NEW EFFICIENT CUSUM CONTROL CHARTS

BY

SANUSI RIDWAN ADEYEMI

A Thesis Presented to the

DEANSHIP OF GRADUATE STUDIES

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN, SAUDI ARABIA

In Partial Fulfillment of the

Requirements for the Degree of

MASTER OF SCIENCE

In

APPLIED STATISTICS

APRIL, 2016

KING FAHD UNIVERSITY OF PETROLEUM & MINERALS

DHAHRAN- 31261, SAUDI ARABIA

DEANSHIP OF GRADUATE STUDIES

This thesis, written by SANUSI Ridwan Adeyemi under the direction of his thesis advisor and approved by his thesis committee, has been presented and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of **MASTER OF SCIENCE IN APPLIED STATISTICS.**

Dr. Muhammad Riaz
(Advisor)

Dr. Hussain Al-Attas
Department Chairman

Dr. Nasir Abbas
(Member)

Dr. Salam A. Zummo
Dean of Graduate Studies

Dr. Munjed Samuh
(Member)

9/5/16
Date

To my parents

ACKNOWLEDGMENTS

All praise and adoration to almighty Allah for giving me the opportunity to be a Muslim, and to complete my master's degree program successfully.

My candid appreciation goes to my parents for their prayers and support. May Almighty Allah give me the ability to take good care of them.

My undiluted appreciation also goes to my supervisor; Dr. Muhammad Riaz, for his belief in me; my co-supervisors; Dr. Samuh Munjed and Dr. Nasir Abbas; and my other lectures, especially Dr. Saddam Akbar Abbasi, Dr. Mohammad H. Omar and Dr. Mu'azu Ramat Abujiya, for their moral and educational advice.

I cannot forget my academic father, Professor Emanuel B. Lucas, no word in the dictionary could be used to appreciate him. Thank you for your support and advice so far.

To my siblings, family members, friends and everybody that have supported me in one way or the other, I say "Jazakumullahu khairan". Finally, my affectionate gratitude goes to my best friend; Bose, for her patience, love and care, despite the distance between us. May almighty Allah guide us to the right path.

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LIST OF ABBREVIATIONS

ARL	:	Average Run Length
ARL_0	:	The *ARL* value when there is no shift in a process
ARL_1	:	The *ARL* value when there is a shift in a process
CL	:	Center Line
CUSUM	:	Cumulative Sum
EWMA	:	Exponentially Weighted Moving Average
FAR	:	False Alarm Rate
FIR	:	Fast Initial Response
LCL	:	Lower Control Limit
UCL	:	Upper Control Limit

ABSTRACT

Full Name : [SANUSI Ridwan Adeyemi]

Thesis Title : [NEW EFFICIENT CUSUM CONTROL CHARTS]

Major Field : [Applied Statistics]

Date of Degree : [April, 2016]

Statistical quality control deals with monitoring of the production/manufacturing processes and control chart is one of its major tools. It is vastly applied in industry to keep the process variability under control. One of the most popular categories of control charts is CUSUM chart which is based on utilizing the information on cumulative sum pattern to detect small shifts. This thesis proposes new efficient CUSUM charts which are based on the utilization of auxiliary information to monitor the location parameter of a study variable. Furthermore, to increase the sensitivity of the proposed charts in detecting moderate to large shifts, the proposed charts are extended to Combined Shewhart CUSUM charts. CUSUM chart for monitoring dispersion parameter is also improved by applying the Fast Initial Response. The average run length performance of the proposed charts is evaluated in terms of shifts in study variable and compared with some recently designed control structures meant for the same purposes. The comparisons revealed that the proposed charts perform really well relative to the other charts under discussion. At last, real life industrial examples are provided to describe the application procedure of the proposed charts.

ملخص الرسالة

الاسم الكامل: سانوسي، رضوان أدييمي

عنوان الرسالة: خرائط المراقبة CUSUM الجديدة والأكثر فعالية

التخصص: إحصاء تطبيقي

تاريخ الدرجة العلمية: نيسان، 2016

يتعامل الضبط الاحصائي للجودة مع مراقبة عمليات الإنتاج/التصنيع، وخريطة المراقبة هي إحدى أدواتها. يتم تطبيق خريطة المراقبة الى حد كبير في الصناعة للحفاظ على تبيان العمليات بالمنتج ضمن المواصفات المطلوبة. إن CUSUM هي واحدة من أهم أصناف خرائط المراقبة وهي مبنية على استخدام المعلومات عن ماهية أو نمط الجمع التراكمي للكشف عن الازاحات الصغيرة. في هذه الرسالة نقترح طريقة CUSUM جديدة وأكثر فعالية مبنية على استخدام معلومات مساعدة للتحكم بمعلمة الموقع الخاصة بالمتغير قيد الدراسة. إضافة الى أن هذه الطريقة المقترحة لها القدرة في الكشف عن الإزاحات المتوسطة الى الكبيرة، الخريطة المقترحة تم توسعتها الى خرائط Shewhart CUSUM المركبة. كذلك في هذه الرسالة تم إجراء تحسين على خريطة CUSUM للتحكم بمعلمة التشتت من خلال تطبيق ما يسمى بالاستجابة الأولية السريعة FIR. تم حساب متوسط طول المدى ARL للخريطة المقترحة من خلال معلمة الازاحة للمتغير قيد الدراسة وتم مقارنتها مع تصاميم أخرى حديثة مماثلة. أظهرت المقارنات بأن أداء الخريطة المقترحة أفضل من الخرائط الأخرى التي شملتها الرسالة. وأخيرا، تم عرض أمثلة واقعية من القطاع الصناعي وذلك كتطبيق على الطريقة الجديدة المقترحة في هذه الرسالة.

CHAPTER 1

INTRODUCTION

Control chart is a statistical chart to observe process quality, it is one of the seven tool kits (Pareto diagram, Cause and effect diagram, Flowcharts, Control chart, Histogram, Scatter diagram and Check sheet) of statistical process control (Montgomery, 2009). Two core types of control chart exist depending on the number of process features or variable to be examined; the univariate control chart and the multivariate control charts. The former is a graphical representation that summarizes one quality characteristic, while the latter describes the characteristics of two or more variable of interest.

Univariate control chart shows the value of the variable of interest over time or against sample number. In addition, three lines exist in a chart; the lower control limit (LCL), the center line (CL) and the upper control limit (UCL). The CL indicates the average value of the in-control process, while the UCL and the LCL give boundaries around the CL for declaring that a process is in-control. These control limits are carefully chosen to ensure that all the study observations are within these boundaries as far as the process remains in-control.

Control charts are used for observing different shifts in a process, these shifts can be a transient shift (memoryless structure) or a persistent shift (memory structure). Shewhart (1924) introduced the Shewhart control chart for detecting transient shifts. This chart monitors sudden shift by using information from the most recent examined samples, consequently, it is not effective in monitoring minute shifts in a process. However, small shifts can be monitored by the memory-type control charts, which are the EWMA chart, developed by Roberts (1959), and the CUSUM control chart, proposed by Page (1954). The Exponentially Weighted Moving Average (EWMA), allot larger weight to the most current data points for detecting small shifts in a process. Also, the CUSUM chart is based on geometric moving average. It detects smaller shifts efficiently by using information from a very long sequence of samples.

In this thesis, CUSUM control chart is considered extensively by proposing new CUSUM charts that are more efficient in detecting smaller to moderate shifts, than the ones in the literature. The efficiency is mainly compared using the average run length (ARL) approach. The proposed charts are compared with existing charts of the same purpose. Efficient estimators used in the field of sampling techniques are used for the construction of the proposed CUSUM charts. The proposed charts detect shifts in location parameter or dispersion parameter in a process, and various statistical properties of the charts are examined.

1.1 CUSUM CONTROL CHART

The CUSUM control chart is used in detecting small shift in a variable (X) of a process, it is a cumulative deviation from the target value μ_0. It is calculated by two statistics which are the

$$C_t^+ = \max\left[0, \left(X_t - \mu_0\right) - K + C_{t-1}^+\right] \tag{1.1}$$

$$C_t^- = \max\left[0, -\left(X_t - \mu_0\right) - K + C_{it1}^-\right] \tag{1.2}$$

upper CUSUM $\left(C_t^+\right)$ and the lower CUSUM $\left(C_t^-\right)$, where t is the observation number and $C_0^+ = C_0^- = 0$ though they can also be set to other values (Headstart values) for fast initial response (FIR) CUSUM (Hawkins and Olwell, 1998). Both C_t^+ and C_t^- are plotted against control limits (*H*). K is the reference value, and it is taken to be half of the shift (δ) to be detected, scaled in standard deviation (σ) unit, under the assumption that the study variable *X* is normally distributed. The lower the value of K, the more sensitive the CUSUM control chart is to small shifts. X_t represents the t^{th} observation for a single sample size ($n = 1$). For a subgroup ($n > 1$), X_t is replaced with the mean of the subgroup in each observation.

CHAPTER 2

LITERATURE REVIEW

In the field of engineering, statistical process control (SPC) is recurrently connected to the use of charting methods for identifying changes in variability or mean of a process. Its activities include Pareto analysis, the experimental design and multivariable analysis, design of sampling and inspection schemes. Base on design structure, we can group control charts into two different aspects; the memoryless control chart (Shewhart-type) and memory control charts. The frequently used memory control charts are the Cumulative Sum (CUSUM) control charts and the Exponentially Weighted Moving Average (EWMA) control chart proposed by Page (1954) and Roberts (1959) respectively. Unlike the Shewhart-type charts that ignores the past information, the CUSUM and the EWMA charts make use of the past information and the current information to give a better performance in detecting small shifts and moderate shifts. The structure of the CUSUM charts and their average run length(*ARL*) performance for various choices of parameters are well explained in Hawkins and Olwell (1998). When fundamental distribution of a process is not normal or unlikely to be normal, nonparametric control charts will be good. Considering small shifts in scatter outliers, Midi and Shabbak (2011) proposed robust EWMA and CUSUM for early detection of the shift in multivariate case. Li et al. (2010) introduced two nonparametric equivalents of the CUSUM and EWMA control charts for detecting shifts in the location parameter of a process, based on the Wilcoxon rank-sum test. The application of robust control chart in CUSUM for detecting shifts in location and dispersion of a process simultaneously was considered by Reynolds and Stoumbos (2010).

Some authors also consider the use of auxiliary variable to increase the efficiency of the study variate, which we also consider in this thesis work. When assessing a control chart's plotting statistic(s), Riaz (2008a) popularized the notion of using auxiliary information. He suggested a control chart which uses a regression-type estimator as the plotting statistic to monitor the process's variability, and showed the supremacy of his chart over the famous Shewhart-type control charts for the same drive. Aiming on small shifts and moderate shifts in the location parameter of a process, Abbas et al. (2014) proposed an EWMA-type control chart which uses one auxiliary variable. The mean in the structure of the proposed chart is estimated using the regression estimation method. It was established that the chart outperformed its univariate and bivariate counterparts. Furthermore, Riaz (2008b) proposed a regression-type estimator to monitor the location of a process. He not only showed the superiority of his proposal over the Shewhart's $\overline{X}$ -chart, but also over the regression charts and the cause-selecting charts.

Due to the advancement in technology and industrial processes, there is need to enhance the sensitivity of CUSUM charts to large shifts. This is done by combining the CUSUM chart with the Shewhart chart, to detects small to large shifts effectively at the same time. Westgard et al. (1977) applied this concept to improve quality control in clinical chemistry. The combination of Shewhart chart and CUSUM chart was observed by Lucas (1982) after which some scholars improved the chart by proposing more efficient charts. Combined Shewhart-CUSUM (hereafter called "CSC") for location parameter can be optimized over the entire mean shift range by adding an extra parameter (w) known as the exponential of the sample mean shift, to the structure of the CSC. This will improve its performance and it will not increase the difficulty level of understanding and implementing the chart (Wu et al., 2008). The CSC, which has a wide range of application, attracts the attention of Environmentalists, and it is the only quality control chart directly recommended by the United States Environment Protection Agency for intra-well monitoring. It has been consistently applied to waste disposal facilities for detection monitoring programs (Gibbons, 1999). Abujiya et al. (2013) replaced the traditional simple random sampling in the plotting statistic of the CSC chart with ranked set sampling.

Control charts monitor the location and (or) dispersion parameter(s) of a process. The location parameter monitoring and its modification is mostly available in the literature, but little work has been done on dispersion monitoring. In detecting shift in process dispersion, CUSUM was applied to subgroup range by Page (1954). Tuprah and Ncube (1987) later compared this procedure with another procedure that was based on sample standard deviation. Using ARL approach, they found that the procedure based on the sample standard deviation detects shift from the target value faster, given that the process variables are normally distributed. Furthermore, one-sided and two-sided CUSUM structures based on logarithmic transformation of process variance was proposed by Chang & Gan (1995) for monitoring shift in process variance, and they also enhanced the performance of the schemes by introducing the Fast Initial Response (FIR) feature. The FIR was first proposed by Roberts (1959) and later improved by Steiner (1999) to reduce the time-varying limits of the first few sample observations. The FIR feature improves the performance of CUSUM chart if there is shift in a process at start-up (Hawkins and Olwell, 1998). The performance of this feature was later improved by using a power transformation with respect to time t (Haq, 2013).

2.1 OBJECTIVES OF THE STUDY

We summarize the main objectives to be achieved in this study:

1. To improve CUSUM control chart that monitor location parameter.

2. To improve CUSUM control chart that monitor dispersion parameter.

3. To extend the proposed charts to combined Shewhart-CUSUM chart.

4. To compare the proposed charts with their counterparts using average run length and some other performance measures.

5. Apply this study to numerous real life dataset.

CHAPTER 3

Efficient CUSUM-Type Control Charts for Monitoring the Process Mean Using Auxiliary Information

Statistical quality control deals with monitoring of the production/manufacturing processes and control chart is one of its major tools. It is vastly applied in industry to keep the process variability under control. One of the most popular categories of control charts is CUSUM chart which is based on utilizing the information on cumulative sum pattern. This article proposes a new two-sided CUSUM charts which are based on the utilization of auxiliary information. The ARL performance of the proposed charts is evaluated in terms of shifts in study variable and compared with some recently designed control structures meant for the same purposes. The comparisons revealed that the proposed charts perform really well relative to the other charts under discussion. At last, a real life industrial example is provided to describe the application procedure of the proposal.

3.1 INTRODUCTION

The output of all the manufacturing processes always includes some amount of variation in it; e.g. in the process of filling two bottles with cooking oil, the amount of oil filled in any of the two bottles will not be exactly the same, and in the process of making tube light rods, the diameter or length of any two rods will not be the same, etc. This inherent part of process is known as common (uncontrollable) cause variation. The variations outside this common cause pattern are called special (controllable) cause variations. These variations are usually large in magnitude, controllable in nature and due to many inescapable causes. Statistical Quality Control (SQC) includes some tools that can be used to discriminate between common and special cause variations. There are seven most commonly referred tools (Montgomery, 2009) and these tools are jointly known as SQC tool-kit. The most important and the most powerful tool of this kit is the control chart which is the graphical display of a quality characteristic plotted against three lines named as Upper Control Limit (UCL), Center Line (CL) and Lower Control Limit (LCL). The two control limits (i.e. UCL and LCL) are basically the parameters of a control chart which are selected in such a way that there is a very small probability, generally referred as False Alarm Rate (FAR) in quality control literature and denoted by (α)) of the in-control data points falling outside these limits.

Control charts are further classified as Shewhart, CUSUM and EWMA-type control charts. The structure of Shewhart-type control charts proposed by Shewhart (1924) is made such that they utilize just the present information and hence, they ignore all the past information which results in less efficiency of these charts for detecting shifts (alterations in a process) that are of smaller magnitude. This drawback of Shewhart-type control charts leads to the proposal of Cumulative Sum (CUSUM) control charts (Page, 1954) and Exponentially Weighted Moving Average (EWMA) control charts (Roberts, 1959). The formation of these control charts is based on utilizing the past information along with the present to improve the performance of control charts for detecting small amount of shifts. The two most commonly named performance measures for control charts are power and average run length (ARL). Power of a control chart is defined as the probability of detecting a shift whereas ARL is defined as average number of samples required to detect a shift. ARL_0 and ARL_1 are the representations of in-control and out-of-control chart $ARLs$ respectively, for a control chart. The $ARLs$ for the Shewhart-type charts (like $\bar{X}$, R, S and S^2) can be obtained by taking the reciprocal of power, as the assumptions of having a geometric run

$$ARL = \frac{1}{Power} = \frac{1}{P(reject \quad null \quad hypothesis \mid null \quad hypothesis \quad is \quad false)}$$

length variable are fulfilled for these charts. For CUSUM and EWMA-type control chart, the ARL values are obtained through averaging the exact run length distribution, as the assumption of geometric run length variable does not hold for these charts.

Auxiliary information is the extra information accessible apart from the information from the sample, at the estimation stage. Ratio, product and regression-type estimators are the most commonly quoted fashions of the exploitation of auxiliary information at the time of estimation (Fuller, 2011). The design of these estimators are structured such that they make use of the sample information and the auxiliary information, hence, they are more efficient than the traditional ones. There is a long history of the use of auxiliary information in the field of survey sampling but Riaz (2008a) popularized the concept of using it at estimation stage in SQC. Riaz (2008a) and Riaz (2008b) proposed the auxiliary based control charts for monitoring the process variability and location respectively where both of these charts are based on regression-type estimators. Furthermore, Riaz and Does (2009) suggested another variability chart based on a ratio-type estimator and showed the dominance of their proposed chart over the one based on regression-type estimator. Following the work of all these authors, several CUSUM-type control charts which are based on auxiliary information are presented in this chapter. The performance of the proposed charts is measured in terms of its ARL values.

The organization of the rest of this chapter is as follows: the design structure of the classical CUSUM control chart is given in Section 3.2; Section 3.3 contains the details regarding the proposed charts (A_XCUSUM charts) and their ARL performance; Section 3.4 gives comparisons of our proposed chart with the other recently developed CUSUM and EWMA-type control charts; Section 3.5 contains an illustrative example in which the application of the proposed charts is shown on a simulated dataset; finally, Section 3.6 concludes the finding of this chapter.

3.2 THE CLASSICAL CUSUM CONTROL CHART

Today, CUSUM control chart proposed by Page (1954) has become one of the most admired algorithms to monitor production processes. There is a close connection between the formation of this chart and the Sequential Probability Ratio Test ($SPRT$) by Wald (1947), which is in agreement with the observation of Fuh (2003) that CUSUM and $SPRT$ form a hidden Markov Chain model. For a two-sided CUSUM chart, two statistics S_i^+ and S_i^- are plotted against single control limit H. These plotting statistics are defined as:

$$\left.\begin{aligned} S_i^+ &= \max[0, (\bar{Y}_i - \mu_0) - K + S_{i-1}^+] \\ S_i^- &= \max[0, -(\bar{Y}_i - \mu_0) - K + S_{i-1}^-] \end{aligned}\right\} \quad (3.1)$$

where i is the sample number, $\bar{Y}$ is the sample mean of study variable Y, μ_0 is the target mean of Y, K is the reference value of CUSUM scheme often taken equal to half of the amount of shift to be detected (Ewan and Kemp, 1960). The starting value for both the plotting statistics is taken equal to zero i.e. $S_0^+ = S_0^- = 0$. Now, these two statistics are plotted against the control limit H and it is concluded that the process mean has moved upward if $S_i^+ > H$ for any value of i, whereas the process mean is said to be shifted downward if $S_i^- > H$ for any value of i. The CUSUM chart is defined by two parameters i.e. K and H which are to be chosen very carefully because, the ARL performance of the CUSUM chart is very sensitive to these parameters (Montgomery, 2009). These two parameters are used in the standardized manner (Montgomery, 2009) given as:

$$K = k \times \sqrt{\mathrm{Var}(\bar{Y})}, \text{ and } \quad H = h \times \sqrt{\mathrm{Var}(\bar{Y})} \tag{3.2}$$

where $\sqrt{\mathrm{Var}(\bar{Y})} = {}^{\sigma_Y}/_{\sqrt{n}}$ and σ_Y is the standard deviation of Y. In the next section, we provide the details regarding the proposed chart, for which we have used the version of the CUSUM given in (3.1).

3.3 THE PROPOSED A$_X$CUSUM CONTROL CHART

Suppose $(y_{i1}, x_{i1}), (y_{i2}, x_{i2}), (y_{i3}, x_{i3}), \ldots$ (where $i = 1,2, \ldots$) represent a sequence of paired observations taken for a quality characteristic Y (which is the study variable) and is also correlated with the auxiliary variable X. Each pair (Y_{ij}, X_{ij}) for $j = 1,2,3, \ldots, n$ is assumed to follow bivariate normal distribution with mean vector μ and variance-covariance matrix Σ given as:

$$\mu = \begin{pmatrix} \mu_0 + \delta\sigma_Y \\ \mu_X \end{pmatrix}, \quad \Sigma = \begin{pmatrix} \sigma_Y^2 & \mathrm{Cov}(Y, X) \\ \mathrm{Cov}(X, Y) & \sigma_X^2 \end{pmatrix} \tag{3.3}$$

where μ_0 is the in-control mean of study variable Y and μ_X are the known mean of auxiliary variable X. σ_Y^2 and σ_X^2 are the population variances of Y and X, respectively, and are assumed to be known. $\mathrm{Cov}(Y, X) = \mathrm{Cov}(X, Y)$ is the covariance between the study variable Y and the auxiliary variable X. δ represents the amount of shift introduced in the study variable Y in σ_Y

units i.e. $\delta = \frac{|\mu_1 - \mu_0|}{\sigma_Y}$, where μ_1 represents the out-of-control mean of Y. Now based on (3.3), there are several estimators in the literature for estimating the population mean (Srivastava (1967), Singh and Tailor (2003), Kadilar and Cingi (2004) and Cochran (1977)). Some of them (along with their expected value and mean square error) are given in Table 3.1.

