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SimReg: A Software Including Some New Developments in Multiple Comparison and Simultaneous Confidence Bands for Linear Regression Models

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Abstract

The problem of simultaneous inference and multiple comparison for comparing means of $k(\geq 3)$ populations has been long studied in the statistics literature and is widely available in statistics literature. However to-date, the problem of multiple comparison of regression models has not found its way to the software. It is only recently that the computational aspects of this problem have been resolved in a general setting. **SimReg** employs this new methodology and provides users with software for multiple regression of several regression models. The comparisons can be among any set of pairs, and moreover any number of predictors can be included in the model. More importantly predictors can be constrained to their natural boundaries, if known.

Computational methods for the problem of simultaneous confidence bands when predictors are constrained to intervals has also recently been addressed. **SimReg** utilizes this recent development to offer simultaneous confidence bands for regression models with any number of predictor variables. Again, the predictors can be constrained to their natural boundaries which results in narrower bands, as compared to the case where no restriction is imposed. A by-product of these confidence bands is a new method for comparing two regression surfaces, that is more informative than the usual partial F test.

Keywords: linear regression, multiple comparison, simultaneous confidence bands, partial F test, statistic software.

1. Introduction

The problem of simultaneous inference and multiple comparison for comparing means of $k(\geq 3)$ populations has been long studied in the statistics literature. [Miller \(1981\)](#), [Hochberg and Tamhane \(1987\)](#), and [Hsu \(1996\)](#) have provided excellent summaries of the work in this area. [Spurrier \(1999\)](#) seems to be the pioneering work to extend this problem to simultaneous comparison of several regression lines. In his work, a set of simultaneous confidence bands for all the contrasts of several simple linear regression lines over the entire range of the single predictor $(-\infty, \infty)$ is constructed when the design matrices of the regression lines are the same. The recent work of [Liu, Jamshidian, and Zhang \(2004\)](#) [hereafter referred to as LJZ (2004)] extends Spurrier’s work in many directions. Specifically, their work allows multiple comparison of several regression models with (1) any number of predictor variables, (2) design matrices that are not necessarily equal, (3) predictors that can be restricted in a finite or infinite range, and (4) comparison of any desired set of pairs of groups. The software **SimReg**, presented in this paper, implements this new methodology and provides a user-friendly interface for data input and analysis.

SimReg also provides simultaneous confidence bands for the linear regression model when predictors are constrained to intervals. This problem has a history going back to [Working and Hotelling \(1929\)](#). [Scheffé \(1953\)](#) provides a simultaneous confidence band when predictors are not constrained, i.e. they range in the interval $(-\infty, \infty)$. Since then there is a fair amount of literature that provide exact simultaneous confidence bands for simple linear regression model with the single predictor constrained to intervals. There are also works that offer conservative simultaneous confidence bands for the cases where predictors are constrained in intervals or other types of regions. [Liu, Jamshidian, and Zhang \(2005a\)](#) [hereafter referred to as LJZ (2005)] seem to be the first paper to offer “exact” simultaneous confidence bands for multiple regression when predictors are constrained in finite intervals. As mentioned, **SimReg** provides these intervals.

The remaining sections are organized as follows: Section 2 describes the mathematical model for the multiple comparison problem, and Section 3 does the same for the simultaneous confidence band problem. Section 4 gives a brief discussion of the algorithm and parameter settings. Section 5 describes how the software can be installed and used. Finally Section 5 gives a few items that are planned for future versions of the software.

2. The multiple comparison problem

Suppose that data are observed for k groups of subjects and let the model

$$\mathbf{Y}_i = X_i \mathbf{b}_i + \mathbf{e}_i, \quad i = 1, \dots, k$$

be the linear regression model for the i th group. Here, for the i th group, $\mathbf{Y}_i^T = (y_{i1}, \dots, y_{in_i})$ denotes the vector of responses, X_i is an $n_i \times (p+1)$ full column rank design matrix with the first column given by $(1, \dots, 1)^T$ and the $l(\geq 2)$ th column given by $(x_{1,l-1}^i, \dots, x_{n_i,l-1}^i)^T$, $\mathbf{b}_i^T = (b_0^i, \dots, b_p^i)$ is the vector of regression coefficients, and $\mathbf{e}_i^T = (e_{i1}, \dots, e_{in_i})$ denotes the vector of regression errors with all the $\{e_{ij}, j = 1, \dots, n_i, i = 1, \dots, k\}$ being iid $N(0, \sigma^2)$. Since $X_i^T X_i$ is non-singular, the least squares estimator of \mathbf{b}_i is given by $\hat{\mathbf{b}}_i = (X_i^T X_i)^{-1} X_i^T \mathbf{Y}_i, i = 1, \dots, k$. Let $\hat{\sigma}^2$ denote the pooled error mean square with degrees of freedom $\nu = \sum_{i=1}^k (n_i - p - 1)$; $\hat{\sigma}^2$ is independent of the $\hat{\mathbf{b}}_i$.

LJZ(2004) proposed a method to construct a set of simultaneous confidence bands for

$$\mathbf{x}^T \mathbf{b}_i - \mathbf{x}^T \mathbf{b}_j = (1, x_1, \dots, x_p) \mathbf{b}_i - (1, x_1, \dots, x_p) \mathbf{b}_j, \quad (i, j) \in \Lambda$$

over a given range $x_l \in [a_l, b_l], l = 1, \dots, p$, where Λ is an index set that determines the comparison of interest. For example, if the pairwise comparison is of interest then $\Lambda = \{(i, j) : 1 \leq i \neq j \leq k\}$; if the comparisons of the second to k th regression models with the first regression model are of interest then $\Lambda = \{(i, j) : 2 \leq i \leq k, j = 1\}$; if the successive comparison of the k regression models is of interest then $\Lambda = \{(i, i + 1) : 1 \leq i \leq k - 1\}$. Specifically, they construct the following set of simultaneous confidence bands

$$\mathbf{x}^T \mathbf{b}_i - \mathbf{x}^T \mathbf{b}_j \in \mathbf{x}^T \hat{\mathbf{b}}_i - \mathbf{x}^T \hat{\mathbf{b}}_j \pm c \hat{\sigma} \sqrt{\mathbf{x}^T \Delta_{ij} \mathbf{x}}, \quad \forall x_l \in [a_l, b_l] \text{ for } l = 1, \dots, p, \text{ and } \forall (i, j) \in \Lambda \quad (2.1)$$

where $\Delta_{ij} = (X_i^T X_i)^{-1} + (X_j^T X_j)^{-1}$, and c is the critical constant required so that the confidence level of this set of simultaneous confidence bands is equal to $1 - \alpha$. The confidence level of the bands in (1.1) is given by $P\{T < c\}$ where

$$T = \sup_{(i,j) \in \Lambda} \sup_{x_l \in [a_l, b_l], l=1, \dots, p} \frac{|\mathbf{x}^T[(\hat{\mathbf{b}}_i - \mathbf{b}_i) - (\hat{\mathbf{b}}_j - \mathbf{b}_j)]|}{\hat{\sigma} \sqrt{\mathbf{x}^T \Delta_{ij} \mathbf{x}}}. \quad (2.2)$$

LJZ (2004) provide a simulation procedure to simulate T and obtain the critical constant. **SimReg** adopts this methodology to obtain c , however, as we will explain in Section 4, we adopt a different algorithm than that proposed by LJZ (2004).

