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## Cold-to-electricity conversion using a piston based engine in cold energy storage (CES) system, part one

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# Accepted Manuscript

Cold-to-electricity conversion using a piston based engine in cold energy storage (CES) system, part one: a theoretical study

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1 **Cold-to-electricity conversion using a piston based engine in cold**  
2 **energy storage (CES) system, part one: a theoretical study**

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7  
8 **Abstract:** This study concerns the cold-to-electricity conversion in a piston based engine. A  
9 theoretical model was developed to predict the power generation rate in the engine.  
10 Parametric study showed the significant effect of p-crank angle, motor speed, gas pressure  
11 and temperature and chamber volume on the power generation by the piston based engine. It  
12 was found that optimal p-crank angle was normally in the range of 55°~76°. The minimum  
13 inner diameter of the valves connected to the engine was a crucial factor that limited the  
14 power capacity of the engine system. Based on the experimental data, the large engine (1900  
15 cm<sup>3</sup>) and small engine (162 cm<sup>3</sup>) had an actual efficiency of 24.1% and -23.6%, respectively.  
16 This implied that the large engine was feasible to be used in CES system for electricity re-  
17 generation during peak hours. However, the induced CES efficiency was 18.9% and 52.1% in  
18 two different situations. The results indicate the importance of engine development in  
19 promoting the CES technology.

20 **Keywords:** Cold-to-electricity conversion; Piston based engine; Power generation rate;  
21 Engine efficiency; CES efficiency.

22

23

## 24 1. Introduction

25 Cold energy storage has attracted more and more attention due to its significant role in  
26 addressing the main energy challenges [1-2]. The CES technology is competitive in dealing  
27 with fluctuation of electricity demand (demand side) [3-4] and intermittency of renewable  
28 resources (supply side) [5-6]. Furthermore, the CES using cryogenics as the storage media  
29 could achieve an exergy efficiency of over 80%, which enables the CES system competitive  
30 compared with thermal energy storage (TES) technologies in the energy discharging process  
31 [7].

32 As the essential component for cold-to-power conversion in a CES system, engine systems  
33 were extensively investigated [8-15]. Based on the thermal analysis of isothermal, adiabatic  
34 and polytropic processes, it was found that isothermal expansion in the engine had the largest  
35 power generation [2]. In reality, feasible solutions for expansion with approximate isothermal  
36 process in engine or turbine have been proposed. These include utilisation of ambient heat [8-  
37 11] and combination of ambient and combustion heat [12-13]. Manning et al [8] developed a  
38 direct drive system using multi-stage expansions with reheating prior to the final stage of  
39 expansion in Brayton cycle. A regenerative device was proposed to improve the heat  
40 efficiency. West et al. [9] used a double acting piston expander to improve the efficiency of  
41 the working fluid. For using environmental heat, Negre [10] (MDI company in France) firstly  
42 designed multistage engine system. Due to the utilisation of ambient heat, the expansion in  
43 the system was approximated to an isothermal process. Marquand [11] (University of  
44 Westminster) developed a two-stage turbine with a heat pipe before the inlet of engine to  
45 make use of environmental heat. The heat transfer area of the heat pipe was up to 1.4 m<sup>2</sup>,  
46 while the heat transfer efficiency was 85%. It was reported that when the inlet pressure was  
47 4.5 MPa, and engine speed was 1000 rpm, the power generation was as high as 25 kW.

48 In the above studies, the processes in engine were considered as multi-stage adiabatic  
49 expansion with middle reheating using the ambient heat. This is favourable for improving  
50 system efficiency and power generation by engine. However, since the inlet temperatures in  
51 the expansion stages of the engine were lower than ambient temperature, system efficiency  
52 and power capacity were significantly restrained. Oxley et al [12] utilized combined heat  
53 from both ambient and combustion in a Stirling engine. Oxygen from the liquefaction and  
54 separation of air was used to enhance the efficiency of fuel combustion. Latter et al [13] used  
55 liquid air as the working fluid in the Rankine cycle. They utilized ambient heat to increase the  
56 liquid air temperature in the multi-stage expansion process. Additional fuel was subsequently  
57 injected to make use of the combustion heat in an internal combustion engine. Comparing  
58 with no utilisation of combustion heat, it was reported that the power generation increased by  
59 50%.

60 Apart from utilisation of environmental heat and combustion heat during the multi-stage  
61 expansion processes, other methods for improving system efficiency and power generation  
62 include controlling of injecting time of high pressure gas [10] and parametric optimization of  
63 engine [14-15]. Negre et al [10] developed an engine system, with high pressure gas injected  
64 into the engine intermittently. In the system, air was absorbed into the chamber of engine  
65 when the piston went down from top dead centre (TDC) to bottom dead centre (BDC). The  
66 absorbed air was then compressed to 20 bar under 400 °C in the movement of piston in the  
67 BDC to TDC process, during which the high pressure gas was injected. As a result, the  
68 temperature of the mixture (expansion gas and the high pressure air) was improved, leading  
69 to an increased power generation by the engine. However, due to the short injecting time, the  
70 thermal energy from the high pressure gas was insufficiently absorbed in the mixing process.  
71 Consequently, Negre [10] designed a special crank published in patent to control time for  
72 mixture. With the new connecting rod, the piston can stay near the top for about 70° of the

73 shaft rotating. As a result, the components from two different injections are fully mixed,  
74 leading to a largely improved power capacity of the engine system.

