ON EFFICIENT APPROACHES TO DESIGN UNIVARIATE AND MULTIVARIATE CONTROL CHARTS FOR PROCESS MONITORING

BY

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To my parents

wife

sisters and brothers

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Firstly, I give gratitude to the almighty Allah, who taught a pen what it knew not. May the peace and blessing of Allah be showered on our noble prophet Mohammed (Sallahu Alayhi Wa Salam) whom Allah sent to guide the mankind from the darkness into light.

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LIST OF ABBREVIATIONS

ARLAverage Run Length

FIR **Fast Initial Response**

MECFIR **Mixed EWMA-CUSUM with modified FIR feature MECHS**

Mixed EWMA-CUSUM with headstart

MECFIRHS **Mixed EWMA-CUSUM with modified FIR and headstart**

SPC Statistical Process Control

CUSUM	**Cumulative Sum**
EWMA	**Exponential Weighted Moving Average**
MCUSUM	**Multivariate Cumulative Sum**
MEWMA	**Multivariate Exponential Weighted Moving Average**
MEC	**Mixed Multivariate EWMA-CUSUM**
MC1	**Multivariate Cumulative Sum 1**
UCL	**Upper Control Limit**
LCL	**Lower Control Limit**
SS-CUSUM	**Sum of Square Cumulative Sum**
SS-EWMA	**Sum of Square Exponential Weighted Moving Average**
Max-CUSUM	**Maximum Cumulative Sum**
Max-EWMA	**Maximum Exponential Weighted Moving Average**
SS-EWMAVAR	**A new sum of square EWMA**
Max-EWMAVAR	**A new maximum EWMA control chart**
Max-CUSUMVAR	**A new maximum CUSUM control chart**
SS-CUSUMVAR	**Anew sum of square CUSUM control chart**

ABSTRACT

Full Name : [Ajadi Jimoh Olawale]

Thesis Title : [On Efficient Approaches to Design Univariate and Multivariate Control Charts for Process Monitoring]

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The control chart is an important statistical technique that is used to monitor the quality of a process mean or dispersion. Shewhart control charts are used to detect larger disturbances in the process parameters, whereas CUSUM and EWMA are meant for smaller and moderate changes. Sometimes, we are interested in monitoring more than one quality characteristics; then we use the multivariate control chart like Hotelling's T^2, MEWMA, MCUSUM and MC1 control charts.

In this thesis, we propose different new univariate and multivariate control charts that monitor location, dispersion or both. The performances of the proposed charts are compared by calculating the run-length properties of each chart.Application examples will also be presented for practical considerations using a real dataset.

ملخص الرسالة

الاسم الكامل: أجدى جيمو**اولاوالي**

عنوان الرسالة: على النهج الفعالة لتصميم المخططات البيانية ذوات أحادي المتغير والمتغيرات المتعددة لرصد عملية

التخصص:الإحصاء التطبيقي

تاريخ الدرجة العلمية:: مايو **٢٠١٥**

مخطط التحكم هو تقنية احصائية مهمة كونها تستخدم لمراقبة جودة متوسط العملية أو التشتت. مخطط شيوارت التحكمي يستخدم للكشف عن الاضطرابات الكبيرة في معايير العمليات حيث أن CUSUM و EWMA تعنى بالتغيرات الصغيرة والمتوسطة. نهتم أحيانا بمراقبة أكثر من واحدة من خصائص الجودة ومن ثم نستخدم مخططات التحكم مثل مخططات Hotelling'sT^2 وMEWMAوMCUSUM و MC1.

في هذه الرسالة تم اقتراح مخططات التحكم الوحيدة والمتعددة المتغيرات والتي تراقب الموقع أو التشتت أو كليهما. تم مقارنة أداء المخطط المقترح عن طريق حساب خصائص طول التشغيل لكل مخطط. أمثلة تطبيقية سوف يتم عرضها لاعتبارات عملية باستخدام مجموعة من البيانات الحقيقية.

CHAPTER 1

INTRODUCTION

Statistical Process Control (SPC) is a collection of useful tools that help us to differentiate between natural and special cause variations. The process is in-control when the natural variation Control chart is the most effective SPC tools. Control chart is used to detect the presence of unnatural variations in the process. In the past, there was no universal agreement on how control charts to be used. Different companies had different rules. Today and with the massive exchange of business among countries with different level of quality, a set of regulatory international standards were developed and widely accepted. This is in addition to the international regulatory standard. Control chart is divided into memoryless and memory type control charts.

1.1 Memoryless Control Charts

Shewhart(1931) is a memoryless type control chart because it is based on only the present information plotting statistic. The major disadvantage of Shewhart control chart is that it is poor in detecting small and moderate shifts in a process parameter.

1.2 Memory-Type Control Charts

Exponential weighted moving average (EWMA) and Cumulative sum (CUSUM) are the two examples of memory-type control charts that use both the past and present information to detect small and moderate shifts in the process parameter. EWMA was first discovered by Robert (1959). The idea of fast initial response (FIR) of EWMA was later developed by Steiner(1999) which lowers the time –varying limits for the first few sample observations. CUSUM was developed by page(1954). Lucas and Crosier [13] improved the CUSUM with the use of Head start which increases the sensitivity of a CUSUM at the beginning of the process. Riaz et al. (2011) also improves the performance of CUSUM charts. Abbas et al. (2013a) proposed a new mixed EWMA-CUSUM chart that combines both EWMA and CUSUM setups. It is very good in detecting small shift in a process but less effective in detecting larger shift as compared to its counterparts. Haq et al.(2014) improved the performance of FIR by Steiner (1999) by using a power transformation with respect to time t. Castagliola et al (2009&2010) propose A New CUSUM-S^2 Control Chart for Monitoring the Process Variance and A Johnson's Type Transformation EWMA-S^2 Control Chart. Abbas et al. (2013b) proposed CS-EWMA Chart for monitoring process dispersion.

Previously, monitoring the process location and dispersion required plotting two different charts separately. Xie(1999) proposed a chart that combine the monitoring process location and dispersion on a single chart in univariate and multivariate control charts and named them Max-EWMA and Max-MEWMA respectively.

1.3 Multivariate Control Charts

Most of the manufacturing or business process has two or more correlated quality characteristics to monitor simultaneously. For example, inner diameter, thickness and length of the tubes can be three correlated quality characteristics will be monitor in the manufacturing process of specific carbon fiber tubing. Though, they can be monitor individually like in the univariate set-up but its drawback is that, it is time consuming and also inflates the probability of false alarm rate of special cause of variation. Hotelling's chi-square (1947) developed the control chart that monitors multivariate quality characteristic. This chart is a direct analog of Shewhart (1931) in univariate set-up. It is only based on presence information. It is insensitive to detect small and moderate shifts in the process parameter.

The most common memory type multivariate control charts are MEWMA, MCUSUM and MC1, they use both the past and present information to detect shift in the process parameter. These control charts are better than Hotelling's T^2 control chart when we are interested in the small and moderate shift in the process parameter.

CHAPTER 2

ON INCREASING THE SENSITIVITY OF MIXED EWMA-CUSUM CONTROL CHARTS FOR LOCATION PARAMETER AND ITS INDUSTRIAL APPLICATION

Control chart is an important statistical technique that is used to monitor the quality of a process. Shewhart control charts are used to detect larger disturbances in the process parameters, whereas CUSUM and EWMA are meant for smaller and moderate changes. In this study, we propose improved mixed EWMA- CUSUM control charts with varying Fast Initial Response (FIR) features and investigate their run length properties. The proposed control charting schemes are compared with the existing counterparts including classical CUSUM, classical EWMA, fast initial response CUSUM, fast initial response EWMA and the mixed EWMA – CUSUM control charts. A case study is presented for practical considerations using a real dataset.

2.1 INTRODUCTION

Statistical Process Control (SPC) is a collection of useful tools that help us to differentiate between two types of variation namely common and special cause variations. A process is declared in-control when natural variationsare present while it is deemed out-of-control in as much as the special cause variationsare also included in it. SPC has seven major tools, including: histogram, check sheet,Pareto chart, cause-and-effect diagram,defect concentration diagram,scatter diagram and control chart (cf. Montgomery (2009)). Control chart is the most effective SPC tool that is used to detect the presence of unusual variations in the process.In the past, there was no universal agreement on how control charts to be used. Each company had different rules. Today and with the massive exchange of business between countries with a different level of quality, a set of regulatory international standards are developed and widely accepted. Such standards like; ISO7870-1 (2014), ISO7870-2 (2013), ISO7870-3 (2012), ISO 7870-4 (2011), ISO7870-5 (2014) and ISO7870-6 (2014). This is in addition to the international regulatory standard ASTM D6299 (2013) and ASTM E2587 (2012).

Control chart is divided into two types of control charts namely memory-less and memory type charts. Shewhart type control chart (Shewhart (1931)) is a memory-less control chart because it is based on only the present information. A major limitation of Shewhart control charts is that it is poor in detecting small and moderate shifts in a process parameter. Exponential weighted moving average (EWMA) and Cumulative sum (CUSUM) are the two examples of memory-type control charts that use both the previous and current information to detect small and moderate shifts in the process parameters. In EWMA charts, the past observations are accounted for, but they are given a smaller weight as they become older, so EWMA chart applies the most weight to the current observations and geometrically decreasing weight to all previous observations. CUSUM charts also give memory by using the information of past history. EWMA was originally discovered by Robert (1959). The idea of fast initial response (FIR) of EWMA was later developed by Steiner (1999) which lowers the time –varying limits for the first few sample observations. CUSUM was developed by page (1954). Lucas and Crosier (1982) improved the CUSUM with the use of Head start, which increases the sensitivity of a CUSUM at the beginning of the process. Abbas et al. (2013) proposed a new mixed EWMA- CUSUM chart that combines both EWMA and CUSUM setups. It is very good at detecting a small shift in a process but less effective at detecting larger shifts as compared with its counterparts. Haq et al. (2014) improved the performance of FIR (cf. Steiner (1999)) by using a power transformation with respect to time t.

In this chapter, we intend to extend the structure of mixed EWMA-CUSUM chart of Abbas et al. (2013)using two extended features namely the head start and modified FIR. Moreover, we also combine headstart andthe Modified FIR feature with the mixed EWMA-CUSUM. The performance of each of these proposed charts is evaluated using run-length performance measure. Average run length (ARL) is an effective measure for evaluating and comparing the performance of the control charts. The in-control ARL of a control chart is denoted by ARL_0, and out-of-control ARL is denoted by ARL_1. For our study purposes, we have used Monte Carlo simulation approach to evaluate ARL performance of different control schemes covered under this study. We run the program in R language 50,000 times and in each time, the run length is calculated; and its average is also calculated for a change at different shifts.

The organization of the rest of the article is as" Section 2.2 describes different memory type control charts; Section 2.3 provides the design structure of the proposed control charts of this study; Section 2.4 offers comparisons of the proposals with the existing counterparts; Section 2.5 includes a case study for our study purposes; Section 2.6 concludes the findings of the study.

2.2 MEMORY TYPE CONTROL CHARTS

In this section we discuss different memory type control charts including CUSUM, EWMA and mixed EWMA-CUSUM control charts.

2.2.1The CUSUM control charts

CUSUM control charts are used in detecting small shifts in a process using cumulative deviation from the target value, μ_0. It is based on two statistics that are upper CUSUM, C_i^+ and lower CUSUM, C_i^- and they are initially set to be zero for classical CUSUM. The sample statistics C_i^+ and C_i^- are plotted against the control limits, H. A reference value k is also used that is taken to be half way of the shifts in the process, that is, $k = \frac{1}{2}\delta\sigma$. The lower the value of k, the more sensitive the CUSUM control chart is to the small shifts. Let X_i represents the ith observation ($n=1$) or the mean of each subgroup when $n>1$. The two CUSUM statistics are defined as:

$$C_i^+ = \max\left[0, (X_i - \mu_0) - k + C_{i-1}^+\right] \quad (2.1) \qquad C_i^- = \max\left[0, -(X_i - \mu_0) - k + C_{i-1}^-\right] \quad (2.2)$$

In the above structure of CUSUM we may also set the initial values at some other levels named headstart for fast initial response. This feature helps in quickly detecting a process that is off-target at the start-up. Table 2.1 gives the ARL values for CUSUM with and without headstart scheme at $ARL_0 = 500$ when $k = 0.5$.

Table 2.1: ARL values for CUSUM with and without Headstart scheme at $\mathbf{ARL_0 = 500}$

	$C = 0$	$C = 1$	$C = 1.5$	$C = 2$
δ	H=5.071	H=5.080	H=5.090	H=5.108
0.00	499.965	502.314	498.086	498.823
0.25	144.561	144.033	140.249	137.126
0.50	39.108	36.764	35.064	32.959
0.75	17.333	15.589	14.459	13.120
1.00	10.506	9.205	8.352	7.486
1.25	7.454	6.421	5.80	5.174
1.50	5.812	4.924	4.43	3.960
1.75	4.777	4.021	3.641	3.254
2.0	4.067	3.422	3.094	2.776

2.2.2 EWMA control charts

EWMA chart was first developed by Roberts (1959) that uses both past and current information. It is defined by the statistic:

$Z_i = \lambda X_i + (1-\lambda) Z_{i-1}$ (2.3) here, λ is the constant that ranges between 0 and 1 (i.e. $0 < \lambda \le 1$). The smaller the value of λ the more sensitive is the chart is to the smaller shifts. The quantity Z_0 is the starting value which is given as target mean μ_0 or the mean of the previous data (from phase 1) The statistic Z_i is plotted against the upper and lower control limits (UCL and LCL respectively) given below:

$$LCL = \mu_0 - L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (2.4)$$

$$UCL = \mu_0 + L\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)} \quad (2.5)$$

where σ is process standard deviation and L is the control limits coefficient that helps in fixing the value of ARL_0 at a pre-specified level.

FIR-EWMA scheme (cf. Steiner (1999)) uses a control chart feature that lowers the time-varying limits. The advantage of using this chart is the detection of the out of control process at early stages. It helps in the reduction of ARL values. Steiner (1999) chose $a = \frac{1}{19}\left[\frac{-2}{\log\left(1-f\right)} - 1\right]$ (where a and f are constants) so that the effect of the FIR feature is minimal after 20. We take the value f=0.5 to behave like 50% headstart normally used in CUSUM. The lower and upper control limits of FIR-EWMA are given as:

$$LCL = \mu_0 - L\sigma\left\{1-\left(1-f\right)^{1+a(i-1)}\right\}\sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)} \quad 2.6)$$

$$UCL = \mu_0 + L\sigma\left\{1-\left(1-f\right)^{1+a(i-1)}\right\}\sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)} \quad (2.7)$$

The ARL values for the classical EWMA control chart for various values of λ, at $ARL_0 = 500$, are given in Table 2.2.The ARL values for FIR-EWMA scheme at $ARL_0 = 500$ and $f = 0.5$ are given in Table 2.3.