3. Simultaneous confidence bands

Consider the multiple linear regression model

$$\mathbf{Y} = X\mathbf{b} + \mathbf{e} \quad (3.1)$$

where $\mathbf{Y}_{n \times 1}$ is the vector of observed responses, $X_{n \times (p+1)}$ is the design matrix with the first column given by $(1, \dots, 1)^T$ and the j th ($2 \leq j \leq p+1$) column given by $(x_{1,j-1}, \dots, x_{n,j-1})^T$, $\mathbf{b}_{(p+1) \times 1} = (b_0, \dots, b_p)^T$ is the vector of regression coefficients, and $\mathbf{e}_{n \times 1}$ be the error vector with $\mathbf{e} \sim N(0, \sigma^2 I)$ and σ^2 unknown. Assume $X^T X$ is non-singular, so the least squares estimator of \mathbf{b} is given by $\hat{\mathbf{b}} = (X^T X)^{-1} X^T \mathbf{Y}$. Let $\hat{\sigma}^2$ denote the mean square error with degrees of freedom $\nu = n - p - 1$; $\hat{\sigma}^2 \sim \sigma^2 \chi_\nu^2 / \nu$ and is independent of the $\hat{\mathbf{b}}$.

It has been argued by several authors that a rectangular \mathcal{X} is often one of the most useful shapes for practical purposes (see e.g., Casella and Strawderman (1980), and Naiman (1987)). In many applications of the linear regression model the experimenter can specify reasonable constraints on the predictor variables in terms of a lower and an upper bound for each of the predictor variables. LJZ (2005) construct a confidence band of the form

$$\mathbf{x}^T \mathbf{b} \in \mathbf{x}^T \hat{\mathbf{b}} \pm c \hat{\sigma} \sqrt{\mathbf{x}^T (X^T X)^{-1} \mathbf{x}} \quad \text{for all } (x_1, \dots, x_p)^T \in \mathcal{X}, \quad (3.2)$$

where \mathcal{X} is a rectangular region given by

$$\mathcal{X} = \{(x_1, \dots, x_p)^T : a_i \leq x_i \leq b_i, i = 1, \dots, p\}.$$

Here $-\infty \leq a_i < b_i \leq \infty, i = 1, \dots, p$ are given constants. The main task is to determine the critical constant c in (3.2) so that the confidence band has a confidence level equal to $1 - \alpha$. The confidence level of the band in (3.2) is given by $P\{T < c\}$, where

$$T = \sup_{x_i \in [a_i, b_i], i=1, \dots, p} \frac{|\mathbf{x}^T(\hat{\mathbf{b}} - \mathbf{b})|}{\hat{\sigma} \sqrt{\mathbf{x}^T(X^T X)^{-1} \mathbf{x}}}. \quad (3.3)$$

The distribution of T does not depend on \mathbf{b} or σ , however, it does depend on the bounds (a_i, b_i) and the design matrix X in a complicated manner. The latter makes it difficult to derive a formula for the distribution of T in this general setting. LJZ (2005) provide a method to determine c via simulating T . **SimReg** uses the active set method described by LJZ (2005) to compute T . A random pivot T can be simulated efficiently, and thus c can be determined as accurately as one wishes by simulating a sufficiently large number of T 's.

A simultaneous confidence band provides information on the whereabouts of the true regression model $\mathbf{x}^T \mathbf{b}$. Any regression model that is contained in the simultaneous confidence band for all $\mathbf{x} \in \mathcal{X}$ is deemed by the confidence band as a plausible candidate for the true model. Of course the true model $\mathbf{x}^T \mathbf{b}$ is included in the confidence band with $1 - \alpha$ probability.

4. Algorithms

The main computing task in the multiple comparison and the simultaneous confidence band problems described in Sections 2 and 3 is that of solving the optimization problems (2.2) and (2.3). In (2.2), the optimization over Λ will be performed by looking at the corresponding objective value for each pair in Λ and obviously choosing the one with the highest value. The sup of the quantity shown in (2.2) over $x_l \in [a_l, b_l]$ for $l = 1, \dots, p$ is less trivial however. LJZ (2004) show that the T in (2.2) is equivalent to

$$\sup_{(i,j) \in \Lambda} \sup_{x_l \in [a_l, b_l], l=1, \dots, p} \frac{|(P_{ij} \mathbf{x})^T \mathbf{Z}_{ij}|}{(\hat{\sigma}/\sigma) \sqrt{(P_{ij} \mathbf{x})^T (P_{ij} \mathbf{x})}} = \sup_{(i,j) \in \Lambda} Q_{ij} \frac{\|\mathbf{Z}_{ij}\|}{(\hat{\sigma}/\sigma)}, \quad (4.1)$$

where

$$Q_{ij} = \sup_{x_l \in [a_l, b_l], l=1, \dots, p} \frac{|(P_{ij} \mathbf{x})^T \mathbf{Z}_{ij}|}{\|P_{ij} \mathbf{x}\| \|\mathbf{Z}_{ij}\|},$$

P_{ij} is the matrix square root of Δ_{ij} , and $\mathbf{Z}_{ij} = P_{ij}^{-1}(\mathbf{Z}_i - \mathbf{Z}_j), 1 \leq i \neq j \leq k$ with $\mathbf{Z}_i \sim N(\mathbf{0}, (X_i^T X_i)^{-1})$. Similarly, LJZ (2005) show that the problem (3.3) is equivalent to

$$T = \sup_{x_i \in [a_i, b_i], i=1, \dots, p} \frac{|(P \mathbf{x})^T \mathbf{Z}|}{(\hat{\sigma}/\sigma) \sqrt{(P \mathbf{x})^T (P \mathbf{x})}} = Q \frac{\|\mathbf{Z}\|}{(\hat{\sigma}/\sigma)}, \quad (4.2)$$

where

$$Q = \sup_{x_i \in [a_i, b_i], i=1, \dots, p} \frac{|(P \mathbf{x})^T \mathbf{Z}|}{\|P \mathbf{x}\| \|\mathbf{Z}\|},$$

P is the square root of $(X^T X)^{-1}$, and $\mathbf{Z} \sim N(0, I_{p+1})$. Thus to simulate a realization of T for the multiple comparison problem a \mathbf{Z}_{ij} is generated, Q_{ij} is computed, and then (4.1) is used to obtain T . This process is similar for the simultaneous confidence band problem where a \mathbf{Z}

is generated, Q is computed and then (4.2) is utilized. The overall process is then to generate a large number of copies of T and use the $100(1 - \alpha)\%$ quantile of the generated values to approximate c .

The problem of obtaining Q_{ij} and Q in (4.1) and (4.2) is mathematically equivalent. So hereafter we will refer to both problems as that of obtaining Q . It turns out that the most time consuming and difficult part of the simulation process described above is that of computing Q . LJZ (2004) took a general approach of applying a smooth optimization technique to obtain Q . As they point out, a shortcoming of their method is that it can lead to local maxima that are not the global solution to the maximization problem. LJZ (2005) provide two other approaches, one which is a branching method, and another which is an active set method. The former is appropriate for cases where the number of predictors is small, say two or three, and the latter works best overall. It can be shown that Q can be obtained by solving a problem of the form

$$Q = \sup_{\mathbf{s} \in \Omega} \frac{|\mathbf{s}^T \mathbf{Z}|}{\|\mathbf{s}\| \|\mathbf{N}\|}, \quad (4.3)$$

where $\Omega = \{\mathbf{s} : \mathbf{s} = \gamma \mathbf{v}, \mathbf{v} \in L, \gamma > 0\}$ and $L = \{P\mathbf{x} : x_i \in [a_i, b_i], i = 1, \dots, p\}$. Furthermore, it can be shown that if $\hat{\mathbf{s}} \in \Omega$ is the solution to

$$\inf_{\mathbf{s} \in \Omega} \|\mathbf{s} - N\|^2, \quad (4.4)$$

then $\hat{\mathbf{s}}$ is also the solution to (4.3). This is a quadratic programming problem for which LJZ (2005) suggest using an active set method. **SimReg** has adopted the algorithm described in LJZ (2005) for both the multiple comparison and the simultaneous confidence band problems.