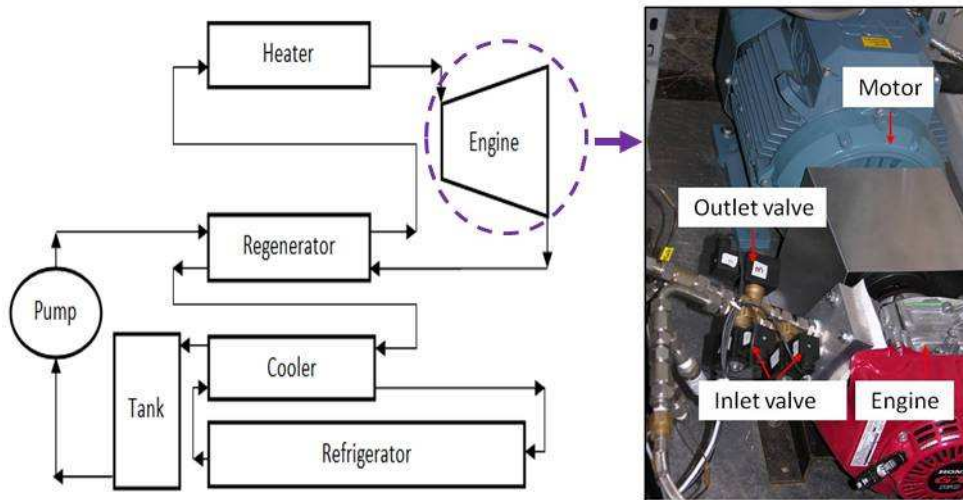
75 Group of Knowlend in University of Washington has done lots of work on parametric  
76 optimization of engines. As claimed by Knowlend et al [14], parametric optimization of  
77 engine was also feasible for improving power generation under the same conditions. Small  
78 radius of chamber, long length of stroke and low engine speed were all favourable for  
79 improving power generation and system efficiency of the engine. Knowlend et al [14] also  
80 designed jagged surfaces for both cylinder block and piston with heater core imbedded inside  
81 expansion chamber. In the design, heat transfer agent was used to increase temperature of  
82 HTF. Knowlend et al [15] analyzed the enhancement of heat transfer rate by this novel  
83 piston-head configuration. They claimed that by imbedding a heater core within the  
84 expansion chamber, the expansion efficiency in the engine was performed as 85% of the ideal  
85 isothermal process.

86 With a new valve scheme connected, a piston based engine system was developed in this  
87 study. By setting an optimal p-crank angle, the maximum power generation rate through gas  
88 expansion in the engine chamber can be achieved. The power generation of the piston based  
89 engine was estimated by establishing a theoretical model. Parameters effects, promotion of  
90 power capacity, engine efficiency and its effect on the CES efficiency were investigated.

## 91 **2. An engine system for cold to power conversion**

92 Cold energy storage technology stores off-time electricity as cold energy in the cold  
93 storage media and regenerates electricity in engine/ turbine system by utilising the cold  
94 exergy through thermodynamic cycle [16]. A regenerator was used in the Rankine cycle for  
95 improving the efficiency of the thermal dynamic cycle and the CES system. Figure 1 showed  
96 the role of engine in the CES system, and a specific piston based engine system used for

97 power/ electricity generation. The referred engine system was mainly composed by an  
 98 engine, a motor/ electrical generator and a new valve scheme with associate controlling  
 99 modules. The advantage of the engine system is the efficient controlling of the opening time  
 100 of the engine inlet, by which high pressure and medium-to-high temperature gas was  
 101 efficiently expanded.

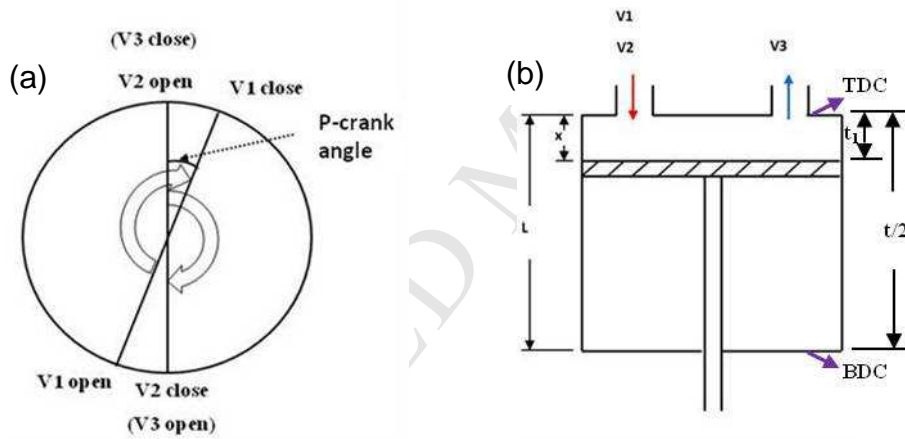


102  
 103

Figure 1. Schematic diagram of the experimental system.

104 Two solenoid valves were used at the inlet of the engine to control the opening time, since  
 105 the expanding gas could only enter the chamber in the condition that two solenoid valves at  
 106 the inlet were open simultaneously. For precisely controlling opening/ closing time of valves,  
 107 count setting method was adopted. Specifically, the circle of the shaft in engine is equally  
 108 divided into 320 parts. Each part is called one count. When the piston is at the top dead centre  
 109 (TDC), an index is marked at the bottom of the shaft where the 0<sup>th</sup> count is scheduled. In the  
 110 controlling modules, opening and closing operations on valves are affected by the counts.  
 111 The influence on valve operations is realized by initiating electrical voltage for all of the 320  
 112 counts in the circle of the shaft. Consequently, by resetting the electrical voltage on different  
 113 counts, the opening and closing time for each valve can be controlled.

114 The piston based engine has two solenoid valves (V1, V2) installed at the inlet and another  
 115 solenoid valve (V3) used at the outlet. The so-called p-crank angle is defined as the rotating  
 116 angles of the shaft in a certain period when both valves at the engine inlet are open  
 117 simultaneously. For instance, V2 is set open from 0<sup>th</sup> count to 180<sup>th</sup> count, while V2 is set  
 118 open from 210<sup>th</sup> count to 30<sup>th</sup> count, as a result, the p-crank angle is  $30 \times \frac{360}{320} \approx 33.8^\circ$  (30  
 119 counts from 0<sup>th</sup> count to 30<sup>th</sup> count), as presented in Figure 2(a). In the opening time ( $t_1$ ),  
 120 piston moves from the TDC to the shown location ( $x$ ); while at half periodic time ( $t/2$ ),  
 121 the piston reaches the bottom dead centre (BDC), as presented in Figure 2(b) where L is the  
 122 length of the engine chamber.



123  
 124 Figure 2. Details of the piston based engine: (a) formation of p-crank angle by count setting;  
 125 (b) piston movement in the engine chamber.