Table2.2.ARL values for the CLASSICAL EWMA scheme at $\mathbf{ARL_0 = 500}$

λ=0.10	λ= 0.25	λ=0.50	λ=0.75

δ	L=2.822	L=3.000	L=3.072	L=3.088
0.00	496.076	499.220	499.771	501.203
0.25	102.779	168.925	254.346	321.150
0.50	28.665	47.119	88.990	139.584
0.75	13.584	19.269	35.577	62.489
1.00	8.163	10.399	17.199	30.585
1.25	5.626	6.698	9.748	16.515
1.50	4.161	4.761	6.245	9.863
1.75	3.254	3.666	4.458	6.366
2.00	2.655	2.939	3.383	4.431

Table 2.3. ARL for the FIR-EWMA scheme at $\mathbf{ARL_0 = 500}$ and, f=0.5

	λ= 0.10	λ= 0.25	λ=0.50	λ=0.75
δ	L=2.913	L=3.078	L=3.149	L=3.167
0.00	498.323	507.747	497.466	503.102
0.25	90.224	152.111	234.905	305.008
0.50	21.701	34.978	66.708	110.508
0.75	8.807	11.751	20.239	38.123
1.00	4.735	5.513	7.802	13.629
1.25	3.031	3.265	3.899	5.651
1.50	2.211	2.313	2.512	3.039
1.75	1.721	1.804	1.868	2.054
2.00	1.459	1.507	1.538	1.606

2.2.3 Mixed Exponentially Weighted Moving Average-Cumulative Sum Charts

Mixed EWMA-CUSUM control chart was introduced by Abbas et. al (2013). The EWMA statistic Z_i, defined as: $Z_i = \lambda X_i + (1-\lambda) Z_{i-1}$, is combinedwith the CUSUM structure. The Mixed EWMA-CUSUM is defined by two statistics which are upper CUSUM, M_i^+, and lower CUSUM, M_i^-.They are initially set to be zero for classical mixed EWMA-CUSUM and their values depend on the EWMA statistic Z_i.

$$M_i^+ = \max\left[0, (\mathrm{Z}_i - \mu_0) - a_i + M_{i-1}^+\right] \quad (2.8)$$

$$M_i^- = \max\left[0, -(\mathrm{Z}_i - \mu_0) - a_i + M_{i-1}^-\right] \quad (2.9)$$

where a_i is a time-varying reference value for the mixed EWMA-CUSUM charting structure and it is given as,

$a_i = a^*\sqrt{\mathrm{var(Z}_i)}$ where $\mathrm{var(Z}_i) = \sigma^2 \dfrac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)$ and a^* is just like k in classical CUSUM; we set $a^* = 0.5$. The control limit for this chart is given as,

$$b_i = b^*\sqrt{\mathrm{var(Z}_i)} = b_i = b^*\sigma\sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)} \quad (2.10)$$

where b^* is a constant like h in classical CUSUM and both M_i^+ and M_i^- are plotted against the control limit b_i. The ARL values for the mixed EWMA-CUSUM scheme at $ARL_0 = 500$ are given in Table 2.4.

Table 2.4. ARL for the Mixed EWMA-CUSUM scheme with a*=0.5 at ARL$_0$=500

	λ= 0.10	λ=0.25	λ=0.50	λ=0.75
δ	$b^* =$ 37.42	20.18	11.20	7.30

0.00	501.234	503.124	506.355	502.166
0.25	80.141	84.314	99.964	119.219
0.50	35.529	30.768	30.553	33.207
0.75	24.028	18.839	16.642	16.443
1.00	18.845	13.890	11.475	10.643
1.25	15.815	11.228	8.866	7.879
1.50	13.787	9.603	7.277	6.306
1.75	12.328	8.429	6.260	5.293
2.0	11.180	7.599	5.521	4.583

2.3 THE PROPOSED CONTROL CHARTS

In this section we develop the design structure of the proposed schemes based on mixed EWMA CUSUM chart of Abbas et al. (2013). We have used two extended features named the head start and modified FIR (cf. Steiner (1999) and Haq et al. (2014)). We have also combined headstart andthe Modified FIR feature with the mixed EWMA-CUSUM. The proposals of this study include Mixed EWMA-CUSUM with headstart (MECHS); Mixed EWMA-CUSUM with modified FIR feature **(**MECFIR);Mixed EWMA-CUSUM with modified FIR and headstart (MECFIRHS). These are described below one by one.

2.3.1 Mixed EWMA-CUSUM with headstart (MECHS)

In the mixed EWMA-CUSUM chart the statistics M_i^+ and M_i^- are given in (2.8) and (2.9) and $b_i = b^* \sigma \sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)}$ as may be seen in Section 2.2.3 above. In order to improve the sensitivity of the mixed EWMA-CUSUM at process start-up we suggest a headstart based structure, namely MECHS,by assigning the initial values of M_0^+ and M_0^- to be 50% of the first value of the control limit (b_1). i.e. $M_0^+ = M_0^- = 0.5b_1$. The headstart of the mixed EWMA-CUSUM may be expressed as:

$$M_0^+ = M_0^- = 0.5b^* \sigma \sqrt{\frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^2\right)}$$

$$= 0.5b^* \sigma \sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-2\lambda+\lambda^2\right)\right)} = 0.5b^* \sigma \sqrt{\frac{\lambda}{2-\lambda}\left(1-1+2\lambda-\lambda^2\right)}$$

$$M_0^+ = M_0^- = 0.5b^* \sigma \sqrt{\frac{\lambda}{2-\lambda}\left(2\lambda-\lambda^2\right)} \quad (2.11)$$

and simplifying it further we have :

$$M_0^+ = M_0^- = 0.5\sigma b^* \lambda \quad (2.12)$$

2.3.2 Mixed EWMA-CUSUM with modified FIR feature (MECFIR)

In this subsection we introduce an improved FIR based mixed EWMA-CUSUM scheme, namely MECFIR,usinga modified FIR adjustments (MFIRadj).We combine the FIR structure of Haq et al. (2014) with the mixed EWMA-CUSUM in the form of MFIRadj. When MFIRadj is integrated in the mixed EWMA-CUSUM control chart, it helps increasing its sensitivity in detecting earlier shifts in the process parameters. The control limit of Mixed EWMA-CUSUM with MFIRadj scheme is defined as:

$$b_i = b^* \sigma \left\{1-\left(1-f\right)^{1+a(i-1)}\right\}^{1+\frac{1}{i}} \sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)} \quad (2.13)$$

Mixed EWMA-CUSUM with MFIRadj scheme is evaluatedby two statistics which are upper CUSUM, M_i^+ and lower CUSUM, M_i^- from equation (2.8) and (2.9) and they are initially set to be zero.

2.3.3 Mixed EWMA-CUSUM with modified FIR and headstart (MECFIRHS)

In this subsection we design the headstart for the mixed EWMA-CUSUM with modified FIR feature based on the above defined structure of MECFIR. We set the initial values at some other headstartlevels toquickly detect the changes in a process that is off-target at the start-up. We develop the FIR based mixed EWMA-CUSUM scheme with headstart, namely MECFIRHS,using the MFIRadj with a head start. We use 50% of the value of b in the first sample point to be the headstart.

From the above sections we know that $b_i = b^* \sigma \left\{1-(1-f)^{1+a(i-1)}\right\}^{1+\frac{1}{i}} \sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^{2i}\right)}$

and $M_0^+ = M_0^- = 0.5 b_1$. Based on these results the headstart of the mixed EWMA-CUSUM with modified FIR feature may be defined as:

$$M_0^- = M_0^+ = 0.5 b^* \sigma \left\{1-(1-f)\right\}^2 \sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-\lambda\right)^2\right)}$$

$$M_0^- = M_0^+ = 0.5 b^* \sigma f^2 \sqrt{\frac{\lambda}{2-\lambda}\left(1-\left(1-2\lambda+\lambda^2\right)\right)}$$

$$M_0^- = M_0^+ = 0.5b^*\sigma f^2 \sqrt{\frac{\lambda}{2-\lambda}\left(2\lambda-\lambda^2\right)} = 0.5b^*\sigma\lambda f^2 \qquad (2.14)$$

The ARL results for the MECHS, MECFIR and MECFIRHS control charting schemesare provided in Tables 2.5, 2.6 and 2.7 respectively.

Table 2.5. ARL values for the mixed EWMA-CUSUM scheme with Headstart at ARL0=500 and a*=0.50

δ	λ=0.10 b^*=37.85	λ=0.25 b^*=20.49	λ=0.5 b^*=11.4	λ=.75 b^*=7.43
0.00	497.73	496.55	505.31	500.27
0.25	73.12	74.43	88.19	106.69
0.50	31.14	24.95	23.13	24.89
0.75	20.95	14.82	11.75	10.98
1.00	16.34	10.86	7.90	6.77
1.25	13.65	8.76	6.09	4.97
1.50	11.83	7.48	5.02	3.96
1.75	10.55	6.57	4.31	3.33
2.00	9.54	5.90	3.81	2.89

Table 2.6. ARL for the MFIRadj Mixed EWMA- CUSUM scheme at ARL_0=500, f=0.50 and a^*=0.50

δ	λ=0.10 b^*=37.70	λ=0.25 b^*=21.18	λ=0.5 b^*=12.50	λ=.75 b^*=8.58
0.00	499.08	499.32	506.04	503.57
0.25	78.69	75.19	74.05	74.64
0.50	34.16	25.40	18.62	14.94
0.75	22.36	13.45	8.14	5.82
1.00	16.29	8.11	4.31	3.03
1.25	12.35	5.08	2.66	2.00
1.50	9.25	3.32	1.89	1.54
1.75	6.77	2.34	1.49	1.31
2.00	4.92	1.74	1.28	1.18

Table 2.7. ARL for the modified FIR feature of mixed

EWMA-CUSUM scheme and Headstart at ARL_0=500, f=0.50 and a^*=0.50

δ	λ=0.10 b^*=39.95	λ=0.25 b^*=22.52	λ=0.5 b^*=13.08	λ=.75 b^*=8.89
0.00	499.87	500.29	497.23	505.01
0.25	72.46	65.22	64.52	69.85
0.50	30.28	21.69	16.04	13.24
0.75	18.96	11.36	6.96	5.07
1.00	13.25	6.71	3.69	2.67
1.25	9.47	4.16	2.30	1.82
1.50	6.74	2.73	1.67	1.41
1.75	4.75	1.93	1.35	1.23
2.00	3.30	1.49	1.19	1.13

2.4.0 COMPARISONS OF THE PROPOSED CHARTS WITH THEIR COUNTERPARTS

In this section, we compare the proposed charts with their existing counterparts including classical CUSUM, classical EWMA, FIR CUSUM, FIR EWMA and mixed EWMA-CUSUM control charting schemes. We use ARL as a performance measure of these charts for comparison purposes.

2.4.1 Proposed charts versus the classical CUSUM and EWMA charts

The ARL tables of the proposed charts are given in Tables2.5-2.7 and those of the classical CUSUM and the classical EWMA in Table 2.1 and 2.2 respectively. It is observed that both MECFIR and MECFIRHS charting schemes perform better than the classical CUSUM and classical EWMA charts at all shifts for varying values of λ, except when it is very small (e.g. when λ=0.10). Moreover, the MECHS control charting scheme offers better performance than the classical CUSUM and EWMA charts at small shifts when λ<0.50. It may be seen in Tables 2.1,2.2,2.5, 2.6 and 2.7. Figures2.1 and 2.2 may also be seen in support of these findings.

2.4.2 Proposed charts versus FIR CUSUM and FIR EWMA charts

The ARL values of the FIR CUSUM are given as a part of Table1 and those of FIR EWMA in Table 3. It is observed that both MECFIR and MECFIRHS are better than FIR CUSUM at all shifts except when λ=0.10(it is effective here at small shift). MECHS performs worse than FIR CUSUM except at a very small shifts when λ <0.75 and perform better at small and moderate shifts when λ =0.75. Moreover, we have noticed that both MECFIR and MECFIRHS schemes are better than FIR EWMA at all shifts when λ=0.50 or 0.75 but inferior at all shifts for λ=0.10 or 0.25, except at a very small shifts. MECHS performs better at small and moderate shifts when λ =0.75 and otherwise it remains on lower end. One may see Tables 2.1,2.3,2.5, 2.6&2.7 and Figures2.3 and 2.4 in support of these findings.

2.4.3 Proposed charts versus the mixed EWMA-CUSUM

The ARL results of the mixed EWMA-CUSUM are given in Table 2.4 above and the corresponding ARL results for the proposed MECHS, MECFIR and MECFIRHS control charting schemesare provided in Tables 2.5-2.7respectively. It is observed that all the proposed charts are better than mixed EWMA-CUSUM at all shifts of the process mean at any value of λ. Tables 2.4-2.7 and Figure 2.5 may be seen in support of these findings.