5. The SimReg software

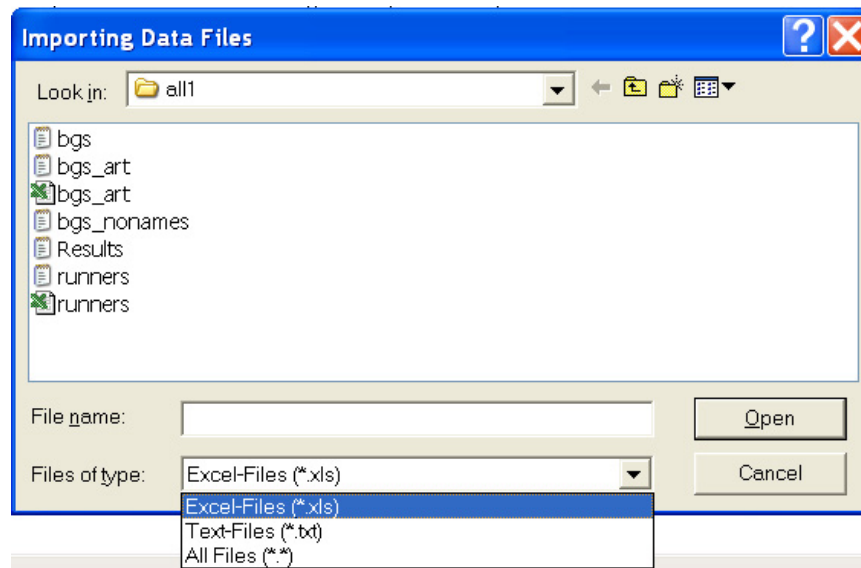
SimReg was developed using MATLAB Version 7.0.1, Release 14 on Windows XP. It can, however, be easily adopted in other platforms. **SimReg** performs two main tasks of performing multiple comparison of several regression lines and obtaining simultaneous confidence bands for a multiple regression model.

What sets **SimReg** apart from the currently existing software is that it performs these tasks with the option of allowing the user to set natural boundaries (constraints) for the predictors. It is clear that a linear regression model often does not hold for the whole range of $(-\infty, \infty)$ range of the predictors, and it is often the case that the predictors have reasonable finite range. Imposing the constraints will result in simultaneous confidence bands that are narrower (see, LJZ 2005), and results in more reliable multiple comparisons. Additionally, for the multiple comparison problem, it allows any type of pair comparison desired by the user.

The software includes a user interface for importing data and setting the desired parameters. It produces output that can be used for inference, and includes graphs for for models with one or two predictors. Below we explain how **SimReg** can be used and what its capabilities are.

5.1. Getting started and data input

SimReg is archived in the file “Simreg.rar” using WinRAR 3.40. Use the following three steps to get started with the **SimReg** program:

Figure 1: **SimReg** window to import data files

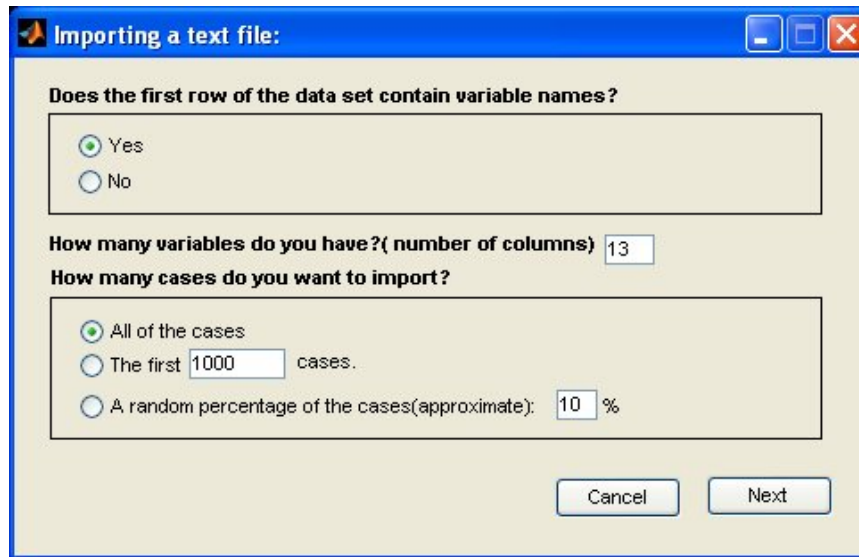
1. Unpack **SimReg** in any desired directory. To unpack, you can download WinRAR from <http://www.rarsoft.com/download.htm>. As an example, suppose that you install the software in the directory “C:\Program Files\Simreg”.
2. Start MATLAB and add the path of the directory where you installed the software to your current MATLAB path. This can be done using the “`addpath`” command in MATLAB. For example, at the MATLAB command prompt you type “`addpath 'C:\Program Files\Simreg'`”.
3. At the MATLAB command prompt, type `simreg`.

When the command `simreg` is issued at the MATLAB command prompt, a window (shown in Figure 1) pops up where the name of the data file to be analyzed should be entered. **SimReg** can import Excel files¹ with the extension “`xls`” or data in ASCII format saved in files with the extension “`txt`”. In either case rows should represent cases and columns represent variables. The first row of the data file can include variable names, or it could be the first case observed. Note that in this version of **SimReg** no missing data handling capability is available, therefore the user should remove all cases with missing data before uploading the data file.

To illustrate the ideas we use, and have included, a modified version of the data used in the Berkeley Guidance Study published by [Tuddenham and Snyder \(1954\)](#). The study was a longitudinal monitoring of 136 boys and girls born in Berkeley, California between January 1928 and June 1929, and followed for at least 18 years. Our modification to this data is addition of an artificial categorical variable “Race” with the aim of including a categorical variable which has more than two levels. One of the races “White”, “Black”, “Asian”, or “Other” was arbitrarily assigned to each case. The modified data set includes a total of 13

¹Excel files can be imported if version 7.x of Matlab is used. This option is not available if version 6.x of Matlab is used.

Figure 2: Options for importing a txt file



variables. The file “bgs_art.txt” is the ASCII version of this data set, and “bgs_art.xls” is the Excel version, both included in the **SimReg** package.