126 For characterizing the p-crank angle, the opening time of the engine inlet is written as in  
 127 Eq. (1):

$$128 \quad \frac{t_1}{t/2} = \alpha \quad (1)$$

129 where  $\alpha$  is the ratio of opening time to half periodic time of the engine. In this case, p-crank  
 130 angle can be expressed as:

$$131 \quad p\text{-crank angle} = 180^\circ \cdot \alpha \quad (2)$$

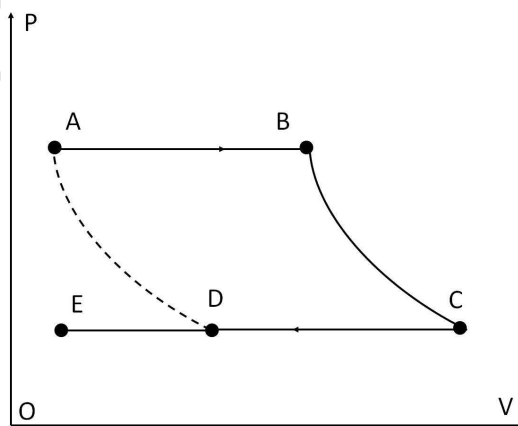


132 In terms of Eqs. (1) ~ (2), it is seen that the increase of the p-crank angle results in the  
 133 prolonging of the opening time of the valves. This indicates more mass flow rate (kg/s) of the  
 134 expanding gas in the engine chamber. On the other hand, the expanding volume is reduced  
 135 due to the continuous movement of the piston in the prolonged opening time, leading to a  
 136 decreased specific power generation (J/kg). This indicates a significant effect of the p-crank  
 137 angle on the power/ electricity generation rate (W) of the piston based engine system.

### 138 3. Theoretical model for predicting power generation by the engine

139 Thermal dynamic cycle of a piston based engine was demonstrated as process A-B-C-E, as  
 140 presented in Figure.3. The process A-B is the gas injection into the engine chamber, during  
 141 which mass of gas increases with time. The process B-C represents the expansion process.  
 142 Since the valves at both inlet and outlet of the engine are closed, gas mass keep constant  
 143 during the process. While the process C-E is the gas elimination process, when the gas mass  
 144 in the chamber is reduced with time. One can see that there is no compression process for the  
 145 engine driven by high pressure gas. As a result, the specific engine power ( $w$ , J/kg) is  
 146 generated in the process B-C, which can be expressed as in Eq. (3):

$$147 \quad w = \int_{P_B}^{P_C} -v \cdot dP \quad (3)$$



148

149

Figure 3. Thermal dynamic cycle in the piston based engine.

150 The gas expansion in the piston based engine can be either isothermal process or adiabatic  
 151 process. As stated in [2], expansion in an isothermal process results in the maximum power  
 152 output, compared with other thermal processes in the engine. However, the isothermal  
 153 condition is difficult to be realized in practical engine tests, since the heat transfer process  
 154 between the expanding gas (i.e. CO<sub>2</sub> or air) and the ambient is inefficient in a short time.  
 155 With a high motor speed (i.e. 2900 rpm), little amount of heat can be transferred to the  
 156 ambient. In this regard, gas expansion (process B-C) can be considered as the adiabatic  
 157 expanding process, which is analysed in the following.

### 158 3.1. Specific power

159 Although the expanding process in the engine is irreversible due to the pressure difference  
 160 between the chamber and the ambient, it is regarded as approximate adiabatic process due to  
 161 the prompt expanding of gas in the chamber. Consequently, the process equation can be  
 162 written as:

$$163 \quad P_B v_B^{k_{po}} = P_C v_C^{k_{po}} \quad (4)$$

164 where  $k_{po}$  is the Poisson factor, which is defined as,

$$165 \quad k_{po} = \frac{C_{p,g}}{C_{v,g}} \quad (5)$$

166 in which  $C_{p,g}$ ,  $C_{v,g}$  are the heat capacity of gas at constant pressure and volume,  
 167 respectively. The technical power by the engine can be expressed as:

$$168 \quad w = \int_{P_{x=x}}^{P_{x=L}} -v \cdot dP \quad (6)$$

169 From Eqs. (4)~(6), the specific power generation in the adiabatic process of an ideal gas is  
 170 calculated as:

$$171 \quad w = \frac{1}{k_{po} - 1} \cdot R_g \cdot T_{in} \cdot \left[ 1 - \left( \frac{P_c}{P_{in}} \right)^{\frac{k_{po} - 1}{k_{po}}} \right] \quad (7)$$

172 In which  $R_g$  is the gas constant ( $J/(kg \cdot K)$ );  $P_{in}$  and  $T_{in}$  are pressure and temperature of the  
 173 expanding gas at the inlet of the piston based engine, respectively;  $P_c$  represents the gas  
 174 pressure after expansion when the piston reaches the BDC of the chamber. In ideal situation,  
 175  $P_c$  is equal to the ambient pressure ( $P_c \approx P_o$ ). However, in the condition that the engine has  
 176 an expansion ratio lower than  $P_{in}/P_c$ ,  $P_c$  is higher than the ambient pressure, indicating a  
 177 significant exergy loss in the expansion process.

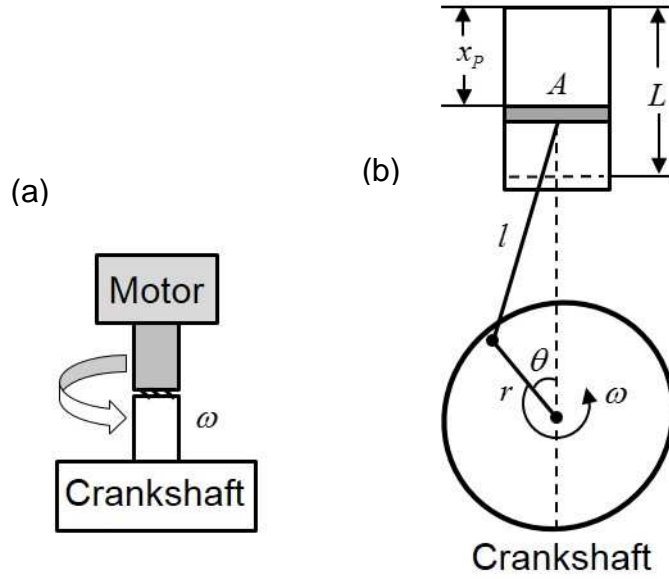
### 178 3.2. Mass flow rate

179 Ideal-state equation for the expansion gas at the engine inlet is as below:

$$180 \quad P_{in} \cdot V_g = m \cdot R_g \cdot T_{in} \quad (8)$$

181 Where  $m$ ,  $V_g$  are the mass and volume of the expansion gas entering the engine chamber at  
 182 the time  $t_1$ , respectively. Elhaj et al [17] developed a theoretical model for predicting the  
 183 piston location during the rotation of the crankshaft. To characterize the movement of the  
 184 piston, Figure 4 shows the schematic diagram of the connections between motor, the  
 185 crankshaft and the piston of the engine.