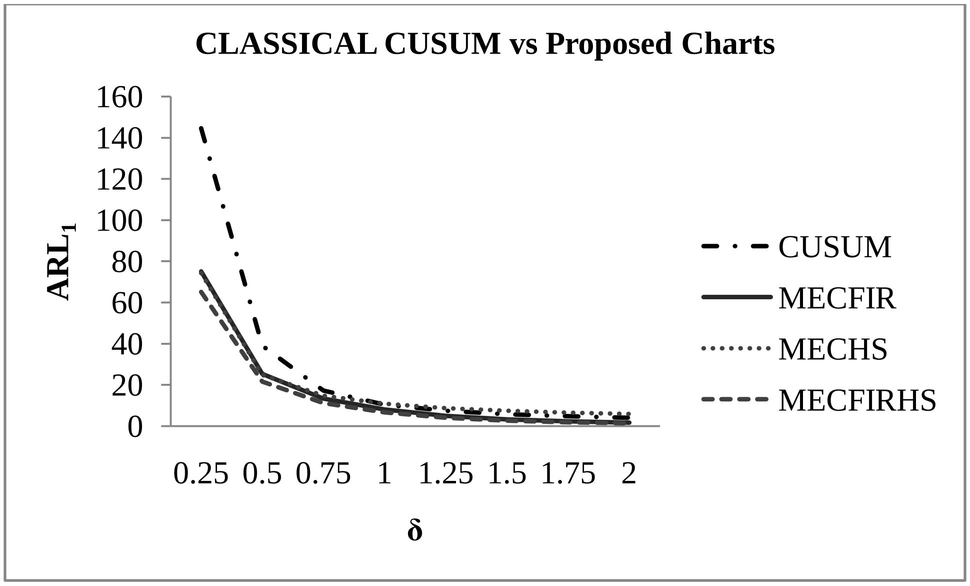

Figure 2.1. ARL curves of the proposed charts and classical CUSUM at ARL_0 =500

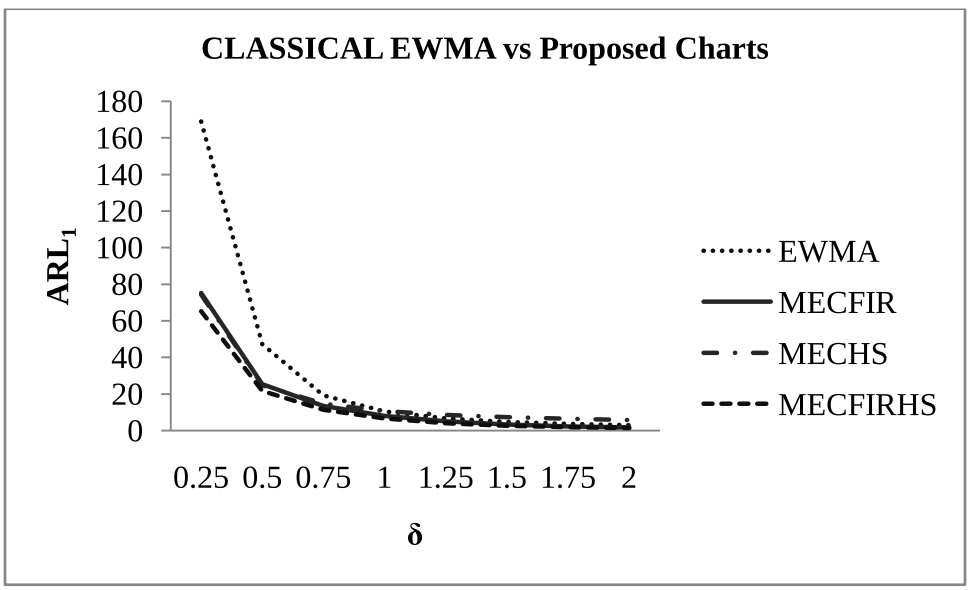

Figure 2.2. ARL curves of the proposed charts and classical EWMA at ARL_0 =500

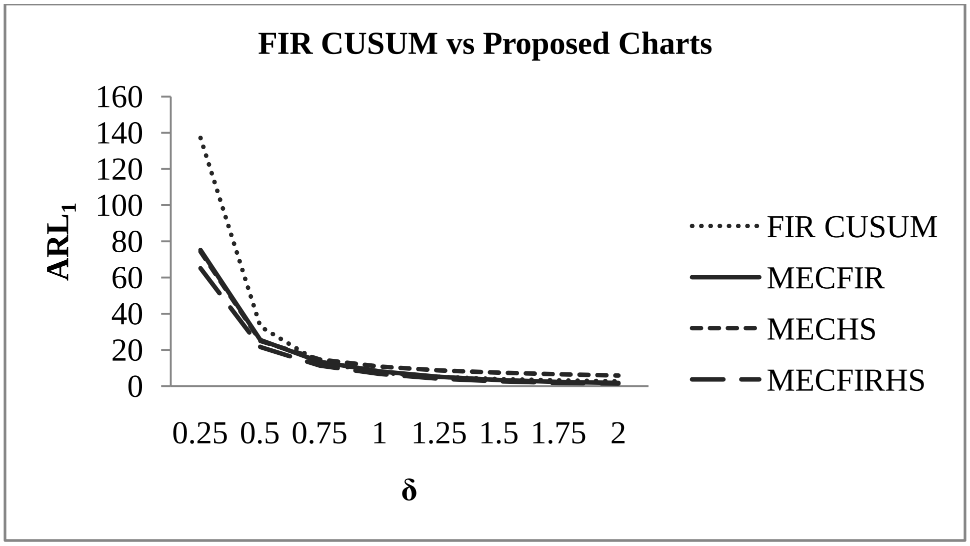

Figure 2.3. ARL curves of the proposed charts and FIR CUSUM at ARL_0 =500

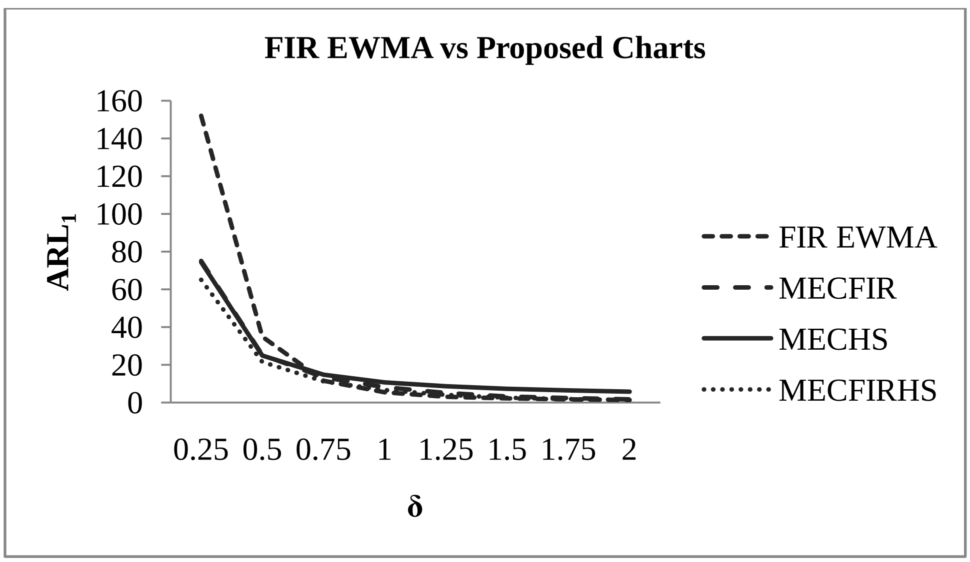

Figure 2.4. ARL curves of the proposed charts and FIR EWMA at ARL_0 =500

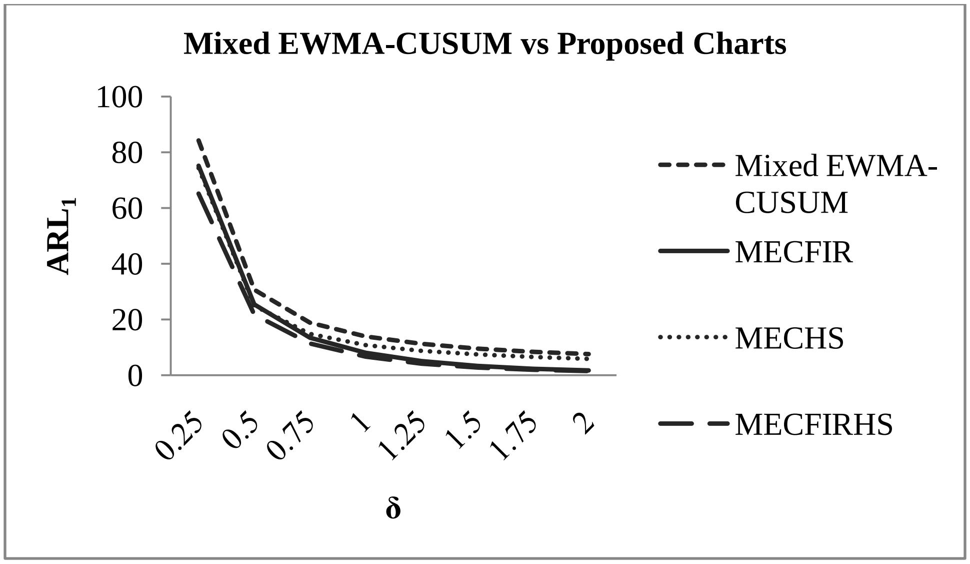

Figure 2.5. ARL curves of the proposed charts and mixed EWMA-CUSUM atARL_0 =500

2.5.0 CASE STUDY

The real life data from the petroleumrefinery laboratory is used as a case study for our study purposes. The description of the data and its analysis are given below.

2.5.1 Description of the data (Purity Analysis of Di-Glycol Amine)

The monitoring of lab analyzer is used for determining concentration of Di-Glycol Amine (DGA) in Spent Amines Samples. DGA is a type of amines compounds used in petroleum refineries to remove sulfur compounds from petroleum gases by using a chemical process known as gas sweetening process. During this gas sweetening process, DGA removes sulfur species from the hydrocarbons and consequently, the DGA is decomposed into variety of DGA degradation products. The strength of the amine (i.e.

DGA) in removing more sulfur from the petroleum gases depends on the concentration of the remaining –none decomposed –DGA.Chemical process engineers send the spent DGA samples to the petroleum refining quality assurance (QA) laboratory for characterizing the spent DGA samples for variety of parameters including the concentration of DGA in spent DGA samples. Based on the strength (i.e. the concentration level) of the remaining DGA in spent DGA samples –expressed as DGA wt%-, the process engineers either adds make-up new DGA or replaces the whole DGA in the chemical process. The quality of DGA test result reported by the lab to the process engineers plays a major role for the chemical process engineer to take the right action.

Potentiometric titration is one of the widely used lab instrumental method for determining the concentration of DGA in spent DGA samples. The sensing part of this DGA lab analyzer is a pH probe which is calibrated by the lab on regular basis. After calibration, the instrument performance is monitoring by using different types of usual control charts. The instructions given by the international standard procedure ASTM D6299 (2013), titled "Applying Statistical Quality Assurance and Control Charting Techniques to Evaluate Analytical Measurement System Performance" are strictly followed. The data used to develop the charts are based on a quality control (QC) DGA sample prepared in-house. This QC DGA standard (30.3wt% amine) is prepared by diluting 2, 2-Aminoethoxy ethanol (98% purity) with deionized water. The lab tests this amine (DGA) control sample once per day. Sufficient data are collected and data adequacy check is conducted for these gathered data prior to developing various control charts as given below.

2.5.2 Analysis of results

The data used to monitor the purity of DGA analyzer performance is used here to construct different control charts covered above in this study. The graphical displays for these different control charts are shown in Figures 2.6-2.12 and the dataset (along with some other quantities for different charts) is shown in Table 2.8. We have constructed control limits of different types of control charts using ARL_0 =500.

Figure 6 (classical CUSUM) shows that sample points 19-41 are found out of control on the increaseside, while on decrease side we have out-of-control signals from sample points 59-84 except 83rd sample.Classical EWMA gives out of control signals at sample 30, 31 and 32 on upper side and also the sample points 60-63 on lower end. The graphical display of classical EWMA is shown in Figure 2.7. It is obvious from Figure 2.8 that the FIR EWMA gives out of control signals at sample 31 and 32 on UCL side while sample points 60-63 on LCL side. The mixed EWMA-CUSUM (cf. Figure 2.9) gives out-of-control signals at sample points 18-56 for theincreaseand sample points 60-84 for decrease. Figure 2.10shows that the MECHS chart gives out of control signals at sample points 14-58 and 60-84for the increase and decrease respectively.Figure 2.11indicates that that the MECFIR chart gives out of control signals at sample 18-55 and 60-84 in the upper and lower sides respectively. Figure 2.12 shows that the MECFIRHS scheme gives out of control signals at sample points 19-55 and 61-84on the upper (increase) and lower (decrease) respectively.It can be observed from figures 2.6-2.12 that the proposed charts of the study detect out-of-control signals more efficiently for this real data collected from the refinery laboratory.

Table 2.8: The output of the DGA using mixed EWMA-CUSUM with modified FIR feature control chart (f=0.5)

Sample#	X_i	Z_i	a_i	M_i^+	M_i^-	b_i
1	30.40	30.127	0.056	0.096	0	0.122
2	29.87	30.062	0.070	0.054	0	0.320
3	30.36	30.137	0.076	0.079	0	0.594
4	30.51	30.230	0.080	0.194	0	0.913
5	30.27	30.240	0.082	0.316	0	1.250
6	30.51	30.308	0.083	0.506	0	1.581
7	30.51	30.358	0.083	0.745	0	1.894
8	29.98	30.264	0.084	0.890	0	2.178
9	30.55	30.335	0.084	1.105	0	2.432
10	30.50	30.376	0.084	1.362	0	2.652
11	30.43	30.390	0.084	1.633	0	2.843
12	30.46	30.407	0.084	1.920	0	3.005
13	30.55	30.443	0.084	2.244	0	3.141
14	30.45	30.445	0.084	2.569	0	3.256
15	29.91	30.311	0.084	2.761	0	3.351
16	30.4	30.333	0.084	2.974	0	3.431
17	30.66	30.415	0.084	3.270	0	3.496
18	30.27	30.379	0.084	3.529	0	3.550
19	30.62	30.439	0.084	3.848^+	0	3.595
20	30.6	30.479	0.084	4.208	0	3.631
21	30.49	30.482	0.084	4.570	0	3.661
…	…	…	…	…	…	…

58	29.09	29.602	0.084	2.674	2.918	3.794
59	29.39	29.549	0.084	2.103	3.319	3.794
60	29.32	29.492	0.084	1.476	3.779	3.794
61	29.31	29.447	0.084	0.803	4.283^{-}	3.794
62	29.55	29.472	0.084	0.156	4.762	3.794
…	…	…	…	…	…	…
83	30.62	30.058	0.084	0	7.431	3.794
84	29.15	29.831	0.084	0	7.551	3.794

3.848^{+} and 4.283^{-} show the out-of-control signal for both upper and lower mixed EWMA-CUSUM with modified FIR feature and headstart respectively.