5.2. Reading the data

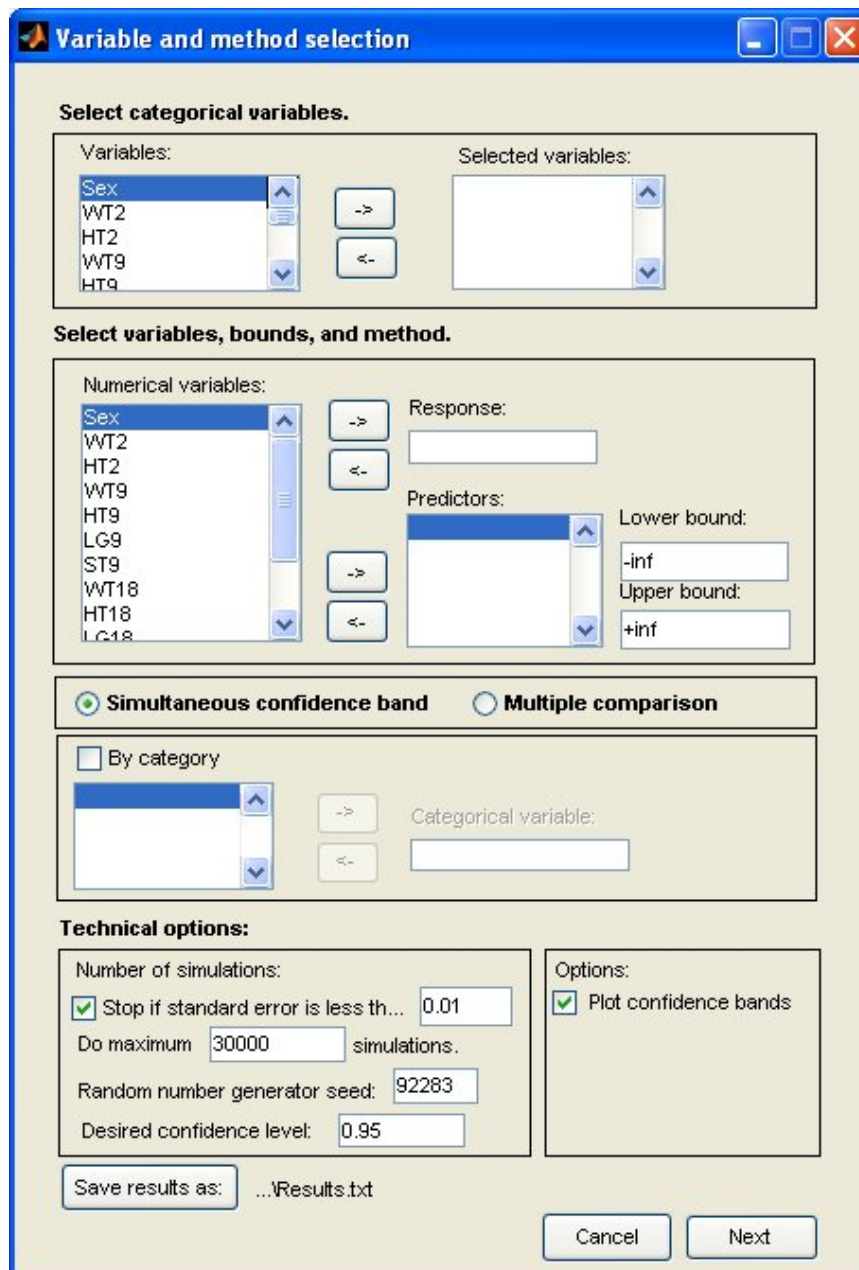
Reading a txt file

If we select a “txt” file (e.g., “bgs_art.txt”) in the *Importing Data Files* menu and click on open, then a second menu titled “*Importing a Text File*” will pop up (see Figure 2). On this menu you must input information such as whether the first row of the data includes variable names, and the number of variables included in your data set (this must be equal to the number of columns in your data set). Additionally you have the option of selecting a subset of data for analysis. In particular, you can use all, a subset of first few cases, or a randomly selected subset of the data. If the first row of your data file does not include variable names, then the program assigns variables names “Var#” where “#” is the column number. Once “Next” is pressed we get a menu titled “*Variable and method selection*”, shown in Figure 3.

Reading an Excel file

As noted earlier, Excel files can be read only in Version 7.x of Matlab and this option is not available in lower versions. To read an Excel file, in the *Importing Data Files* menu choose the option of “*.xls” and select your input file and press open. This will lead directly to the menu titled “*Variable and method selection*”, shown in Figure 3. Note that when importing an Excel file, the first row of the data must contain variable names. Additionally, the menu “*Importing a Text File*” and the options in that menu for subsetting the data are not available, and all the data in the file are used for analysis.

Figure 3: Menu to select regression variables and the method of analysis



5.3. Analysis of the data

The menu “*Variable and method selection*”, shown in Figure 3, is used to specify the model and select one of the options of “simultaneous confidence band” or “multiple comparison”. The first step, however, consists of specifying the categorical variables in the study, if any. Initially, the menus on the right hand side of the first and the second panels of the menu in Figure 3 consist of all the names of the variables read. On the top panel, the user selects the name of the categorical variables. If multiple comparison is to be performed, obviously, at least one categorical variable must exist. For our example, two categorical variables of “Sex” and “Race” were selected, as shown on Figure 4. Once the categorical variables are selected, then these variables are added to the fourth panel where the option “*By category*” appears and they are removed from the second panel titled “*Numerical variables*”. As shown on Figure 4, for our example the variables “Sex” and “Race” were removed from the middle panel, and added to the fourth panel.

The next step is to specify the regression model. The response and the predictor variables for the model are selected in the second panel. In the example shown “HT18” is selected as response and “WT9” and “HT9” are selected as predictors. At this stage lower and upper bounds for the predictors can be set. The default is $(-\infty, \infty)$, which is written as “-inf” and “+inf”. For our example we have set the bound on WT9 to (10, 60) and that for Ht9 to (120, 160).

On the third panel of the menu shown on the Figure 4, the user will choose the type of analysis to be performed. As mentioned earlier, **SimReg** provides two methods of simultaneous confidence bands and multiple comparison.

The confidence band option

When the “Simultaneous confidence band” option in the third panel of the “*Variable and method selection*” menu is selected, then the critical value c for the simultaneous confidence band (3.2) is computed. The user has the option of performing this analysis for each level of a selected categorical variable, by choosing the “By category” option and selecting the desired categorical variable. If no category is selected, then the analysis will be performed for the whole data set, ignoring categories. If a category is selected, then a separate simultaneous confidence band will be given for each of the levels of the selected category.

As shown in Figure 4, for our example, we request a simultaneous confidence band for the regression model for each level of the category “Sex”. When the button “Next” is pressed, then the software runs a simulation and when the computation is done a menu pops up which states “please see Result.txt for the result.” If the plot option is selected (available only if $p = 1, 2$), then plots of the confidence bands and the regression plane (line) will also be given.

Table 1 shows the content of the file “Results.txt” for our example. In general, the output consists of summary statistics, regression coefficients and standard T-test and p-values, and the residual mean square for all observed values or for each level of the selected categorical variable, if one was selected. Most importantly, it gives the critical value c in (3.2) for the confidence band. Finally, if the plot option is selected, a plot of confidence bands, the regression line (or plane) and the observed data is produced for all the data or each category level, if a categorical variable is selected. For our example, these are given for the two levels of the variable “Sex”, namely “Female” and ‘Male’. Additionally, the number of simulations performed and the lower and upper bounds for the predictors are echoed. For our example,

Figure 4: Categorical, response, predictors, and their bounds selected.