186



187

188 Figure 4. Schematic diagram of the motor and the piston based engine: (a) connection  
 189 between motor and the crankshaft; (b) connection between the crankshaft and the piston.

190 Taking the TDC as the reference position, the piston location ( $x_p$ ) can be formulated as the  
 191 function of the rod length ( $l$ ), crank radius ( $r$ ) and the rotating angle ( $\theta$ ):

$$192 \quad x_p = r(1 - \cos \theta) + l \left[ 1 - \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2 \theta} \right] \quad (9)$$

193 Since the motor speed ( $\omega$ ) is controlled as constant values in the experimental system, the  
 194 rotating angle can be written as:

$$195 \quad \theta = \omega t \quad (10)$$

196 The volume of the gas in the engine chamber at the time  $t_1$  can be expressed as:

$$197 \quad V_g = x_p \cdot A \quad (11)$$

198 in which  $A$  is the sectional area of the engine chamber. In consideration of one rotation  
 199 period ( $t$ ), according to Eq. (8) ~ (11), the average mass flow rate ( $\bar{m}_{in}$ ) at the inlet is  
 200 obtained:

$$201 \quad \bar{m}_{in} = \frac{m}{t} = \frac{P_{in}}{T_{in}} \cdot \frac{1}{2\pi R_g} \cdot \omega \cdot A \cdot \left\{ r[1 - \cos(\omega t_1)] + l \left[ 1 - \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2(\omega t_1)} \right] \right\} \quad (12)$$

202 Since it takes half of the rotation period for the piston moving from TDC to BDC, from Eq.  
 203 (9), the relation of the crankshaft radius and the length of the engine chamber is established:

$$204 \quad x_p = 2r = L \quad (\text{when } \theta = \pi) \quad (13)$$

### 205 3.3. Power generation rate

206 The power generation rate of the piston based engine is calculated as the product of the  
 207 specific power and the average mass flow rate. Therefore, based on Eq. (7) and Eq. (12), the  
 208 average engine power rate can be calculated:

$$209 \quad \bar{W} = \frac{1}{k_{po} - 1} \cdot \frac{1}{2\pi} \cdot P_{in} \cdot \omega \cdot A \cdot \left[ 1 - \left( \frac{P_c}{P_{in}} \right)^{\frac{k_{po} - 1}{k_{po}}} \right] \cdot \left\{ r[1 - \cos(\omega t_1)] + l \left[ 1 - \sqrt{1 - \left( \frac{r}{l} \right)^2 \sin^2(\omega t_1)} \right] \right\} \quad (14)$$

210 According to Eq. (1) and (2), the opening time ( $t_1$ ) can be expressed as:

$$211 \quad t_1 = \frac{p - \text{crank angle}}{180^\circ} \cdot \frac{\pi}{\omega} \quad (15)$$

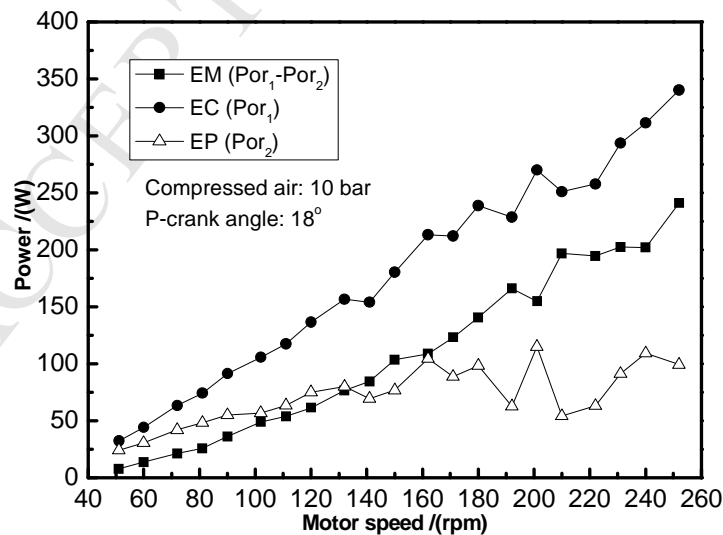
212 The above analysis indicates that the average engine power rate is determined by the thermal  
 213 physical parameters of the expansion gas (i.e. pressure and Possion factor), geometric  
 214 parameters (i.e. the volume, sectional area and length of the chamber, the rod length, and the  
 215 crank radius) and the operating conditions including the p-crank angle, the expansion ratio  
 216 and the motor rotating speed. However, since  $P_c$  is affected by the key parameters such as the  
 217 p-crank angle, rotating speed and the chamber volume, optimal parameters exist for achieving  
 218 the maximum power generation by the engine.

219 In the practical engine system, the valves installed at the inlet and outlet of the engine has  
 220 their minimum inner diameters. For example, for small valve that has a reflection time of  
 221 15ms for opening and closing operations, the minimum inner diameter ( $D_v$ ) is 1.8 mm. Due  
 222 to the gas flowing velocity in the valve is restrained below the sound velocity ( $u_m = 340m/s$   
 223 ), the volume flow rate at the inlet of the piston based engine becomes a limited value (

224  $\bar{V}_{in} \leq \frac{\pi}{4} D_v^2 \cdot u_m$ ). In the period that the piston moves from the TDC to the BDC, the average  
 225 volume flow rate can be written as:

$$226 \quad \bar{V}_{in} = \frac{1}{\pi} \cdot \omega \cdot A \cdot \left\{ r[1 - \cos(\omega t_1)] + l \left[ 1 - \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2(\omega t_1)} \right] \right\} \quad (16)$$

227 Based on the above equations, the maximum rotating speed is evaluated as 160 rpm. This  
 228 indicates that the power generation rate is affected by the motor rotating speed when it is less  
 229 than the maximum value (160 rpm). However, due to the limitation of the volume flow rate  
 230 of the expansion gas, the power generation rate is not significantly influenced with further  
 231 increasing of the rotating speed. Figure 5 was the experimental results showing the effect of  
 232 motor speed on the engine power generation. In the figure, EP (written as  $Pow_2$ ) represented  
 233 the contribution of the engine for power production, EC (written as  $Pow_1$ ) was the electricity  
 234 consumption, while EM (written as  $Pow_1 - Pow_2$ ) was the net electricity output of the  
 235 system. It was shown that although there was small fluctuation under a motor speed beyond  
 236 160 rpm, the power generation by engine (EP) became approximately stable.