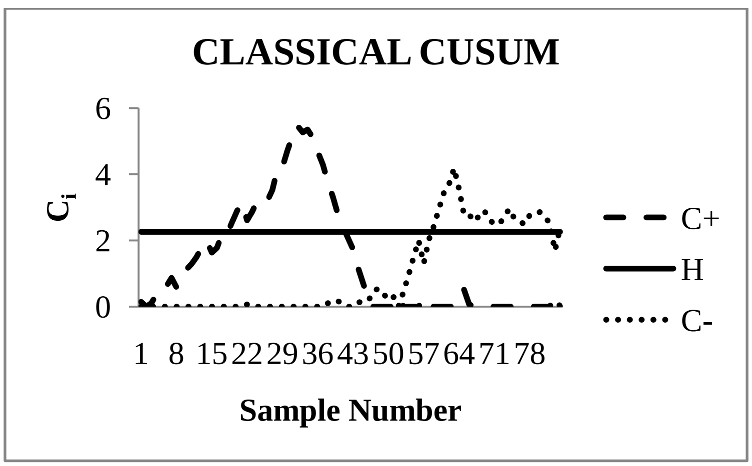

Figure 2.6. Control chart of classical CUSUM when k=0.5σ, H=5.071σ

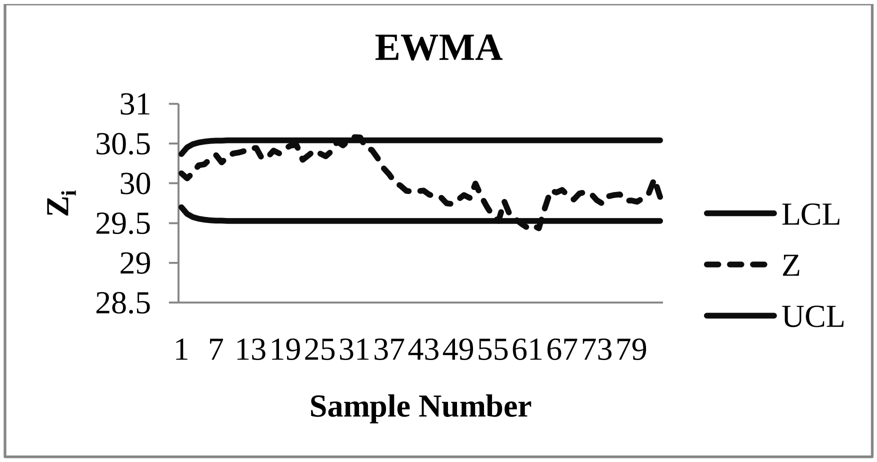

Figure 2.7. Control chart of classical EWMA control chart when $\lambda = 0.25$ and L=3

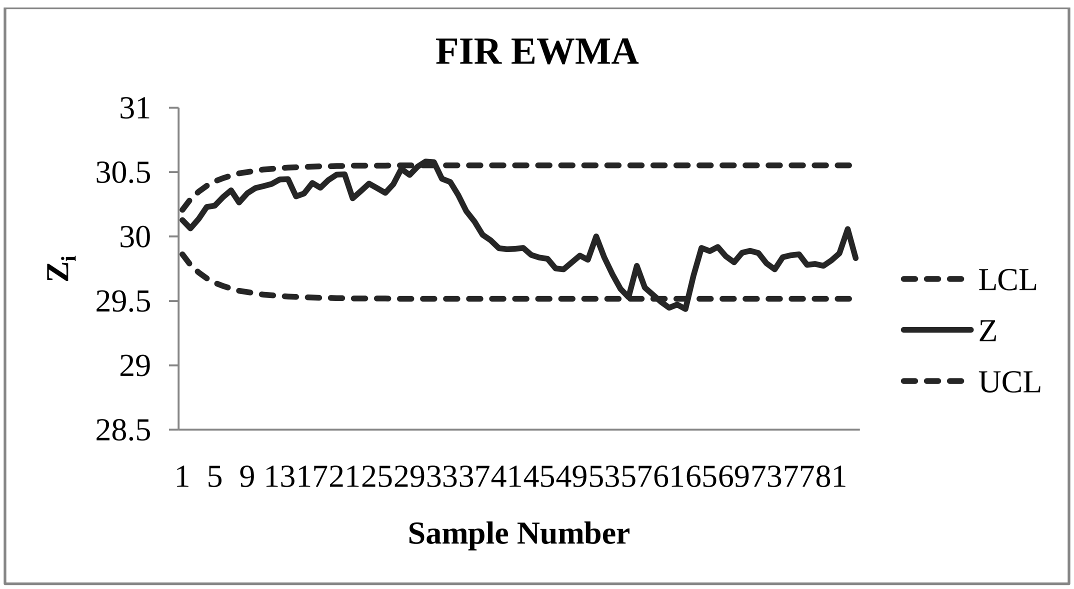

Figure 2.8. Control chart of FIR EWMA chart when $\lambda = 0.25$, $L = 3.0781$ and $f = .5$

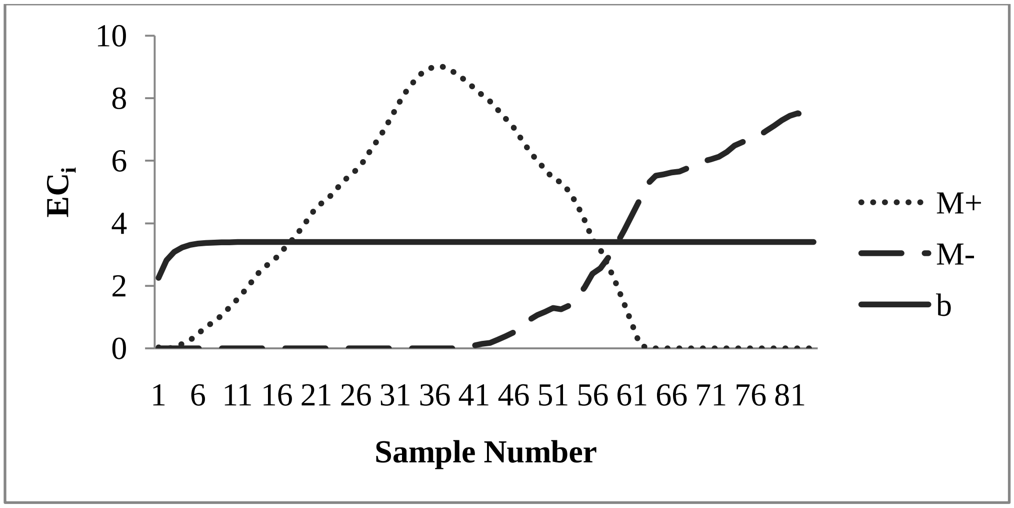

Figure 2.9. Control chart of mixed EWMA-CUSUM when $\lambda = 0.25$**,** $a^* = 0.5$ **and**

$b^* = 20.18$

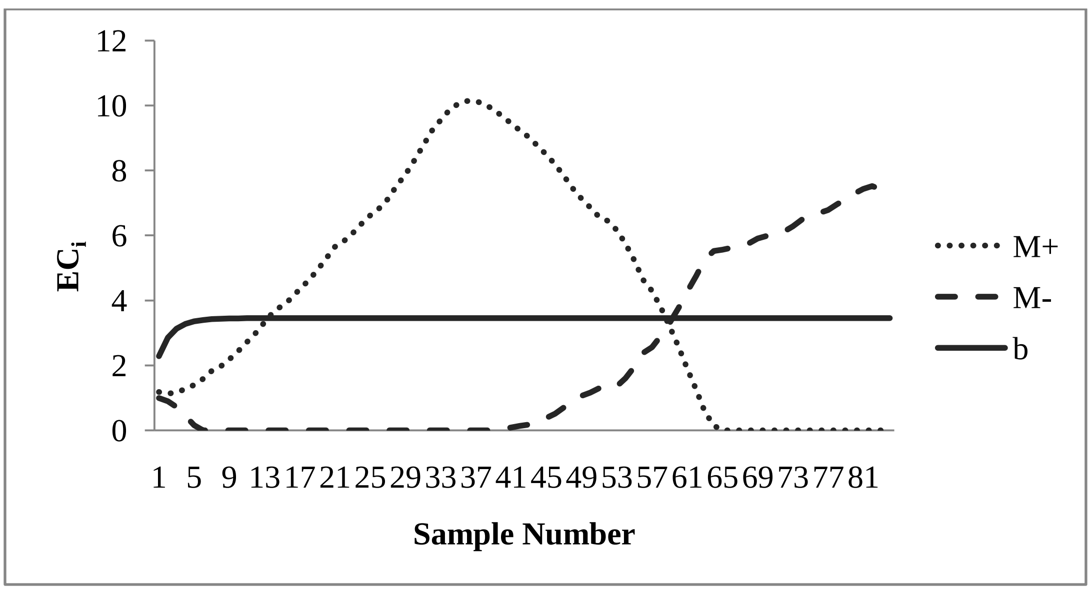

Figure 2.10. Control chart of MECHS ($M_0^+ = M_0^-$ **=2.561),** $\lambda = 0.25$ **,** $a^* = 0.5$ **and**

$b^* = 20.49$

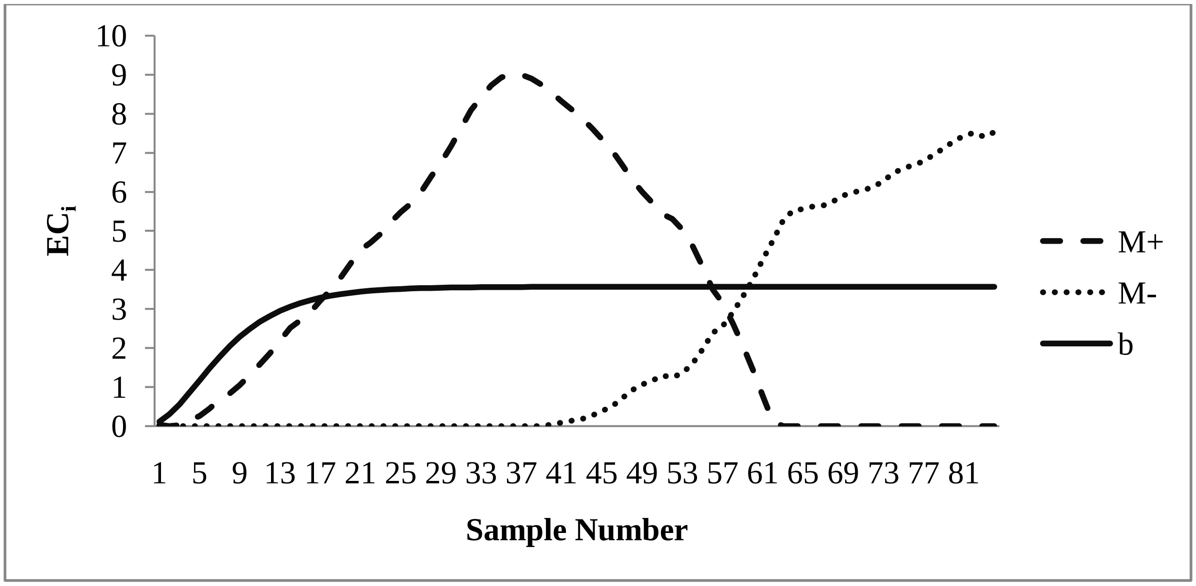

Figure 2.11. Control chart of MECFIR using $f = 0.5$**,** $\lambda = 0.25$ **,** $a^* = 0.5$ **and**

$b^* = 21.18$

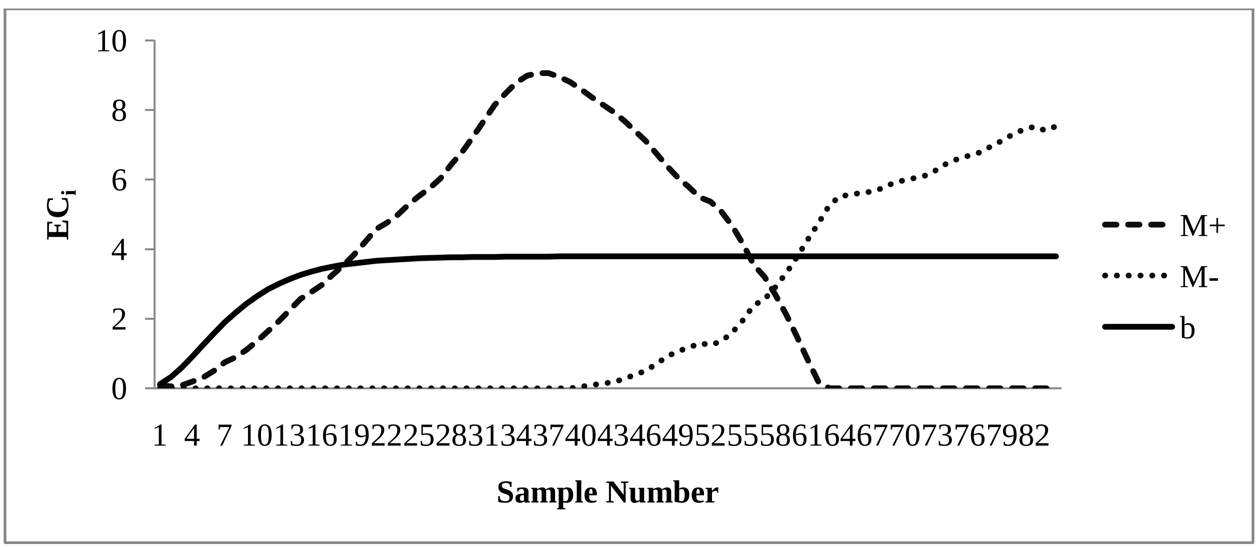

Figure 2.12. Control chart of MECFIRHS using $f = 0.5$ **,** $\lambda = 0.25$**,** $a^* = 0.5$**,** $b^* = 22.52$ **and Head start (**$M_0^+ = M_0^-$ **=0.137)**

2.6 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In this study, we have proposed some improvements on the mixed EWMA-CUSUM control charts with varying FIR features in the form of MECHS, MECFIR and MECFIRHS control charting schemes. We have investigatedARL properties of the proposed schemes and compared them with the existing counterparts including classical CUSUM, classical EWMA, FIR EWMA and FIR CUSUM. We have observed that the proposals of the studyimprove the detection ability of the mixed EWMA-CUSUM chart for the processes that are off-target at the start-up.The comparisons showed that the proposed schemes are really good at detecting shifts (especially of smaller magnitude) in the process relative tom the other existing schemes covered in this study. The scope of this study may be extended for dispersion charts and also in the multivariate setups for an improved and efficient monitoring of process parameters.