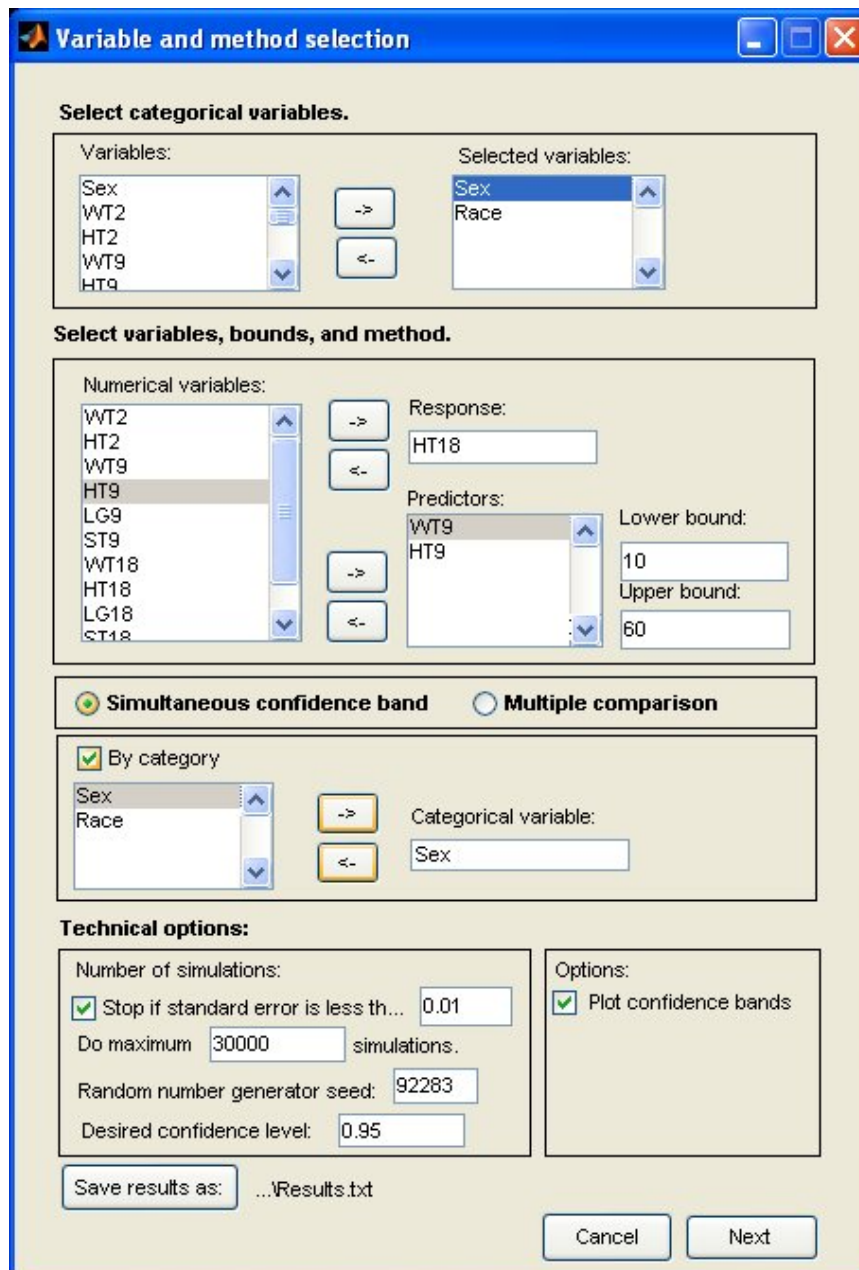


Table 1: The output for the simultaneous confidence band example

```

Summary statistics and results of the analysis
=====
Summary statistics:
All observed values:

Variable   N     Min      Mean      Median   Std. Dev.   Max
HT18    136   153.600   172.579   172.500    8.844     195.100
WT9     136   19.900    31.626    30.900    5.969     66.800
HT9     136   121.400   135.493   135.700    5.496     152.500

=====

Summary statistics:
Group:      Female

Variable   N     Min      Mean      Median   Std. Dev.   Max
HT18     70   153.600   166.544   166.750    6.075     183.200
WT9      70   22.000    31.621    30.650    5.824     47.400
HT9      70   121.400   135.120   135.700    5.613     152.500

=====

Regression Analysis
Group:      Female

Response:   HT18
Residual Mean Square = 11.0927

Variable    Coeff      s.e.      T-stat     p-value
Constant   23.1721    11.9682    1.936      0.0285
WT9        -0.3585    0.1004    -3.572     0.0003
HT9         1.1450    0.1041    10.995     0.0000

=====

Group:      Female

Number of simulations = 30000

The lower and upper bounds imposed on the predictors are:
WT9 Lower Bound = 10.000 Upper Bound = 60.000
HT9 Lower Bound = 120.000 Upper Bound = 160.000

The simulated critical value for 0.950 confidence level = 2.8768
The standard error = 0.0096

=====

Summary statistics:
Group:      Male

Variable   N     Min      Mean      Median   Std. Dev.   Max
HT18     66   160.900   178.979   178.900    6.517     195.100
WT9      66   19.900    31.632    31.000    6.164     66.800
HT9      66   122.000   135.889   135.600    5.385     147.500

=====

Regression Analysis
Group:      Male

Response:   HT18
Residual Mean Square = 9.3265

Variable    Coeff      s.e.      T-stat     p-value
Constant   20.7883    11.0985    1.873      0.0328
WT9        -0.2055    0.0809    -2.540     0.0068
HT9         1.2120    0.0926    13.087     0.0000

=====

Group:      Male

Number of simulations = 30000

The lower and upper bounds imposed on the predictors are:
WT9 Lower Bound = 10.000 Upper Bound = 60.000
HT9 Lower Bound = 120.000 Upper Bound = 160.000

The simulated critical value for 0.950 confidence level = 2.8640
The standard error = 0.0116

=====

The pooled variance is 10.2368

The degrees of freedom is 130
=====

```

the critical value c for females is 2.8768 with standard error .0096, and that for the “Male” is 2.8640 with standard error .0116. Finally, Figure 5 shows the 95% confidence bands, and regression plane superimposed by a scatter of the observed data for each of the levels of the variable Sex for our example. These plots can be navigated by the tools available in MATLAB.

The multiple comparison option

To perform multiple comparison, this option must be selected in the “*Variable and method selection*” menu and the user must choose a categorical variable, using the “*By category*” option, whose levels will be compared. To give an example, we will perform a multiple comparison of the same regression model that we specified in the previous subsection, but using the levels of the categorical variable “Race”. Figure 6 shows the relevant specifications. In this case when the button “next” is pressed, then the “*Pair comparison selection*” menu, shown in Figure 7, pops up.

The “*Pair comparison selection*” menu allows the user to provide the set Λ , defined in Section 2. The user has the option of selecting (comparing) either all pair comparison in the top panel, comparison to a specific control level in the middle panel, or any arbitrary pairs on the bottom panel. In the example shown in Figure 7, comparison to a control is selected, and specifically we are making comparison of the level `White` to all other levels of the variable `Race`. Once again, when the computation is complete, a window pops-up which states “Please see Results.txt for the result”. Additionally, if the plot option is on, plots will be generated as well.

The content of the file `Results.txt` for our example is given in Tables 2 and 3. This output in general consists of summary statistics for the observed variables, and result of the fitted regression model for each levels of selected categorical variable. More importantly, it provides the critical value c in Section 2.2, the statistics T in (2.2), and its corresponding p -value for the multiple comparison specified. For our example, the critical value is $c = 3.1947$ with the standard error of 0.0107, and the test statistics is $T = 11.917$ with the p -value nearly zero, indicating a significant difference between the pairs.

In addition to multiple comparison, the test statistics and p -values for single pair comparison of the pairs specified in the “*Pair comparison selection*” menu (Λ) is given. For our example, single pair comparisons indicate a significant difference between the pairs (Asian, White) and (Other, White), but not a significant difference between the pair (Black, White). Corresponding to each of these tests of significance of difference of pairs, the program generates plots that can be used for this purpose. These plots are more informative than simply the p -values (see Liu, Jamshidian, Zhang, Bertz, and Han (2005b)). Specifically, the plots generated are simultaneous confidence bands for pair differences; that is, they test the equality of the regression model between single pairs of categorical levels selected. If the bands cross the zero line (or plane), then the hypothesis of equality of regression model for the pair indicated on the plot is rejected. Figures 8 and 9 shows a snapshot of the plots just mentioned for our example. As expected (from the p -values observed earlier), the bands for the pairs (Asian, White) and (Other, White) intersect the zero plane, but that for the pair (Black, White) does not intersect the zero plane.