237

238

Figure 5. Effect of the motor speed (with small valves).

#### 239 4. Optimal p-crank angle

240 As aforementioned, the optimal p-crank angle exists since the mass flow rate of the  
 241 expansion gas increases while the specific power generation is reduced with increasing of the  
 242 p-crank angle. For the gas expansion process, ideal-gas equation can be described as  
 243 Eqs.(17)~(18):

$$244 \quad P_{in} x^k = P_c L^k \quad (17)$$

$$245 \quad \frac{x}{L} = \left(\frac{P_c}{P_{in}}\right)^{\frac{1}{k}} \quad (18)$$

246 In terms of Eq. (14), (17) and (18), the average power rate of the piston based engine can be  
 247 written as:

$$248 \quad \bar{W} = \frac{1}{k_{po} - 1} \cdot \frac{1}{2\pi} \cdot P_{in} \cdot \omega \cdot A \cdot [1 - f(\theta)^{(k_{po}-1)}] \cdot f(\theta) \cdot 2r \quad (19)$$

249 In which,

$$250 \quad f(\theta) = \frac{r[1 - \cos(\theta)] + l[1 - \sqrt{1 - (\frac{r}{l})^2 \sin^2(\theta)}]}{2r} \quad (\theta = \pi\alpha) \quad (20)$$

251 For achieving the maximum power rate, the following equations are resulted:

$$252 \quad \left\{ f(\theta) \cdot [1 - f(\theta)^{(k_{po}-1)}] \right\}' = 0 \quad (21)$$

$$253 \quad f(\theta) = k_{po}^{-\frac{1}{k_{po}-1}} \quad (22)$$

254 As a result, the optimal p-crank angle is obtained. In a case study, the radius of the crankshaft  
 255 is the same as the length of the rod ( $r = l$ ), consequently,

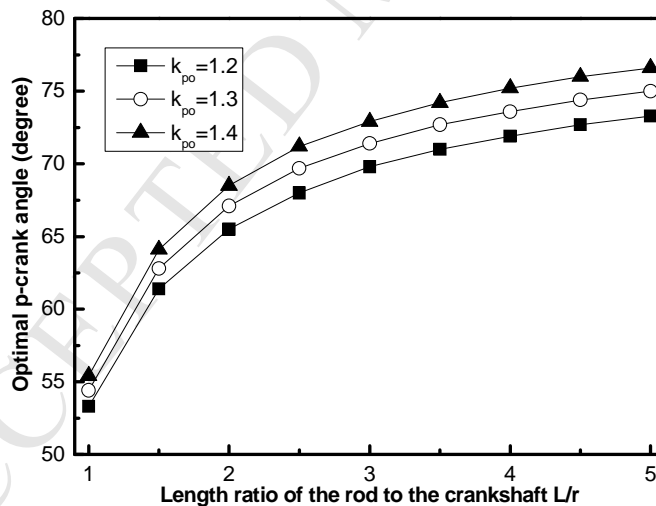
$$256 \quad \theta = \arccos[1 - k_{po}^{-\frac{1}{k_{po}-1}}] \quad (\text{in Rad}) \quad (23a)$$

$$257 \quad \alpha = \frac{\arccos[1 - k_{po}^{-\frac{1}{k_{po}-1}}]}{\pi} \quad (23b)$$

$$258 \quad p - \text{crank angle} = \frac{180}{\pi} \arccos[1 - k_{po}^{-\frac{1}{k_{po}-1}}] \quad (\text{in Degree}) \quad (23c)$$

259 For compressed air under different temperatures, the Possion factor  $k_{po}$  is in the range of  
 260 1.2~1.4. According to the above equations, the optimal p-crank angle in the case study is  
 261 calculated as approximately  $55^\circ$ . The value of  $\alpha$  is evaluated as around 0.307, indicating the  
 262 opening time represents 15.4% of the motor rotation period for achieving the maximum  
 263 power rate of the piston based engine. The case study also shows that the ratio of the piston  
 264 location to the chamber length is 42.7% by the end of the opening time. This allows the gas to  
 265 be expanded with the maximum mean power rate.

266 Figure 6 shows the influence of the length ratio of the rod to the crankshaft ( $L/r$ ) on the  
 267 optimal p-crank angle. The figure indicates that the optimal p-crank angle increases from  
 268 approximately  $55^\circ$  to  $76^\circ$  with a variation of the length ratio from 1 to 5. Furthermore, with  
 269 the expansion gas that has a higher Possion factor, the p-crank angle for achieving the  
 270 maximum power generation by the engine is normally  $1^\circ\sim 4^\circ$  larger.

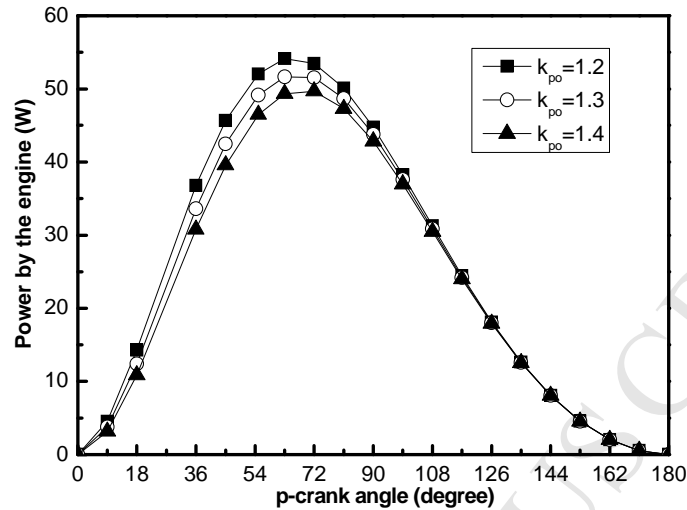


271  
 272 Figure 6. Variation of the optimal p-crank angle due to the change of the length ratio of the  
 273 rod to the crankshaft.

274 The effect of the p-crank angle on the power generation by the engine is shown in Figure 7.  
 275 With the increase of the p-crank angle, the power contribution by the engine increases firstly  
 276 and decreases subsequently. The highest power by engine is obtained under the optimum p-



277 crank angle. It is shown that under different Possion factor, the optimum p-crank angle is  
 278 slightly changed.