Table 3.1 : Definition and properties of some estimators for estimating population mean

Estimators ($\bar{Y}_p, p = 1,2,\ldots,10$)	$E(\bar{Y}_p)$	$MSE(\bar{Y}_p)$
$\bar{Y}_1 = \frac{\sum_{j=1}^{n} Y_j}{n}$	μ_0	$\frac{\sigma_Y^2}{n}$
$\bar{Y}_2 = \bar{Y} + b_{YX}(\mu_X - \bar{X})$	$\mu_0 - \text{Cov}(\bar{X}, b_{YX})$	$\frac{\sigma_Y^2}{n}(1 - \rho_{YX}^2)$
$\bar{Y}_3 = \bar{Y}\left(\frac{\mu_X}{\bar{X}}\right)$	$\mu_0 + \frac{\mu_Y(C_X^2 - \rho_{YX}C_Y C_X)}{n}$	$\frac{\mu_Y^2(C_Y^2 + C_X^2 - 2\rho_{YX}C_Y C_X)}{n}$
$\bar{Y}_4 = \bar{Y}\left(\frac{\mu_X + \rho_{YX}}{\bar{X} + \rho_{YX}}\right)$	$\mu_0 + \frac{\mu_Y g(gC_X^2 - \rho_{YX}C_Y C_X)}{n}$	$\frac{\mu_Y^2(C_Y^2 + g^2 C_X^2 - 2g\rho_{YX}C_Y C_X)}{n}$
$\bar{Y}_5 = [\bar{Y} + b_{YX}(\mu_X - \bar{X})]\left(\frac{\mu_X}{\bar{X}}\right)$	$\mu_0 + \frac{\mu_X C_X^2}{n}$	$\frac{\mu_X^2[C_X^2 + C_Y^2(1 - \rho_{YX}^2)]}{n}$
$\bar{Y}_6 = [\bar{Y} + b_{YX}(\mu_X - \bar{X})]\left(\frac{\mu_X + C_X}{\bar{X} + C_X}\right)$	$\mu_0 + \frac{\mu_X C_X^2}{n}\left(\frac{\mu_X}{\mu_X + C_X}\right)^2$	$\frac{\mu_X^2\left[\left(\frac{\mu_X}{\mu_X + C_X}\right)^2 C_X^2 + C_Y^2(1 - \rho_{YX}^2)\right]}{n}$
$\bar{Y}_7 = [\bar{Y} + b_{YX}(\mu_X - \bar{X})]\left(\frac{\mu_X + \beta_{2(X)}}{\bar{X} + \beta_{2(X)}}\right)$	$\mu_0 + \frac{\mu_X C_X^2}{n}\left(\frac{\mu_X}{\mu_X + \beta_{2(X)}}\right)^2$	$\frac{\mu_X^2\left[\left(\frac{\mu_X}{\mu_X + \beta_{2(X)}}\right)^2 C_X^2 + C_Y^2(1 - \rho_{YX}^2)\right]}{n}$
$\bar{Y}_8 = [\bar{Y} + b_{YX}(\mu_X - \bar{X})]\left(\frac{\mu_X\beta_{2(X)} + C_X}{\bar{X}\beta_{2(X)} + C_X}\right)$	$\mu_0 + \frac{\mu_X C_X^2}{n}\left(\frac{\mu_X\beta_{2(X)}}{\mu_X\beta_{2(X)} + C_X}\right)^2$	$\frac{\mu_X^2\left[\left(\frac{\mu_X\beta_{2(X)}}{\mu_X\beta_{2(X)} + C_X}\right)^2 C_X^2 + C_Y^2(1 - \rho_{YX}^2)\right]}{n}$
$\bar{Y}_9 = [\bar{Y} + b_{YX}(\mu_X - \bar{X})]\left(\frac{\mu_X C_X + \beta_{2(X)}}{\bar{X}C_X + \beta_{2(X)}}\right)$	$\mu_0 + \frac{\mu_X C_X^2}{n}\left(\frac{\mu_X C_X}{\mu_X C_X + \beta_{2(X)}}\right)^2$	$\frac{\mu_X^2\left[\left(\frac{\mu_X C_X}{\mu_X C_X + \beta_{2(X)}}\right)^2 C_X^2 + C_Y^2(1 - \rho_{YX}^2)\right]}{n}$
$\bar{Y}_{10} = \bar{Y}\left(\frac{\mu_X}{\bar{X}}\right)^\alpha$	$\mu_0 + \frac{\mu_Y\left(\frac{\alpha(\alpha-1)}{2}C_X^2 - \alpha\rho_{YX}C_Y C_X\right)}{n}$	$\frac{\mu_Y^2(C_Y^2 + \alpha^2 C_X^2 - 2\alpha\rho_{YX}C_Y C_X)}{n}$

Some of the quantities in Table 3.1 are defined as: $b_{YX} = \frac{s_{YX}}{s_X^2}$ is the sample regression coefficient where $s_{YX} = \frac{1}{n-1}\sum_{j=1}^{n}(Y_j - \bar{Y})(X_j - \bar{X})$ and $s_X^2 = \frac{1}{n-1}\sum_{j=1}^{n}(X_j - \bar{X})^2$; $\beta_{YX} = \frac{\sigma_{YX}}{\sigma_X^2}$ is the population regression coefficient; ρ_{YX} is the population correlation coefficient between the variables X and Y; $C_Y = \frac{\sigma_Y}{\mu_Y}$ and $C_X = \frac{\sigma_X}{\mu_X}$ are the population coefficient of variation for variables Y and X, respectively; $g = \frac{\mu_X}{\mu_X + \rho_{YX}}$; $\beta_{2(X)}$ is the population coefficient of kurtosis for variable X; the optimal value for α (that minimizes the mean square error) $\alpha = -\rho_{YX}\frac{C_Y}{C_X}$.

In this section, we have utilized the efficiency of the estimators in Table 3.1 to design a CUSUM-type structure and tried to study the effect of these efficient estimators on the ARL performance of CUSUM chart. Now the plotting statistics of the proposed chart (which is based on the estimators given in Table 1) is given as:

$$\left.\begin{aligned} T_i^+ &= \max\left[0, \left(\bar{Y}_{p,i} - E(\bar{Y}_p)\right) - K_p + T_{i-1}^+\right] \\ T_i^- &= \max\left[0, -\left(\bar{Y}_{p,i} - E(\bar{Y}_p)\right) - K_p + T_{i-1}^-\right] \end{aligned}\right\} \quad (3.4)$$

Initial values for the statistics given in (3.4) are taken equal to zero i.e. $T_0^+ = T_0^- = 0$. The decision rule for the proposed chart is given as: the statistics T_i^+ and T_i^- are plotted against the control limit H_p. For any value of i, if the value of T_i^+ exceeds the value of H_p then the process mean is declared to be shifted upward and if the value of T_i^- exceeds the value of H_p then the process mean is said to be moved downward. K_p and H_p are defined as:

$$K_p = k_p \times \sqrt{MSE(\bar{Y}_p)} \text{ and } H_p = h_p \times \sqrt{MSE(\bar{Y}_p)} \quad (3.5)$$

where k_p and h_p are the design parameters of the proposed A_XCUSUM chart. The values of k_p and h_p need to be selected very carefully because the ARL properties of the proposed chart mainly depend on these two constants (along with the value of ρ_{YX}). For some selected values of

Table 3.2: Design parameters $(\boldsymbol{k_p}, \boldsymbol{h_p})$ of the proposed A$_X$CUSUM for $\boldsymbol{ARL_0} \cong \mathbf{370}$

ρ_{YX}	Estimator									
	$\bar{Y}_1$	$\bar{Y}_2$	$\bar{Y}_3$	$\bar{Y}_4$	$\bar{Y}_5$	$\bar{Y}_6$	$\bar{Y}_7$	$\bar{Y}_8$	$\bar{Y}_9$	$\bar{Y}_{10}$
0.25	(0.25,8.008)	(0.25,8.082)	(0.25,8.018)	(0.25,8.048)	(0.25,8.048)	(0.25,8.094)	(0.25,8.085)	(0.25,8.083)	(0.25,8.040)	(0.25,8.091)
	(0.50,4.774)	(0.50,5.099)	(0.50,4.787)	(0.50,4.782)	(0.50,4.782)	(0.50,5.088)	(0.50,5.101)	(0.50,5.088)	(0.50,5.059)	(0.50,5.100)
	(0.75,3.339)	(0.75,3.860)	(0.75,3.340)	(0.75,3.348)	(0.75,3.348)	(0.75,3.860)	(0.75,3.850)	(0.75,3.863)	(0.75,3.831)	(0.75,3.864)
	(1.00,2.516)	(1.00,3.180)	(1.00,2.512)	(1.00,2.522)	(1.00,2.522)	(1.00,3.177)	(1.00,3.169)	(1.00,3.175)	(1.00,3.150)	(1.00,3.194)
0.50	(0.25,8.008)	(0.25,8.083)	(0.25,8.000)	(0.25,8.004)	(0.25,8.004)	(0.25,8.078)	(0.25,8.083)	(0.25,8.078)	(0.25,8.084)	(0.25,8.135)
	(0.50,4.774)	(0.50,5.060)	(0.50,4.775)	(0.50,4.762)	(0.50,4.762)	(0.50,5.070)	(0.50,5.084)	(0.50,5.066)	(0.50,5.086)	(0.50,5.138)
	(0.75,3.339)	(0.75,3.860)	(0.75,3.329)	(0.75,3.330)	(0.75,3.330)	(0.75,3.838)	(0.75,3.836)	(0.75,3.834)	(0.75,3.840)	(0.75,3.894)
	(1.00,2.516)	(1.00,3.180)	(1.00,2.508)	(1.00,2.499)	(1.00,2.499)	(1.00,3.146)	(1.00,3.145)	(1.00,3.145)	(1.00,3.145)	(1.00,3.217)
0.75	(0.25,8.008)	(0.25,8.084)	(0.25,8.014)	(0.25,7.995)	(0.25,7.995)	(0.25,8.075)	(0.25,8.050)	(0.25,8.067)	(0.25,8.072)	(0.25,8.108)
	(0.50,4.774)	(0.50,5.065)	(0.50,4.775)	(0.50,4.760)	(0.50,4.760)	(0.50,5.045)	(0.50,5.040)	(0.50,5.039)	(0.50,5.043)	(0.50,5.108)
	(0.75,3.339)	(0.75,3.845)	(0.75,3.342)	(0.75,3.332)	(0.75,3.332)	(0.75,3.780)	(0.75,3.772)	(0.75,3.772)	(0.75,3.773)	(0.75,3.885)
	(1.00,2.516)	(1.00,3.168)	(1.00,2.513)	(1.00,2.506)	(1.00,2.506)	(1.00,3.070)	(1.00,3.070)	(1.00,3.069)	(1.00,3.066)	(1.00,3.210)
0.90	(0.25,8.008)	(0.25,8.030)	(0.25,8.010)	(0.25,7.984)	(0.25,7.984)	(0.25,8.049)	(0.25,8.041)	(0.25,8.041)	(0.25,8.035)	(0.25,8.043)
	(0.50,4.774)	(0.50,5.066)	(0.50,4.768)	(0.50,4.744)	(0.50,4.744)	(0.50,4.936)	(0.50,4.946)	(0.50,4.942)	(0.50,4.939)	(0.50,5.100)
	(0.75,3.339)	(0.75,3.840)	(0.75,3.338)	(0.75,3.320)	(0.75,3.320)	(0.75,3.630)	(0.75,3.636)	(0.75,3.640)	(0.75,3.626)	(0.75,3.882)
	(1.00,2.516)	(1.00,3.163)	(1.00,2.512)	(1.00,2.500)	(1.00,2.500)	(1.00,2.890)	(1.00,2.900)	(1.00,2.901)	(1.00,2.880)	(1.00,3.192)

Table 3.3: ARL values for the proposed A$_X$CUSUM chart with estimator $\bar{Y}_2$

ρ_{YX}	k_2	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	371.8	31.91	12.36	7.65	5.58	3.70	2.85	2.26	2.02	1.80	1.12
	0.5	370.9	28.56	8.88	4.95	3.46	2.25	1.76	1.35	1.08	1.00	1.00
	0.75	370.7	82.32	14.79	6.47	4.12	2.46	1.87	1.46	1.13	1.00	1.00
	1	370.6	141.90	23.53	7.72	4.33	2.37	1.71	1.28	1.06	1.00	1.00
0.5	0.25	371.2	26.95	10.77	6.76	4.96	3.33	2.56	2.07	1.98	1.46	1.01
	0.5	369.7	35.61	9.66	5.40	3.80	2.46	1.97	1.62	1.21	1.00	1.00
	0.75	369.3	64.13	11.55	5.37	3.52	2.19	1.68	1.24	1.04	1.00	1.00
	1	369.4	115.40	16.82	6.08	3.59	2.08	1.49	1.12	1.02	1.00	1.00
0.75	0.25	371.4	18.39	7.84	5.06	3.78	2.60	2.03	1.93	1.51	1.01	1.00
	0.5	369.3	20.58	6.52	3.90	2.83	1.99	1.52	1.08	1.01	1.00	1.00
	0.75	369.9	32.70	6.79	3.61	2.53	1.71	1.16	1.01	1.00	1.00	1.00
	1	367.6	60.24	8.26	3.70	2.45	1.53	1.08	1.01	1.00	1.00	1.00
0.9	0.25	372.2	10.81	4.98	3.34	2.56	1.98	1.47	1.02	1.00	1.00	1.00
	0.5	369.5	9.79	3.83	2.48	1.98	1.22	1.01	1.00	1.00	1.00	1.00
	0.75	370.7	11.67	3.55	2.21	1.69	1.04	1.00	1.00	1.00	1.00	1.00
	1	370.1	17.16	3.63	2.10	1.50	1.02	1.00	1.00	1.00	1.00	1.00

Table 3.4: ARL values for the proposed A$_X$CUSUM chart with estimator $\bar{Y}_3$

ρ_{YX}	k_3	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.4	32.96	12.76	7.90	5.75	3.80	2.91	2.34	2.04	1.83	1.20
	0.5	370.1	42.25	11.52	6.24	4.31	2.73	2.09	1.77	1.43	1.02	1.00
	0.75	367.3	59.42	13.09	6.04	3.84	2.30	1.71	1.32	1.09	1.00	1.00
	1	368.5	80.22	16.72	6.55	3.78	2.08	1.47	1.15	1.02	1.00	1.00
0.5	0.25	370.3	24.52	9.97	6.30	4.66	3.14	2.38	2.03	1.94	1.25	1.00
	0.5	369.8	28.41	8.39	4.78	3.40	2.24	1.80	1.38	1.08	1.00	1.00
	0.75	368.8	38.85	8.71	4.37	2.93	1.87	1.35	1.07	1.00	1.00	1.00
	1	371.8	53.81	10.30	4.40	2.75	1.63	1.17	1.02	1.00	1.00	1.00
0.75	0.25	371.7	15.33	6.72	4.38	3.32	2.25	1.97	1.64	1.11	1.00	1.00
	0.5	369.5	14.67	5.16	3.18	2.37	1.71	1.15	1.00	1.00	1.00	1.00
	0.75	369.1	17.88	4.77	2.72	1.98	1.26	1.01	1.00	1.00	1.00	1.00
	1	369.1	23.85	4.91	2.52	1.74	1.11	1.00	1.00	1.00	1.00	1.00
0.9	0.25	368.9	8.76	4.16	2.84	2.15	1.77	1.04	1.00	1.00	1.00	1.00
	0.5	368.8	7.09	3.00	2.05	1.60	1.01	1.00	1.00	1.00	1.00	1.00
	0.75	370.3	7.06	2.56	1.66	1.18	1.00	1.00	1.00	1.00	1.00	1.00
	1	371.3	8.02	2.35	1.42	1.07	1.00	1.00	1.00	1.00	1.00	1.00

Table 3.5: $\boldsymbol{ARL}$ values for the proposed A$_X$CUSUM chart with estimator $\boldsymbol{\bar{Y}_4}$

ρ_{YX}	k_4	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	368.9	32.99	12.76	7.90	5.76	3.81	2.91	2.34	2.04	1.83	1.20
	0.5	370.1	41.79	11.47	6.23	4.30	2.72	2.09	1.76	1.42	1.02	1.00
	0.75	369.2	59.38	13.13	6.03	3.84	2.31	1.71	1.32	1.08	1.00	1.00
	1	369.7	80.63	16.73	6.54	3.78	2.09	1.47	1.15	1.03	1.00	1.00
0.5	0.25	370.7	24.47	9.98	6.29	4.64	3.14	2.38	2.02	1.94	1.25	1.00
	0.5	371.6	28.20	8.32	4.76	3.38	2.24	1.80	1.37	1.08	1.00	1.00
	0.75	369.6	38.83	8.70	4.34	2.92	1.86	1.35	1.07	1.00	1.00	1.00
	1	368.6	53.29	10.26	4.38	2.73	1.62	1.17	1.02	1.00	1.00	1.00
0.75	0.25	370.7	15.21	6.70	4.37	3.30	2.24	1.97	1.62	1.11	1.00	1.00
	0.5	369.4	14.64	5.13	3.16	2.36	1.70	1.14	1.00	1.00	1.00	1.00
	0.75	368.3	17.90	4.75	2.71	1.98	1.25	1.01	1.00	1.00	1.00	1.00
	1	368.9	23.96	4.88	2.51	1.73	1.11	1.00	1.00	1.00	1.00	1.00
0.9	0.25	369.7	8.71	4.13	2.83	2.14	1.77	1.04	1.00	1.00	1.00	1.00
	0.5	370.6	7.03	2.98	2.04	1.59	1.01	1.00	1.00	1.00	1.00	1.00
	0.75	370.3	7.03	2.54	1.65	1.18	1.00	1.00	1.00	1.00	1.00	1.00
	1	368.6	7.94	2.34	1.42	1.07	1.00	1.00	1.00	1.00	1.00	1.00

Table 3.6: ARL values for the proposed A_XCUSUM chart with estimator $\bar{Y}_5$

ρ_{YX}	k_5	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.9	48.38	17.44	10.45	7.50	4.84	3.64	2.97	2.49	2.01	1.88
	0.5	370.4	77.64	18.95	9.30	6.15	3.70	2.71	2.19	1.94	1.42	1.05
	0.75	369.7	132.55	28.83	10.88	6.27	3.41	2.41	1.95	1.64	1.11	1.01
	1	370.1	195.24	51.18	15.53	7.40	3.44	2.32	1.81	1.44	1.05	1.00
0.5	0.25	369.0	44.28	16.09	9.72	7.00	4.55	3.43	2.81	2.32	1.99	1.76
	0.5	369.3	69.29	16.89	8.49	5.63	3.45	2.54	2.09	1.87	1.27	1.02
	0.75	369.0	119.63	24.61	9.59	5.66	3.13	2.25	1.84	1.50	1.06	1.00
	1	371.4	179.62	42.55	13.05	6.42	3.11	2.15	1.67	1.31	1.03	1.00
0.75	0.25	368.5	36.20	13.65	8.39	6.09	4.00	3.06	2.47	2.09	1.92	1.34
	0.5	369.3	52.62	13.29	7.00	4.77	2.99	2.24	1.93	1.64	1.08	1.00
	0.75	370.1	89.75	17.47	7.36	4.56	2.66	1.98	1.59	1.23	1.01	1.00
	1	370.2	140.21	27.82	9.06	4.87	2.56	1.83	1.39	1.12	1.00	1.00
0.9	0.25	369.5	29.78	11.69	7.26	5.32	3.54	2.73	2.17	2.00	1.67	1.06
	0.5	369.9	39.11	10.56	5.79	4.03	2.58	2.01	1.71	1.31	1.01	1.00
	0.75	370.5	63.31	12.45	5.71	3.69	2.25	1.71	1.28	1.06	1.00	1.00
	1	368.3	97.75	17.62	6.43	3.73	2.10	1.49	1.14	1.03	1.00	1.00

Table 3.7: $\boldsymbol{ARL}$ values for the proposed A_XCUSUM chart with estimator $\bar{\boldsymbol{Y}}_{\boldsymbol{6}}$

ρ_{YX}	k_6	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.4	48.75	17.53	10.46	7.49	4.84	3.64	2.97	2.49	2.01	1.88
	0.5	369.7	78.62	19.04	9.38	6.15	3.71	2.71	2.19	1.94	1.43	1.05
	0.75	371.1	28.89	10.89	6.27	3.41	2.41	1.95	1.63	1.11	1.01	1.00
	1	371.6	195.45	51.14	15.64	7.41	3.44	2.31	1.80	1.44	1.05	1.00
0.5	0.25	370.8	43.92	16.07	9.71	6.99	4.55	3.43	2.81	2.32	1.99	1.76
	0.5	370.6	68.85	16.79	8.48	5.65	3.45	2.54	2.08	1.87	1.27	1.02
	0.75	370.8	118.37	24.44	9.55	5.63	3.13	2.25	1.84	1.50	1.06	1.00
	1	368.2	177.64	41.94	12.95	6.42	3.12	2.14	1.67	1.31	1.03	1.00
0.75	0.25	370.2	36.10	13.67	8.39	6.09	4.01	3.06	2.47	2.09	1.92	1.34
	0.5	369.8	52.90	13.35	7.02	4.78	2.99	2.24	1.93	1.64	1.08	1.00
	0.75	368.1	90.01	17.61	7.38	4.56	2.65	1.98	1.59	1.23	1.01	1.00
	1	370.7	141.26	27.93	9.09	4.90	2.57	1.83	1.39	1.12	1.01	1.00
0.9	0.25	370.6	29.57	11.62	7.24	5.31	3.53	2.72	2.17	2.00	1.66	1.06
	0.5	368.4	38.80	10.49	5.79	4.03	2.58	2.02	1.71	1.31	1.01	1.00
	0.75	369.9	62.45	12.46	5.72	3.68	2.25	1.71	1.28	1.06	1.00	1.00
	1	370.1	97.87	17.48	6.42	3.72	2.11	1.50	1.15	1.03	1.00	1.00

Table 3.8: ARL values for the proposed A_XCUSUM chart with estimator $\bar{Y}_7$

ρ_{YX}	k_7	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.9	48.25	17.27	10.35	7.39	4.79	3.60	2.94	2.46	2.01	1.00
	0.5	369.6	77.42	18.63	9.21	6.04	3.66	2.68	2.16	1.93	1.39	1.04
	0.75	371.5	133.50	28.33	10.71	6.20	3.35	2.38	1.93	1.62	1.10	1.01
	1	370.9	195.31	50.13	15.27	7.25	3.40	2.28	1.78	1.42	1.05	1.00
0.5	0.25	368.9	43.07	15.81	9.57	6.89	4.48	3.38	2.78	2.29	1.98	1.73
	0.5	369.5	67.11	16.33	8.30	5.54	3.39	2.50	2.07	1.85	1.24	1.02
	0.75	369.4	115.99	23.71	9.29	5.50	3.08	2.22	1.82	1.47	1.05	1.00
	1	368.4	175.94	40.76	12.54	6.25	3.05	2.11	1.64	1.29	1.02	1.00
0.75	0.25	370.4	35.18	13.38	8.23	5.98	3.94	3.01	2.43	2.07	1.90	1.29
	0.5	369.7	51.00	12.95	6.84	4.67	2.94	2.21	1.90	1.61	1.06	1.00
	0.75	368.4	87.10	16.77	7.12	4.43	2.60	1.95	1.56	1.20	1.01	1.00
	1	368.7	136.89	26.63	8.69	4.73	2.51	1.80	1.36	1.10	1.00	1.00
0.9	0.25	369.7	28.68	11.32	7.08	5.18	3.46	2.66	2.13	1.99	1.60	1.04
	0.5	370.6	37.15	10.15	5.62	3.93	2.52	1.99	1.67	1.26	1.01	1.00
	0.75	371.0	60.03	11.86	5.49	3.58	2.21	1.67	1.25	1.05	1.00	1.00
	1	371.3	94.47	16.52	6.15	3.60	2.05	1.46	1.12	1.02	1.00	1.00

