48
Multiple wheels, subgrade case 2	1.5	197	169
Multiple wheels, subgrade case 3	1.5	320	290
Multiple wheels, subgrade case 3*	1.5	340	320

*Groundwater level at base of pavement

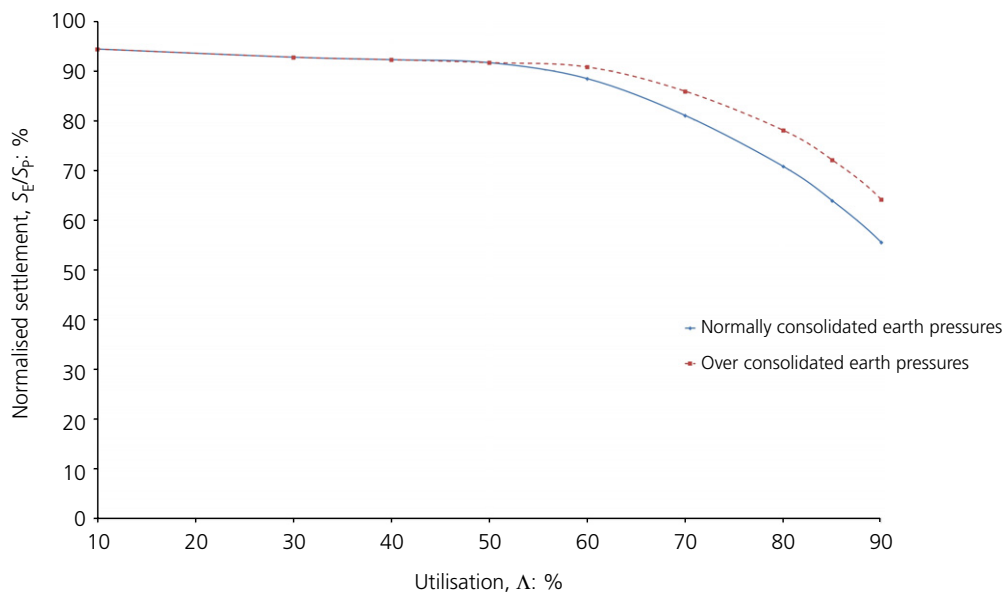


Figure 12. Normalised settlement plotted against utilisation plots for Mohr–Coulomb single-wheel models of subgrade case 3* (i.e. groundwater at base of pavement) with pavement depth of 0.25 m and varying coefficients of lateral earth pressure in the overconsolidated portion of the subgrade layer

behaviour can be described by assuming an influence depth equal to the vehicle width, as proposed by Foye *et al.* (2008) for shallow foundations. The pavement and upper overconsolidated subgrade layers are less influential for large multi-wheeled vehicles.

- Principal stress rotation from moving wheel loads is known to accelerate degradation significantly. This will complicate the degradation regime under a large heavy vehicle; soil at depth is expected to be less affected by stress rotation than surface soils due to the higher self-weight stresses. However, if loads close to the static bearing capacity are transported, a passive wedge begins to mobilise, resulting in significant principal stress rotations extending to depth. As the passive wedge provides the final restraint against failure, rapid strength degradation here could result in sudden collapse. Further research into soil under such extreme conditions is required.

- Limiting the subgrade to small-strain behaviour to avoid degradation under repeated transient load may be unfeasible for a temporary road; the large factors of safety necessary (in the region of 3 to 10) may not be economically justifiable.
- Conventional investigation and design, while useful for designing pavement layers to resist rutting along wheel tracks, is governed by the properties of the subgrade surface and does not give adequate understanding of deeper-seated risks.

In contrast to the conventional design method, it is recommended that deep investigation (similar to that used for large shallow foundations) and in-service monitoring of deep subsoils are conducted to understand the real-time risk of strength degradation. A design that allows repeated application of medium to large strains is expected to result in significant changes to the subgrade. If the interaction between