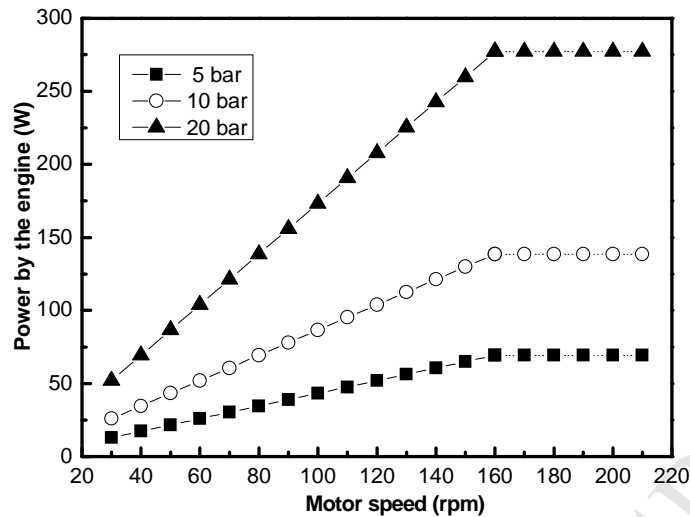
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280 Figure 7. The effect of p-crank angle on the engine power generation (compressed air: 10 bar;  
 281 motor speed: 60 rpm).

## 282 5. Power capacity of the engine under high operating parameters

283 This section aims to estimate the power capacity of the piston based engine under different  
 284 operating parameters, including the motor speed, gas pressure and temperature. The  
 285 parametric study was conducted under the optimal p-crank angle for obtaining the maximum  
 286 power generation in theory. Figure 8 shows the effect of motor speed in the condition of  
 287 providing compressed air to a small engine (with a chamber volume of  $162 \text{ cm}^3$ ) for  
 288 expansion. When motor speed was lower than 160 rpm, the power generation by engine  
 289 increased linearly with increasing of the motor speed. However, due to the restriction by the  
 290 smallest section area of the flowing passage in the valves, the power generation became  
 291 constant with a motor speed beyond 160 rpm.

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Figure 8. Effect of the motor speed on the power generation by the small engine.

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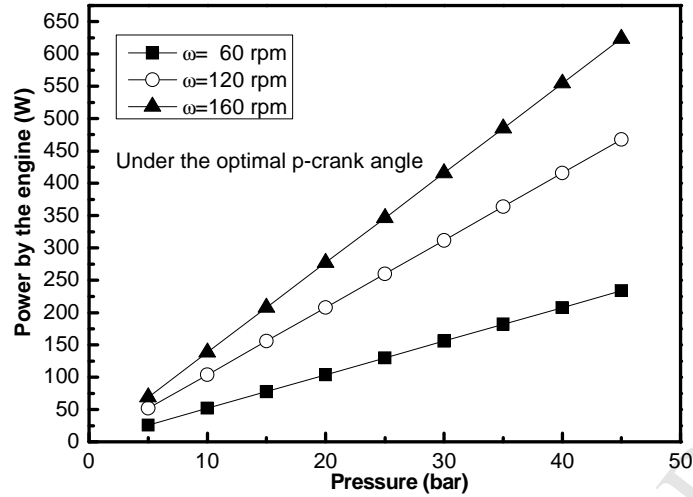
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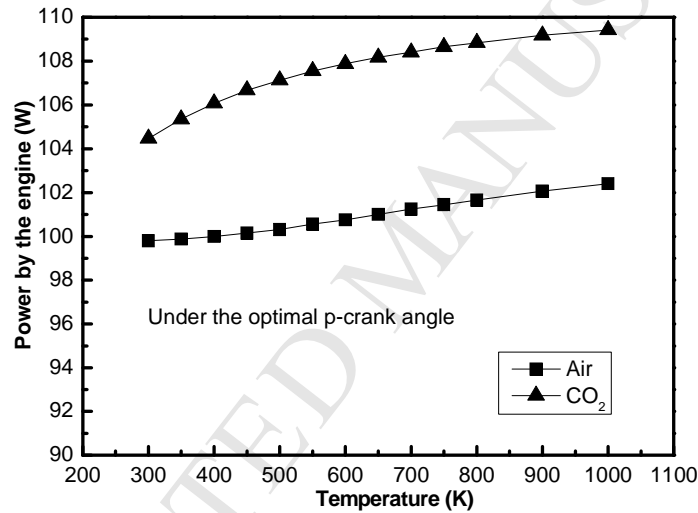
Figure 9 and Figure 10 presents the effects of pressure and temperature on the power capacity of the small engine (chamber volume of  $162 \text{ cm}^3$ ), respectively. As can be seen in Figure 9, power generation increased linearly with the increase of the gas pressure. It was noted that the positive effect of gas pressure on promotion of the power generation became more significant under a relatively higher motor speed (within 160 rpm). However, the effect of the gas temperature was featured differently with changed provision of the expansion gas to the engine. Figure 10 indicated the temperature influence was not obvious. This is because the multiplier  $[1 - (\frac{P_c}{P_{in}})^{\frac{k_{po}-1}{k_{po}}}]$  in Eq. (14) is relatively small for the expansion gas with a temperature in the range of 300~1000 K. Compared with the compressed air,  $\text{CO}_2$  led to more power generation by the engine. Consequently,  $\text{CO}_2$  is regarded as a better option as the expansion gas in the engine for power generation.



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Figure 9. Effect of the gas pressure on the power generation by engine.



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Figure 10. Effect of the gas temperature on the engine power generation.