Table 3.9: ARL values for the proposed A_XCUSUM chart with estimator $\bar{Y}_8$

ρ_{YX}	k_8	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.4	48.63	17.53	10.48	7.49	4.84	3.63	2.96	2.49	2.01	1.88
	0.5	371.1	78.15	19.04	9.37	6.14	3.70	2.71	2.19	1.94	1.42	1.05
	0.75	369.1	133.91	29.03	10.95	6.29	3.40	2.41	1.95	1.63	1.11	1.01
	1	369.4	196.36	51.60	15.74	7.45	3.45	2.31	1.80	1.44	1.05	1.00
0.5	0.25	372.4	44.07	16.12	9.70	6.99	4.55	3.43	2.81	2.32	1.99	1.76
	0.5	371.1	69.13	16.85	8.49	5.65	3.45	2.54	2.09	1.87	1.28	1.02
	0.75	369.3	118.74	24.54	9.53	5.63	3.14	2.25	1.84	1.50	1.06	1.00
	1	368.3	177.81	42.25	12.94	6.41	3.11	2.14	1.67	1.31	1.03	1.00
0.75	0.25	370.9	36.12	13.69	8.39	6.09	4.00	3.05	2.47	2.09	1.92	1.34
	0.5	369.5	52.73	13.26	7.01	4.77	2.99	2.24	1.92	1.64	1.08	1.00
	0.75	370.4	89.25	17.45	7.37	4.55	2.65	1.98	1.59	1.23	1.01	1.00
	1	369.9	138.86	27.82	9.09	4.88	2.56	1.83	1.38	1.12	1.00	1.00
0.9	0.25	370.4	29.61	11.65	7.26	5.31	3.53	2.72	2.17	2.00	1.66	1.06
	0.5	370.5	38.87	10.55	5.80	4.03	2.58	2.02	1.71	1.31	1.01	1.00
	0.75	368.7	62.75	12.39	5.72	3.69	2.25	1.71	1.28	1.06	1.00	1.00
	1	371.1	96.74	17.36	6.40	3.71	2.10	1.49	1.14	1.02	1.00	1.00

Table 3.10: ARL values for the proposed A$_X$CUSUM chart with estimator $\bar{Y}_9$

ρ_{YX}	k_9	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	370.6	33.06	12.74	7.85	5.72	3.78	2.91	2.32	2.03	1.85	1.17
	0.5	369.2	78.15	19.04	9.37	6.14	3.70	2.71	2.19	1.94	1.42	1.05
	0.75	371.4	133.91	29.03	10.95	6.29	3.40	2.41	1.95	1.63	1.11	1.01
	1	369.4	148.39	25.33	8.21	4.51	2.44	1.76	1.33	1.08	1.00	1.00
0.5	0.25	370.1	28.13	11.14	6.98	5.12	3.42	2.64	2.11	1.99	1.56	1.03
	0.5	370.3	38.08	10.19	5.64	3.94	2.54	2.01	1.70	1.28	1.01	1.00
	0.75	368.1	68.31	12.24	5.64	3.66	2.25	1.73	1.30	1.06	1.00	1.00
	1	371.3	121.56	18.26	6.44	3.76	2.16	1.56	1.16	1.02	1.00	1.00
0.75	0.25	369.7	19.66	8.29	5.32	3.97	2.73	2.07	1.97	1.68	1.02	1.00
	0.5	369.4	52.73	13.26	7.01	4.77	2.99	2.24	1.92	1.64	1.08	1.00
	0.75	370.6	36.70	7.37	3.85	2.67	1.80	1.24	1.02	1.00	1.00	1.00
	1	370.9	138.86	27.82	9.09	4.88	2.56	1.83	1.38	1.12	1.00	1.00
0.9	0.25	368.9	12.28	5.55	3.68	2.83	2.02	1.78	1.11	1.00	1.00	1.00
	0.5	369.7	11.66	4.35	2.76	2.11	1.47	1.02	1.00	1.00	1.00	1.00
	0.75	369.7	14.95	4.13	2.46	1.88	1.13	1.00	1.00	1.00	1.00	1.00
	1	368.7	96.74	17.36	6.40	3.71	2.10	1.49	1.14	1.02	1.00	1.00

Table 3.11: $\boldsymbol{ARL}$ values for the proposed A$_X$CUSUM chart with estimator $\boldsymbol{\bar{Y}_{10}}$

ρ_{YX}	k_{10}	δ										
		0	0.25	0.5	0.75	1	1.5	2	2.5	3	4	5
0.25	0.25	369.4	36.21	12.88	7.87	5.73	3.78	2.92	2.30	2.02	1.89	1.15
	0.5	370.2	66.81	12.00	6.23	4.25	2.69	2.07	1.81	1.40	1.01	1.00
	0.75	369.0	162.02	15.51	6.15	3.80	2.27	1.73	1.28	1.04	1.00	1.00
	1	370.1	332.20	25.81	7.07	3.79	2.06	1.44	1.10	1.01	1.00	1.00
0.5	0.25	370.2	25.18	9.97	6.28	4.64	3.13	2.37	2.02	1.95	1.22	1.00
	0.5	370.1	31.45	8.41	4.76	3.37	2.22	1.81	1.36	1.06	1.00	1.00
	0.75	368.4	49.61	8.99	4.33	2.90	1.87	1.33	1.05	1.00	1.00	1.00
	1	371.2	75.31	11.16	4.40	2.72	1.61	1.14	1.01	1.00	1.00	1.00
0.75	0.25	369.2	15.23	6.71	4.37	3.31	2.25	1.97	1.63	1.11	1.00	1.00
	0.5	370.9	14.39	5.14	3.18	2.36	1.70	1.15	1.01	1.00	1.00	1.00
	0.75	369.4	16.98	4.75	2.72	1.98	1.26	1.02	1.00	1.00	1.00	1.00
	1	370.1	21.49	4.85	2.52	1.74	1.11	1.01	1.00	1.00	1.00	1.00
0.9	0.25	370.1	8.73	4.16	2.84	2.16	1.75	1.06	1.00	1.00	1.00	1.00
	0.5	370.2	6.99	3.01	2.05	1.59	1.02	1.00	1.00	1.00	1.00	1.00
	0.75	369.2	6.82	2.57	1.66	1.20	1.00	1.00	1.00	1.00	1.00	1.00
	1	369.7	7.41	2.36	1.44	1.09	1.00	1.00	1.00	1.00	1.00	1.00

k_p (0.25, 0.50, 0.75, and 1.00), ρ_{YX} (0.25, 0.50, 0.75 and 0.90) and fixed $ARL_0 = 370$, the corresponding h_p are guessed by running 10^5 simulations in R software (R Core Team, 2014). These constants are given in table 2 where we fixed $ARL_0 = 370$ Based on the constants in Table 3.2, the ARL values of the proposed A_XCUSUM chart (for all the estimators) are given in Tables 3.3 – 3.11.

From Tables 3.1 – 3.11, the chief findings about the proposed A_XCUSUM control chart is presented as follows:

i. The use of auxiliary variable with the control structure of CUSUM chart is really advantageous in terms of ARL_1 (The *ARL* value when there is a shift in a process) values if the value of ρ_{YX} is reasonably large (cf. Tables 3.3 – 3.11).

ii. For a fixed value of ARL_0, the ARL_1 values decrease rapidly with increase in the values of either or both ρ_{YX} and $|\delta|$ (cf. Tables 3.3 – 3.11).

iii. For all values of ρ, h ranges from $(7.984 \text{ to } 8.135), (4.744 \text{ to } 5.138), (3.320 \text{ to } 3.894)$ and $(2.499 \text{ to } 3.194)$ for k equals $0.25, 0.5, 0.75$ and 1 respectively (cf. Table 3.2).

iv. For weak positive correlation between the Y and X, A_2CUSUM (i.e. the proposed CUSUM with estimator $\bar{Y}_2$) chart outperform other proposed charts, over the whole range of δ, when $k \in (0.25,0.5)$ (cf. Tables 3.3 – 3.11).

v. When $k \in (0.75,1)$ and there is small positive value of ρ_{YX}, then A_3CUSUM and A_4CUSUM charts give the best performance in the cases of small to moderate shifts, while A_{10}CUSUM chart is the best in detecting large shift (cf. Tables 3.3 – 3.11).

vi. For $\rho_{YX} = 0.5$, A_3CUSUM and A_4CUSUM charts give the best performance (followed by A_{10}CUSUM chart) when $\delta \in (0.25,0.5)$ i.e. small shifts (cf. Tables 3.3 – 3.11).

vii. For $\rho_{YX} = 0.5$, A_4CUSUM and A_{10}CUSUM charts give the best performance (followed by A_3CUSUM chart) when $\delta \in (0.75,5)$ i.e. moderate and large shifts (cf. Tables 3.3 – 3.11).

viii. For $\rho_{YX} = 0.75$, A_3CUSUM and A_4CUSUM charts precede A_{10}CUSUM chart in outperforming other proposed charts in detecting small shift (cf. Tables 3.3 – 3.11).

ix. For a strong positive correlation $\rho_{YX} \geq 0.75$, A_3CUSUM, A_4CUSUM and A_{10}CUSUM charts are the best preceded by A_2CUSUM chart, in detecting moderate to large shift (i.e. $\delta \geq 0.75$) (cf. Tables 3.3 – 3.11).

3.4 COMPARISONS

Generally, ARL is used to compare the performance of two charts. Wu et al. (2009) highlighted some of the drawbacks of ARL as it gives the performance of a control chart for a specific shift size. Hence, they recommended some measures which evaluate the performance of a control chart over a range of δ values. These measures are named as extra quadratic loss (EQL) and ratio of average run lengths ($RARL$) which are defined as:

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta \tag{3.6}$$

$$RARL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \frac{ARL(\delta)}{ARL_{\text{benchmark}}(\delta)} d\delta \tag{3.7}$$

Another performance measure named as performance comparison index (PCI) given by Ou et al. (2012) is defined as:

$$PCI = \frac{EQL}{EQL_{\text{benchmark}}} \tag{3.8}$$

where $ARL_{\text{benchmark}}$ and $EQL_{\text{benchmark}}$ are evaluated for the benchmark chart (taken as the best chart in this section).

Table 3.12: Performance comparison of classical EWMA, classical CUSUM and A_XCUSUM charts with fixed $ARL_0 = 370$

	EWMA		CUSUM									
	λ	Classical	Classical	A_2 ($\rho = 0.5$)	A_2 ($\rho = 0.75$)	A_3 ($\rho = 0.5$)	A_3 ($\rho = 0.75$)	A_4 ($\rho = 0.5$)	A_4 ($\rho = 0.75$)	A_{10} ($\rho = 0.5$)	A_{10} ($\rho = 0.75$)	k
EQL	0.05	6.067	6.249	6.553	5.373	6.255	4.612	6.242	**4.583**	6.245	4.599	0.25
RARL		1.347	1.407	1.498	1.163	1.408	1.005	1.406	**1.000**	1.542	1.220	
PCI		1.324	1.364	1.430	1.172	1.365	1.006	1.362	**1.000**	1.003	1.363	
EQL	0.14	4.556	4.477	5.116	3.708	4.485	3.197	4.467	**3.186**	4.480	3.200	0.50
RARL		1.479	1.497	1.728	1.215	1.500	1.004	1.493	**1.000**	1.810	2.240	
PCI		1.430	1.405	1.605	1.164	1.407	1.003	1.402	**1.000**	1.004	1.406	
EQL	0.25	3.946	3.881	4.768	3.486	3.881	2.931	3.873	2.927	3.965	**2.925**	0.75
RARL		1.515	1.546	2.028	1.340	1.544	1.008	1.540	1.006	1.646	**1.000**	
PCI		1.349	1.327	1.630	1.192	1.327	1.002	1.324	1.001	1.356	**1.000**	
EQL	0.38	3.835	3.864	5.248	3.711	3.856	2.898	3.842	2.897	4.067	**2.876**	1.0
RARL		1.559	1.619	2.464	1.541	1.611	1.018	1.602	1.018	1.784	**1.000**	
PCI		1.334	1.344	1.825	1.291	1.341	1.008	1.336	1.007	1.414	**1.000**	

In this current study, we have used the sensitivity parameter of CUSUM chart $k = 0.25, 0.5, 0.75$ and 1 which are the optimal choices for detecting a shift of size $\delta = 0.5, 1, 1.5$ and 2, respectively. For the same values of δ, we have found the optimal choices for the sensitivity parameter (λ) of EWMA chart to be $\lambda = 0.05, 0.14, 0.25$ and 0.38 for $\delta = 0.5, 1, 1.5$ and 2, respectively, using the technique of Crowder (1989). Finally, the comparisons of all the charts under discussion in the form of EQL, $RARL$ and PCI are provided in Table 3.12 where the in-control ARL for all the charts is fixed at 370. In Table 3.12, smaller value of EQL shows a better performance of a chart, and the best chart in every situations is taken as the benchmark chart, indicated by bold value. The best charts in Table 3.12 are A4CUSUM for k=0.25,0.5 and A10CUSUM for k=0.75,1. Similarly, the value of $RARL$ (or PCI) greater than 1 means that the benchmark chart has a superior overall performance and vice versa. It can be clearly seen from Table 3.12 that AXCUSUM is outperforming the classical EWMA and the classical CUSUM charts.

3.5 ILLUSTRATIVE EXAMPLE

In this section, we provide an illustrative example to show the implementation of our proposed chart in real situation. For this purpose, we have considered the bivariate data by Constable and Parker (1988) on the measurements of a component part for an automobile's braking system, containing the study variable $Y =$ BAKEWT and the auxiliary variable $X =$ ROLLWT. 45 data points are taken from the in-control process and are used to estimate the population parameters. These estimates came out to be $\widehat{\mu_0} = 201.18$, $\widehat{\mu_X} = 210.24$, $\widehat{\sigma_Y} = 1.17$, $\widehat{\sigma_X} = 1.23$ and $\widehat{\rho_{YX}} = 0.54$. Considering these estimates as the known parameters, we have generated two datasets from bivariate normal distribution. Dataset 1 with $\mu_1 = 201.7$, $\mu_X = 210.24$, $\sigma_Y = 1.17$, $\sigma_X = 1.23$ and $\rho_{YX} = 0.54$ contains 15 paired observations which refer to an out-of-control situation with $\delta = \frac{(\mu_1-\mu_0)}{\sigma_Y/\sqrt{n}} = \frac{(201.7-201.18)}{1.17/\sqrt{1}} \cong 1$. Similarly, Dataset 2 with $\mu_1 = 200.6$, $\mu_X = 210.24$, $\sigma_Y = 1.17$, $\sigma_X = 1.23$ and $\rho_{YX} = 0.54$ contains 15 paired observations which refer to an out-of-control situation with negative shift i.e. $\delta = \frac{(\mu_1-\mu_0)}{\sigma_Y/\sqrt{n}} = \frac{(200.6-201.18)}{1.17/\sqrt{1}} \cong -1.1$. The inspiration of generating dataset in such manner is taken from Singh and Mangat, (1996, pp. 221).

According to the findings of section 3.3 A_2CUSUM, A_4CUSUM and A_{10}CUSUM are generally performing best in most of the situations. So we have applied the classical CUSUM, A_2CUSUM, A_4CUSUM and A_{10}CUSUM (with $k = 0.5$) to the generated datasets. The chart output for all the charts when there is a positive shift in the process location is given in Figure 3.1, while Figure 3.2 contains the display of all the charts when the process location is shifted downwards.

Figure 3.1 shows that the classical CUSUM detects the shift at sample # 10, A_2CUSUM detects the shift at sample # 13, A_4CUSUM detects the shift at sample # 9 and A_{10}CUSUM also detects the shift at sample # 9. Similarly for the negative shift in the process parameter, classical CUSUM detects the shift at sample # 10, A_2CUSUM detects the shift at sample # 12, A_4CUSUM detects the shift at sample # 9 and A_{10}CUSUM detects the shift at sample # 5. These findings of the illustrative example are also authenticating the findings of Section 3.3 where we said the superiority order is A_{10}CUSUM (the best), followed by A_4CUSUM, classical CUSUM and A_2CUSUM.

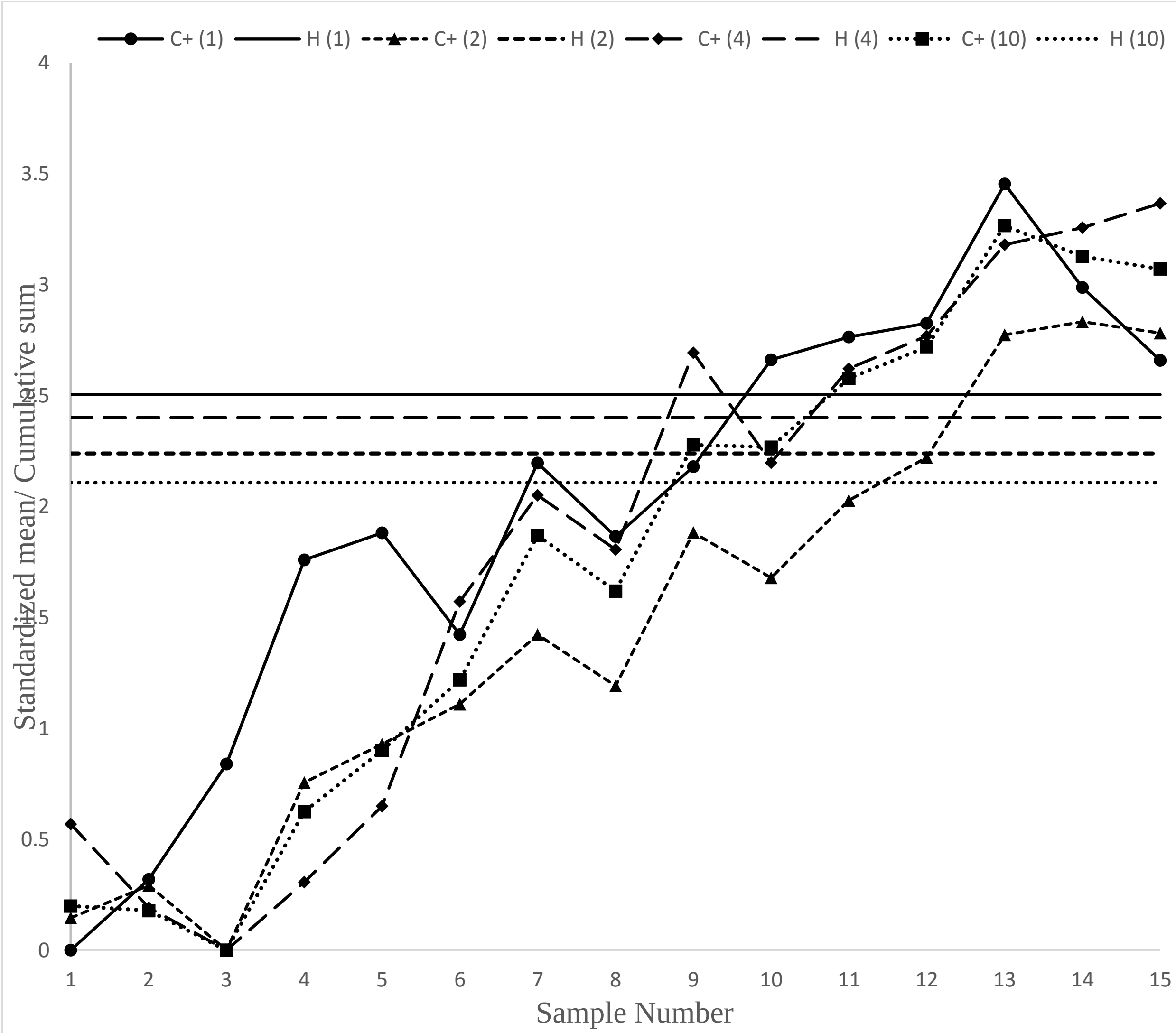

Figure 3.1: Graphical display of the classical CUSUM, A_2CUSUM, A_4CUSUM and A_{10}CUSUM charts for dataset 1

3.6 SUMMARY AND CONCLUSIONS

Quality of manufactured products and services are always important for the management department of a firm or industry. SQC provides some suitable tools to monitor and improve the quality of products by reducing the undesirable variation in their output. Control chart is the most important tool of SQC which is further categorized into Shewhart, CUSUM and EWMA-type control charts. Shewhart-type control charts are

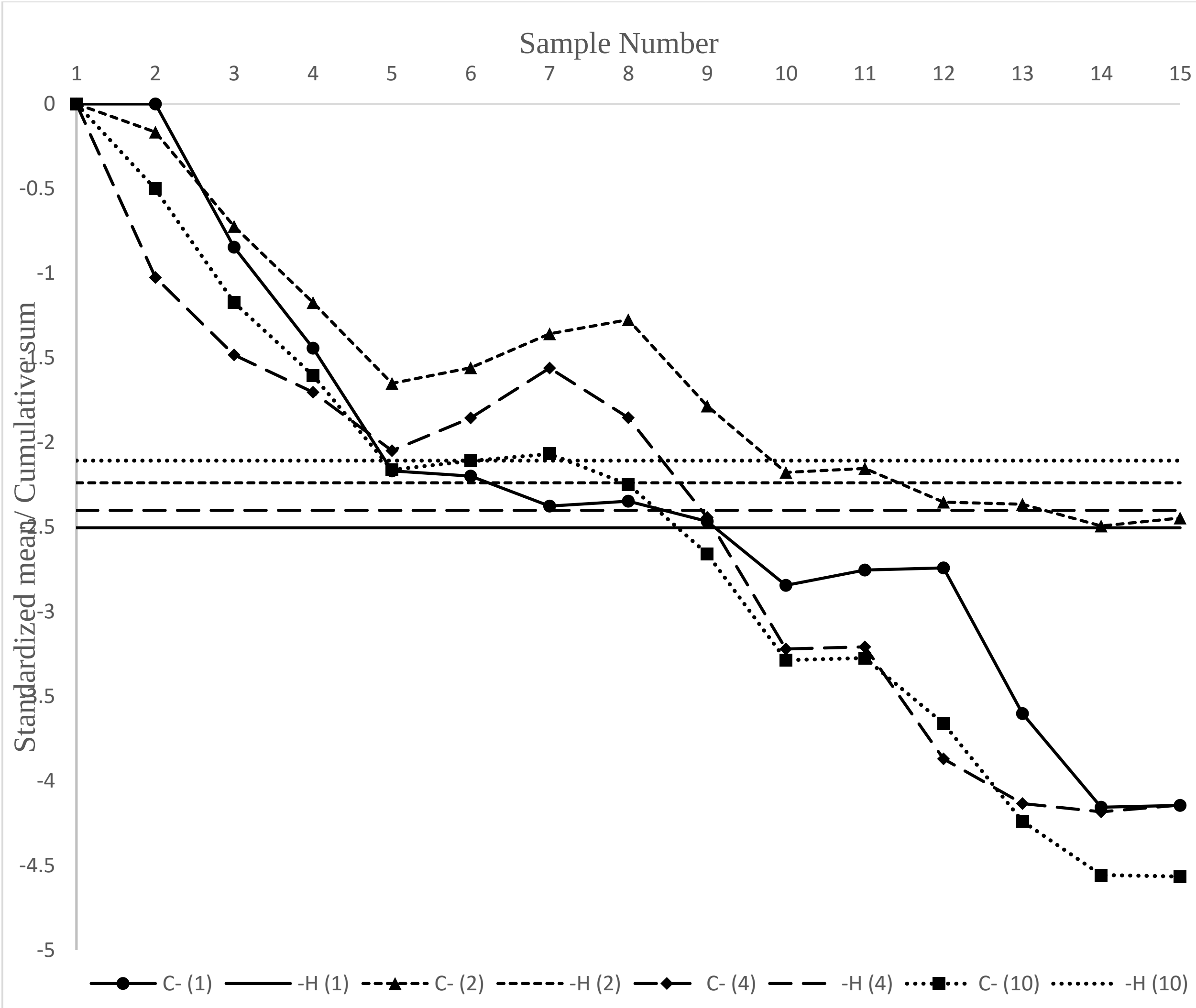

Figure 3.2: Graphical display of the classical CUSUM, A_2CUSUM, A_4CUSUM and A_{10}CUSUM charts for dataset 2

built to detect large shifts in the process while CUSUM and EWMA-type control charts are designed to give better performance against small and moderate shifts. This chapter proposes a new two-sided CUSUM-type control chart named as A_XCUSUM control chart for monitoring the mean of a process. The proposed chart is based on the information of auxiliary variable and different estimators are used to exploit the auxiliary information. The study revealed that the proposed chart is generalized form of the classical CUSUM chart and its performance is also better than the classical CUSUM and the classical EWMA charts.