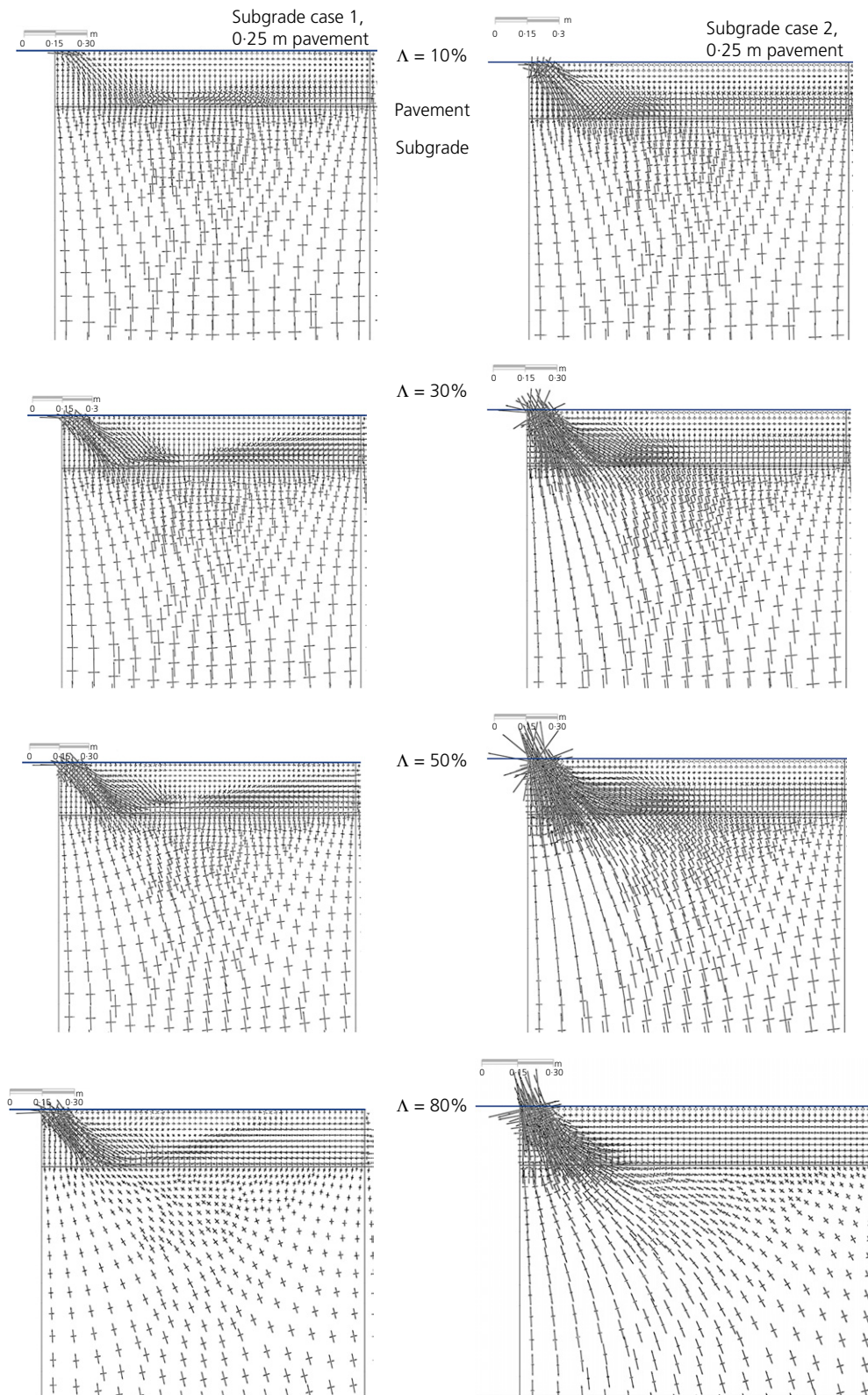


Figure 13. Principal stress vectors (i.e. line length = stress magnitude, rotation = stress direction) for single-wheel Mohr–Coulomb models with varying degrees of utilisation (Λ). A vector aligned horizontally and vertically but where the horizontal vector is larger indicates a rotation of 90° (i.e. full principal stress reversal)

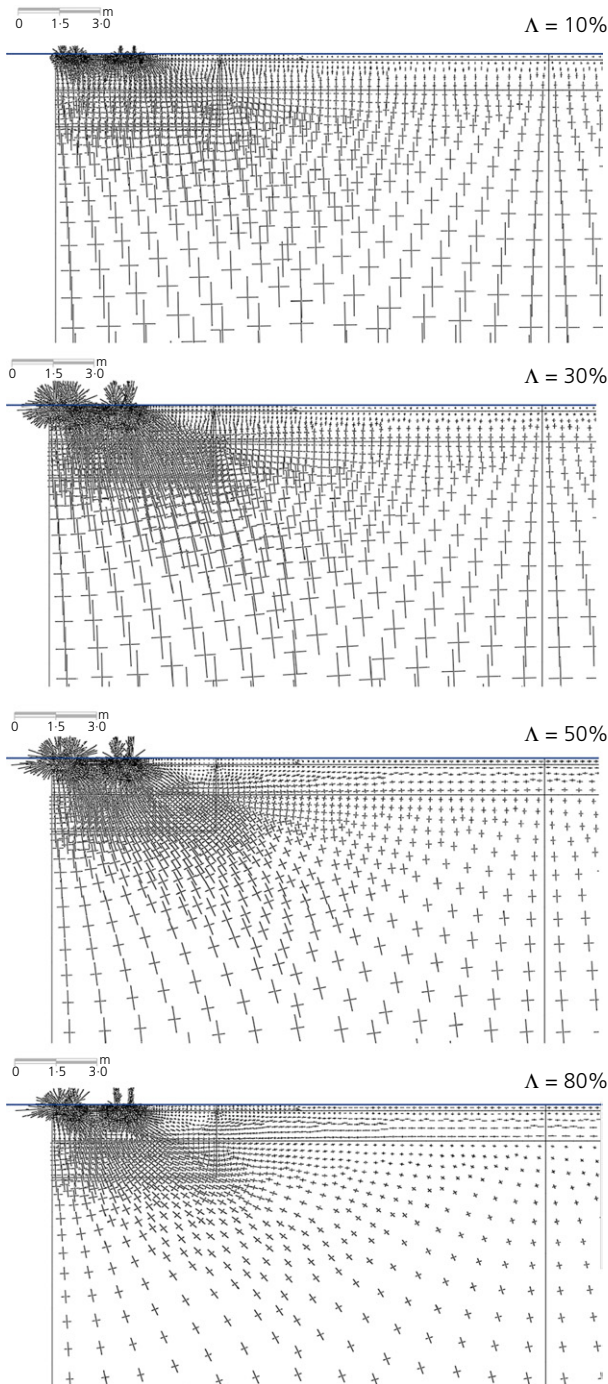


Figure 14. Principal stress vectors (i.e. line length = stress magnitude, rotation = stress direction) for multi-wheel Mohr–Coulomb model with subgrade case 3 and a 1.5 m pavement depth, with varying degrees of utilisation (Λ). A vector aligned horizontally and vertically but where the horizontal vector is the larger indicates a rotation of 90° (i.e. full principal stress reversal)

degradation and consolidation-related strengthening over time is understood and the risks are well managed, significant construction cost benefits are possible.

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