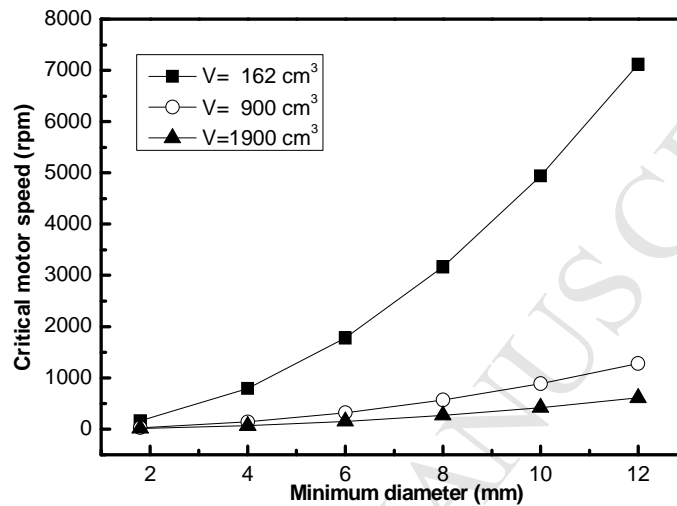
311 As mentioned in the above section, power generation by the piston based engine is  
 312 restricted by the minimum inner diameter of the valves. Theoretically, the velocity of gas is  
 313 less than the sound velocity 340 m/s. As a result, the critical motor speed ( $\omega_{cr}$ ) is induced,  
 314 which indicates the maximum power capacity of engine under certain parameters:

$$315 \quad \omega_{cr} = \frac{\pi^2}{4} \cdot \frac{340 D_{v,\min}^2}{V} \quad (24)$$

316 Figure 11 shows the calculated critical motor speed for different volume engines with a given

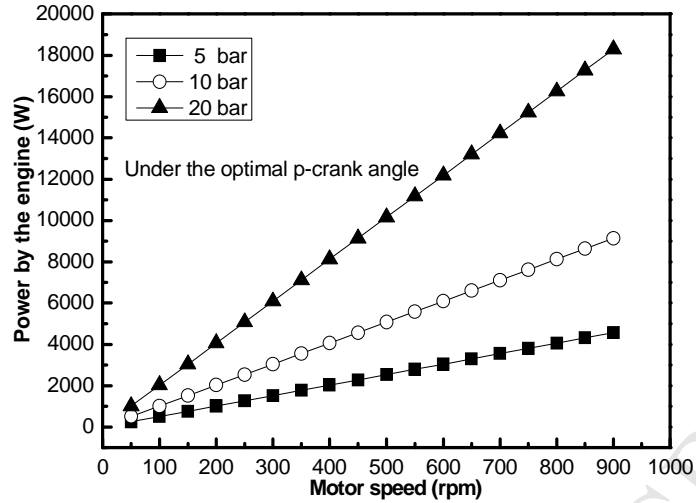
317  $D_{v,\min}$ . In the case studies in which valves with the minimum diameter of 1.8 mm and 12 mm

318 are used, the critical motor speed is estimated as approximately 160 rpm and 600 rpm,  
 319 respectively. Since high motor speed results in large power generation by the engine, large  
 320 valves with increased  $D_{v,\min}$  is suggested to be used for enlarging the power capacity of the  
 321 engine system.



322  
 323 Figure 11. The critical motor speed affected by the minimum diameter of the valve and the  
 324 engine volume.

325 The power capacity of the large engine (1900 cm<sup>3</sup>) using large valves ( $D_{v,\min}=12$  mm) was  
 326 predicted, as shown in Figure 12. Compressed air with pressure of 5, 10 and 20 bar was  
 327 expanded in the engine respectively. It was found that the maximum power generations at the  
 328 motor speed of 600 rpm in the above cases were as high as 3 kW , 6 kW and 12 kW ,  
 329 respectively. Beyond the motor speed of 600 rpm, the power generation was restrained by the  
 330 fixed  $D_{v,\min}$  of the valves. With larger valves used, improved maximum power by the engine  
 331 can be achieved under a higher critical motor speed (>600 rpm). However, the large time  
 332 delay of the large valves may cause failure in controlling the opening and closing operations  
 333 under high motor speed.



334

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Figure 12. The prediction of power capacity of the large engine (1900 cm<sup>3</sup>).

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## 6. Further discussions on the piston based engine

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In theory, the power generation by the engine is affected by the p-crank angle, motor speed, gas pressure and temperature, chamber volume and the minimum inner diameter of the valves used at the inlet and outlet of the piston based engine. However, in practical cases, the engine power generation is also significantly influenced by the performance of engine in the gas expansion process. This section concerns the efficiency of the engine and its effect on the energy storage efficiency of the CES system.

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### 6.1 Engine efficiency

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The engine efficiency refers to the exergy efficiency that is defined as the ratio of actual power output to the theoretical engine power based on the thermal analysis. In the case study of the small engine (162 cm<sup>3</sup>), the practical power generation by experiments was expressed as  $36.2 \times Tor_2 \cdot \omega$ , where  $\omega$  is the motor speed, while  $Tor_2$  represented the torque percentage by engine contribution. Consequently, the engine efficiency can be formulated:

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$$\eta_{engine} = \frac{36.2 \times Tor_2 \cdot \omega}{\frac{1}{k_{po}-1} \cdot \frac{1}{\pi} \cdot P_{in} \cdot \omega \cdot A \cdot \left[1 - \left(\frac{P_c}{P_{in}}\right)^{\frac{k_{po}-1}{k_{po}}}\right] \cdot \left\{ r[1 - \cos(\omega t_1)] + l \left[1 - \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2(\omega t_1)}\right] \right\}} \quad (25a)$$

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351 In the experiments, electricity consumption (EC) and electricity output on the motor (EM)

352 were measured, therefore, the engine efficiency can be expressed as:

$$353 \quad \eta_{engine} = \frac{EC - EM}{\frac{1}{k_{po} - 1} \cdot \frac{1}{\pi} \cdot P_{in} \cdot \omega \cdot A \cdot \left[1 - \left(\frac{P_c}{P_{in}}\right)^{\frac{k_{po} - 1}{k_{po}}}\right] \cdot \left\{r[1 - \cos(\omega t_1)] + l\left[1 - \sqrt{1 - \left(\frac{r}{l}\right)^2 \sin^2(\omega t_1)}\right]\right\}} \quad (25b)$$

354 In a case study with a large engine (1900 cm<sup>3</sup>), the inlet pressure of the compressed air was

355 20 bar, the inlet angle was set as 18° while the motor speed was 60 rpm. The measured

356 electricity consumption (EC) on the motor was 60 W, while the maximum net electricity

357 output of the motor (EM) was -160 W. Therefore, the engine power (EP=EC-EM) generation

358 was as high as 220 W. Based on the given parameters in the case study, the theoretical engine

359 power was evaluated as 664 W. According to Eq.(25), the engine efficiency for the large

360 engine under the conditions was approximately 33.1%. For a small engine (162 cm<sup>3</sup>) under

361 the same operating conditions, EC and EM were measured as 45 W and 15 W, respectively.