CHAPTER 3

MIXED MULTIVARIATE EWMA-CUSUM CONTROL CHARTS

FOR AN IMPROVED PROCESS MONITORING

Multivariate exponential weighted moving average and cumulative sum charts are two most common memory type multivariate control charts. They make use of the present and past information to detect small shifts in the process parameter(s).In this article, we propose two new multivariate control charts using a mixed version of their design setups. The plotting statistics of the proposed charts are based on the cumulative sum of the multivariate exponentially weighted moving averages. The performances of the proposed schemes are evaluated in terms of average run length. The proposals are compared with their existing counterparts including Hotelling's T^2, MCUSUM, MEWMA and MC1 charts. An application example is also presented for practical considerations using a real dataset.

3.1 INTRODUCTION

Most of the manufacturing or business processes have two or more correlated quality characteristics to be monitored simultaneously. For instant, inner diameter, thickness and length of the tubes may be three correlated quality characteristics that might be monitored in the manufacturing process of specific carbon fiber tubing. Though, they can be monitored individually like in the univariate set-up but it has drawbacks such as: it misses the important information on correlation structures, it consumes more time and also inflates the probability of false alarm rate of special cause of variation. In these situations, we move towards monitoring of process vectors or matrices using multivariate control charts.

Hotelling's (1947) developed the control chart that monitors multivariate quality characteristics. This chart is a direct analog of Shewhart (1931) control chart that is based on the present information and is insensitive to detect small and moderate shifts in the process parameter(s). They are better than memory type control charts in detecting large shift in the process parameter. The memory type multivariate control charts like Multivariate Exponentially Weighted Moving Average Control Chart (MEWMA) proposed by Lowry et al. (1992), Multivariate Cumulative Sum (MCUSUM) introduced by Crosier (1988) and Multivariate CUSUM 1 (MC1) developed by Pignatiello and Runger (1990) use both the previous and current data to detect shiftin the process parameters. These control charts are better than Hotelling T^2 control chart when we are interested in the smaller/moderate shifts in the process parameters.

Abbas et al. (2013 a&b) used the idea of merging the structures of EWMA and CUSUM charts for location and dispersion parameters. Later, Zaman et al. (2014) extended this idea in a reverse mixing pattern. In this study, we propose two new multivariate control charts based on combining the effects of MEWMA and MCUSUM charts for mean vector. We named these two multivariate charts as mixed multivariate EWMA-CUSUM Control chart 1(MEC1) and mixed multivariate EWMA-CUSUM Control chart 2 (MEC2).

In our current study, we have used Average run length (ARL) as a performance measure that is an effective measure of comparing the performance of the control charts. The in-control ARL of a control chart is denoted by $ARL_0$and out-of-control ARL by ARL_1.

3.2 MULTIVARIATE CONTROL CHARTS

Let X_{i1}, X_{i2}, X_{i3}, …, X_{ip} be the i^{th} observation of p quality characteristics, for $i = 1, 2, 3, ...n$, where n is the total number of samples to be monitored. X_{ip} may be an individual observation or mean of subgroup of the observation collected at a time. We make an assumption of normality for the distribution of X_{ip}, that is, $X_{ip} \sim N(\mu, \Sigma)$, where μ and Σ are the mean and covariance matrix of the distribution respectively. In order to monitor the mean vector μ we have a variety of charts to detect large and small shifts. For larger shifts, we have memory-less charts such as χ^2 control chart and the smaller shifts are detected by memory type control charts like MEWMA, MC1 and MCUSUM.

The χ^2 control chart is based on the statistic given as:

$$\chi_i^2 = X_i' \Sigma^{-1} X_i , \qquad (3.1)$$

For the control limit, say $h_1 > 0$, anytime $\chi_i^2 > h_1$ the process is declared out of control.

We use Hotelling T^2 control chart when $\sum$ is unknown but for the case of the known parameter, one may use χ^2 control chart, (the details in this regard may be seen in Montgomery (2009)).

MCUSUM was introduced by Crosier (1988) and is based on the statistic given below:

$$C_i = \{(S_{i-1} + X_i)' \Sigma^{-1} (S_{i-1} + X_i)\}^{\frac{1}{2}}$$

$$where\ S_i = (S_{i-1} + X_i)\left(1 - \frac{k}{C_i}\right)\ if\ C_i > k;\ S_i = 0, if\ C_i \leq k,$$

$$for\ i = 1,2, \ldots, and\ \ S_0 = 0$$

Here $k > 0$ is the reference value; it is taken to be equal to 0.5 throughout this article. Furthermore, we calculate the following statistic for monitoring purposes:

$$U_i = \{S_i' \Sigma^{-1} S_i\}^{\frac{1}{2}} \ (3.2)$$

For the control limit, say h_2 (cf. Table 3.1), and anytime $U_i > h_2$ the process is declared out of control.For different amounts of shifts (δ), the ARL results of MCUSUM chart for different correlated quality characteristics (p) such as p= 2, 3 and 4 are given in Table 3.1 at ARL_0 =200. It is to be mentioned that δ is defined as (in the form of non-centrality parameter): $\delta = \left(\mu' \Sigma^{-1} \mu\right)^{1/2}$.

MEWMA proposed by Lowry et al. (1992) follows a direct multivariate extension of the univariate EWMA control chart (as introduced by Roberts (1959)).

The MEWMA chart has the statistic:

$$Z_i = \lambda X_i + (1-\lambda) Z_i (3.3)$$

where $\Sigma_{z_i} = \frac{\lambda}{2-\lambda}(\Sigma)$ and based on Z_i we have the following statistic

$$T_i^2 = Z_i' \Sigma_{Z_i}^{-1} Z_i (3.4)$$

where $0 < \lambda \leq 1$ and $Z_0 = 0$. For out-of-control signals we compare the statistic with the corresponding control limit (say h_3) and receive the signals if points fall outside i.e.

$$T^2 = Z_i' \Sigma_z^{-1} Z_i > h_3 .$$

The control limit h_3 and the ARL values of the MEWMA control chart for different values of λ (at ARL_0 =200 when p=2) are given in Table 3.2for different values of δ .

Pignatiello and Runger (1990) proposed two multivariate CUSUM control charts for location monitoring. Theone with the better performance, MC1, is based on the vectors of cumulative sums as follows:

$$S_i = \sum_{j=i-n_i+1}^{i} (\overline{X}_i - \mu_0) \quad (3.5)$$

and the consequent statistic is given as:

$$V_i = \max\left\{0, \left(S_i' \Sigma^{-1} S_i\right)^{1/2} - kn_i\right\} \qquad (3.6)$$

where k>0 and $n_i = \begin{cases} n_{i-1} + 1 & \text{if } V_{i-1} > 0 \\ 1 & \text{otherwise} \end{cases}$

The control limit, say h_4 , is given in Table 3.3 and anytime $V_i > h_4$, the process is deemed out of control. The ARL values of the MC1 control chart for p= 2, 3 and 4 at ARL_0 =200 are given in Table 3.3 at varying choices of δ .

Table 3.1: ARL values for MCUSUM scheme when k=0.5

p=2	p=3	p=4	
δ	h_2=5.50	h_2=6.88	h_2=8.19
0.00	201.00	200.20	200.49
0.25	83.53	87.11	89.41
0.50	29.57	31.74	33.47
0.75	15.14	16.75	18.34
1.00	9.87	11.17	12.34
1.25	7.31	8.40	9.43
1.50	5.79	6.69	7.62
1.75	4.84	5.65	6.39
2.00	4.12	4.84	5.49
2.25	3.62	4.27	4.86
2.50	3.25	3.81	4.35
2.75	2.94	3.46	3.94
3.00	2.70	3.18	3.62

Table 3.2: ARL values for MEWMA scheme for p=2.

δ	λ=0.10 h_3=8.66	λ=0.25 h_3=9.89	λ=0.50 h_3=10.44	λ=0.75 h_3=10.60
0.00	201.51	200.88	202.99	201.89
0.25	76.60	105.15	135.85	161.30
0.50	28.08	39.01	62.67	90.12
0.75	15.21	18.06	29.71	46.94
1.00	10.11	10.72	15.91	25.88
1.25	7.63	7.24	9.44	14.94
1.50	6.11	5.42	6.36	9.43
1.75	5.11	4.34	4.62	6.23
2.00	4.42	3.63	3.62	4.43
2.25	3.88	3.14	2.96	3.39
2.50	3.49	2.78	2.52	2.70
2.75	3.18	2.50	2.20	2.22
3.00	2.93	2.29	1.95	1.91

Table 3.3: ARL values for MC1 scheme when k=0.5

δ	p=2 h_4=4.75	p=3 h_4=5.52	p=4 h_4=6.18
0.00	195.89	200.48	200.22
0.25	89.63	98.71	104.77
0.50	30.95	34.84	36.51
0.75	14.83	16.30	17.18
1.00	9.40	10.12	10.74

1.25	6.70	7.26	7.69
1.50	5.22	5.69	6.08
1.75	4.31	4.75	5.02
2.00	3.69	4.07	4.32
2.25	3.22	3.53	3.81
2.50	2.88	3.19	3.41
2.75	2.62	2.88	3.10
3.0	2.41	2.64	2.84

3.3.0 THE PROPOSED CHARTS

In this section, we propose two new multivariate control charts, namely MEC 1 and MEC 2 charts, based on combining the effects of MEWMA and MCUSUM charts. The inspiration of this approach is taken from Abbas et al. (2013 a&b) and Zaman et al. (2014). This section is divided into two parts for the two proposals separately.

3.3.1 MIXED MULTIVARIATE EWMA-CUSUM CONTROL CHART 1(MEC1)

Let $X_1, X_2, X_3, \ldots, X_p$ be the samples of p quality characteristics to be monitored. The proposed MEC1 control chart is developed by transforming the samples into MEWMA statistic given as: $Z_i = \lambda X_i + \left(1-\lambda\right) Z_{i-1}$. We integrate it into MCUSUM as given below:

$MEC_i = \max\left(0, MEC_{i-1} + (Z_i - \mu_0) - k^*\right)$ (3.7) where $MEC_0 = 0$ and k^* is defined as:

$$k^* = k\frac{\left(MEC_{i-1} + Z_i - \mu_0\right)}{\left[\left(MEC_{i-1} + Z_i - \mu_0\right)' \Sigma_{Z_i}^{-1}\left(MEC_{i-1} + Z_i - \mu_0\right)\right]^{1/2}} \quad \text{where } k > 0$$

if $k^* \geq MEC_{i-1} + (Z_i - \mu_0)$ then $MEC_i = 0$

which implies that

$$k\frac{\left(MEC_{i-1} + Z_i - \mu_0\right)}{\left[\left(MEC_{i-1} + Z_i - \mu_0\right)' \Sigma_{Z_i}^{-1}\left(MEC_{i-1} + Z_i - \mu_0\right)\right]^{1/2}} \geq \left(MEC_{i-1} + Z_i - \mu_0\right)$$

$$k \geq \left[\left(MEC_{i-1} + Z_i - \mu_0\right)' \Sigma_{Z_i}^{-1}\left(MEC_{i-1} + Z_i - \mu_0\right)\right]^{1/2}$$

Therefore, if $k \geq \left[\left(MEC_{i-1} + Z_i - \mu_0\right)' \Sigma_{Z_i}^{-1}\left(MEC_{i-1} + Z_i - \mu_0\right)\right]^{1/2}$ then $MEC_i = 0$

Otherwise $MEC_i = MEC_{i-1} + (Z_i - \mu_0) - k^*$

Finally, we calculate the statistic $MEC1_i = MEC_i' \Sigma_z^{-1} MEC_i$,

The control limit, say h_5 , is used to take decision such that anytime $MEC1_i > h_5$, the process is out of control. The control limit h_5 and ARL values of the MEC1 control chart for different values of λ at ARL_0 =200 when p=2, are given in Table 3.4 at varying choices of δ. Moreover, the ARL values of MEC1 control chart for p=2, 3 and 4 at ARL_0 =200, are given Table 3.5. Also, the Standard Deviation Run Lengths (SDRLs) of MEC1 are given in Table 3.8.

Table 3.4: ARL values for MEC1 scheme when k=0.5

δ	λ=0.10 h_5 =38.57	λ=0.25 h_5 =21.76	λ=0.50 h_5 =12.20	λ=0.75 h_5 =7.95
0.00	197.97	201.43	199.51	197.51
0.25	66.23	65.50	69.28	75.29
0.50	33.83	28.97	27.17	27.57
0.75	23.85	18.64	16.03	15.12
1.00	19.11	14.11	11.44	10.31
1.25	16.23	11.54	8.97	7.82
1.50	14.34	9.95	7.50	6.39
1.75	12.93	8.82	6.50	5.43
2.00	11.86	7.97	5.76	4.73
2.25	10.97	7.29	5.18	4.19
2.50	10.28	6.78	4.77	3.82
2.75	9.67	6.33	4.41	3.50
3.00	9.17	5.97	4.13	3.24

Table 3.5: ARL values for MEC1 scheme when k=0.5 and λ=0.25

δ	p=2 h_5 =21.76	p=3 h_5 =28.31	p=4 h_5 =34.91
0.00	201.43	199.05	203.98
0.25	65.50	70.83	77.24
0.50	28.97	33.08	37.60
0.75	18.64	21.68	24.93
1.00	14.11	16.47	18.96
1.25	11.54	13.63	15.59

1.50	9.95	11.70	13.45
1.75	8.82	10.33	11.84
2.00	7.97	9.33	10.64
2.25	7.29	8.54	9.75
2.50	6.78	7.96	9.04
2.75	6.33	7.39	8.42
3.00	5.97	6.96	7.93

Table3.8:SDRLs values for MEC1 scheme at ARL$_0$=200

	38.57	21.76	12.2	7.95
δ	λ=0.10	λ=0.25	λ=0.50	λ=0.75
0.00	163.03	180.90	187.62	191.11
0.25	37.13	44.86	55.34	65.26
0.50	11.56	13.30	15.78	18.63
0.75	5.73	6.29	6.99	7.81
1.00	3.57	3.69	3.98	4.30
1.25	2.49	2.49	2.63	2.76
1.50	1.93	1.87	1.92	2.00
1.75	1.55	1.47	1.47	1.53
2.00	1.29	1.19	1.18	1.21
2.25	1.08	0.99	0.96	0.98
2.50	0.95	0.86	0.82	0.83
2.75	0.84	0.76	0.72	0.72
3.00	0.75	0.68	0.64	0.62

3.3.2 MIXED MULTIVARIATE EWMA-CUSUM CONTROL CHART 2 (MEC2)

Following the inspiration and guidelines of Section 3.1 we integrate MEWMA statistic in MC1 and nameit as MEC2 control chart. The MEWMA statistic $Z_i = \lambda X_i + (1-\lambda) Z_{i-1}$ is transformed into the vectors of cumulative sums of MC1 (cf. Section 3.2, equations (3.5& 3.6)) as given below. The statistic Z_i is distributed with mean of μ_0 and covariance matrix Σ_{Z_i} defined as: $\Sigma_{Z_i} = (\lambda / 2 - \lambda)(\Sigma)$. Let we define the vectors of cumulative sums as given below:

$$S_i = \sum_{j=i-n_i+1}^{i} (Z_i - \mu_0) \quad (3.8)$$

and the resulting statistic based on S_i is given as:

$$MEC2_i = \max\left\{0, \left(S_i^{'} \Sigma_{Z_i}^{-1} S_i\right)^{1/2} - k_1 n_i\right\} \quad (3.9)$$

where $k_1 = k\left(\left(\mu_1 - \mu_0\right)^{'} \Sigma_{Z_i}^{-1}\left(\mu_1 - \mu_0\right)\right)^{1/2}$ for k>0.