Technical options

The “*Variable and method selection*” menu, shown in Figure 3, consists of a “*Technical options*”

Figure 5: The observed data, simultaneous confidence bands, and the regression plane for Females and Males

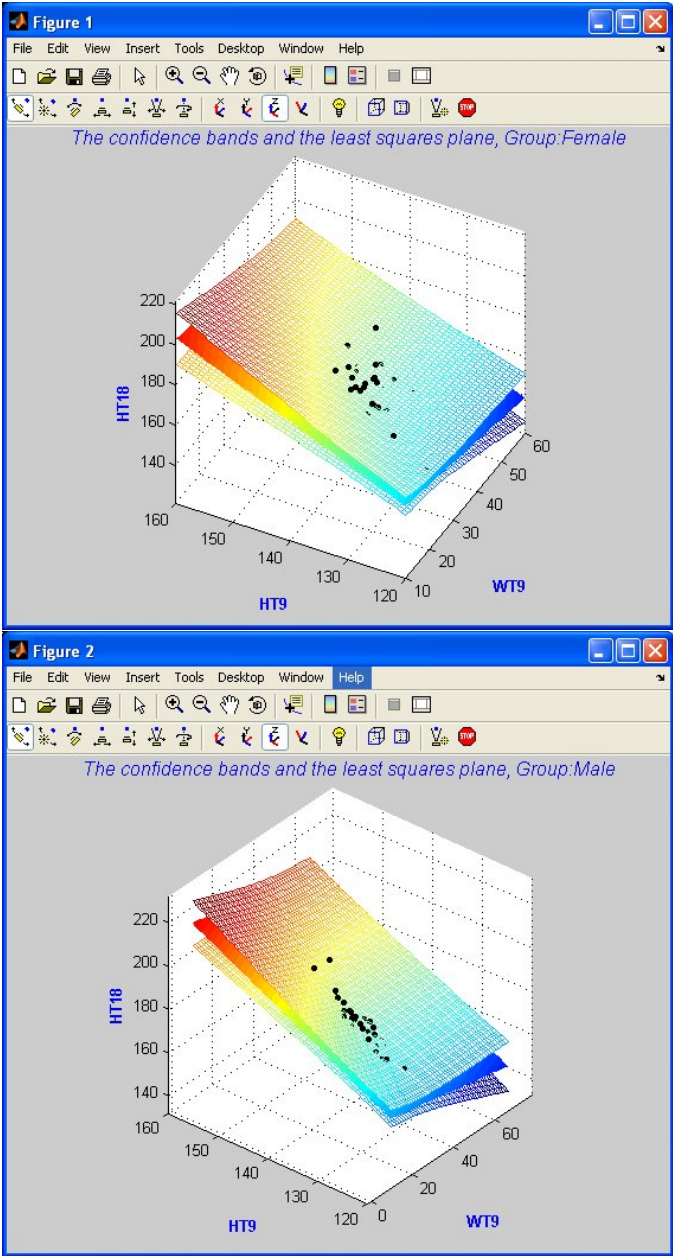


Figure 6: A snapshot of the “variable and method selection menu”, specifying multiple comparison