CHAPTER 4

Combined Shewhart CUSUM Charts using Auxiliary Variable

Control chart is an important tool for monitoring disturbances in a statistical process, and it is richly applied in the industrial sector, the health sector, the agricultural sector, among others. The Shewhart chart and the cumulative sum (CUSUM) chart are traditionally used for detecting large shifts and small shifts, respectively, while the Combined Shewhart CUSUM (CSC) monitors both small and large shifts. Using auxiliary information, we propose new CSC (M_iCSC) charts with more efficient estimators (the Regression-type estimator, the Ratio estimator, the Singh and Tailor estimator, the power ratio-type estimator, and the Kadilar and Cingi estimators) for estimating the location parameter. We compare the charts using average run length, standard deviation run length and extra quadratic loss, with other existing charts of the same purpose, and found out that some of the M_iCSC charts outperform their counterparts. At last, a real-life industrial example is provided.

4.1 INTRODUCTION

The most widely known quality control chart, Shewhart chart, was proposed by (Shewhart, 1924). It detects shifts in a production process by signaling when a process goes beyond some particular threshold limits known as control limits. Shewhart chart makes use of the information when the process goes out of the control limits and ignores the information when the process is within the control limits, i.e. in-control. Due to this fact, the chart is sensitive for detecting large shifts (or disturbance) in a process. Roberts (1959) and Page (1954) proposed Exponentially Weighted Moving Average (EWMA) chart and Cumulative Sum (CUSUM) chart, respectively, which make use of the information when the process gets out of control and even when the process is in-control, hence, these charts are sensitive to small and moderate shifts in a process. Other modifications of these charts have been proposed to increase their efficiency in terms of time, cost, and simplicity of usage and expression.

The plotting statistic of CUSUM chart assumes normality. What if the plotting statistic is not normally distributed or its normality is altered? Nazir et al., (2013) answered these questions by suggesting some charts which are not normally distributed or their normality has been altered. They aimed at finding charts that perform practically well under normal, contaminated normal, non-normal, and special cause contaminated parent cases. Based on mean, median, Hodge-Lehman, midrange and trimean statistics, they proposed different CUSUM charts for phase II monitoring of location parameter and computed their performance measure using the average run length (ARL) approach. Abujiya et al. (2015) suggested the use of well-structured sampling techniques such as the double ranked set sampling, the median-double ranked set sampling, and the double-median ranked set sampling, to significantly improve the performance of the CUSUM chart, without inflating the false alarm rate. They compared their proposed charts with some existing charts and found out that their charts perform better.

Due to the advancement in technology and industrial processes, emphasis has been made on the implementation of CUSUM chart to existing Levey-Jennings or Shewhart control charts to improve their performance. These can be done manually using control charts or in a computerized quality control systems. Westgard et al. (1977) applied this concept to improve quality control in clinical chemistry. The combination of Shewhart chart and CUSUM chart was observed by Lucas (1982), after which some scholars improved the chart by proposing more efficient charts. Combined Shewhart-CUSUM (hereafter called "CSC") for location parameter can be optimized over the entire mean shift range by adding an extra parameter (w) known as the exponential of the sample mean shift, to the structure of the CSC. This will improve its performance and it will not increase the difficulty level of understanding and implementing the chart (Wu et al., 2008). The CSC, which has a wide range of application, attracts the attention of Environmentalists, and it is the only quality control chart directly recommended by the United States Environment Protection Agency for intra-well monitoring. It has been consistently applied to waste disposal facilities for detection monitoring programs (Gibbons, 1999). Abujiya et al. (2013) replaced the traditional simple random sampling in the plotting statistic of the CSC, with ranked set sampling.

The control statistics of the classical Shewhart, CUSUM, and CSC charts for monitoring location parameter are based on the usual unbiased simple mean estimator

$\left(\bar{x} = (1/n)\sum_{i=1}^{n} x_i\right)$ for estimating the population mean. However, in the field of sample survey, different scholars have suggested many estimators other than the simple mean in terms of their mean square error (MSE). Some of these estimators requires the use of auxiliary variable(s) which are cheap, easy and affordable to get, and also, with known population parameters (Cochran, 1953). According to Cochran (1953), the correlation between the study variable and the auxiliary variable will serves as an advantage to increase the precision of estimation. Sukhatme & Sukhatme (1970) proposed regression estimator for estimating the mean, while power ratio-type estimator and modified ratio-type estimator were suggested by Srivastava (1967) and Ahmad et al. (2014) respectively. Interested reader can see H. P. Singh & Tailor (2003), Kadilar & Cingi (2004), Kadilar & Cingi (2006a), Kadilar & Cingi (2006b), Gupta & Shabbir (2008) and Adebola et al. (2015) for different forms of a transformed ratio estimator.

G. Zhang (1992) suggested the cause-selecting control chart, while Riaz (2008b) popularised the use of auxiliary information at the estimation stage, for monitoring dispersion parameter. He concluded that the chart is better than the R chart, the S chart and the S^2 chart. Furthermore, Riaz (2008a) suggested similar chart for location parameter estimation, which was also superior to the Shewhart chart, the regression chart and the cause-selecting control chart. Assuming stability of parameters, Ahmad et al. (2014) proposed new Shewhart charts based on auxiliary information for non-cascading processes. The charts monitor a dispersion parameter in an efficient way. The superiority of the charts over competing charts was shown using the ARL, relative average run length (RARL) and extra quadratic loss (EQL) under *t* and normal distributed process environment. Similar work was also done for location parameter monitoring, and it was found out that there is an improvement in the detection ability of Shewhart chart base on the level of correlation between the concerned variables (Riaz, 2015).

Since most of the estimators are more efficient than the simple mean estimator based on simple random sample, their introduction to the plotting statistic(s) of the Shewhart chart, the CUSUM chart, and the CSC chart would results to efficient control charts. Hence, this study aims at optimizing the CSC chart by introducing some efficient estimators to its plotting statistics. These estimators use auxiliary information in the sampling stage. This is helpful whenever there is no information about the population of the variable of interest, but there is information about a closely related variable(s) which is cheap and affordable to get.

The rest of this article is organized as follows: Location estimators and their properties are explained in the next section; The general structure of the proposed charts is explained in Section 4.3; Section 4 explains the performance measures for evaluating the proposed charts and compares the proposed charts with their existing counterparts; Section 4.5 gives an illustrative example; and finally, conclusions and recommendations are given in Section 4.6.

4.2 LOCATION ESTIMATORS AND THEIR PROPERTIES

We assume that a process has a quality characteristic of interest X and an auxiliary quality characteristic A. Let the population parameters of X and A, respectively, be represented as $\overline{X}$ and $\overline{A}$ for the means; σ_X^2 and σ_A^2 for the variances; $C_X = \sigma_X / \overline{X}$ and $C_A = \sigma_A / \overline{A}$ for the coefficient of variations; $\beta_{2(X)}$ and $\beta_{2(A)}$ for the coefficient of kurtoses; σ_{XA} for the covariance between X and A; and ρ_{XA} for the correlation coefficient. Let the sample statistics of X and A, respectively, be represented as $\bar{x}$ and $\bar{a}$ for the means; s_x^2 and s_a^2 for the variances; c_x and c_a for the coefficient of variations; s_{xa} for the covariance; and r_{xa} for the correlation coefficient. Let x_i and (x_i, a_i) be univariate and bivariate sample respectively, where $i = 1, 2, \ldots, n$ and $n =$ sample size. From the sample statistics, we have $\bar{x} = \sum_{i=1}^{n} x_i / n$, $\bar{a} = \sum_{i=1}^{n} a_i / n$, $s_x^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1)$, $s_a^2 = \sum_{i=1}^{n} (a_i - \bar{a})^2 / (n-1)$, $c_x = s_x / \bar{x}$, $c_a = s_a / \bar{a}$ and $r_{xa} = s_{xa} / s_x s_a$. Based on this introduction, some efficient estimators with one auxiliary variable for estimating the mean of a quality process characteristic, assuming sampling with replacement, are presented with their respective bias (*B*) and *MSE*.

i) The Simple Random Sampling Estimator (Cochran, 1953)

$$M_1 = \sum_{i=1}^{n} x_i / n \tag{4.1}$$

with $B(M_1) = 0$ and $MSE(M_1) = \sigma_X^2 / n$.

ii) The Regression-Type Estimator (Difference Estimator) (Cochran, 1953)

$$M_2 = \bar{x} + b_{XA}(\bar{A} - \bar{a}) \tag{4.2}$$

where $b_{XA} = -\rho_{XA}\sigma_X / \sigma_A$, with $B(M_2) = 0$ and $MSE(M_2) = \sigma_X^2 (1 - \rho_{XA}^2) / n$.

The bias and the MSE of the next estimators are given up to the first order approximation.

iii) The Ratio Estimator (Cochran, 1953)

$$M_3 = \bar{x}\bar{A}/\bar{a} \tag{4.3}$$

with $B(M_3) = \bar{X}\left(C_A^2 - \rho_{XA}C_X C_A\right)$ and $MSE(M_3) = \bar{X}^2\left(C_X^2 + C_A^2 - 2\rho_{XA}C_X C_A\right)$

iv) The Singh and Tailor Estimator (H. P. Singh & Tailor, 2003)

$$M_4 = \bar{x}\left(\frac{\bar{A} + \rho_{XA}}{\bar{a} + \rho_{XA}}\right) \tag{4.4}$$

with $B(M_4) = \bar{X}g\left(gC_A^2 - \rho_{XA}C_X C_A\right)/n$ and

$MSE(M_4) = \bar{X}^2\left(C_X^2 + g^2C_A^2 - 2g\rho_{XA}C_X C_A\right)/n$, where $g = \bar{A}/\left(\bar{A} + \rho_{XA}\right)$.

v) *The Power Ratio-Type Estimator (Srivastava, 1967)*

$$M_5 = \bar{x}(\bar{A}/\bar{a})^{\alpha} \tag{4.5}$$

where $\alpha = -\rho_{XA}C_X/C_A$, with $B(M_5) = \left(\bar{X}/n\right)\left(\alpha(\alpha+1)/2\right)C_A^2 - \alpha\rho_{XA}C_X C_A$ and

$MSE(M_5) = \left(\sigma_X^2/n\right)\left(1 + \alpha^2\left(C_A^2/C_X^2\right) - 2\alpha\rho_{XA}C_X C_A\right)$.

vi) The Kadilar and Cingi Estimator's Series 1 (Kadilar & Cingi, 2004)

$$M_6 = \left[\bar{x} + b_{XA}\left(\bar{A} - \bar{a}\right)\right]\bar{A}/\bar{a} \tag{4.6}$$

with $B(M_6) = \bar{X}C_A^2/n$ and $MSE(M_6) = \bar{X}^2\left[C_A^2 + C_X^2\left(1 - \rho_{XA}^2\right)\right]/n$

vii) *The Kadilar and Cingi Estimator's Series 2 (Kadilar & Cingi, 2004)*

$$M_7 = \left[\bar{x} + b_{XA}\left(\bar{A} - \bar{a}\right)\right]\frac{\bar{A} + C_A}{\bar{a} + C_A} \quad (4.7)$$

with $B(M_7) = \left(\bar{X}/n\right)C_A^2\left[\left(\bar{A}/\left(\bar{A} + C_A\right)\right)^2\right]$ and

$$MSE(M_7) = \left(\bar{X}^2/n\right)\left[\left(\bar{A}/\left(\bar{A} + C_A\right)\right)^2 C_A^2 + C_X^2\left(1 - \rho_{XA}^2\right)\right].$$

viii) The Kadilar and Cingi Estimator's Series 3 (Kadilar & Cingi, 2004)

$$M_8 = \left[\bar{x} + b_{XA}\left(\bar{A} - \bar{a}\right)\right]\frac{\bar{A} + \beta_{2(A)}}{\bar{a} + \beta_{2(A)}} \quad (4.8)$$

with $B(M_8) = \left(\bar{X}/n\right)C_A^2\left(\bar{A}/\left(\bar{A} + \beta_{2(A)}\right)\right)^2$ and

$$MSE(M_8) = \left(\bar{X}^2/n\right)\left[\left(\bar{A}/\left(\bar{A} + \beta_{2(A)}\right)\right)^2 C_A^2 + C_X^2\left(1 - \rho_{XA}^2\right)\right].$$

ix) *The Kadilar and Cingi Estimator's Series 4 (Kadilar & Cingi, 2004)*

$$M_9 = \left[\bar{x} + b_{XA}\left(\bar{A} - \bar{a}\right)\right]\frac{\bar{A}\beta_{2(A)} + C_A}{\bar{a}\beta_{2(A)} + C_A} \quad (4.9)$$

with $B(M_9) = \left(\bar{X}/n\right)C_A^2\left(\bar{A}\beta_{2(A)}/\left(\bar{A}\beta_{2(A)} + C_A\right)\right)^2$ and

$$MSE(M_9) = \left(\bar{X}^2/n\right)\left[\left(\bar{A}\beta_{2(A)}/\bar{A}\beta_{2(A)} + C_A\right)^2 C_A^2 + C_X^2\left(1 - \rho_{XA}^2\right)\right].$$

x) *The Kadilar and Cingi Estimator's Series 5 (Kadilar & Cingi, 2004)*

$$M_{10} = \left[\bar{x} + b_{XA}\left(\bar{A} - \bar{a}\right)\right]\frac{\bar{A}C_A + \beta_{2(A)}}{\bar{a}C_A + \beta_{2(A)}} \quad (4.10)$$

with $B(M_{10}) = \left(\bar{X}/n\right)C_A^2\left(\bar{A}C_A/\left(\bar{A}C_A + \beta_{2(A)}\right)\right)^2$ and

$$MSE(M_{10}) = \left(\bar{X}^2/n\right)\left[\left(\bar{A}C_A/\left(\bar{A}C_A + \beta_{2(A)}\right)\right)^2 C_A^2 + C_X^2\left(1 - \rho_{XA}^2\right)\right].$$

4.3 GENERAL STRUCTURE OF THE PROPOSED CHARTS

The CSC is a combination of the Shewhart chart and the CUSUM chart, where the Shewhart chart is responsible for early detection of a large shift while the CUSUM chart detects small to moderate shifts in a quality control process. The addition of Shewhart chart limits to CUSUM chart will improve the performance of CUSUM in detecting a large shift, which is an advantage over ordinary CUSUM chart, though there will be payoff in the CUSUM structure, as well as in the Shewhart structure, by widening the control limits of the two charts. According to Henning et al. (2015), the CSC is the probabilistic combination of two charts to form a new one by adjusting their control limits, and taking the sensitivity of false alarm rates to the new scheme into consideration. This has large scope of application {Westgard et al. (1977), Lucas (1982), Wu et al. (2008), Montgomery (2009), Abujiya et al. (2013) and Henning et al. (2015)}. Like the CUSUM chart, the CSC chart is not difficult to construct and use (Lucas, 1982).

In this study, a bivariate setup from a normal distribution such that $(X, A) \sim N_2\left(\mu_X, \mu_A, \sigma_X^2, \sigma_A^2, \rho_{XA}\right)$ is assumed in proposing some improved CSC control charts, using the location estimators $M_i, i = 2, 3, \ldots, 10$. Let $Z_t = \left(M_{i,t} - \mu_{M_i}\right) / \sigma_{M_i}$ be the standardized transformation of the estimators $M_i, i = 2, 3, \ldots, 10$, for the n-subgroup t^{th} sample, where $\mu_{M_i} = \bar{X} + B(M_i)$ and $\sigma_{M_i}^2 = MSE(M_i)$. Hence, the general control charting structure of the proposed charts is presented. The CUSUM's plotting statistics are given as

$$\begin{aligned} C_t^+ &= \max(0, Z_t - k + C_{t-1}^-); \quad C_0^+ = 0 \\ C_t^- &= \max(0, -Z_t - k + C_{t-1}^-); \quad C_0^- = 0 \end{aligned} \tag{4.11}$$

and the Shewhart's plotting statistic is given as Z_t with upper control limit $(UCL) = L$ and lower control limit $(LCL) = -L$. A process is declared out of control if $C_t^+ > h$ or $C_t^- > h$ or $|Z_t| > L$, where h is the control limit of the CUSUM chart, predetermined based on the desired false alarm rate and k is one-half of the magnitude of the shift (δ) we are interested in, which is expressed as $k = \delta/2$ (Montgomery, 2009).

After the plotting statistics of the proposed charts have been stated, it is necessary to distinguish between the two states of control; in-control and out-of-control. A process is in-control if the population parameters of the study variable in a quality process have target mean value μ_0 and true variance σ_0^2, but if the parameters are altered to new values μ_1 and σ_1^2, the process is said to be out-of-control. Since our focus is on monitoring the shift in location parameter, we are concerned with the alteration of the population mean from μ_0 to μ_1 with shift $\delta = (\mu_1 - \mu_0)/(\sigma_X / \sqrt{n})$. Therefore, if $(X, A) \sim N_2(\mu_X, \mu_A, \sigma_X^2, \sigma_A^2, \rho_{XA})$ for the in-control case, we have $(X, A) \sim N_2(\mu_X + \delta, \mu_A, \sigma_X^2, \sigma_A^2, \rho_{XA})$ for the out-of-control case.

Based on the purpose of this work, any of the sensitizing rules given in quality control literatures (Abbas et al, 2011) may be used. Specifically, we use the first rule (one-out-of-one) which is the most popular to detect an out-of-control process. To explain the rule with respect to the proposed charts, generate n samples from a bivariate normal distribution $(x_i, a_i) \sim N_2(\mu_X, \mu_A, \sigma_X^2, \sigma_A^2, \rho_{XA})$, estimate the mean of the samples using the estimators $M_i, i = 2, 3, \ldots, 10$ and construct the plotting statistics. According to the first rule, once the plotting statistics fall outside the process control limits, the process is declared as out-of-control, to indicate a shift in the location parameter of the variable of interest.

4.3.1 SPECIAL CASES

Let $M_i CSC, i = 1, 2, 3, \ldots, 10$ represents the proposed chart.

i. It is worthy of note that $M_1 CSC$ chart is the **classical CSC** chart.

ii. If h approaches infinity, we have the **Shewhart chart.**

iii. If L approaches infinity, we have the **CUSUM chart**.

4.4 PERFORMANCE MEASURES

In this section, following the works of some authors {Zhang et al. (2012); Riaz, (2015)}, performance of the $M_i CSC$ charts $(i = 2, 3, \ldots, 10)$ using the ARL and the standard deviation run length (SDRL) for each shift (δ) is done. In addition, evaluation of the overall precision of the charts over the entire shift is carried out using extra quadratic loss (EQL) in order to make an accurate and reliable conclusion about the relative effectiveness of the $M_i CSC$ charts $(i = 2, 3, \ldots, 10)$. Below is a brief description of these measures.

ARL is the average number of points (samples) plotted until a point indicates an out-of-control signal (Montgomery, 2009). It is a popular measure for measuring the effectiveness of a control chart. ARL can be categorized into ARL_0 and ARL_1. ARL_0 is the ARL value when a process is stable i.e. in an in-control state ($\delta = 0$) while ARL_1 is the ARL value when a process is unstable i.e. in an out-of-control state ($\delta \neq 0$). It is expected that ARL_0 has a large value while ARL_1 has a small value (Ahmad et al., 2014). This idea is often used to measure the effectiveness of a chart and to compare the performance of different charting structures. Interested reader should see Jamali et al. (2006), Riaz & Does (2008), Cox (2010), Abbasi et al. (2012), Busaba et al. (2012) and the references therein.