Also, $n_i = \begin{cases} n_{i-1} + 1 & \text{if } MEC2_{i-1} > 0 \\ 1 & \text{otherwise} \end{cases}$

The control limit, h_6, is given in Table 3.6, and anytime $MEC2_i > h_6$, the process is out of control. The ARL values of the MEC2 control chart for different values of λ at ARL_0 =200 when p=2, are given in Table 6.The ARL values of MEC2 control chart for 2, 3 and 4 correlated quality characteristics (p) at ARL_0 =200, are given Table 3.7 at varying choices of δ. Lastly, the Standard Deviation Run Lengths (SDRLs) of MEC2 are given in Table 3.9.

Table 3.6: ARL values for MEC2 scheme at ARL_0=200 when p=2

	λ=0.10	λ= 0.25	λ=0.50	λ=0.75
	h_6 =1.81	h_6 =6.25	h_6 =6.50	h_6 =5.65
δ	k1=2.18	k1=1.32	k1=0.87	k1=0.65

0.00	201.29	200.56	203.30	202.04
0.25	75.07	85.63	91.42	92.73
0.50	27.81	30.39	31.98	31.97
0.75	15.31	15.39	15.35	15.17
1.00	10.56	10.18	9.71	9.46
1.25	8.18	7.77	7.16	6.87
1.50	6.73	6.40	5.74	5.39
1.75	5.80	5.56	4.89	4.54
2.00	5.10	4.92	4.27	3.90
2.25	4.57	4.45	3.83	3.45
2.50	4.17	4.09	3.49	3.12
2.75	3.84	3.79	3.23	2.85
3.00	3.56	3.54	3.01	2.62

Table 3.7: ARL values for MEC2 scheme when λ=0.25 at ARL_0=200

	p=2	p=3	p=4
δ	h_6 =6.25	h_6 =8.21	h_6 =9.91
0.00	200.56	198.02	201.61
0.25	85.63	94.00	102.71
0.50	30.39	33.37	36.96

0.75	15.39	16.89	18.38
1.00	10.18	11.31	12.00
1.25	7.77	8.64	9.24
1.50	6.40	7.10	7.67
1.75	5.56	6.19	6.70
2.00	4.92	5.52	5.94
2.25	4.45	5.00	5.40
2.50	4.09	4.59	4.99
2.75	3.79	4.28	4.62
3.00	3.54	3.99	4.35

Table 3.9:SDRLs values for MEC2 scheme at ARL_0=200

δ	λ=0.10 h_6=1.81	λ=0.25 h_6=6.25	λ=0.50 h_6=6.5	λ=0.75 h_6=5.65
0.00	190.40	196.82	200.92	200.28
0.25	64.93	79.37	85.81	88.57
0.50	18.75	23.59	25.99	26.33
0.75	7.83	9.06	9.94	10.10
1.00	4.21	4.54	4.93	5.06
1.25	2.73	2.74	2.95	3.08
1.50	1.94	1.86	1.97	2.05
1.75	1.48	1.41	1.47	1.53
2.00	1.19	1.11	1.13	1.18
2.25	0.97	0.90	0.90	0.94
2.50	0.84	0.79	0.77	0.80
2.75	0.74	0.70	0.66	0.70
3.00	0.65	0.61	0.59	0.62

It is to be mentioned thatwe have used the Monte Carlo simulation approach to evaluate the ARL measures provided in this study. We have performed these simulations by developing a code in R language and executing ita reasonable number of times, say 10^4times (for a relevant discussion about the number of simulations needed in control charting studies, one may see Kim (2005), Schaffer and Kim (2007), Mundform et al. (2011)).

3.4 RESULTS DISCUSSION AND COMPARATIVE ANALYSIS

In this section, we provide a discussion about the performance of the two proposed MEC1 and MEC2 charts. We also provide comparisons of these proposals with their counterparts including MCUSUM, MEWMA, MC1 and Hotelling T^2/χ^2 control charts.For different amounts of shifts, δ, the ARL values of the proposed charts and the other competing charts are provided in Tables 3.1-3.7. The SDRLs of the proposed charts are also given in Tables 3.8-3.9. These results are based on 10^4Monte Carlo simulations, at each run, for our study purposes.For a comparative analysis of the proposed charts with their existing counterparts, we have listed the comparative results in Tables 3.10 and 3.11. We have also created Figures 3.1 and 3.2 to serve purpose of ease in comparison and discussion.

These results advocate that:

- the proposals of the study are quite efficient at detecting smaller shifts in process location.
- as the value of p increases, the efficiency of the chart to detect shifts in the process parameter reduces in general.
- MEC1 is better than MEC2 if $\delta \le 0.25$ for λ=0.10, when $\delta \le 0.50$ for λ= 0.25 and λ=0. 50; and finally when $\delta \le 0.75$ for λ=0.75.

- MEC1 chart is better than MCUSUM and MC1 when there are smaller shift in process mean(i. e. $\delta \leq 0.50$) for all values of λ except when it is very small (e.g. when λ=0.1).
- MEC1 chart is also better than MEWMA when $\delta \leq 0.25$ for λ=0.10 , when $\delta \leq 0.50$ for λ= 0.25, when $\delta \leq 1.25$ for λ=0. 50 and when $\delta \leq 1.75$ for λ=0.75.
- MEC1 is better than χ^2 control chart for small and moderate shifts in the process mean for all value of λ.
- MEC2 control chart is better than MEWMA and χ^2 control chart for small and moderate shifts in the process mean for all value of λ except when λ is very small (e.g. when λ=0.10).
- MEC2 chart is better than MCUSUM and MC1 when $\delta \leq 0.50$ for λ=0.10 and $\delta \leq 0.25$ for $\lambda \geq 0.25$.

Table 10: Comparison of MEC1, MCUSUM, MC1, MEWMA and χ^2 Control Charts

	MEC1	MCUSUM	MC1	MEWMA	χ^2
δ	h_5=12.20	h_2=5.50	h_4=4.75	h_3=10.45	h_1=10.60

0.00	199.51	201.00	195.49	202.99	199.59
0.25	69.28	83.53	89.63	135.85	171.42
0.50	27.17	29.57	30.95	62.67	117.61
0.75	16.03	15.14	14.83	29.71	70.11
1.00	11.44	9.87	9.40	15.91	42.49
1.25	8.97	7.31	6.68	9.44	25.36
1.50	7.50	5.79	5.22	6.36	15.71
1.75	6.50	4.84	4.31	4.62	10.32
2.00	5.76	4.12	3.69	3.62	6.93
2.25	5.18	3.62	3.22	2.96	4.89
2.50	4.77	3.25	2.88	2.52	3.56
2.75	4.41	2.94	2.62	2.20	2.71
3.00	4.13	2.70	2.41	1.95	2.17

Table 3.11:Comparison of MEC2, MCUSUM, MC1, MEWMA and χ^2 Control Chart.

	MEC2	MCUSUM	MC1	MEWMA	χ^2
δ	h_6=1.81	h_2=5.50	h_4=4.75	h_3=8.66	h_1=10.60

0.00	201.29	201.00	195.49	201.51	199.59
0.25	75.07	83.53	89.63	76.60	171.42
0.50	27.81	29.57	30.95	28.08	117.61
0.75	15.31	15.14	14.83	15.21	70.11
1.00	10.56	9.87	9.40	10.11	42.49
1.25	8.18	7.31	6.68	7.63	25.36
1.50	6.73	5.79	5.22	6.11	15.71
1.75	5.80	4.84	4.31	5.11	10.32
2.00	5.10	4.12	3.69	4.42	6.93
2.25	4.57	3.62	3.22	3.88	4.89
2.50	4.17	3.25	2.88	3.49	3.56
2.75	3.84	2.94	2.62	3.18	2.71
3.00	3.56	2.70	2.41	2.93	2.17

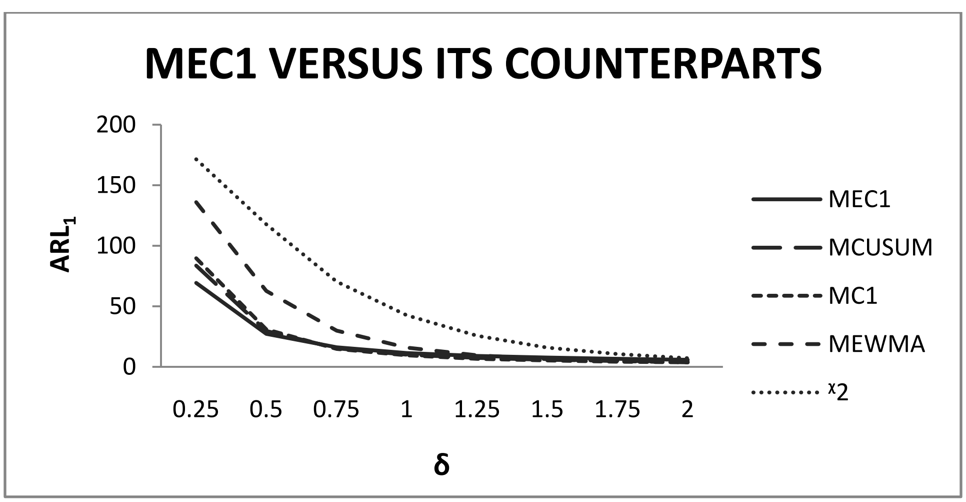

Figure 3.1. ARL Curves of MEC1 and its counterparts at ARL_0 =200

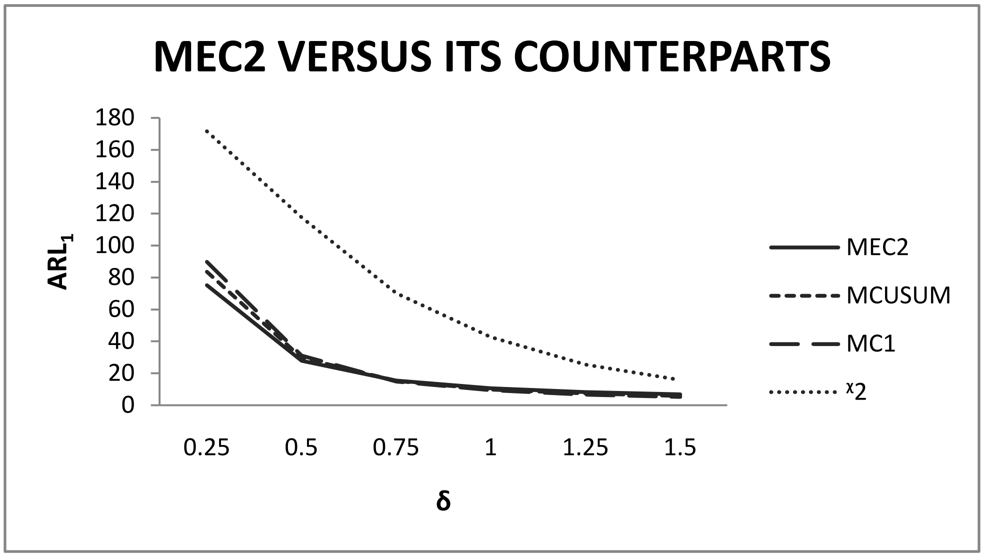

Figure 3.2. ARL Curves of MEC2 and its counterparts at ARL_0 =200

3.5 ILLUSTRATIVE EXAMPLE

Wind turbines are used in a lot industrial applications for instant, road signage, remote telemetry, mobile base stations and also marine applications, off-grid systems, and so on. There is a need to monitor the wind turbines output which rely on the wind speed and the height it is being placed.

In this section, we provide an illustrative example for practical demonstration of our proposed and other competing counterparts. We have used a real data set obtained by measuring the wind speedcollected in the year 2007 at Juaymah meteorological station in Saudi Arabia. A large sample comprises of 4465 observations with ten minutes averaged wind speed data at 10m, 20m, 30m and 40m above ground level were used during the phase 1 stage. The in-control mean and covariance matrix are calculated after all the out-of-control samples have been removed and they are given below as:

$$\mu = \begin{bmatrix} 4.69 \\ 5.41 \\ 5.98 \\ 6.56 \end{bmatrix} \text{ and } \Sigma = \begin{bmatrix} 2.89 & 2.90 & 2.81 & 2.69 \\ 2.90 & 3.01 & 3.01 & 2.96 \\ 2.81 & 3.01 & 3.13 & 3.16 \\ 2.69 & 2.96 & 3.16 & 3.28 \end{bmatrix}$$

The parameters above are used to monitor the subsequent observations.

We now try to monitor another sample of 120 observations, using the mean and covariance derived in phase 1, by the proposed charts and their counterparts. We have constructed all the control charts under discussion (using k=0.5, λ=0.25 where needed) at ARL_0 =200. The resulting control charts are displayed in Figures 3.3- 3.7. Moreover, Table 3.12 provides the dataset and other related statistic for different charts.