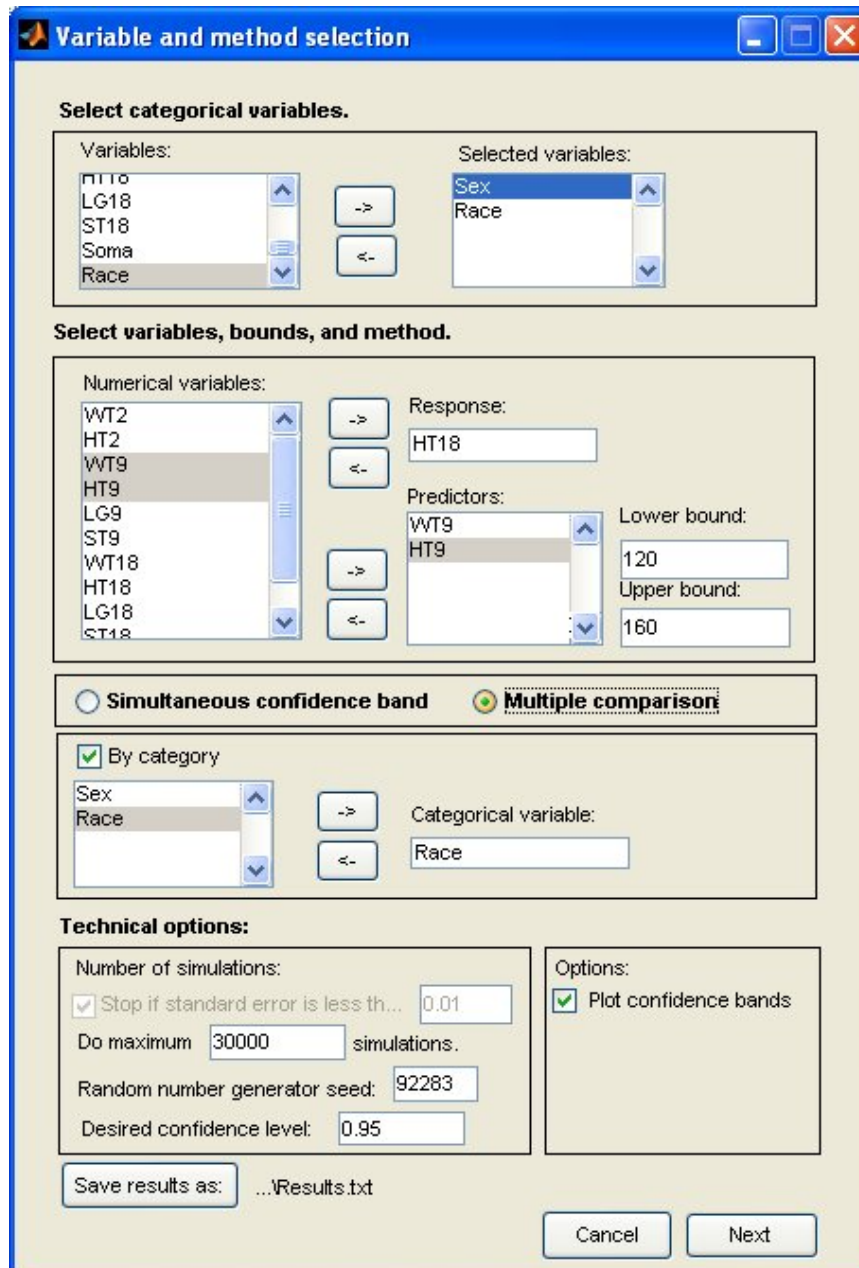


Figure 7: A snapshot of the “Pair Comparison Selection” menu

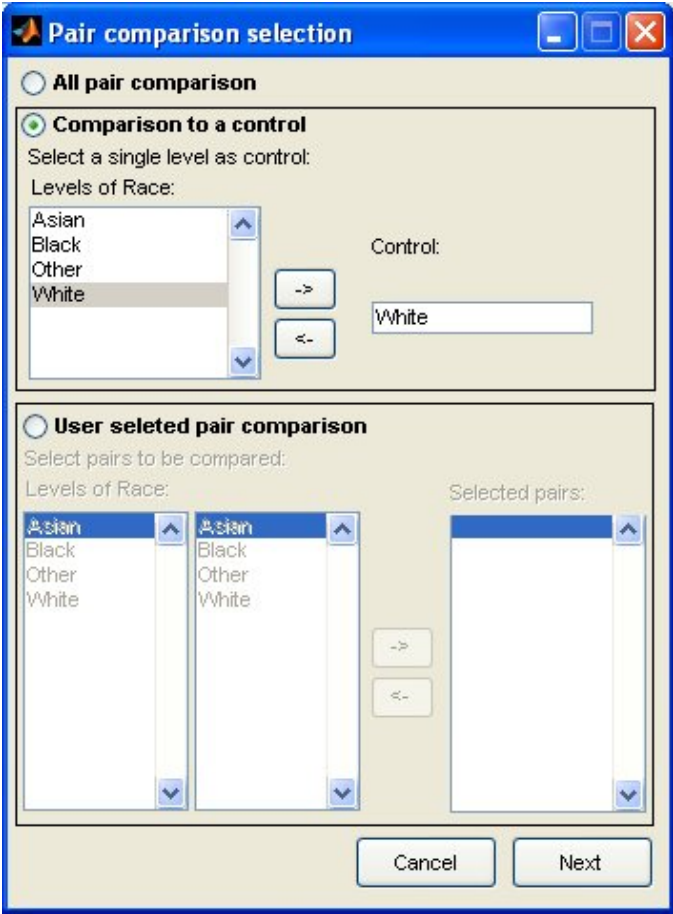


Table 2: Output for the multiple comparison of **White** with other levels of the variable **Race**.

```

Summary statistics and results of the analysis
=====

Summary statistics:
Group:      Asian

Variable    N      Min      Mean      Median      Std. Dev.      Max
HT18       45     154.600   168.804   168.100     7.875          188.000
WT9        45     22.200   32.364    30.800     7.906          66.800
HT9        45     123.200  135.556   135.800     6.010          152.500

=====

Regression Analysis
Group:      Asian

Response:      HT18
Residual Mean Square = 29.0842

Variable      Coeff      s.e.      T-stat      p-value
Constant     33.0710    23.9793    1.379       0.0876
WT9          -0.0381    0.1539    -0.248     0.4027
HT9          1.0104     0.2024     4.992     0.0000

=====

Summary statistics:
Group:      Black

Variable    N      Min      Mean      Median      Std. Dev.      Max
HT18       24     160.900   178.096   177.850     6.945          194.300
WT9        24     19.900   30.642    30.900     4.102          43.200
HT9        24     122.000  135.129   134.300     5.218          147.400

=====

Regression Analysis
Group:      Black

Response:      HT18
Residual Mean Square = 8.0487

Variable      Coeff      s.e.      T-stat      p-value
Constant     8.1853    16.1141    0.508       0.3084
WT9          -0.1775    0.1737    -1.022     0.1592
HT9          1.2976     0.1366     9.502     0.0000

=====

Summary statistics:
Group:      Other

Variable    N      Min      Mean      Median      Std. Dev.      Max
HT18       34     153.600   166.706   166.750     5.790          177.500
WT9        34     22.000   31.538    30.900     5.485          42.400
HT9        34     121.400  135.100   135.650     5.387          144.800

=====

Regression Analysis
Group:      Other

Response:      HT18
Residual Mean Square = 10.5821

Variable      Coeff      s.e.      T-stat      p-value
Constant     24.7002    16.8182    1.469       0.0760
WT9          -0.4657    0.1424    -3.271     0.0013
HT9          1.1598     0.1449     8.002     0.0000

=====

Summary statistics:
Group:      White

Variable    N      Min      Mean      Median      Std. Dev.      Max
HT18       33     167.000   179.764   180.200     6.538          195.100
WT9        33     24.400   31.427    31.000     4.439          43.100
HT9        33     125.400  136.079   136.000     5.259          146.000
    
```


Table 3: Table 2 (Continued)

```

Regression Analysis
Group:      White

Response:   HT18
Residual Mean Square = 9.6432

Variable      Coeff      s.e.      T-stat      p-value
Constant     12.4400    16.8338    0.739      0.2328
WT9          -0.4061    0.1772    -2.291     0.0146
HT9          1.3234    0.1496     8.845     0.0000
=====

The pooled variance is 16.1927

The degrees of freedom is 124
=====
Number of simulations = 30000

The lower and upper bounds imposed on the predictors are:
WT9 Lower Bound = 10.000 Upper Bound = 60.000
HT9 Lower Bound = 120.000 Upper Bound = 160.000

The simulated critical value for 0.950 confidence level = 3.1947
The standard error = 0.0107

=====
Observed statistics and p-values for single pair comparison:
-----
Pair { Asian, White} Observed Statistics = 11.0953 p-value = 0.0000
Pair { Black, White} Observed Statistics = 1.0176 p-value = 1.0000
Pair { Other, White} Observed Statistics = 11.9170 p-value = 0.0000

Observed statistics and p-values for all selected pairs comparison:
-----
Observed Statistics = 11.9170 p-value = 0.0000
=====