362 As a result, the engine power (EP) was equal to 30 W. However, the theoretical engine power

363 was calculated as 57 W under the conditions, so the engine efficiency of the small engine was

364 as high as 52.6%. It was noted that the large engine induced a lower engine efficiency. This is

365 because more exergy loss is caused due to the inadequate gas expansion in the large engine

366 chamber. However, it doesn't indicate small engine is better, since the engine system results

367 in a net electricity consumption (EM) indicating the non-feasibility of the engine in a real

368 CES system.

369 **6.2 Influence of the engine efficiency on energy storage efficiency**

370 Although the efficiency of the large engine is relatively lower than that of the small engine,

371 large engine is still recommended to be used in a CES system. This is because large engine

372 can result in a net power generation that can be sent back to the electrical grid for peaking  
 373 shifting purpose.

374 From the point of view of energy storage and utilisation, the contribution of the engine  
 375 system lies in the net output power by the system. Therefore, the actual engine efficiency can  
 376 be expressed as:

$$377 \quad \eta'_{engine} = \frac{-EM}{\frac{1}{k_{po}-1} \cdot \frac{1}{\pi} \cdot P_{in} \cdot \omega \cdot A \cdot [1 - (\frac{P_c}{P_{in}})^{\frac{k_{po}-1}{k_{po}}}] \cdot \left\{ r[1 - \cos(\omega t_1)] + l[1 - \sqrt{1 - (\frac{r}{l})^2 \sin^2(\omega t_1)}] \right\}} \quad (26)$$

378 As a result, in the above case study, the large engine system had an actual efficiency of  
 379 24.1%. In contrast, for the small engine system, the actual engine efficiency for power  
 380 generation was -26.3%.

381 In the CES system as demonstrated in [2, 16], the energy storage efficiency of the CES  
 382 system can be formulated as:

$$383 \quad \eta_{CES} = \frac{W_e - W_p}{E_{heat} + E_{cold}} \quad (27)$$

384 where  $W_e$  represents the power generation by the engine system;  $W_p$  is the electricity  
 385 consumption for pumping the working fluid in the Rankine cycle;  $E_{cold}$  represents the  
 386 electricity consumption for storing cold energy in the cold storage media;  $E_{heat}$  is the  
 387 electricity consumption by heaters in the Rankine cycle. From the above analysis, the storage  
 388 efficiency is written as:

$$389 \quad \eta_{CES} = \frac{-EM - W_p}{E_{heat} + E_{cold}} \quad (28)$$

390 A case study was presented below to show the influence of the actual engine efficiency on  
 391 the energy storage efficiency of the CES system. In the case study, the expansion gas was

392 heated up to 200 °C by the heater in the Rankine cycle. Since pump was not used in the actual  
 393 operation, the electricity consumption by the pump was zero. With a mass flow rate of 0.0426  
 394 kg/s, the theoretical engine power was:

$$395 \quad EP_{theo} = 0.0426 \times \frac{1}{k_{po} - 1} \cdot R_g \cdot T_{in} \cdot \left[ 1 - \left( \frac{P_c}{P_{in}} \right)^{\frac{k_{po} - 1}{k_{po}}} \right] \quad (29)$$

396 which was calculated as 8659 W under the above operating conditions. In terms of the above  
 397 engine efficiency of 33.1%, the actual engine power (EP) by the large engine was 2866 W.  
 398 However, in consideration of the electricity consumption (EC) by the system, the net output  
 399 of the engine system was further reduced to 2084 W. In the experiment, the electricity  
 400 consumption by the refrigerator ( $E_{cold}$ ) for cold energy storage was 4000 W, while the  
 401 electricity consumption in the super heater ( $E_{heat}$ ) for heating the working fluid to 200 °C was  
 402 7000 W (electricity consumed) or 0 W (waste heat from diesel engine used). As a result, the  
 403 CES efficiency was estimated as approximately 18.9% and 52.1%, respectively. This  
 404 indicates that for storing 10 kW of cold energy by consuming cheap electricity in off-peak  
 405 time, the CES system has a capacity of feeding back electricity of 1.9~5.2 kW that can be  
 406 later used in peak time. Therefore, engine efficiency in practical operations is still a big  
 407 challenge for promoting the CES technology development.

## 408 7. Concluding remarks

409 Theoretical study indicates the significant effect of p-crank angle on the power generation  
 410 by a piston based engine. Under different operating conditions and the geometric parameters  
 411 of the engine (i.e. the length ratio  $L/r$ ), the optimal p-crank angle which leads to the  
 412 maximum power generation varies in the range of 55° and 76°. Apart from that, motor speed,  
 413 gas pressure and temperature, and chamber volume have positive influence in increasing the  
 414 power generation by the piston based engine. However, the promotion of the power capacity



415 is limited by the minimum inner diameter of the valves at both inlet and outlet of the engine  
416 system.

417 Although large engines have a lower engine efficiency compared with small engines due to  
418 the more exergy loss in the chamber, it is still competitive since a net power generation is  
419 induced, showing its feasibility in a real CES system. Considering the power consumptions  
420 by the motor, the actual efficiency of the large engine was calculated as 24.1% in a case  
421 study, while in contrast, it was -26.3% for the small engine in the case. Due to the low  
422 engine efficiency, the energy storage efficiency of the CES system in two different situations  
423 was estimated as approximately 18.9% (when electricity consumed in the super heater) and  
424 52.1% (when waste heat from diesel engine is recovered), respectively. Therefore, there is a  
425 long way for improving the engine performance. However, from the point of view of  
426 electricity saving in off-peak time, cold energy storage is still worthwhile.

#### 427 **Acknowledgement**

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- Under different operating conditions and the geometric parameters of the engine (i.e. the length ratio  $L/r$ ), the optimal p-crank angle which leads to the maximum power generation varies in the range of  $55^\circ$  and  $76^\circ$ .
- Although large engines have a lower engine efficiency compared with small engines due to more exergy loss in the chamber, it is still competitive since a net power generation is induced, showing its feasibility in a real CES system. Considering the power consumptions by the motor, the actual efficiency of the large engine was calculated as 24.1% in a case study, while in contrast, it was -23.6% for the small engine in the case. Due to the low engine efficiency, the energy storage efficiency of the CES system was estimated as 18.9% and 52.1% in two different situations.