On the other hand, SDRL is the standard deviation of points (samples) plotted until a point indicates an out-of-control signal. It is also used to compare different charts and examine their response to shift in parameter(s). The smaller the SDRL, the better the performance of a control chart (Abujiya et al., 2015). There is also EQL, which is the weighted average ARL over all shifts considered in a control process. It measures the effectiveness of a chart over all range of shifts, unlike ARL that deals with a specific shift. In the work of Wu et al. (2008), Wu et al. (2009), Ou et al. (2012) and Abujiya et al. (2015), EQL and its other forms were used to measure the effectiveness of control charts over a range of process shifts. The mathematical expression of EQL is given as

$$EQL = \frac{1}{\delta_{\max} - \delta_{\min}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta \qquad (4.12)$$

where $\delta_{\min}$ and $\delta_{\max}$ are the minimum and maximum values of the shifts considered in a process; and $ARL(\delta)$ is the ARL at a particular shift (δ). The EQL values are computed with numerical integration approach. A particular CSC chart could have different combinations of *h* and *L*, and the combination with the lowest EQL will give the optimum choice of *h* and *L*.

The ARL of the M_iCSC charts $(i = 2, 3, \ldots, 10)$ are given in Tables (4.1 – 4.4) and the value of the best chart at each magnitude of shift is written in bold fonts. Also presented in Tables (4.1 – 4.4) are the EQL values. Furthermore, the SDRL results for the M_iCSC charts $(i = 2, 3, \ldots, 10)$ are presented in Tables (4.5 – 4.8).

Table 4.1 : ARL values of the proposed charts with $\rho_{XA} = 0.25$ and *k* = 0.25

L	3.20	4.13	3.20	3.20	2.60	3.95	4.10	4.20	4.00	4.10
H	9.200	9.670	9.180	9.20	6.551	10.050	9.480	9.170	9.780	9.900
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	370.59	371.25	371.53	368.65	370.02	370.14	367.98	373.32	368.83	369.74
0.25	**27.11**	37.44	36.50	36.68	**27.56**	58.95	57.63	54.67	58.43	39.78
0.50	10.68	14.39	13.78	13.86	**10.06**	20.92	19.97	19.2	20.63	15.12
0.75	6.22	8.83	8.13	8.12	**5.88**	12.47	12.02	11.51	12.38	9.3
1.00	3.95	6.42	5.44	5.43	**3.73**	8.96	8.54	8.16	8.8	6.68
1.50	1.74	3.88	2.65	2.66	**1.7**	5.56	5.35	5.19	5.46	4.08
2.00	1.11	2.28	1.47	1.48	**1.1**	3.73	3.72	3.65	3.74	2.4
2.50	**1.01**	1.34	1.10	1.09	**1.01**	2.37	2.49	2.48	2.41	1.39
3.00	**1.00**	1.06	1.01	1.01	**1.00**	1.49	1.59	1.61	1.53	1.07
4.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	1.04	1.05	1.05	1.04	**1.00**
5.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
EQL	6.360	8.137	7.186	7.185	6.298	10.955	10.987	10.822	10.959	8.362

Table 4.2: ARL values of the proposed charts with $\rho_{XA} = 0.25$ and *k* = 0.5

L	3.20	4.20	3.25	3.20	2.70	4.40	4.20	4.40	4.30	4.30
h	9.200	4.060	5.247	5.40	3.463	5.330	5.558	5.354	5.360	5.510
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	371.01	371.28	368.43	371.18	368.74	368.48	371.84	366.72	368.76	369.76
0.25	**31.31**	52.33	46.49	47.83	35.75	82.74	89.22	83.19	85.51	53.55
0.50	8.90	12.66	12.17	12.47	**8.62**	19.87	20.73	19.72	19.98	13.06
0.75	4.91	6.87	6.52	6.52	**4.54**	9.73	10.14	9.77	9.84	6.98
1.00	3.21	4.71	4.33	4.35	**3.02**	6.37	6.62	6.29	6.46	4.76
1.50	1.65	2.9	2.37	2.32	**1.64**	3.82	3.95	3.79	3.85	2.93
2.00	1.11	1.92	1.47	1.45	**1.11**	2.74	2.81	2.72	2.76	1.99
2.50	1.01	1.31	1.1	1.09	**1.01**	2.07	2.04	2.05	2.05	1.39
3.00	**1.00**	1.07	1.01	1.01	**1.00**	1.59	1.49	1.54	1.54	1.10
4.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	1.08	1.05	1.07	1.07	**1.00**
5.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	1.01	**1.00**	**1.00**	1.01	**1.00**
EQL	6.190	7.562	6.962	6.950	6.183	9.865	9.911	9.776	9.853	7.703

Table 4.3: ARL values of the proposed charts with $\rho_{XA} = 0.75$ and k = 0.25

L	3.200	4.000	3.150	3.100	4.300	3.625	4.000	4.000	3.9.00	4.200
H	5.350	10.100	9.692	10.000	11.370	4.620	9.183	9.258	9.640	9.500
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	370.59	367.75	370.81	373.083	368.91	372.01	367.96	369.31	367.73	368.35
0.25	27.11	22.57	**15.75**	17.72	20.84	22.17	40.86	40.02	43.26	22.72
0.50	10.68	9.47	**5.27**	6.96	9.03	8.28	15.20	14.97	15.91	9.50
0.75	6.22	5.94	**2.94**	3.46	5.66	5.07	9.35	9.22	9.65	6.01
1.00	3.95	4.04	1.87	**1.84**	3.71	3.72	6.69	6.61	6.9	4.29
1.50	1.74	1.71	1.08	**1.05**	1.47	2.40	4.11	4.03	4.15	2.08
2.00	1.11	1.06	**1.00**	**1.00**	1.03	1.62	2.50	2.42	2.46	1.14
2.50	**1.01**	**1.00**	**1.00**	**1.00**	**1.00**	1.15	1.50	1.44	1.42	1.01
3.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	1.01	1.12	1.10	1.10	**1.00**
4.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
5.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
EQL	6.360	6.260	5.554	5.628	6.113	6.639	8.534	8.402	8.528	6.425

Table 4.4: ARL Values of the proposed charts with $\rho_{XA} = 0.75$ and *k* = 0.5

L	3.20	4.15	3.20	3.15	4.30	4.00	4.00	4.20	4.00	4.20
H	5.350	4.06	5.355	5.490	6.700	5.507	5.497	5.300	5.577	5.680
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	371.01	370.25	368.66	371.36	370.64	370.31	371.63	367.64	369.27	369.08
0.25	31.31	22.85	**16.06**	16.35	20.62	60.13	59.59	54.91	60.47	25.09
0.50	8.90	7.05	**5.32**	5.35	6.87	14.18	14.43	13.55	14.6	7.59
0.75	4.91	4.15	**2.92**	**2.92**	4.09	7.55	7.50	7.12	7.57	4.45
1.00	3.21	2.96	1.83	**1.8**	2.84	5.08	5.09	4.86	5.12	3.16
1.50	1.65	1.62	**1.06**	**1.06**	1.41	3.09	3.1	2.99	3.12	1.75
2.00	1.11	1.08	**1.00**	**1.00**	1.03	2.06	2.07	2.05	2.07	1.12
2.50	1.01	1.01	**1.00**	**1.00**	**1.00**	1.39	1.39	1.43	1.40	1.01
3.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	1.11	1.11	1.13	1.11	**1.00**
4.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
5.00	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**
EQL	6.190	5.998	5.549	5.548	5.883	7.921	7.922	7.834	7.955	6.116

Table 4.5: SDRL values for the proposed charts with $\rho_{XA} = 0.25$ and *k* = 0.25

L	3.20	4.13	3.20	3.20	2.60	3.95	4.10	4.20	4.00	4.10
H	9.200	9.670	9.180	9.20	6.551	10.050	9.480	9.170	9.780	9.900
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	358.20	360.25	352.77	356.51	364.25	364.09	357.81	363.34	363.31	356.60
0.25	14.66	13.46	10.90	22.77	16.46	40.47	39.61	37.87	40.06	23.47
0.50	4.32	5.17	3.99	5.90	4.17	9.10	8.89	8.30	9.08	5.59
0.75	2.66	3.46	2.96	3.29	2.46	4.31	4.10	3.91	4.19	2.76
1.00	2.07	2.08	1.68	2.46	1.88	2.73	2.56	2.46	2.63	1.81
1.50	1.04	0.97	0.85	1.61	0.97	1.70	1.53	1.40	1.65	1.38
2.00	0.35	0.39	0.34	0.81	0.34	1.48	1.33	1.22	1.41	1.22
2.50	0.09	0.20	0.17	0.31	0.09	1.27	1.20	1.14	1.24	0.68
3.00	0.00	0.08	0.00	0.11	0.01	0.79	0.82	0.81	0.8.0	0.27
4.00	0.00	0.00	0.00	0.00	0.00	0.2	0.23	0.23	0.20	0.04
5.00	0.00	0.00	0.00	0.00	0.00	0.05	0.06	0.06	0.05	0.02

Table 4.6: SDRL Values for the proposed charts with $\rho_{XA} = 0.25$ and k = 0.5

L	3.20	4.20	3.25	3.20	2.70	4.40	4.20	4.40	4.30	4.30
H	9.200	4.060	5.247	5.400	3.463	5.330	5.558	5.354	5.360	5.510
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	358.20	368.03	370.10	361.17	363.61	367.80	368.31	366.08	360.52	365.63
0.25	14.66	44.53	39.87	40.86	30.24	75.98	80.33	75.94	76.79	45.85
0.50	4.32	6.55	7.04	7.13	4.87	13.04	13.60	12.82	12.95	6.99
0.75	2.66	2.65	2.99	3.04	2.03	4.74	4.78	4.67	4.77	2.78
1.00	2.07	1.54	1.85	1.90	1.28	2.49	2.57	2.42	2.54	1.57
1.50	1.04	0.88	1.15	1.16	0.75	1.18	1.22	1.14	1.18	0.86
2.00	0.35	0.75	0.71	0.71	0.33	0.82	0.87	0.79	0.83	0.72
2.50	0.09	0.51	0.33	0.31	0.1	0.68	0.77	0.69	0.71	0.53
3.00	0.00	0.26	0.11	0.11	0.02	0.58	0.60	0.57	0.59	0.30
4.00	0.00	0.04	0.00	0.00	0.01	0.27	0.23	0.25	0.25	0.06
5.00	0.00	0.01	0.00	0.00	0.01	0.08	0.07	0.06	0.08	0.03

Table 4.7: SDRL Values for the proposed charts with $\rho_{XA} = 0.75$ and *k* = 0.25

L	3.2	4	3.15	3.1	4.3	3.625	4	4	3.9	4.2
H	5.35	10.1	9.692	10	11.37	4.62	9.183	9.258	9.64	9.5
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	367.01	365.04	363.77	364.27	368.51	359.04	364.22	363.1	354.66	362.83
0.25	24.22	17.61	10.03	8.2	8.53	13.61	25.46	24.62	26.79	10.27
0.50	4.61	3.14	2.3	3.16	2.57	3.14	6.06	5.9	6.37	2.87
0.75	2.15	1.5	1.33	2.06	1.67	1.5	2.98	2.92	3.12	1.58
1.00	1.46	0.95	0.93	1.15	1.53	0.95	1.97	1.94	2.13	1.27
1.50	0.84	0.67	0.28	0.23	0.78	0.67	1.4	1.4	1.53	1.06
2.00	0.35	0.56	0.04	0.03	0.17	0.56	1.22	1.2	1.27	0.39
2.50	0.09	0.36	0.01	0	0.04	0.36	0.76	0.72	0.71	0.11
3.00	0.02	0.12	0	0	0.01	0.12	0.36	0.32	0.32	0.04
4.00	0	0	0	0	0.01	0	0.07	0.05	0.05	0.02
5.00	0	0	0	0	0	0	0.01	0.01	0.01	0.01

Table 4.8: SDRL Values for the proposed charts with $\rho_{XA} = 0.75$ and *k* = 0.5

L	3.200	4.15	3.20	3.15	4.30	4.00	4.00	4.20	4.00	4.20
h	5.350	4.060	5.355	5.490	6.700	5.507	5.497	5.300	5.577	5.680
δ	M_1	M_2	M_3	M_4	M_5	M_6	M_7	M_8	M_9	M_{10}
0.00	367.01	366.21	370.72	373.70	369.75	373.24	374.11	357.65	367.11	360.22
0.25	24.22	15.62	10.39	10.40	12.82	53.08	51.31	48.41	52.35	17.48
0.50	4.61	2.77	2.36	2.41	2.49	8.10	8.20	7.76	8.44	3.04
0.75	2.15	1.30	1.38	1.41	1.26	3.21	3.18	2.94	3.20	1.41
1.00	1.46	0.88	0.94	0.94	1.00	1.78	1.80	1.68	1.79	0.94
1.50	0.84	0.65	0.25	0.24	0.63	1.02	0.99	0.91	1.01	0.72
2.00	0.35	0.28	0.02	0.04	0.18	0.83	0.83	0.74	0.83	0.34
2.50	0.09	0.08	0.00	0.00	0.05	0.058	0.058	0.56	0.59	0.1
3.00	0.02	0.02	0.00	0.00	0.03	0.032	0.032	0.34	0.32	0.04
4.00	0.00	0.01	0.00	0.00	0.01	0.07	0.06	0.07	0.07	0.01
5.00	0.00	0.01	0.00	0.00	0.00	0.00	0.01	0.02	0.01	0.00

We have also presented the ARL curve of the proposed control schemes for a visual comparison. Figures (4.1 – 4.4) present the ARL curves for M_iCSC charts $(i = 2, 3, \ldots, 10)$ for monitoring changes in the process mean using different values of *k* and ρ_{XA} with *n* = 5 and $ARL_0 = 370$.

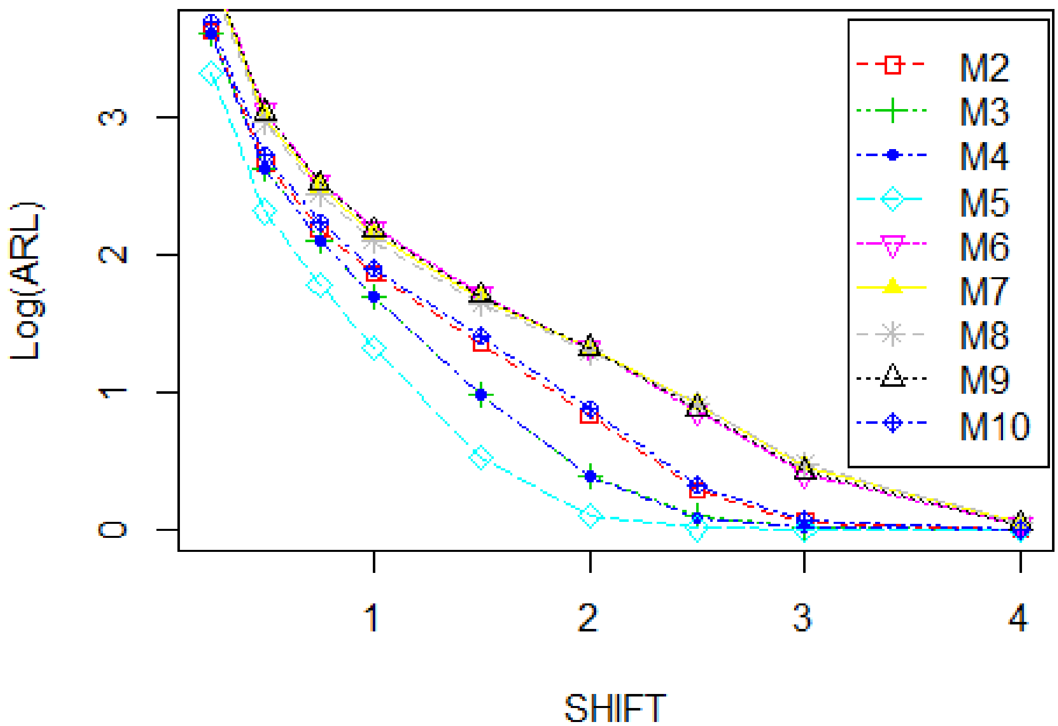

Figure 4.1: ARL curve of the proposed charts with ρ_{XA} = 0.25, k = 0.25 and ARL_0 = 370.

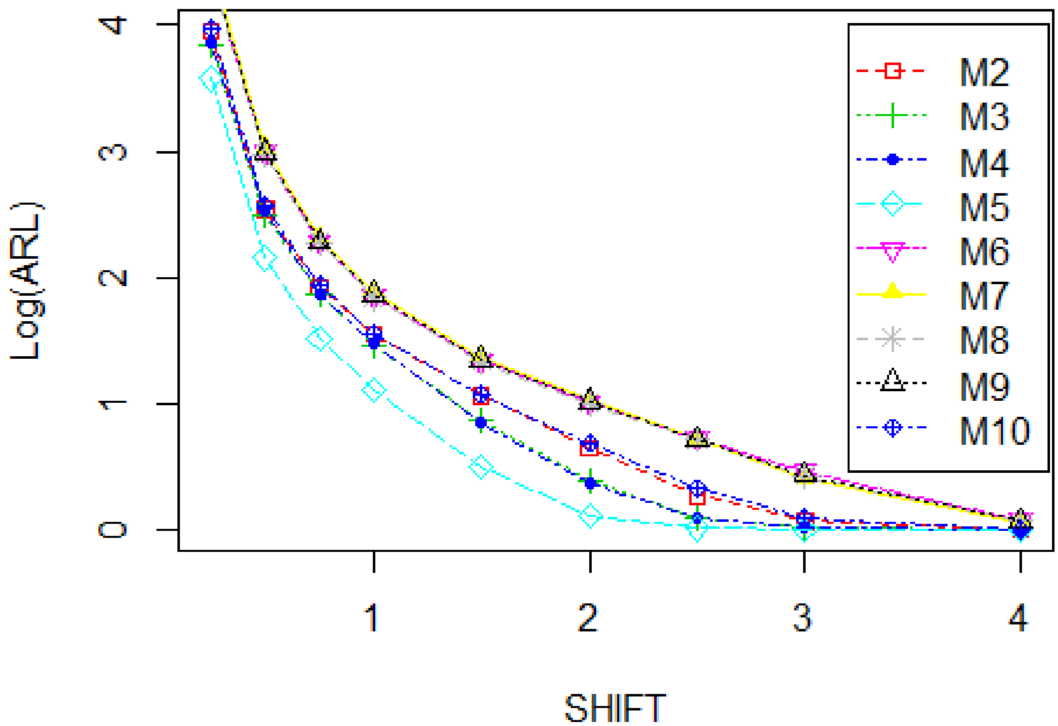

Figure 4.2: ARL curve of the proposed charts with ρ_{XA} = 0.25, k = 0.50 and ARL_0 = 370.

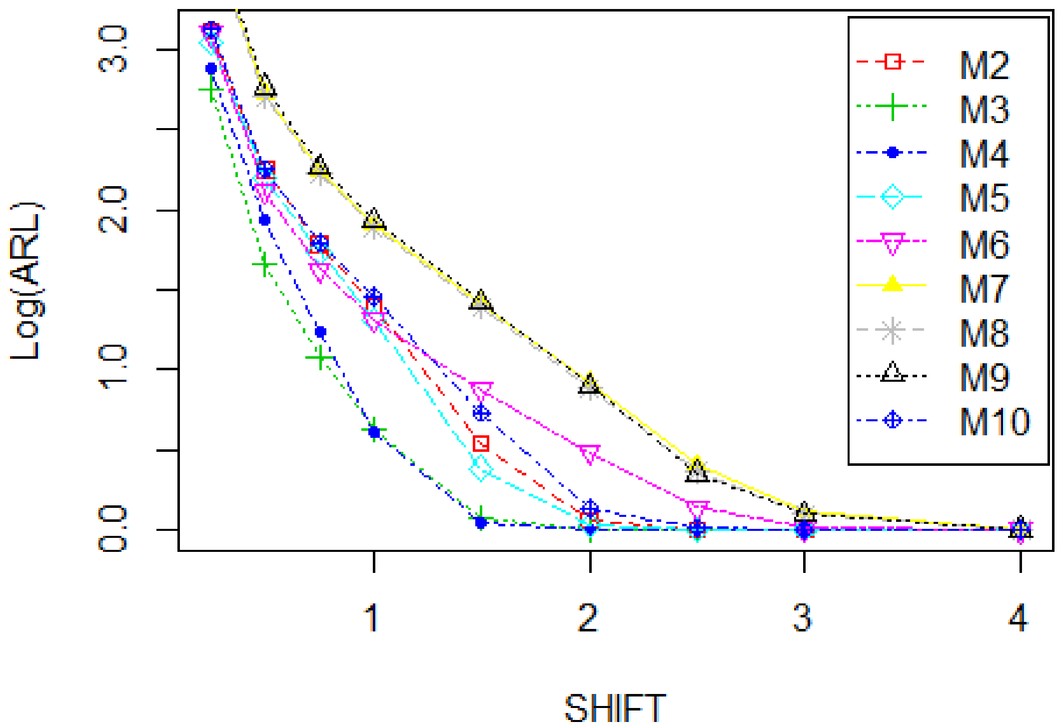

Figure 4.3: ARL curve of the proposed charts with ρ_{XA} = 0.75, k = 0.25 and ARL_0 = 370.

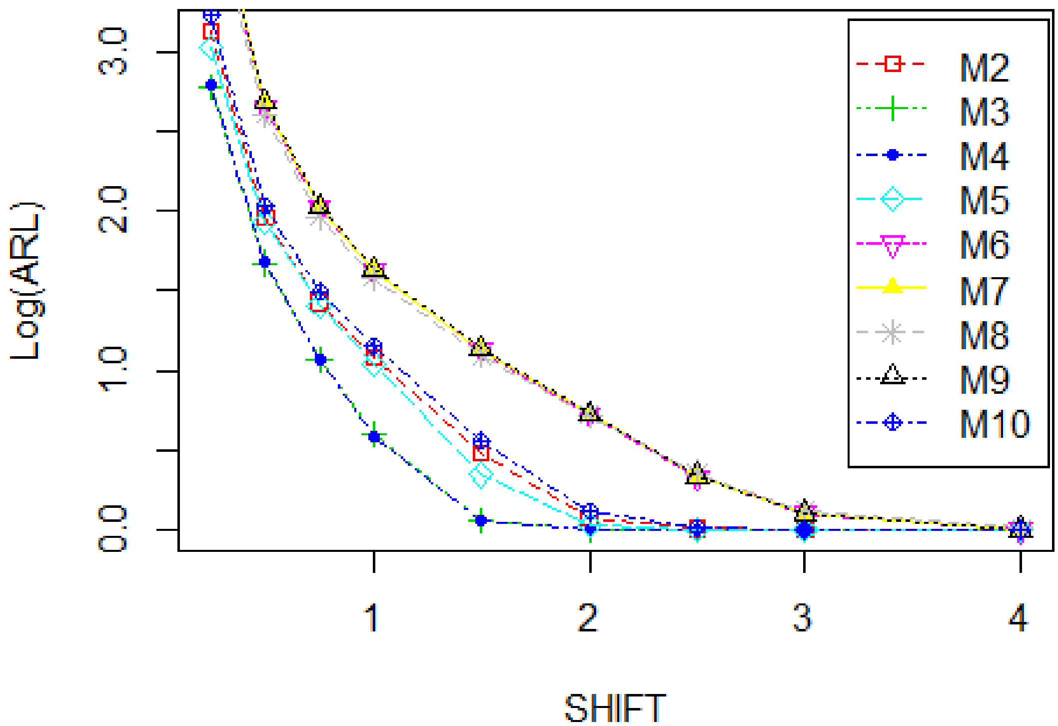

Figure 4.4: ARL curve of the proposed charts with ρ_{XA} = 0.75, k = 0.50 and ARL_0 = 370.