The following detection abilities are examined for different charts used in this study:

- Chi-square control chart gives only one out-of-control signal at sample point 105.
- MEWMA control chart generates 29 out-of-control signals at the sample points 27, 42, 76 and 95-120.
- MC1 chart offers 26 out-of-control signals, at sample points95-120.
- MEC1charttriggers 43 out-of controlpoints, at sample points 44-61 and 96-120.
- MEC2 chart detects 38 out-of-controlpoints, at sample points 49-56 and 91-120.

Table 3.12. Numerical Example of the Proposed Charts and their Counterparts

S/no	WS10	WS20	WS30	WS40	$MEC1_i$	$MEC2_i$	x^2	T_i	V_i
1	5.4	6.5	7.5	8.5	0.57	0.00	2.62	2.62	1.12
2	5.6	6.6	7.7	8.6	2.02	0.66	3.81	5.76	2.37
3	6.2	7.1	8.1	9.2	4.66	2.48	7.83	12.49	4.43
4	3	3.8	4.1	4.4	5.08	2.05	7.89	1.54	1.84
5	3	3.7	3.9	4.3	4.35	0.40	6	2.8	0.56

6	2.6	2.9	3.6	4	3.63	0.00	9.16	2.38	0
7	3.2	3.3	3.6	4.2	3.89	1.25	9.19	6.73	2.53
8	3.6	4.6	5.5	6.2	4.92	1.78	1.89	3.94	1.55
9	4	5.1	5.8	6.3	5.27	1.38	6	3.47	0.57
10	3.8	4.9	5.8	6.6	6.14	1.23	1.96	4.5	1.02
11	3.4	3.8	3.9	4.3	6.82	1.28	3.15	3.65	1.02
12	3.1	4.2	5.2	5.6	8.77	2.43	11.44	10.25	2.9
13	3.3	4.3	5.2	6	11.09	3.81	1.58	9.33	3.41
14	3.3	4.4	5.4	6.5	13.21	4.90	3.21	7.69	3.53
15	3.8	4.9	5.9	7	15.09	5.69	3.04	7.88	3.73
16	4	5.1	6.1	7.1	17.00	6.50	2.06	8.72	4.19
17	3.7	4.8	5.8	6.8	19.12	7.51	2.15	10	4.89
18	3.5	4.6	5.5	6.5	21.46	8.77	2.51	11.31	5.71
19	6.1	6.5	6.6	6.8	22.32	8.68	2.3	2.86	3.93
20	6.6	7	7.1	7.3	21.85	7.33	2.73	0.73	1.99
21	6.5	6.9	7.1	7.4	20.42	5.03	2.24	1.38	0.04
22	6.5	7.1	7.3	7.5	18.64	2.40	3.09	3.53	0
23	6.6	7	7.2	7.4	16.30	0.00	2.47	5.37	1.07
24	6.9	7.2	7.4	7.6	13.27	1.52	3.73	8.06	2.45
25	6.8	7.3	7.6	7.8	10.31	3.33	2.63	9.9	3.47
26	6.8	7.2	7.4	7.5	7.76	5.54	3.43	12.65	4.77
27	6.9	7.3	7.5	7.6	6.62	8.09	3.55	15.24	6.13
28	3.3	4.4	5.1	5.7	8.09	9.20	5.57	8.12	4.99
29	3.3	4.2	5	5.7	9.47	9.30	1.2	4.01	3.92
30	3.3	4.3	4.9	5.5	11.26	8.93	4.19	5.76	3.48
31	3.3	4.3	5	5.6	13.39	8.48	3.32	8.27	3.51
32	3.5	4.4	5.2	5.8	15.46	8.05	1.77	8.05	3.45
33	3.5	4.4	5.1	5.8	17.29	7.51	1.02	7.07	3.07
34	3.3	4.2	5	5.8	18.64	6.57	1.04	5.67	2.28
35	2.7	3.6	4.4	5.3	19.55	5.07	2.08	5.74	1.26
36	2.7	3.6	4.4	5.3	20.29	3.26	2.08	6.74	0.48
37	2.9	4	4.7	5.4	22.26	2.63	5.26	10.66	1.84
38	2.6	3.5	4.3	5.2	24.01	1.87	2.2	10.61	1.55
39	2.7	3.5	4.4	5.3	25.27	0.69	2.84	9.68	0.82
40	2.4	3.2	4.1	5	26.39	0.00	3.11	10.97	0.57
41	2.8	3.9	4.7	5.4	28.69	2.11	4.53	11.76	2.13
42	3.3	4.3	5	5.4	31.87	4.37	6.91	14.02	4.04
43	5.3	6.1	6.8	7.3	34.20	5.84	0.97	8.67	3.78
44	5.1	6	6.8	7.3	36.24	7.06	2.1	8.17	4.09
45	5.8	6.8	7.5	8	38.19	8.17	2.66	8.63	4.59
46	6	6.9	7.6	8.2	39.59	8.83	1	6.58	4.24
47	5.8	6.6	7.4	7.9	40.56	9.31	2.36	6.8	4.05
48	5.9	6.8	7.4	7.9	41.57	9.82	1.52	6.78	4.24

49	5.8	6.7	7.3	7.9	42.44	10.16	1.01	5.84	4.08
50	6.5	7.4	8	8.5	43.26	10.55	1.91	7.07	4.2
51	6.1	7	7.7	8.4	43.74	10.66	1.08	6.03	3.65
52	6	6.9	7.7	8.2	44.49	11.21	2.41	7.44	4.08
53	7	8	8.7	9.2	45.61	12.23	3.47	10.61	4.77
54	3.7	4.3	4.8	5.5	45.94	12.15	1.3	3.12	3.36
55	3.5	4.2	4.7	5.3	46.19	11.71	0.74	1.07	2.93
56	4.1	4.6	5.1	5.8	45.39	10.14	2.36	0.26	1
57	4.5	5	5.5	6.2	43.72	7.70	2.52	1.84	0
58	3.5	4	4.5	5.1	41.83	4.94	1.44	2.77	0.7
59	3.3	3.7	4.1	4.7	39.54	1.64	2.62	5.17	1.8
60	2.3	2.8	3.1	3.7	37.50	0.00	3.58	6.93	2.81
61	1.6	1.9	2.2	2.7	35.59	2.06	5.18	11.48	4.51
62	2.3	2.6	2.9	3.3	34.05	4.44	3.69	13.84	5.8
63	4.7	5.9	7.1	8.4	32.84	5.88	6.55	9.35	5.37
64	4.4	5.5	6.7	7.8	32.14	6.93	4.28	9.4	5.38
65	4.4	5.4	6.5	7.5	31.86	8.00	3.25	10.76	5.73
66	4.4	4.6	4.7	4.9	31.13	8.92	2.74	5.82	5.6
67	4.5	4.8	5.1	5.2	30.39	9.25	3.77	4.73	5.24
68	4.5	4.7	4.8	5	29.27	9.61	2.72	5.91	5.54
69	4.6	4.9	5.1	5.2	28.30	9.73	2.94	7.28	5.5
70	4.8	5.2	5.3	5.4	27.54	9.42	3.04	7.7	5.07
71	5.1	5.4	5.5	5.6	26.86	9.23	2.62	9.57	5.2
72	5.2	5.6	5.6	5.8	26.18	8.97	3.12	10.12	5.06
73	5.1	5.5	5.6	5.7	25.99	8.84	2.89	11.91	5.18
74	5.1	5.5	5.6	5.8	26.12	9.00	2.07	12.33	5.46
75	9.5	10.6	11.3	11.9	25.06	7.50	9.4	9.46	2.92
76	9.4	10.4	11	11.5	23.40	5.12	8.55	14.61	1.22
77	5.8	6.8	7.6	8.4	21.93	2.71	1.34	10.17	0
78	5.3	6.4	7.3	8.2	20.60	0.00	1.75	7.61	0.82
79	4.6	5.7	6.7	7.4	20.26	1.20	2.71	6.37	1.54
80	4.7	5.6	6.3	7.2	19.46	1.86	1.71	4.35	1.55
81	4.9	5.8	6.5	7.3	18.58	2.39	0.76	3.96	1.76
82	5.3	6.3	7.2	8.2	17.25	3.13	2.66	5.66	2.65
83	5.1	6.3	7.2	8.1	16.75	4.36	2.61	7.33	3.56
84	5.2	6.3	7.1	7.9	17.06	5.78	1.78	8.07	4.22
85	4.8	5.9	6.7	7.8	17.42	7.54	4.62	11.26	5.53
86	2.3	2.4	2.6	2.8	16.96	6.97	6.29	1.65	2.96
87	6.5	7.6	8.4	9.1	17.33	7.15	2.69	2.33	3.45
88	6.3	7.2	7.8	8.5	17.83	7.68	1.64	3.56	3.86
89	5.2	5.9	6.3	6.9	18.22	8.01	0.95	3.41	3.74
90	5.5	6.6	7.4	8.1	19.40	8.67	2.07	4.35	4.25
91	5.2	6.5	7.4	8.2	21.64	9.92	4.66	8.18	5.44

92	4.3	5.4	6.4	7	24.02	10.81	4.39	8.75	5.54
93	4.1	5	5.8	6.4	26.08	11.32	1.24	7.52	5.37
94	4	5	5.9	6.4	28.32	11.67	4.49	10.69	5.6
95	4.4	5.6	6.6	7.1	31.45	12.60	8.22	18.66	6.87
96	4.1	5.3	6.2	6.8	35.23	14.25	5.79	22.97	8.29
97	4.3	5.4	6.4	6.9	39.34	16.27	6.6	27.35	9.56
98	4.5	5.6	6.6	7.1	43.75	18.73	6.42	30.97	10.97
99	3.9	5	5.9	6.6	48.04	21.37	2.64	26.82	11.96
100	3.1	4	4.9	5.6	51.65	23.49	2.02	21.04	12.22
101	2.7	3.6	4.4	5	55.04	25.40	2.94	19.8	12.8
102	2.3	3.2	4	4.5	58.60	27.37	5.38	22.92	13.8
103	2.9	3.8	4.6	5	62.51	29.66	6.32	27.19	15.12
104	3.6	4.8	5.6	5.9	67.78	33.27	15.28	40.16	17.97
105	3.7	4.8	5.8	6.4	73.06	37.10	4.99	38.22	19.55
106	3.9	5	6	6.7	78.03	40.82	3.12	32.87	20.74
107	3.5	4.6	5.6	6.2	83.11	44.70	5.26	33.74	22.39
108	3.6	4.8	5.7	6.3	88.46	48.93	6.54	35.69	24.35
109	4.3	5.3	6.2	6.7	93.62	53.01	4.21	33.41	25.69
110	4.3	5.5	6.4	7.1	98.63	57.04	3.97	30.6	27.11
111	4.2	5.4	6.5	7.1	103.94	61.40	6.96	34.19	29.13
112	4.5	5.8	6.8	7.5	109.35	65.93	5.88	35.01	31
113	4.5	5.7	6.7	7.5	114.29	70.06	2.81	29.84	32.09
114	4.8	5.9	6.8	7.6	118.49	73.51	1.5	22.85	32.62
115	4.9	5.9	7	7.7	122.26	76.53	3.99	20.17	33.31
116	4.6	5.7	6.8	7.5	126.05	79.56	3.92	21.34	34.49
117	4.4	5.6	6.7	7.4	130.17	82.91	4.77	24.06	36.11
118	4.4	5.4	6.3	7.1	133.57	85.57	0.95	17.76	36.34
119	5	6.2	7.2	7.9	137.23	88.49	3.71	19.28	37.71
120	5.4	6.8	7.8	8.5	141.54	92.07	7.86	24.72	39.78
			Control Limits		**34.91**	**9.91**	**14.86**	**13.86**	**6.18**