```

Figure 8: Single pair comparison plots

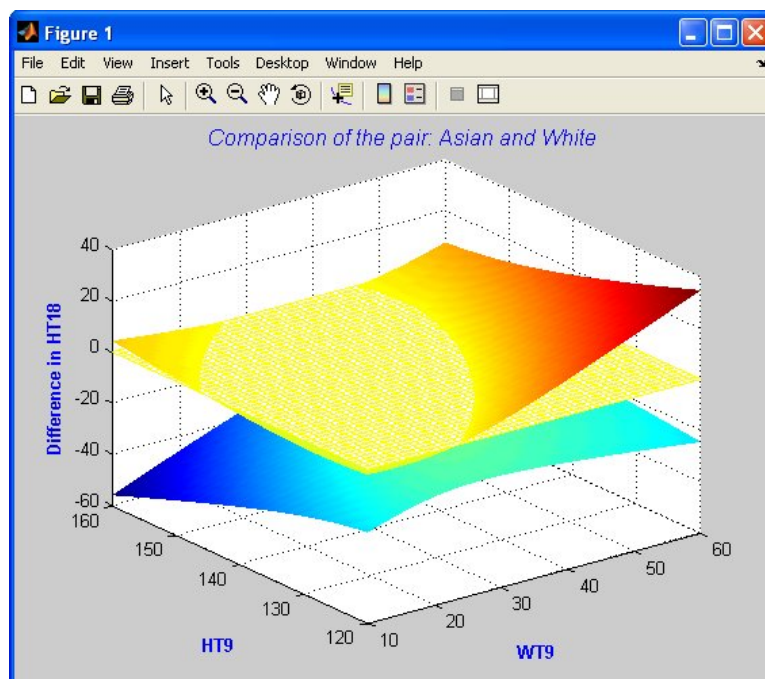
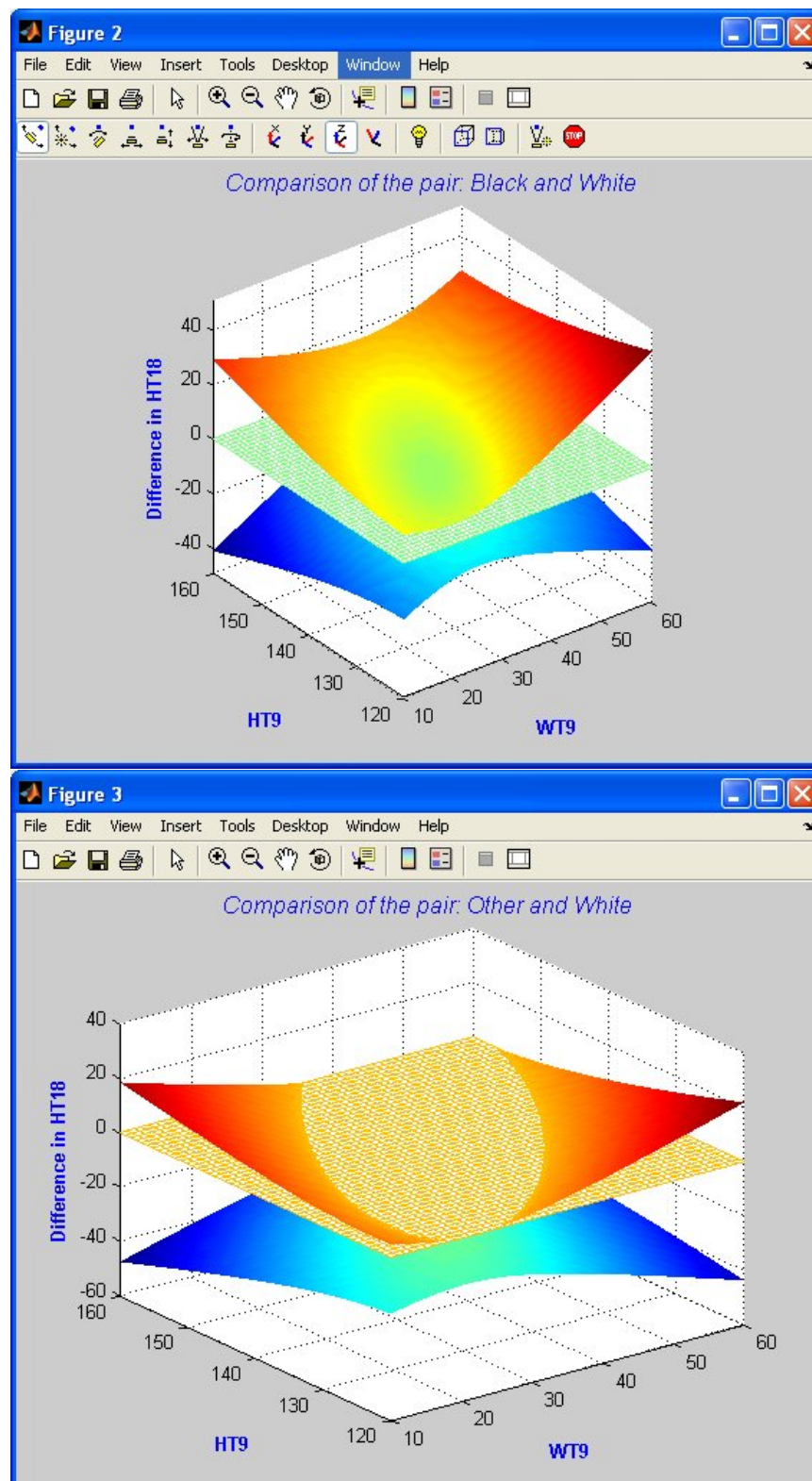


Figure 9: Single pair comparison plots



section, where some technical parameters for the simulation process can be set. The user can specify the maximum number of simulations desired (see LJZ 2004 and 2005 for guidelines), or the simulation process can optionally be stopped by setting a threshold for the tolerance level of the standard error of the estimated critical value. The standard error is computed using the methods given in LJZ (2005). The standard error option is not available when multiple comparison is selected (see Figure 6). Also in this panel a seed for the random number generator can be set, and the desired confidence level, a value between 0.5 and 1, can be input. If the number of predictors is less than or equal to 2, then there is a plot option, which is on by default, and results in the plots explained above. Finally, the button “Save results as” allows the user to input a filename where the results of the analysis will be stored in. The default filename for the results is “Results.txt” and it is stored in the current directory. This file is a txt file, and it can be best viewed either using the MATLAB editor or the Notepad. If Notepad is used, it would be best to set the font to “Courier New” with 12 point font size.

5.4. Access to a few useful variables

The user may be interested to perform further analyses after running the software. **SimReg** provides two files `useful_vars_scb.mat` and `useful_vars_mc.mat` that contain a few useful variables for this purpose. The file `useful_vars_scb.mat` is created when the simultaneous confidence band option is selected, and the `useful_vars_mc.mat` is created when the multiple comparison option is run. These files are stored in the current directory, where the user is running **SimReg**. The MATLAB’s load `filename` command can be used to upload the content into the workspace area.

The following variables are stored in the file `useful_vars_scb.mat`, if no categorical variable is selected:

B This matrix contains $(X^T X)^{-1}$.

RMS The residual mean square from the regression analysis.

se Standard error of c .

beta_ls Regression coefficient from the regression analysis.

c The critical constant for the simultaneous confidence band.

n Number of cases.

p Number of predictors.

pred The predicted values from the regression model.

Predictor_names A vector of strings containing predictor names.

resid The residual values from the regression model.

response_name A string containing the name of the response variable.

se Standard error of c .

y A vector containing all the observations on the response variable.

The following variables are stored in the file `useful_vars_scb.mat`, if a categorical variable is selected:

B_all This matrix contains $(X^T X)^{-1}$ for all levels of the selected categorical variable. If there are k levels, then the size of the matrix has $k \times (p + 1)$ rows and $(p + 1)$ columns.

RMS_all The residual mean square from the regression analysis for each of the levels of the selected categorical variable.

c_all This is a vector containing the critical constants for the simultaneous confidence bands for each of the levels of the selected categorical variables.

n_all The number of cases in each of the levels of the selected categorical variable.

Predictor_names A vector of strings containing predictor names.

se_all This is a vector containing the standard error for the critical constants for the simultaneous confidence bands for each of the levels of the selected categorical variables.

The following variables are stored in the file `useful_vars_mc.mat`:

B_all This matrix contains $(X_i^T X_i)^{-1}$ for all levels of the selected categorical variable. If there are k levels, then the size of the matrix has $k \times (p + 1)$ rows and $(p + 1)$ columns.

Bij_all This matrix contains $(X_i^T X_i)^{-1} + (X_j^T X_j)^{-1}$ for each pair (i, j) in a row of Λ in order. If Λ has ℓ rows, then **Bij_all** consists of $\ell * (p + 1)$ rows and $p + 1$ columns.

lambda A two-column matrix with rows consisting of the pairs of levels of the categorical variable selected for multiple comparison.

RMS_all The residual mean square from the regression analysis for each of the levels of the selected categorical variable.

beta_all The regression coefficient from the regression analysis for each of the levels of the selected categorical variable.

c The critical value for multiple comparison.

lb A vector containing the lower-bound restrictions on predictors.

n_all The number of cases in each of the levels of the selected categorical variable.

Predictor_names A vector of strings containing predictor names.

response_name A string containing the name of the response variable.

rnd_seed The seed used for random number generation.

se Standard error of c .

sighat The pooled standard deviation $\hat{\sigma}$.

tmax A vector containing all simulated values of T .

ub A vector containing the upper-bound restrictions on predictors.

5.5. Future improvements of the software and copyright

The following includes a list of improvements that are planned to be made in **SimReg** in the future versions:

- Handle missing data.
- Add a context sensitive help to each of the menus.
- Add a “back” button to each menu.
- Provide html output.
- Provide one sided confidence bands.

This software may not be replicated, copied, or used in any form for any commercial purpose without the written consent from the first author.

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