Based on the results in Tables (4.1 – 4.8) and Figures (4.1 – 4.4), we summarize our major findings from the proposed charts as follow:

I. The proposed M_5CSC chart has smaller ARL values than all other charts when $\rho_{XA} = 0.25$ for all values of *k*. This means that the chart is able to detect all magnitudes of the shift faster than other proposed charts when there is a weak positive correlation between the study variable and the auxiliary variable. A point equally supported by the SDRL (cf. Table 4.6). On the overall performance in terms of EQL, the proposed M_5CSC still dominates all other charts (Tables 4.1 - 4.2, Figures 4.1 – 4.2).

II. For $\rho_{XA} = 0.75$, the proposed M_3CSC chart and M_4CSC chart have smaller ARL values than all other charts when $\delta \leq 2$ for all values of *k*. This means that the charts are able to detect small to moderate shifts faster than other proposed charts when there is a strong positive correlation between the study variable and the auxiliary variable. On the overall performance in terms of EQL, the proposed M_3CSC and M_4CSC still dominate all other charts (Tables 4.3 – 4.4, Figures 4.3 – 4.4).

III. Almost all the charts have good performance in detecting large shifts, over all values of k (Tables 4.1 – 4.4, Figures 4.1 – 4.4).

IV. The proposed charts are ARL unbiased for all the different values of ρ_{XA} and δ, i.e., ARL_0 is always greater than ARL_1 for any choice of δ (Tables 4.1 – 4.8).

V. For $\delta = 0$, there is no significant difference between the ARL and the SDRL of the proposed charts. In addition, the ARL and SDRL values approach 1 and 0, respectively, as shift increases (Tables 4.1 – 4.8).

4.5 COMPARISONS WITH EXISTING CHARTS

4.5.1 M_iCSC charts $(i = 2, 3, \ldots, 10)$ vs. Classical CSC chart (M_1CSC)

Most of the proposed charts outperform the classical CSC chart except for few cases of detecting small and large shifts when there is weak positive correlation between the study variable and the auxiliary variable. For example, in Tables 4.1 – 4.2, M_1CSC chart shows the best performance for shift of *0.25*, while M_5CSC chart (closely followed by M_1CSC chart) shows the best performance for other values of shifts. This implies that our proposed charts will perform better than the classical CSC when there is a high value of ρ_{XA}, irrespective of the value of *k* This is evident from the low values of ARL_1 and EQL of the proposed charts (Tables 4.1 – 4.4).

4.5.2 M_iCSC charts $(i = 2, 3, \ldots, 10)$ vs. CUSUM charts based on Median, Mid-range, Hodges-Lehman (HL), and Trimean (TM) estimators under unconterminated Normal distribution.

Nazir et al., (2013) proposed robust CUSUM charts that are effective in detecting small shifts when the parameters of the underlying normal distribution of a process are contaminated. Assuming no contamination in the parameters of the normal distribution of a process, most of our proposed charts outperform their charts in detecting all magnitudes of shift, over all values of *k*. Specifically, M_5CSC chart (when $\rho_{XA} = 0.25$), M_3CSC chart (when $\rho_{XA} = 0.75$) and M_4CSC chart (when $\rho_{XA} = 0.75$) perform better than their proposed charts, and this is evident from the low ARL_1 values of M_3CSC chart, M_4CSC chart and M_5CSC chart (cf. Tables 4.9 – 4.10).

*

Table 4.9: Some selected proposed charts versus existing CUSUM charts based on different estimators (Median, Mid-range, Hodges-Lehmann [HL] and Trimean [TM]), when k = 0.25.

L	3.15	3.10	2.60				
h	9.692	10.000	6.551	8.030	8.030	8.030	8.030
δ	M_3(rho=.75)	M_4(rho=.75)	M_5(rho=.25)	Median	Mid-range	HL	TM
0.00	370.81	373.08	370.02	372.50	370.75	373.12	373.93
0.25	15.75	17.72	27.56	31.59	29.82	25.83	27.59
0.50	5.27	6.96	10.06	12.38	11.68	10.44	10.92
0.75	2.94	3.46	5.88	7.70	7.31	6.55	6.90
1.00	1.87	1.84	3.73	5.60	5.35	4.81	5.07
1.50	1.08	1.05	1.70	3.73	3.55	3.25	3.39
2.00	1.00	1.00	1.10	2.85	2.73	2.48	2.60

Table 4.10: Some selected proposed charts versus existing CUSUM charts based on different estimators (Median, Mid-range and Hodge Lehman), when k = 0.5.

L	3.20	3.15	2.70				
H	5.355	5.490	3.463	4.774	4.774	4.774	4.774
δ	M_3(rho=.75)	M_4(rho=.75)	M_5(rho=.25)	Median	Mid-range	HL	TM
0.00	368.66	371.36	368.74	374.28	370.11	367.10	368.02
0.25	16.06	16.35	35.75	41.83	37.53	29.99	32.52
0.50	5.32	5.35	8.62	11.27	10.27	8.79	9.36
0.75	2.92	2.92	4.54	6.07	5.71	5.00	5.25
1.00	1.83	1.80	3.02	4.21	3.97	3.52	3.70
1.50	1.06	1.06	1.64	2.67	2.53	2.31	2.39
2.00	1.00	1.00	1.11	2.07	1.99	1.85	1.91

4.6 ILLUSTRATIVE EXAMPLE

In this section, we provide an illustrative example to show the implementation of our proposed charts in real situation. For this purpose, we have considered the bivariate data by Constable and Parker (1988) on the measurements of a component part for an automobile's braking system, containing the study variable $X =$ BAKEWT and the auxiliary variable $A =$ ROLLWT. The 45 data points, which are taken from the in-control process, are used to estimate the population parameters. These estimates came out to be $\bar{x} = 201.18$, $\bar{a} = 210.24$, $s_x = 1.17$, $s_a = 1.23$ and $r_{xa} = 0.54$. Considering these estimates as the known parameters, we have generated dataset from bivariate normal distribution with $\bar{X} = 201.18$, $\bar{A} = 210.24$, $\sigma_X = 1.17$, $\sigma_A = 1.23$ and

$\rho_{XA} = 0.54$ containing 15 paired observations but the last seven observations refer to an out-of-control situation with $\delta = \left(\bar{X}_1 - \bar{X}_0\right)/\left(\sigma_X / \sqrt{n}\right) = (203 - 201.18)/\left(1.17/\sqrt{1}\right) \cong 3.47$ where $\bar{X}_0$ and $\bar{X}_1$ are the in-control mean and the out-of-control mean respectively. The inspiration of generating dataset in such a manner is taken from Singh and Mangat (1996).

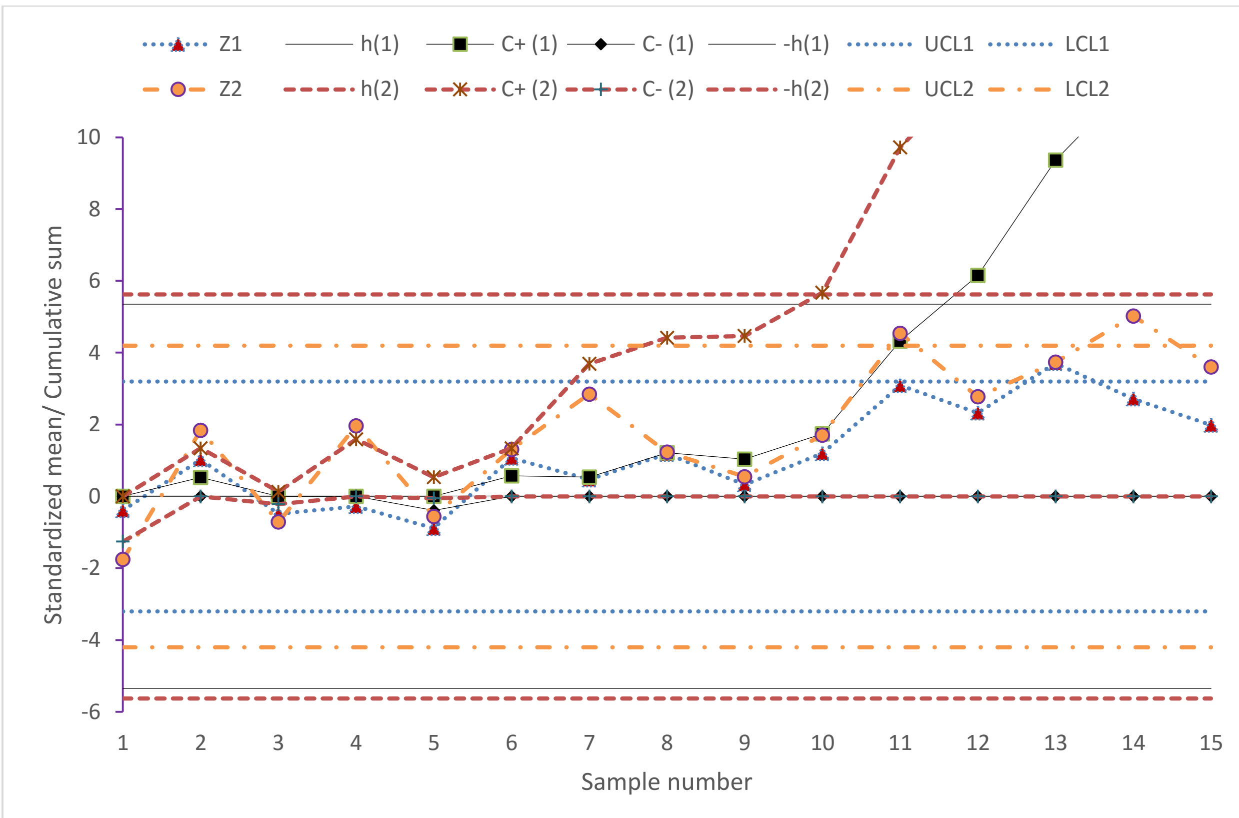

Figure 4.5: Graphical display of the M_iCSC $(i = 1,2)$ charts.

The classical CUSUM (M_1CSC) and some selected M_iCSC $(i = 2,4,10)$ charts with $k = 0.5$ are applied to the generated dataset. The chart outputs for M_iCSC $(i = 1,2)$ charts and M_iCSC $(i = 4,5)$ charts are respectively given in Figure 4.5 and Figure 4.6.

They are splited into two figures to aid visually. The M_iCSC $(i = 1,2,4,5)$ charts signal a shift in the process when either of the Shewhart or CUSUM detects a shift. In accordance with our findings, the proposed charts show their superiority. M_1CSC detects the shift at sample #12 (cf Figure 4.5), M_2CSC detects the shift earlier at sample #10 (cf Figure 4.5) while M_4CSC and M_5CSC detect the shift at sample #11 (cf Figure 4.6).

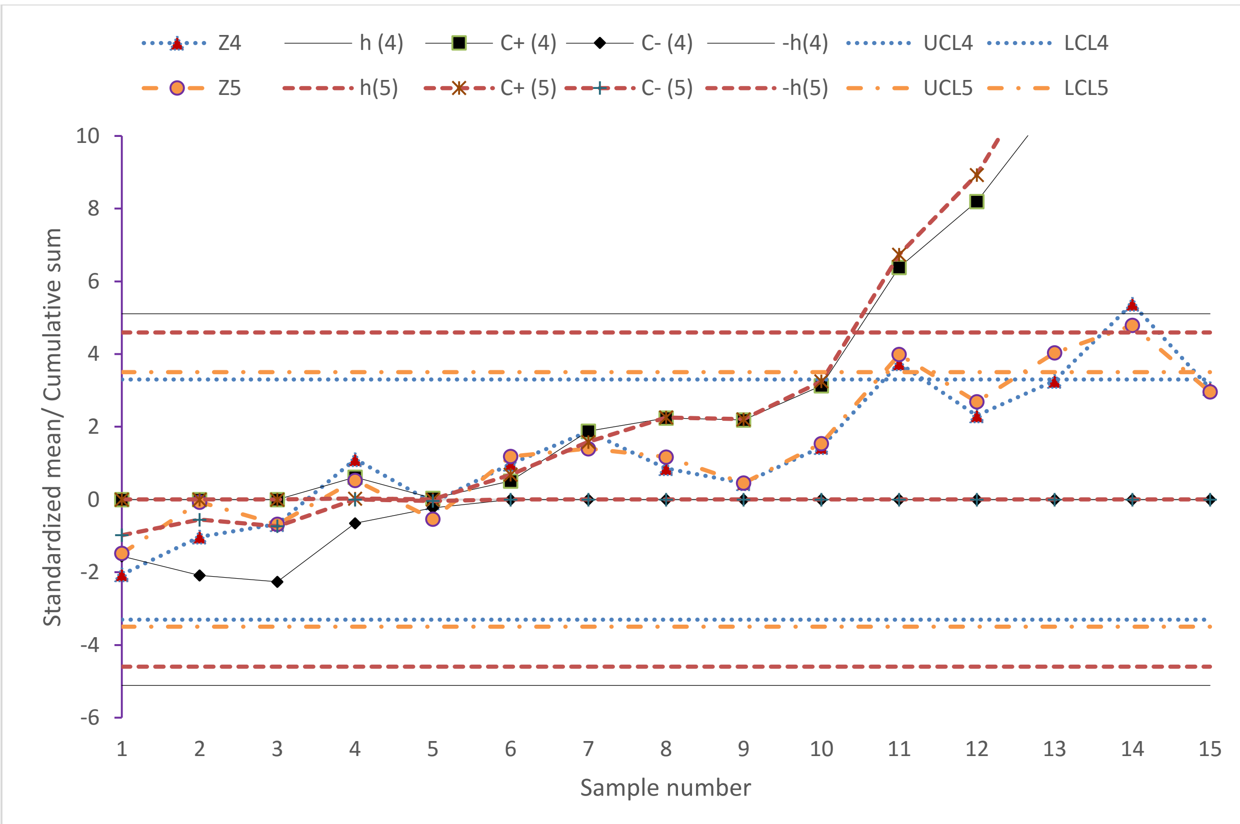

Figure 4.6. Graphical display of the M_iCSC $(i = 4,5)$ charts.

4.7 CONCLUSIONS AND RECOMMENDATIONS

Shewhart chart is traditionally used for detecting large shifts, while CUSUM chart is used for detecting small shifts. CSC chart was proposed to monitor small and large shifts simultaneously. We study the effect of introducing some efficient estimators to CSC chart, and observed that some of the proposed charts with the following estimators; the Ratio estimator, the Singh and Tailor estimator, and the Power ratio-type estimator give a better performance than the classical CSC chart and some existing CUSUM charts, in detecting small, moderate and large shifts.

We hereby recommend that if there is a weak positive correlation between a study variable and an auxiliary variable, M_5CSC chart (with the Power ratio-type estimator) should be preferred, while M_3CSC chart (with the Ratio estimator) or M_4CSC chart (with the Singh and Tailor estimator) should be preferred over their counterparts in detecting small, moderate and large shifts.

CHAPTER 5

USING FIR TO IMPROVE CUSUM CHARTS FOR MONITORING PROCESS DISPERSION

Statistical process control deals with monitoring process to detect disturbances in the process. These disturbances may be from the process mean or variance. In this study, we propose some charts that are efficient for detecting early shifts in dispersion parameter, by applying the First Initial Response feature. Performance measures such as average run length, standard deviation run length, extra quadratic length, relative average run length and performance comparison index are used to compare the proposed charts with their existing counterparts including the Shewhart R, the Shewhart S chart, the Shewhart S chart with warning lines, the CUSUM of the range R, CUSUM of the standard deviation S, the EWMA of $\ln S^2$, the CUSUM of $\ln S^2$, the $P_\sigma\ CUSUM$, the $\chi - CUSUM$ and the $CP\ CUSUM$. The proposed charts do not only detect early shifts in process dispersion faster, but also have better overall performance than their existing counterparts.

5.1 INTRODUCTION

Statistical process control (SPC) is a collection of useful tools for detecting alteration in a process. It has wide application in the industrial field, the medical field, and other fields where variation is being monitored. The variation may be a natural cause variation or a special cause variation. The natural cause variation is always small, random, tolerable, acceptable, harmless, uncontrollable and unassignable. A process with this type of variation is sad to be in-control. Inversely, special cause variation has properties that are direct opposite of the natural cause variation, hence, a process with this kind of variation is said to be out-of-control. SPC has seven major tools namely Histogram, Cause-and-effect diagram, Pareto Chart, Check Sheets, Defect concentration diagram, Scatter plot and Control chart (Montgomery, 2007). Control chart is the most useful, the most effective and the most commonly used tool among the other tools. There are generally accepted rules on how control charts are to be used in companies, unlike in the past when there is no universally acceptable rules on the usage of control charts. Some of the universally acceptable international regulatory standards being used, due to the rapid rate of business exchange between different countries, are ISO 7870-4:2011, ISO 7870-3:2012, ISO 7870-2:2013, ASTM E2587:2012, ASTM D6299:2013, ISO 7870-1:2014, ISO 7870-5:2014 and ISO 7870-6:2014.

Control charts monitor the location and (or) dispersion parameter(s) of a process. The location parameter monitoring and its modification is mostly available in the literature, but little work has been done on dispersion monitoring. There are two good reasons to monitor dispersion parameter; increase in process variance above the required level may imply increase in the number of defective unit in a process; and decrease in process variance below the required level may imply that process units are closer to their target value, leading to high process capability (Acosta-Mejia et al. 1999). The control charts for location and dispersion monitoring can be broadly divided into two; the memory control chart and the memoryless control chart, which are respectively good for early detection of small and large shifts. The Shewhart chart proposed by Shewhart (1931) is the traditional memoryless control chart, while the traditional memory control charts are the Cumulative Sum (CUSUM) chart and the Exponential Moving Average (EWMA) chart proposed by Page (1954) and Roberts (1959) respectively. CUSUM and EWMA get memory from past information. Many authors have studied their structures and also suggested several modifications to improve their structures for monitoring process mean, but less attention has been given to the monitoring of process variance.

In detecting shift in process dispersion, CUSUM was applied to subgroup range by Page (1954). Tuprah and Ncube (1987) later compared this procedure with another procedure that was based on sample standard deviation. Using average run length (ARL) approach, they found that the procedure based on the sample standard deviation detects shift from the target value faster, given that the process variables are normally distributed. Furthermore, EWMA structure based on subgroup range was suggested by Ng (1988), while natural logarithmic transformation of subgroup variance was introduced to a one-sided EWMA structure to monitor process standard deviation (Crowder and Hamilton, 1992). Similarly, one-sided and two-sided CUSUM structures based on logarithmic transformation of process variance was proposed by Chang & Gan (1995) for monitoring shift in process variance, and they also enhanced the performance of the schemes by introducing Fast Initial Response (FIR). The FIR feature was first proposed by Roberts

(1959) and later improved by Steiner (1999) to reduce the time-varying limits of the first few sample observations. The FIR feature improves the performance of CUSUM chart if there is shift in a process at start-up (Hawkins and Olwell, 1998). The performance of this feature was later improved by using a power transformation with respect to time *t* (Haq, 2013).

This chapter focuses on using the FIR feature to improve the work of Acosta-Mejia et al. (1999), where they monitor increase and decrease in the variance of a normal process using CUSUM structures based on the chi-squared $(\chi - CUSUM)$ transformation, the inverse normal transformation $(P_{\sigma}\ CUSUM)$, and the CUSUM structure derived from the likelihood ratio test for the change point of a normal process $(CP\ CUSUM)$.

The rest of this chapter is organized as follows; the general structure of the proposed charts, and the FIR feature are explained in Section 5.2; Section 5.3 contains the performance evaluations and comparisons; and finally, summary and conclusions are given in Section 5.4.

5.2 THE PROPOSED CHARTS

5.2.1 CUSUM chart for monitoring process mean

CUSUM chart for monitoring process mean is good for early detection of small shift in a process. It has different structures, one of them is the standardized two-sided CUSUM structure. Let $\bar{X}_i$ be the i^{th} mean of sample observation of size n from a normal distribution with mean μ_0 and standard deviation σ_0, and $Z_i = \left(\overline{X}_i - \mu_0\right) / \left(\sigma_0 / \sqrt{n}\right)$, then the standardized two-sided CUSUM is given as

$$\begin{aligned} C_i^+ &= \max\left(0, Z_i - k_1 + C_{i-1}^+\right) \\ C_i^- &= \max\left(0, -Z_i - k_2 + C_{i-1}^-\right) \end{aligned} \tag{5.1}$$

where $C_i^+, C_i^- \geq 0$, $C_i^+ = C_i^- = 0$ and $k_1(k_2)$ is the upper (lower) reference value. Mostly, $k_1 = k_2 = k$. $k = (1/2)\delta\sigma_0 / \sqrt{n}$ is taken to be half of the mean shift (δ) to be detected, scaled in standard deviation unit. The plotting statistics C_i^+ and C_i^- are respectively plotted against the control limits h_1 and h_2. The process detects an upward shift when either of the plotting statistics plot outside its respective control limit. In most cases, $h_1 = h_2 = h$, that is, the control limits for the plotting statistics may be the same.

5.2.2 CUSUM chart for monitoring process dispersion

Let $X_i \sim N\left(\mu_0, \sigma_0^2\right)$ be the i^{th} observation of the study variable in a process. Suppose there is a disturbance in the variance of the process, the distribution of X_i becomes $X_i \sim N\left(\mu_0, \lambda^2\sigma_0^2\right)$, where λ represents a shift in standard deviation . $\lambda = 1$ implies no shift, $\lambda > 1$ implies positive shift while $\lambda < 1$ implies negative shift. We now show that an out-of-control ARL when $\lambda \neq 1$ can be calculated directly from the in-control ARL. Let

$$T_i = \frac{\left(X_i - \mu_0\right)}{\sigma_0} \sim N\left(0, \lambda^2\right) \tag{5.2}$$

$$Z_i = \frac{T_i}{\lambda} = \frac{\left(X_i - \mu_0\right)}{\lambda\sigma_0} \sim N(0,1)$$

$$\Rightarrow \quad T_i = \lambda Z_i \text{ or } Z_i = T_i/\lambda$$

The CUSUM structure for T_i is given as

$$\begin{aligned} C_0^+ &= C_0^- = 0 \\ C_i^+ &= \max\left(0, T_i - k_1 + C_{i-1}^+\right) \\ C_i^- &= \max\left(0, T_i - k_2 + C_{i-1}^-\right) \end{aligned} \tag{5.3}$$

Designing the standardized CUSUM structure for Z_i, we have

$$\begin{aligned} U_0^+ &= U_0^- = 0 \\ U_i^+ &= \max\left(0, Z_i - k_1/\lambda + U_{i-1}^+\right) \\ U_i^- &= \max\left(0, Z_i - k_2/\lambda + U_{i-1}^-\right) \end{aligned} \tag{5.4}$$

Comparing equations (5.3) and (5.4), we have $C_i^+ = \lambda U_i^+$ and $C_i^- = \lambda U_i^-$. Accordingly, $C_i^+ > h_1$ iff $U_i^+ > \lambda h_1$, and $U_i^- > h_2$ iff $U_i^- > \lambda h_2$. This implies that;

- The ARL of the CUSUM $S_i^+\left(S_i^-\right)$ to the control limit $h_1\left(h_2\right)$ is equivalent to the ARL of the CUSUM $U_i^+\left(U_i^-\right)$ to the control limit $h_1/\lambda\left(h_2/\lambda\right)$.
- The CUSUM U_i^+ and U_i^- are CUSUM of standard unshifted $N(0,1)$ with reference vales k_1/λ and k_2/λ respectively and control limits h_1/λ and h_2/λ respectively.

We now briefly introduce the P_σ *CUSUM*, the $\chi - CUSUM$ and the *CP CUSUM* for process dispersion.

P_σ *CUSUM*: Let S_i^2 be the subgroup variance of X_i $(i = 1,2,...,n)$ observed from a normal distribution with variance σ^2. Applying the inverse normal transformation to S_i^2 and assuming that $\sigma = \sigma_0$, we have

$$P_{\sigma i} = \phi^{-1}\left\{F_{\chi^2_{n-1}}\left[\frac{(n-1)S_i^2}{\sigma_0^2}\right]\right\}$$

where $P_\sigma \sim N(0,1)$, $\phi(\ )$ is the cumulative distribution from a standard normal distribution and $F_{\chi^2_{n-1}}(\ )$ is the cumulative distribution from a chi-squared distribution with $(n-1)$ degree of freedom. Monitoring the mean of $P_{\sigma i}$ is equivalent to monitoring the variance of X_i. As a result, we could replace Z_i by $P_{\sigma i}$ in equation (5.1) to monitor process variance. The reference values k_1 and k_2, and the control limits h_1 and h_2 that fix a particular ARL could be guessed by a search method or by simulation.

$\chi - CUSUM$**:** Wilson & Hilferty (1931) proved that $\sqrt[3]{\chi_n^2/n}$ is approximately $N((1-2/(9n)), 2/(9n))$. For $iid \quad N(\mu,\sigma)$, when $\sigma = \sigma_0$ we have

$$\chi_i = \frac{\left(\left(S_i^2/\sigma_0^2\right)^{1/3} - \left(1 - \frac{2}{9(n-1)}\right)\right)}{\sqrt{\frac{2}{9(n-1)}}} \approx N(0,1).$$

Monitoring the mean of χ_i is equivalent to monitoring the variance of X_i. Hence, replacing Z_i by χ_i in equation (1) gives the $\chi - CUSUM$. h_1 and h_2 that fix a particular ARL could be guessed by a search method or by simulation. To determine k_1 and k_2, let $\sigma_1 > \sigma_0$ for upward shift and $\sigma_2 > \sigma_0$ for downward shift be the process standard deviation to be monitored, then

$$k_1 = \frac{1}{2}\{E(\chi_i \mid \sigma_0) + E(\chi_i \mid \sigma_1)\}$$

$$= \frac{1}{2}\left\{\left[\left(\sigma_1^2 / \sigma_0^2\right)^{1/3} - 1\right]\left[1 - \frac{2}{9(n-1)}\right] \Big/ \sqrt{\frac{2}{9(n-1)}}\right\}$$

and

$$k_2 = \frac{1}{2}\{E(-\chi_i \mid \sigma_0) + E(-\chi_i \mid \sigma_2)\}$$

$$= \frac{1}{2}\left\{\left[1 - \left(\sigma_1^2 / \sigma_0^2\right)^{1/3}\right]\left[1 - \frac{2}{9(n-1)}\right] \Big/ \sqrt{\frac{2}{9(n-1)}}\right\}.$$

CP CUSUM **:** This is derived from the likelihood ratio test for the change point of a normal process variance to monitor process dispersion. The *CP CUSUM* structure is given as

$$C_0^+ = C_0^- = 0$$
$$C_i^+ = \max\left(0, Z_i^2 - nk_1 + C_{i-1}^+\right)$$
$$C_i^- = \max\left(0, -Z_i^2 + nk_2 + C_{i-1}^-\right)$$

where $Z_i^2 = \sum_{m=1}^{n} Z_{im}^2$. $Z_{im} = \left(X_{im} - \mu_0\right)/\sigma_0$ represents the *mth* standardized observation in subgroup i. The reference values are defined as

$$k_1 = \frac{\ln \lambda_1}{\left(1 - \left(1/\lambda_1\right)\right)}; \quad k_2 = \frac{\ln \lambda_2}{\left(1 - \left(1/\lambda_2\right)\right)}$$

where $\lambda_1 = \sigma_1/\sigma_0$ and $\lambda_2 = \sigma_2/\sigma_0$ are the relative increase and decrease in process standard deviation.

The P_σ *CUSUM*, the $\chi - CUSUM$ and the *CP CUSUM* were shown by Acosta-Mejia et al. (1999) to detect shifts in process variance quickly. If there is an out-of-control point at the start of a process, it could be detected at the earliest time by introducing a head start to CUSUM structure.

5.2.3 FAST INITIAL RESPONSE (FIR)

FIR CUSUM feature is designed by given a process a head start. Head start enables the CUSUM structure to start off at a point other than the usual zero-point. The CUSUM structure works by accumulating small shift until the shift is large enough to be noticed. The FIR feature would enable a CUSUM chart to give signal as early as possible if there is a shift at the start of a process, hence, reducing the time to signal. To maintain the same in-control $ARL\left(ARL_0\right)$ of a CUSUM chart, the h value of the corresponding FIR CUSUM must be increased by small amount. Ironically, the out-of-control $ARL\,(ARL_1)$ of the FIR CUSUM would be lesser than that of its corresponding CUSUM chart. In the work of Lucas & Crosier (1982), the ARL_1 of FIR CUSUM is 30% to 40% shorter than the corresponding ARL_1 of CUSUM chart, in monitoring location parameter. Using their recommended head start, we make $C_0 = h/2$ in our CUSUM schemes, and we focus on one-sided FIR CUSUM scheme.

5.3 PERFORMANCE EVALUATION AND COMPARISON

In the work of Abujiya et al. (2015), performance measures such as ARL, standard deviation run length (SDRL), extra quadratic length (EQL), relative average run length (RARL) and performance comparison index (PCI) were used in determining and explaining the efficiency of their proposed chart. In the same manner, we consider the same approach in this section.

ARL: is the average number of samples observed until the first out-of-control signal (false alarm) is detected in a process. ARL_0 represents the ARL when there is no shift in a process parameter (dispersion parameter in our case) while ARL_1 represents the ARL when there is shift in a process parameter (dispersion parameter in our case). It is desirable to have high value of ARL_0 but low value of ARL_1 to efficiently monitor process parameter(s) (Riaz et al. 2014).

SDRL: is the standard deviation of the number of samples observed until a false alarm is detected in a process. It is often used to evaluate the performance measure of a chart, and the ability of the chart to respond to shift in its parameter (Abbasi et al. 2012). The chart with a better performance have a smaller SDRL.

QUANTILE: The 0.05, 0.25, 0.50, 0.75 and 0.95 quantiles (denoted as q5, q25, q50, q75 and q95) are estimated to determine the pattern of the run length distribution of an in-control process.

EQL: gives the efficiency of a chart over the entire shifts considered in a process. The chart with the lowest EQL is said to be the most efficient chart. It is calculated using numerical computation, with the formula;

$$EQL = \frac{1}{\delta_{\min} - \delta_{\max}} \int_{\delta_{\min}}^{\delta_{\max}} \delta^2 ARL(\delta) d\delta .$$

RARL: gives the overall effectiveness of a chart with respect to a benchmark (bmk) chart. A benchmark chart is usually the best chart (with the lowest EQL) or the chart been compared with. It uses ARL values to determine how close a chart is to the benchmark chart. RARL equals to one for the benchmark chart, and greater than one for the inferior chart (to the benchmark chart) (Zhao et al. 2005).

$$RARL = \frac{1}{\delta_{\min} - \delta_{\max}} \int_{\delta_{\min}}^{\delta_{\max}} \frac{ARL(\delta)}{ARL_{bmk}(\delta)} d\delta$$

Table 5.1: SDRL and Quantile points of the proposed charts for upward shifts in σ at ARL_0 = 200.

% increase in σ		0	10	20	30	40	50	100
SDRL	A	215.47	36.93	12.85	6.23	3.89	2.67	1.04
	B	211.58	36.74	12.6	6.18	3.86	2.66	1.03
	C	218.23	29.95	9.34	4.74	2.87	2.09	0.83
q5	A	4	2	2	1	1	1	1
	B	4	2	2	1	1	1	1
	C	5	2	1	1	1	1	1
q25	A	49	8	4	3	2	2	1
	B	46	8	4	3	2	2	1
	C	47	7	4	2	2	2	1
q50	A	136	23	8	5	4	3	2
	B	131	22	8	5	4	3	2
	C	133	18	7	4	3	3	1
q75	A	283	48	17	9	6	5	2
	B	278.25	49	17	9	6	5	2
	C	282	39	13	7	5	4	2
q95	A	628.05	110	39	20	13	9	4
	B	620	108	37	19	12	9	4
	C	625.05	88	29	15	10	7	3

A = $P_\sigma\ CUSUM + FIR$ **, B =** $\chi - CUSUM + FIR$ **and C =** $CP\ CUSUM + FIR$

Table 5.2: SDRL and Quantile points of the proposed charts for downward shifts in σ at ARL_0 = 200.

	Shift	1	0.9	0.8	0.7	0.6
SDRL	A	214	35.27	8.34	3.08	1.53
	B	214.38	34.81	8.19	3.04	1.5
	C	214.02	30.49	6.84	2.21	0.99
q5	A	6	4	3	2	2
	B	6	4	3	2	2
	C	5	3	3	2	2
q25	A	48	10	5	4	3
	B	44	9	5	4	3
	C	48	8	4	3	3
q50	A	134	21	8	5	4
	B	132	22	8	5	4
	C	134	20	7	4	3
q75	A	285	47	14	7	5
	B	282	46	14	7	5
	C	284	42	11	6	4
q95	A	631	106	28	12	7
	B	629.05	104	27	12	7
	C	632.05	92	22	9	5

A = $P_\sigma\ CUSUM + FIR$, B = $\chi - CUSUM + FIR$ and C = $CP\ CUSUM + FIR$

PCI: is the ratio of the EQL of a chart to the EQL of a benchmark chart under the same condition. The best chart (benchmark chart) has PCI = 1, while the worst chart, as compared to the benchmark chart, has the highest value of PCI (Ou et al., 2012).

$$PCI = \frac{EQL}{EQL_{bmk}}$$

Table 5.3: EQL, RARL and PCI of the proposed charts.

	Upward shift in σ			Downward shift in σ		
	A	B	C	A	B	C
EQL	28.62225	28.22307	25.29412	28.24351	28.06285	26.76816
RARL	1.208851	1.198675	1	1.180846	1.168157	1
PCI	1.131577	1.115796	1	1.055116	1.048367	1

A = P_σ $CUSUM + FIR$, B = $\chi - CUSUM + FIR$ and C = $CP\ CUSUM + FIR$

Based on the result presented in Tables 5.1 – 5.5 and Figures 5.1 – 5.2, the basic findings are summarized as follows;

I. For $\lambda = 0$, there is no significant difference between the ARL and the SDRL of the proposed charts (Tables 5.1, 5.2, 5.4 and 5.5).

II. For $\lambda \neq 0$, the ARL and the SDRL of the proposed charts decrease rapidly (Tables 5.1, 5.2, 5.4 and 5.5).

III. The FIR feature does not only improve the charts ability to detect out-of-control signal at process start-up, but also improve the detection ability of the charts for any shift in process standard deviation. (Tables 5.4 – 5.5 and Figures 5.1 – 5.2).

IV. The quantile points show that the run length distribution of the proposed charts are positively skewed (Tables 5.1 – 5.2).

Table 5.4 : ARL comparison of dispersion charts for positive shift in process standard deviation.

Dispersion charts (n = 5)	Percentage increase in standard deviation						
	0	10	20	30	40	50	100
Shew. R (UCL = 4.88)	200.18	68.75	30.72	16.55	10.20	6.96	2.40
Shew. S (UCL =1.93)	200.10	65.10	28.30	15.10	9.20	6.30	2.40
Shew.[1] S (h_1 = 1.53, h_2 = 2.03)	200.00	58.90	24.60	13.00	8.10	5.70	2.20
EWMA ln S^2 (k = 1.06, λ =0.05)	200.00	43.00	18.10	11.00	7.60	6.00	3.20
$CUSUM\ \ln S^2$ (k = 0.068, h = 2.66)	199.93	42.94	18.07	10.75	7.63	5.98	3.18
$CUSUM\ R$ (k = 2.56, h = 4.88)	201.80	40.4	17.60	10.82	7.81	6.13	3.13
$\chi\ CUSUM$ (k = 0.38, h = 4.28)	200.70	41.04	17.17	10.23	7.26	5.66	2.90
$P_\sigma\ CUSUM$ (k = 0.38, h = 4.28)	201.10	41.04	17.15	10.21	7.24	5.65	2.98
$CUSUM\ S$ (k = 1.034, h = 1.90)	200.60	38.80	16.85	10.36	7.50	5.85	3.01
$CP\ CUSUM$ (k = 1.193, h = 18.45)	200.76	34.60	14.14	8.42	5.93	4.58	2.20
$P_\sigma\ CUSUM$ + FIR (k = 0.38, h = 4.403)	203.26	34.80	12.90	6.99	4.82	3.64	1.86
$\chi\ CUSUM$ + FIR (k = 0.38, h = 4.398)	198.28	34.66	12.59	6.95	4.75	3.63	1.86
$CP\ CUSUM$ + FIR (k = 1.193, h = 18.95)	201.76	28.22	9.96	5.76	3.94	3.06	1.59

[1] Shewhart chart with lower warning limit h_1r and lower action limit h_2r.

Table 05.5: ARL comparison of dispersion charts for negative shift in process standard deviation.

Dispersion charts (n = 5)	Percentage decrease in standard deviation				
	0	10	20	30	40
Shew. R (LCL = 0.55)	200.28	133.61	85.37	51.75	29.41
Shew. S (LCL = 0.23)	200.01	133.34	85.37	51.65	29.24
Shew.[1] S (h_1 = 0.47, h_2 = 0.06)	200	101.24	49.48	23.56	11.16
EWMA ln S^2 (k = 2.22, , λ = 0.10)	201	50.01	20.67	11.87	7.89
$CUSUM\ \ln S^2$ (k = 0.43, h = 5.49)	200.01	47.47	18.96	10.78	7.17
P_σ $CUSUM$ (k = 0.23, h = 5.76)	201.1	44.69	17.58	10.14	6.94
χ $CUSUM$ (k = 0.23, h = 5.75)	201.2	44.35	17.41	10.05	6.92
$CUSUM\ R$ (k = 2.093, h = 4.34)	200.95	45.25	17.41	9.95	6.88
$CUSUM\ S$ (k = 0.846, h = 1.70)	200.15	44.63	17.01	9.7	6.7
$CP\ CUSUM$ (k = 0.793, h = 11.66)	199.64	38.38	14.15	8.24	5.96
P_σ $CUSUM$ + FIR (k = 0.23, h = 6.085)	201.33	34.08	11.03	5.93	3.97
χ $CUSUM$ + FIR (k = 0.23, h = 5.94)	200.33	33.86	10.78	5.86	3.96
$CP\ CUSUM$ + FIR (k = 0.793, h = 11.99)	201.77	30.1	8.9	4.75	3.39

[1] Shewhart chart with lower warning limit h_1r and lower action limit h_2r.

V. The 0.50 quantile (median) of the run length is lesser than the fixed ARL_0 of 200, meaning that there is 50% chance of the median producing a false alarm in the first 134 samples (approximately) while a false alarm occurs on the average of every 200 samples (Tables 5.1 – 5.2).

VI. Generally, the performance measures indicate a substantial gain in efficiency of the proposed charts.

VII. CP CUSUM with FIR feature is the most efficient charts among the proposed charts in detecting small shift (increase or decrease) in process dispersion.

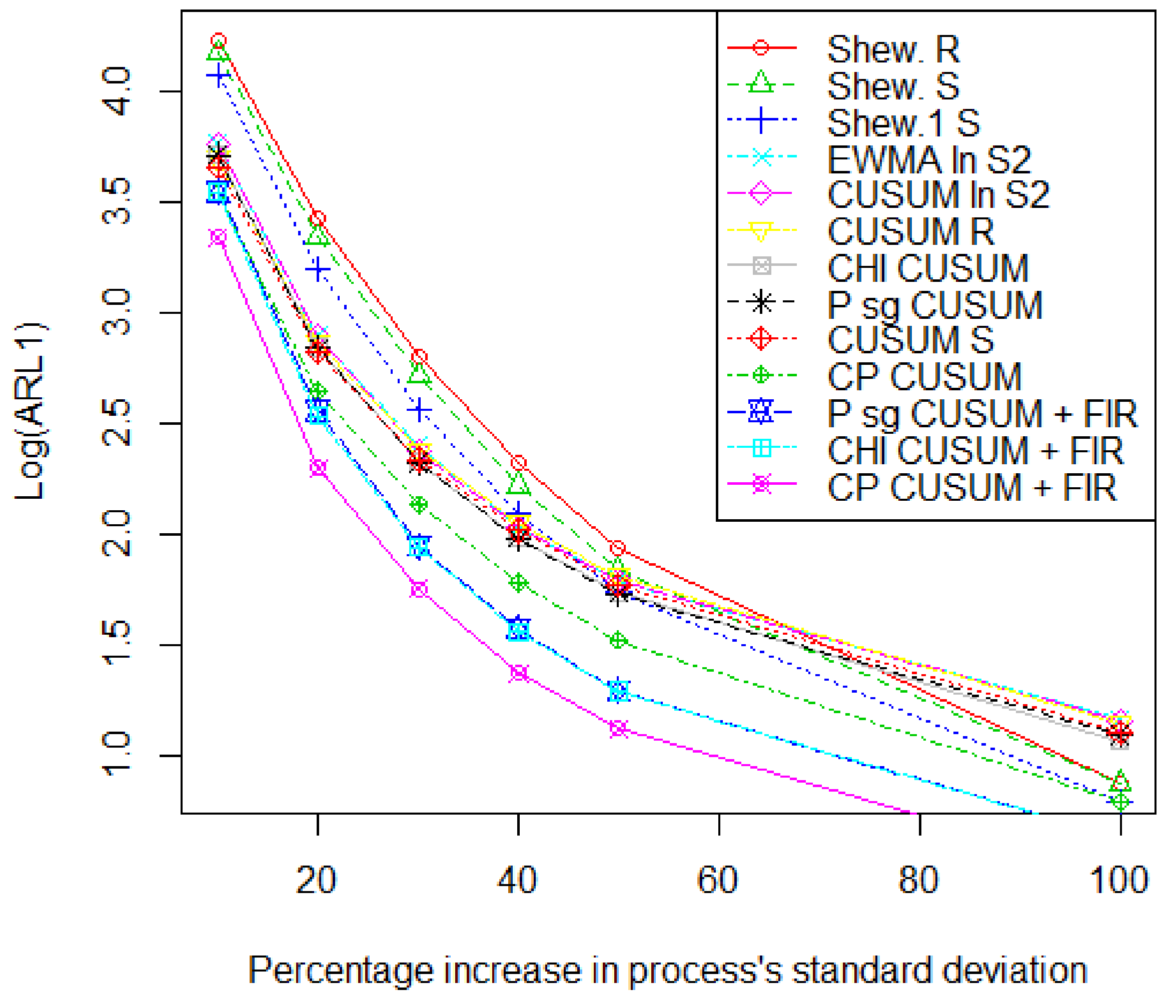

Figure 5.1: ARL curves of the proposed charts and some existing charts for positive shift in standard deviation.

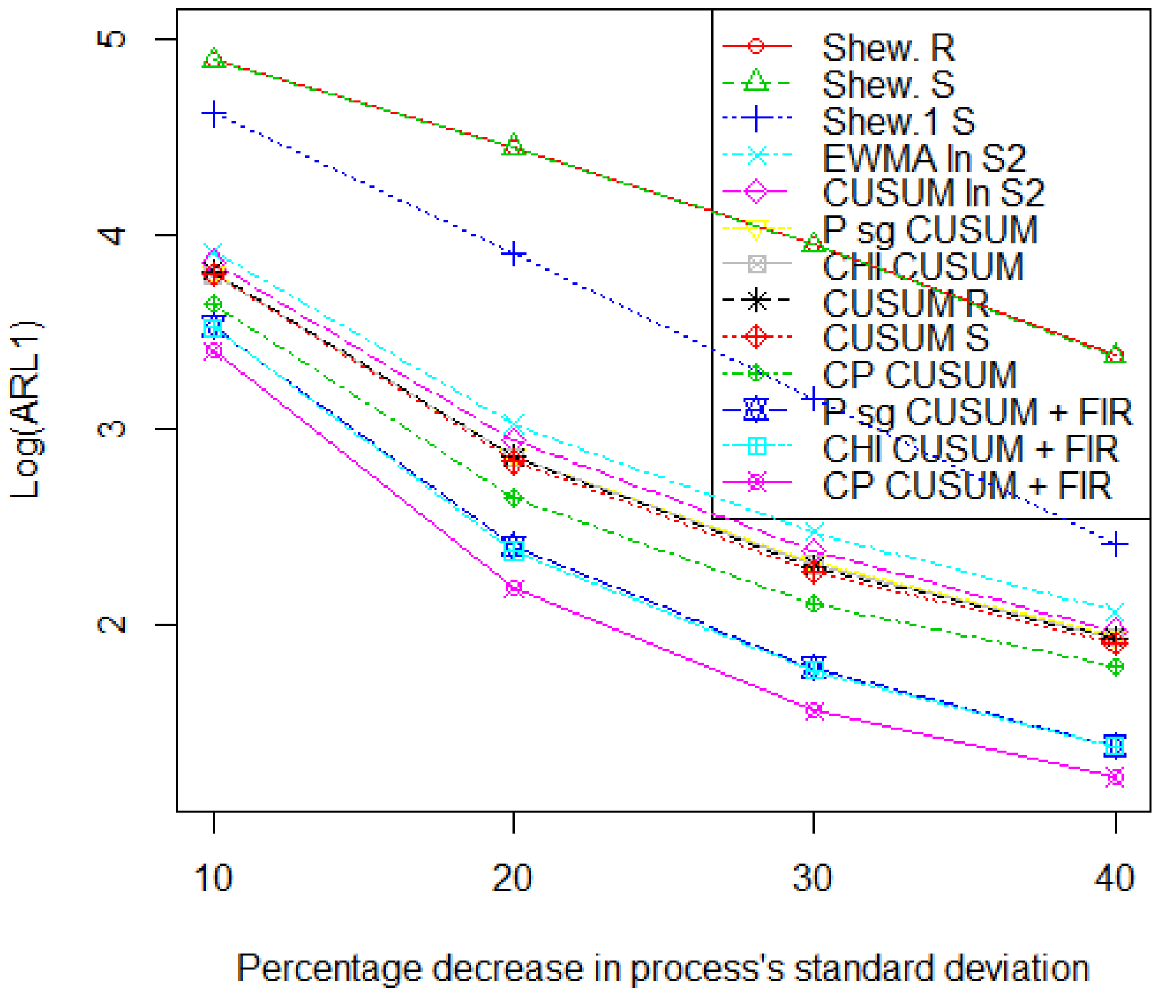

Figure 5.2: ARL curves of the proposed charts and some existing charts for negative shift in standard deviation.

Using the discussed measures, the proposed charts are compared with some existing charts for detecting shift in process dispersion. One-sided CUSUM structures are considered with a target ARL_0 of 200. The shift in the process dispersion is considered in terms of the percentage change in the process standard deviation, while the process mean is assumed stable. The existing charts taken into consideration are ; the Shewhart R, the Shewhart S chart, the Page's (1963) Shewhart S chart with warning lines, the CUSUM of the range R, CUSUM of the standard deviation S, the EWMA of $\ln S^2$ (Crowder & Hamilton, 1992), the CUSUM of $\ln S^2$ (Chang & Gan, 1995), and the P_σ *CUSUM*, the $\chi - CUSUM$ and the *CP CUSUM* of Acosta-Mejia et al. (1999).

The reference values (*k*) and the plotting statistics of the charts considered are standardized to be independent of any value of σ_0. In the CUSUM S chart, we have

$$C_i^+ = \max\left\{0, \left(S_i/\sigma_0\right) - k_1 + C_{i-1}^+\right\}$$

where $k_1 = c_4\left\{1 + \left(\sigma_1/\sigma_0\right)\right\}/2$, for the upper one-sided plotting statistic. Similarly, the upper one-sided plotting statistic of the CUSUM of R chart is given as

$$C_i^+ = \max\left\{0, \left(R_i/\sigma_0\right) - k_1 + C_{i-1}^+\right\}$$

where $k_1 = d_2\left\{1 + \left(\sigma_1/\sigma_0\right)\right\}/2$.

Table 5.4 (and Figure 5.1) presents the ARL comparison of the charts in detecting 20% increase in process standard deviation, with subgroup of size 5. The charts are arranged in ascending order of their respective performance. Supplementing the Shewhart S chart with warning line gives a better performance than the Shewhart S chart and the Shewhart R chart. EWMA $\ln S^2$ chart gives a better performance than the Shewhart's charts in detecting increase in process's standard deviation, but it is outperformed by the *CUSUM* $\ln S^2$ chart. However, the *CUSUM R*, the $\chi - CUSUM$, the P_σ *CUSUM*, the *CUSUM S* and the CP CUSUM all outperformed the *CUSUM* $\ln S^2$ chart, but perform worse than the $P_\sigma\ CUSUM$ with FIR, the $\chi - CUSUM$ with FIR and the CP CUSUM with FIR.

Table 5.5 (and Figure 5.2) gives the ARL performance of the charts in detecting 20% decrease in process standard deviation, with fixed ARL_0 200 and subgroup of size 5. The one-sided plotting statistic of the CUSUM S chart in detecting decrease in process standard deviation is given as

$$C_i^- = \max\left\{0, k - \left(R_i / \sigma_0\right) + C_{i-1}^-\right\}$$

while the one-sided plotting statistic of the CUSUM R chart for detecting decrease in process standard deviation is given as

$$C_i^- = \max\left\{0, k - \left(S_i / \sigma_0\right) + C_{i-1}^-\right\}.$$

The CUSUM R gains advantage over the $\chi - CUSUM$ and the $P_\sigma\ CUSUM$ in detecting decrease in σ (unlike when detecting increase in σ). The $P_\sigma\ CUSUM$, the $\chi - CUSUM$, the CUSUM R and the CUSUM S are comparable and show better performance than the Shewhart's charts, the EWMA $\ln S^2$ chart and the CUSUM $\ln S^2$ chart, but perform worse than the CP CUSUM, the $P_\sigma\ CUSUM$ with FIR, the $\chi - CUSUM$ with FIR and the CP CUSUM with FIR. Consistently, the charts with FIR features give the best performance, with CP CUSUM + FIR having the overall best performance.

5.4 SUMMARY AND CONCLUSION

Control chart is one of the tools of quality control to monitor production process, and to distinguish between assignable causes and chance causes of variation. The variation may be due to change in location parameter and/or dispersion parameter of a process. Few works has been done on monitoring dispersion parameter of a process. The work of Acosta-Mejia et al. (1999) was improved to efficiently detect shift in dispersion parameter at start-up by applying the First Initial Response proposed by Lucas and Crosier (1982). The proposed charts do not only detect shifts in process dispersion faster, but also have better overall performance than their charts and some other existing charts for monitoring process dispersion. Performance measures such as ARL, SDRL, quantile, EQL, RQRL and PCI are used for comparison.

CHAPTER 6

SUMMARY AND CONCLUSION

A new two-sided CUSUM charts which are based on the utilization of auxiliary information are proposed. The ARL performance of the proposed charts is evaluated in terms of shifts in study variable and compared with some recently designed control structures meant for the same purposes. The comparisons revealed that the proposed charts perform really well relative to the other charts under discussion, and a real life industrial example is provided to describe the application procedure of the proposal.

Furthermore, the Shewhart chart and the cumulative sum (CUSUM) chart are traditionally used for detecting large shifts and small shifts respectively, while the Combined Shewhart CUSUM (CSC) monitors small shifts and large shifts simultaneously. Using auxiliary information, new CSC (M_iCSC, i = 2, 3, …, 10) charts with more efficient estimators (the Regression-type estimator, the Ratio estimator, the Singh and Tailor estimator, the power ratio-type estimator and the Kadilar and Cingi estimators) for estimating location parameters are proposed. The charts are compared using Average Run Length (ARL), Standard Deviation Run Length (SDRL) and Extra Quadratic Loss (EQL), with other existing charts of the same purpose, and it is shown that some of the M_iCSC charts outperform their counterparts. A real-life industrial example is provided to show the efficiency and the application of the proposed charts.

In addition, it is known that statistical process control deals with monitoring process to detect disturbance in the process. The disturbance may be from the process mean or variance. We assume that the process mean is stable and propose some charts that are efficient for detecting early shifts in dispersion parameter, by applying the First Initial

Response (FIR) feature. Performance measures such as average run length (ARL), standard deviation run length (SDRL), extra quadratic length (EQL), relative average run length (RARL) and performance comparison index (PCI) are used to compare the proposed charts with their existing counterparts including the Shewhart R, the Shewhart S chart, the Shewhart S chart with warning lines, the CUSUM of the range R, CUSUM of the standard deviation S, the EWMA of $\ln S^2$, the CUSUM of $\ln S^2$, the $P_\sigma\ CUSUM$, the $\chi - CUSUM$ and the $CP\ CUSUM$. The proposed charts do not only detect early shifts in process dispersion faster, but also have better overall performance than their existing counterparts.

References

Abbas, N., Riaz, M., & Does, R. J. M. M. (2011). Enhancing the performance of EWMA charts. *Quality and Reliability Engineering International*, *27*(6), 821–833.

Abbas, N., Riaz, M., & Does, R. J. M. M. (2014). An EWMA-Type Control Chart for Monitoring the Process Mean Using Auxiliary Information. *Communications in Statistics - Theory and Methods*, *43*(16), 3485–3498. doi:10.1080/03610926.2012.700368

Abbasi, S. A., Riaz, M., & Miller, A. (2012). Enhancing the performance of CUSUM scale chart. *Computers & Industrial Engineering*, *63*(2), 400–409.

Abujiya, M. R., Lee, M. H., & Riaz, M. (2015). Increasing the Sensitivity of Cumulative Sum Charts for Location. *Quality and Reliability Engineering International*, *31*(6), 1035–1051. doi:10.1002/qre.1661

Abujiya, M. R., Riaz, M., & Lee, M. H. (2013). Improving the Performance of Combined Shewhart–Cumulative Sum Control Charts. *Quality and Reliability Engineering International*, *29*(8), 1193–1206.

Acosta-Mejia, C. a., Pignatiello, J. J., & Venkateshwara Rao, B. (1999). A comparison of control charting procedures for monitoring process dispersion. *IIE Transactions*, *31*(6), 569–579. doi:10.1080/07408179908969859

Adebola, F. B., Adegoke, N. A., & Sanusi, R. A. (2015). A Class of Regression Estimator with Cum-Dual Ratio Estimator as Intercept. *International Journal of Probability and Statistics*, *4*(2), 42–50.

Ahmad, S., Abbasi, S. A., Riaz, M., & Abbas, N. (2014). On efficient use of auxiliary information for control charting in SPC. *Computers and Industrial Engineering*, *67*(1), 173–184. doi:10.1016/j.cie.2013.11.004

Ahmad, S., Riaz, M., Abbasi, S. A., & Lin, Z. (2014). On efficient median control charting. *Journal of the Chinese Institute of Engineers*, *37*(3), 358–375.

Busaba, J., Sukparungsee, S., Areepong, Y., & Mititelu, G. (2012). Analysis of average run length for CUSUM procedure with negative exponential data. *Chiang Mai Journal of Science*, *39*(2), 222–230.

Chang, T. C., & Gan, F. F. (1995). Cumulative sum control chart for monitoring process variance. *Scholarbank.nus.edu.sg*.

Cochran, W. G. (1953). Sampling techniques.

Constable, C. G., & Parker, R. L. (1988). Statistics of the geomagnetic secular variation for the past 5 m.y. *Journal of Geophysical Research*, *93*(B10), 11569. doi:10.1029/JB093iB10p11569

Cox, M. A. A. (2010). Average run lengths of control charts for monitoring observations from a Burr distribution. *The Journal of Risk Finance*, *11*(5), 508–514.

Crowder, S. V. (1989). Design of exponentially weighted moving average schemes. *Journal of Quality Technology*, *21*(3), 155–162.

Crowder, S. V, & Hamilton, M. D. (1992). An {EWMA} for monitoring a process standard deviation. *Journal of Quality Technology*, *24*(1), 12–21. Retrieved from http://cat.inist.fr/?aModele=afficheN&cpsidt=5111232

Ewan, W. D., & Kemp, K. W. (1960). Sampling inspection of continuous processes with no autocorrelation between successive results. *Biometrika*, *47*(3/4), 363–380.

Fuh, C.-D. (2003). SPRT and CUSUM in hidden Markov models. *Annals of Statistics*, 942–977.

Fuller, W. A. (2011). *Sampling statistics* (Vol. 560). John Wiley & Sons.

Gibbons, R. D. (1999). Use of Combined Shewhart-CUSUM Control Charts for Ground Water Monitoring Applications. *Wiley Online Library*, *37*(5), 682–691.

Gupta, S., & Shabbir, J. (2008). On improvement in estimating the population mean in simple random sampling. *Journal of Applied Statistics*, *35*(5), 559–566. doi:10.1080/02664760701835839

Haq, A. (2013). A new hybrid exponentially weighted moving average control chart for monitoring process mean. *Quality and Reliability Engineering International*, *29*(7), 1015–1025.

Hawkins, D. M., & Olwell, D. H. (1998). *Cumulative sum charts and charting for quality improvement*. Springer Science & Business Media.

Henning, E., Samohyl, R. W., Walter, O. M. F. C., & Konrath, A. C. (2015). Performance of a Combined Cusum-Shewhart Chart for Binomial Data for Large Shifts in the Process Mean. *Int. Journal of Engineering Research and Application*, *5*(8), 235–243.

Jamali, A. S., Jinlin, L., & Durad, M. H. (2006). Average run length performance of Shewhart control charts with interpretation rules. In *Industrial Informatics, 2006 IEEE International Conference on* (pp. 1329–1333). IEEE.

Kadilar, C., & Cingi, H. (2004). Ratio estimators in simple random sampling, *151*, 893–902. doi:10.1016/S0096-3003(03)00803-8

Kadilar, C., & Cingi, H. (2006a). Improvement in estimating the population mean in simple random sampling. *Applied Mathematics Letters*, *19*(1), 75–79.

Kadilar, C., & Cingi, H. (2006b). New ratio estimators using correlation coefficient. *Inter Stat*, *4*(March), 1–11.

Li, S.-Y., Tang, L.-C., & Ng, S.-H. (2010). Nonparametric CUSUM and EWMA control charts for detecting mean shifts. *Journal of Quality Technology*, *42*(2), 209.

Lucas, J. M. (1982). Combined Shewhart-CUSUM quality control schemes. *Journal of Quality Technology*, *14*(2).

Lucas, J. M., & Crosier, R. B. (1982). Fast initial response for CUSUM quality-control schemes: give your CUSUM a head start. *Technometrics*, *24*(3), 199–205.

Midi, H., & Shabbak, A. (2011). Robust multivariate control charts to detect small shifts in mean. *Mathematical Problems in Engineering*, *2011*.

Montgomery, D. (2009). *Introduction to statistical quality control*. *John Wiley & Sons Inc.* doi:10.1002/1521-3773(20010316)40:6<9823::AID-ANIE9823>3.3.CO;2-C

Montgomery, D. C. (2007). *Introduction to statistical quality control*. John Wiley & Sons.

Nazir, H. Z., Riaz, M., Does, R. J. M. M., & Abbas, N. (2013). Robust CUSUM Control Charting. *Quality Engineering*, *25*(3), 211–224. doi:10.1080/08982112.2013.769057

Ng, C. H. (1988). Development and evaluation of control charts using Exponentially Weighted Moving Averages. Oklahoma State University.

Ou, Y., Wu, Z., & Tsung, F. (2012). A comparison study of effectiveness and robustness of control charts for monitoring process mean. *International Journal of Production Economics*, *135*(1), 479–490.

Page, E. S. (1954). Continuous inspection schemes. *Biometrika*, 100–115.

Page, E. S. (1963). Controlling the standard deviation by CUSUMS and warning lines. *Technometrics*, *5*(3), 307–315.

R Core Team (2014). R: A language and environment for statistical computing. *R Found*

ation for Statistical Computing, *Vienna*, *Austria*. URL http://www.R-project.org/.

Reynolds, M. R., & Stoumbos, Z. G. (2010). Robust CUSUM charts for monitoring the process mean and variance. *Quality and Reliability Engineering International*, *26*(5), 453–473.

Riaz, M. (2008a). Monitoring process mean level using auxiliary information. *Statistica Neerlandica*, *62*(4), 458–481.

Riaz, M. (2008b). Monitoring process variability using auxiliary information. *Computational Statistics*, *23*(2), 253–276.

Riaz, M. (2015). Control charting and survey sampling techniques in process monitoring. *Journal of the Chinese Institute of Engineers*, *38*(3), 342–354.

Riaz, M., Abbasi, S. A., Ahmad, S., & Zaman, B. (2014). On efficient phase II process monitoring charts. *The International Journal of Advanced Manufacturing*

Technology, *70*(9-12), 2263–2274.

Riaz, M., & Does, R. J. M. M. (2008). An alternative to the bivariate control chart for process dispersion. *Quality Engineering*, *21*(1), 63–71.

Riaz, M., & Does, R. J. M. M. (2009). A process variability control chart. *Computational Statistics*, *24*(2), 345–368.

Roberts, S. W. (1959). Control chart tests based on geometric moving averages. *Technometrics*, *1*(3), 239–250. doi:10.1080/00401706.1959.10489860

Shewharrt, W. A. (1931). *Economic Control of Quality of Manufactured Products*. New York: Macmillan.

Shewhart, W. A. (1924). Some applications of statistical methods to the analysis of physical and engineering data. *Bell System Technical Journal*, *3*(1), 43–87.

Singh, H. P., & Tailor, R. (2003). Use of known correlation coefficient in estimating the finite population mean. *Statistics in Transition*, *6*(4), 555–560.

Singh, R., & Mangat, N. S. (1996). *Elements of Survey Sampling* (Vol. 15). doi:10.1007/978-94-017-1404-4

Srivastava, S. K. (1967). An estimator using auxiliary information in sample surveys. *Calcutta Statistical Association Bulletin*, *16*(62-63), 121–132.

Steiner, S. H. (1999). Exponentially weighted moving average control charts with time varying control limits and fast initial response. *Journal of Quality Technology*, *31*(0), 1.

Sukhatme, P. V, & Sukhatme, B. V. (1970). *Sampling theory of surveys with applications*. London:[sn].

Tuprah, K., & Ncube, M. (1987). A comparison of dispersion quality control charts. *Sequential Analysis*, *6*(2), 155–163.

Wald, A. (1973). *Sequential analysis*. Courier Corporation.

Westgard, J. O., Groth, T., Aronsson, T., & de Verdier, C. H. (1977). Combined Shewhart-CUSUM control chart for improved quality control in clinical chemistry. *Clinical Chemistry*, *23*(10), 1881–7. Retrieved from http://www.ncbi.nlm.nih.gov/pubmed/902415

Wilson, E. B., & Hilferty, M. M. (1931). The distribution of chi-square. *Proceedings of the National Academy of Sciences*, *17*(12), 684–688.

Wu, Z., Jiao, J., Yang, M., Liu, Y., & Wang, Z. (2009). An enhanced adaptive CUSUM control chart. *IIE Transactions*, *41*(7), 642–653.

Wu, Z., Yang, M., Jiang, W., & Khoo, M. B. C. (2008). Optimization designs of the combined Shewhart-CUSUM control charts. *Computational Statistics and Data*

Analysis, *53*(2), 496–506. doi:10.1016/j.csda.2008.08.032

Zhang, C. W., Xie, M., & Jin, T. (2012). An improved self-starting cumulative count of conforming chart for monitoring high-quality processes under group inspection. *International Journal of Production Research*, *50*(March 2015), 7026–7043. doi:10.1080/00207543.2011.649305

Zhang, G. (1992). *Cause-selecting Control Chart and Diagnosis: Theory and Practise*. aarhus School of Business.

Zhao, Y., Tsung, F., & Wang, Z. (2005). Dual CUSUM control schemes for detecting a range of mean shifts. *IIE Transactions*, *37*(11), 1047–1057. doi:10.1080/07408170500232321

Vitae

Name :SANUSI, Ridwan Adeyemi

Nationality :Nigerian

Date of Birth :7/17/1989

Email :amhigher2010@yahoo.com

Address :A216, Ifelodun street, Ajagba, Wakajaye, Iyana-Church, Iwo-Road, Ibadan, Nigeria

Academic Background :Click here to enter text.

EDUCATION

2014 – 2016 **King Fahd University of Petroleum and Minerals, Department of Mathematics**, Dhahran, Saudi Arabia. M.Sc. in Statistics [CGPA: 3.929/4.000]

2008 – 2012 **University of Ibadan, Department of Mathematics & Statistics**, Ibadan, Nigeria. B.Sc. (Hons) in Statistics [First Class]

2007 – 2009 **University of Ibadan, Department of Mathematics & Statistics**, Ibadan, Nigeria. Professional Diploma in Statistics [Credit]

PEER-REVIEWED ARTICLES

- **Ridwan A. Sanusi,** Muhammad Riaz, Nasir Abbas and Mu'azu Ramat Abujiya. 'Using FIR to improve CUSUM charts for monitoring process dispersion'. (Under preparation).

- **Ridwan A. Sanusi,** Mu'azu Ramat Abujiya, Muhammad Riaz and Nasir Abbas. 'Combined Shewhart CUSUM Charts using Auxiliary Variable'. *Quality and Reliability Engineering International* (Under review).

- **Ridwan A. Sanusi,** Muhammad Riaz and Nasir Abbas 'An Efficient CUSUM-Type Control Chart for Monitoring the Process Mean Using Auxiliary Information'. *Quality Technology and Qualitative Management* (Under review).

- **Sanusi, RA**, Adebola, FB, Adegoke, NA. (2016). "Cases of Road Traffic Accident in Nigeria: A Time Series Approach." *Mediterranean Journal of Social Sciences 7.2 S1 (2016): 542.*
- Adebola, FB, **Sanusi, RA**, Adegoke, NA. (2015). 'Consequences of road traffic accident in Nigeria: Time series approach.' *International Journal of Computer Applications Technology and Research,* 4 (4), pp. 262-273.
- Adebola, FB, Adegoke, NA, **Sanusi, RA.** (2015). 'A Class of Regression Estimator with Cum-Dual Ratio Estimator as Intercept.' *International Journal of Probability and Statistics*, 4(2), pp. 42-50. doi: 10.5923/j.ijps.20150402.02.

CONFERENCE ARTICLE

- **Sanusi, RA,** Riaz, M, Adegoke, NA. (2016). 'A New EWMA-Type Control Chart for Monitoring Location Parameter Using Auxiliary Information In The Form of Ratio Estimator'. *UAE MATH DAY 2016, New York University, Abu Dhabi, UAE*.