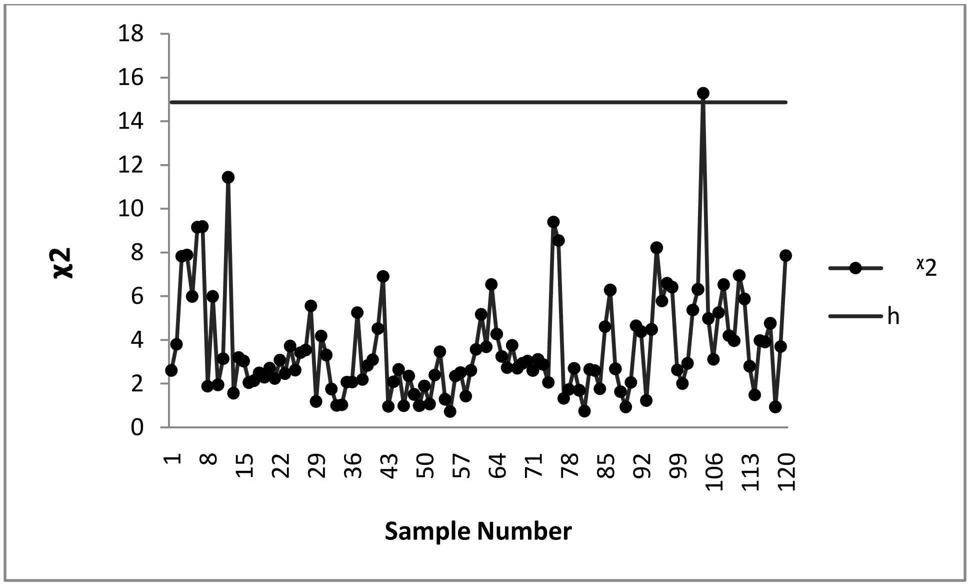

Figure 3.3. χ^2 Control Chart when h=14.86 at ARL_0 =200

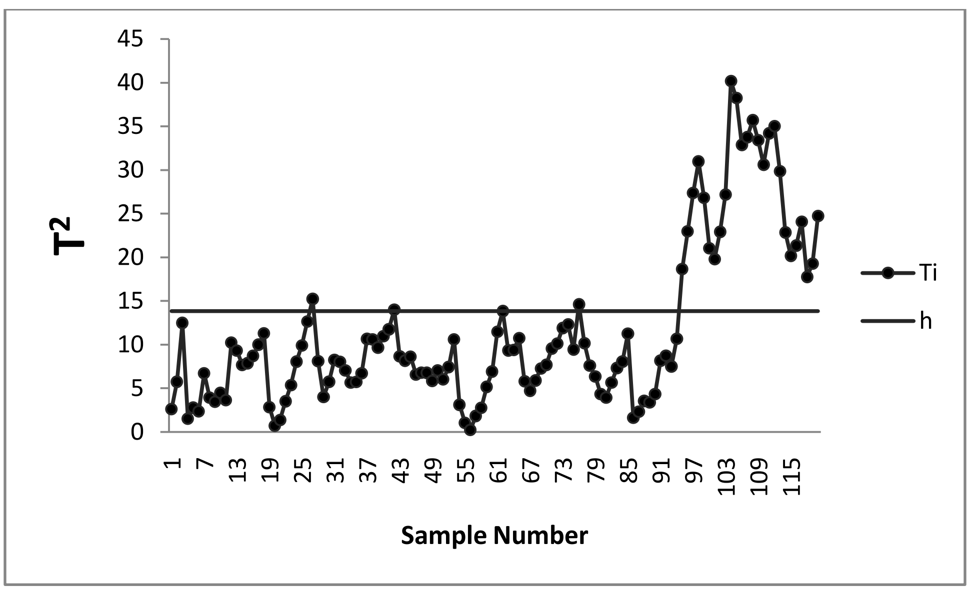

Figure 3.4. MEWMA Chart when h=13.86 and λ=0.25 at ARL_0 =200

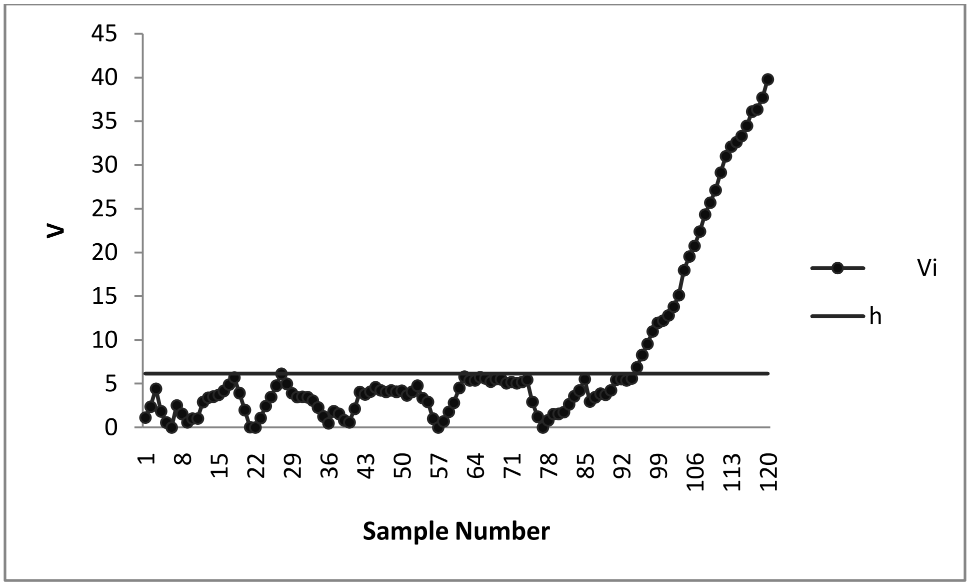

Figure 3.5. The graph of MC1 when k=0.5, h=6.18 at ARL_0 =200

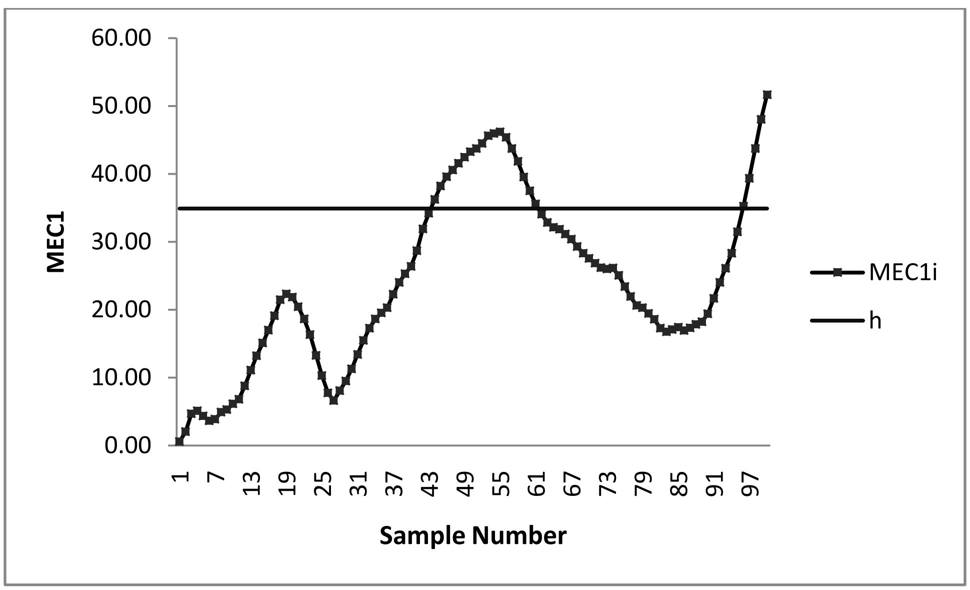

Figure 3.6. MEC1 Chart when k=0.5, h=34.91 and λ=0.25 at ARL_0 =200

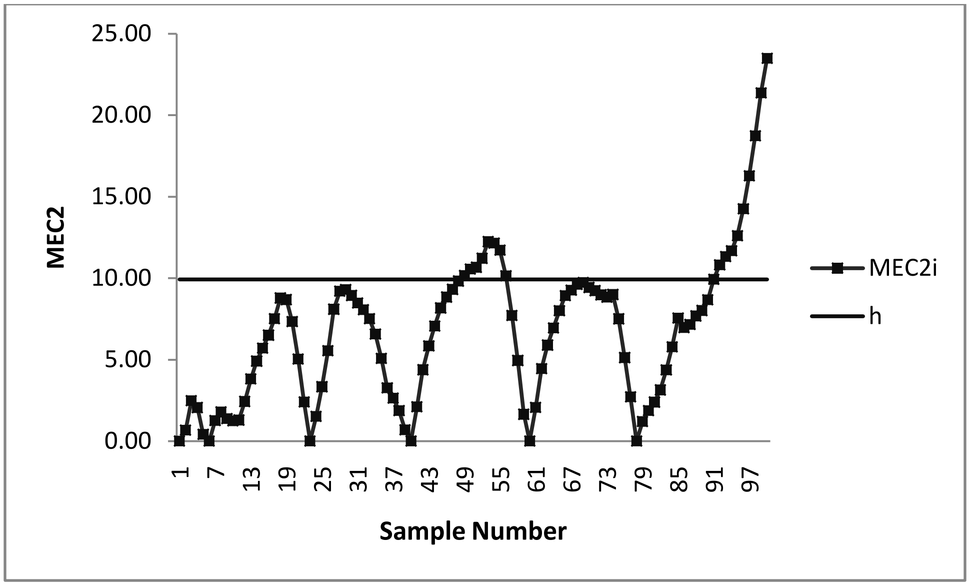

Figure 3.7. MEC2 Chart when k_1 =0.87, h=9.91 and λ=0.25 at ARL_0 =200

From the above analysis of detection abilities it is evident that the proposals of the study are quite effective at detecting shifts in process mean vector, especially of smaller magnitude.

3.6 CONCLUSIONS AND RECOMMENDATIOS

We have proposed two multivariate mixed EWMA-CUSUM control charts,in the form of MEC1 and MEC2 control charts, to monitor changes in the process mean vector. The performance of the proposed schemes is evaluated in terms of ARL and compared with other competing charts like MCUSUM, MEWMA, MC1 and Hotelling T^2/χ^2 control charts. The comparisons showed that the proposed schemes are really good at detecting the small shifts in the process as compared with the other schemes under study.

The scope of this study may be extended to mixed EWMA-CUSUM control chart for monitoring process dispersion in the form of variance-covariance matrix. Moreover, non-normal environments may be investigated in search of mixed robust design structures for mean vector and variance-covariance matrix.

CHAPTER 4

NEW MEMORY-TYPE CONTROL CHARTS FOR MONITORING PROCESS MEAN AND DISPERSION.

Control chart is widely used to monitor the quality of the products of industrial or business processes. Max-CUSUM and Max-EWMA are based on memory-type control charts that monitor the process mean and standard deviation simultaneously. This chapterintroduces seven new control charts that monitor the process mean and dispersion simultaneously. The proposed control charting schemes are compared with the existing counterparts including Max-EWMA, Max-CUSUM, $SS-EWMA$ and $SS-CUSUM$. A case study is presented for practical considerations using a real dataset.

4.1 INTRODUCTION

All the manufacturing and non-manufacturing processes are bound to vary. These process variations consist of two types, namely natural and special cause variations. The natural variation can be background noise, which cannot be controlled but the assignable cause variation is due to many artificial factors like faulty machines, operator mistakes, defective items and so on. Statistical Process Control (SPC) is a collection of useful techniques that helps to differentiate between the two sources of variations. Control chart is a powerful tool of SPC to detect the special cause variation within a process.

Control chart is divided into two types namely memory-less and memory type charts. Shewhart type control charts are memoryless control chart because it is based on only the present information. Shewhart (1931) introduced the fundamental control charts to monitor process behaviors in terms of location and dispersion parameters. $\bar{X}$ chart is extensively employed in the industries to monitor process mean while R, S and S^2 control charts are used for monitoring process dispersion/variability. A major limitation of Shewhart control charts is that it is poor in detecting small and moderate shifts in a process parameter. Exponential weighted moving average (EWMA) and Cumulative sum (CUSUM) are the two examples of memory type control charts that use both the past and present information to detect small and moderate shifts in the process parameters. In EWMA charts the past observations are accounted for, but they are given a smaller weight as they become older, so EWMA chart applies the most weight to the current observations and geometrically decreasing weight to all previous observations. CUSUM charts also give memory by using the information of past history. The basic structures of CUSUM and EWMA were presented bypage (1954) and Robert (1959) respectively.

One popular approach of monitoring the process location and dispersion requiresdesigning two different charts separately. An alternative approach is in the form of Max chart developed by Chen and Cheng (1998). It can monitor both process mean and standard deviation in a single chart, but it has the deficiency of detecting small shifts in the process parameters. Xie (1999) introduced Max-EWMA, SS-EWMA, EWMA-

Max and EWMA-SC control charts to overcome this challenge. Following the inspiration of Xie (1999) for process monitoring, Thaga (2009) and Cheng and Thaga (2010) developed Max-CUSUM and SS- CUSUM respectively, which are also memory-type control charts. These control charts use both present and previous information about the process; they are very effective in detecting small shifts than Max-Chart.

Sixnew charts are proposed in this chapter to aid in detecting small shifts than the existing memory-type univariate control charts that monitor both location and dispersion. Two of the proposed charts are based on combining the effects of Max-EWMA and SS-EWMA respectively with Max-CUSUM and SS-CUSUM control charts. The other four charts are developed by replacing the statistic thatmonitors the process dispersion of Max-EWMA, Max-CUSUM, SS-EWMA and SS-CUSUM with the three parameters logarithmic transformation to S^2 which was suggested by Castagliola (2005).

The organization to the rest of this chapter is as" Section 4.2 describes different memory-type control charts that monitor both location and dispersion simultaneously. Section 4.3 provides the design structure of the proposed control charts of this study; Section 4.4 offers comparisons of the proposals with the existing counterparts; Section 4.5 includes a case study for our study purposes; Section 4.6 concludes the findings for the study.

4.2.0 Memory Type Control Charts for Monitoring Process Mean and Dispersion

In this section we provide a brief description of the design structures of some useful charts considered in this study. We focus on the structures of simultaneous control charts.

The commonly used memory-type univariate control charts for simultaneous monitoring of the process mean and dispersion are: Max-CUSUM, Max-EWMA, SS-CUSUM and SS-EWMA. The details of these control charts are given below one by one.

4.2.1 Max-EWMA Control Chart

Let $X_i = X_{i1},...,X_{in}$, i=1,2,3,… represent a sequence of samples of size n.The samples are independent and identical normal distributed.

Let $\bar{X}_i = (X_{i1}+...+X_{in})/n$ and $S_i^2 = \frac{\sum_{j=1}^{n}(X_{ij}-\bar{X})^2}{n-1}$ denote the mean and variance for the i^{th} sample respectively. These two statistics are the minimum variance unbiased estimator of μ_0 and σ_0^2 correspondingly. Max-EWMA chart is defined by following statistics:

$$Z_i = \sqrt{n}\frac{(\bar{X}_i-\mu_0)}{\sigma_0} \qquad (4.1)$$

$$Y_i = \Phi^{-1}\left\{H\left[\frac{(n-1)S_i^2}{\sigma_0^2}; n-1\right]\right\} \qquad (4.2)$$

In equation (4.2), $\Phi^{-1}(.)$ represents inverse standard normal cumulative distribution function and $H(.,n-1)$ is a Chi-square distribution function with n-1 degrees of freedom. The functions Z_i and Y_i follow the standard normal distribution and are independent statistics. The two EWMA statistics derived from Z_i and Y_i are given below:

$$U_i = (1-\lambda)U_{i-1} + \lambda Z_i \qquad (4.3)$$

$$V_i = (1-\lambda)V_{i-1} + \lambda Y_i \qquad (4.4)$$

$M_i = max(|U_i|, |V_i|)$ (4.5)

The highest absolute value (M_i) of both statistics U_i *and* V_i is calculated and it is compare with the upper control limit (UCL) given in Equation 4.6.

$$n_i = \begin{cases} n_{i-1} + 1 & if \;\; MEC2_{i-1} > 0 \\ 1 & otherwise \end{cases} \quad (4.6)$$

The expected value and variance of M_i are derive through the numerical computation and they are given to be $E(M_i)$=1.12379 and $Var(M_i)$=0.363381 respectively.

Therefore, UCL=1.128379+0.602811L.

The ARL results for the Max- EWMA control charting schemes for various shifts in the process mean (a) and dispersion (b) are provided in Table 4.1.

Table4.1. ARL values for Max- EWMA control charting schemes at $ARL_0 = 250$, $L = 2.785$ *and* $\lambda = 0.10$

	a								
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	250.68	24.88	8.82	5.32	3.85	3.07	2.57	2.23	2.04
1.25	17.70	13.20	7.88	5.27	3.86	3.10	2.60	2.27	2.06
1.50	7.35	6.94	5.87	4.66	3.76	3.08	2.62	2.30	2.06
1.75	4.80	4.66	4.32	3.87	3.38	2.95	2.57	2.27	2.05
2.00	3.61	3.59	3.45	3.23	2.96	2.69	2.44	2.21	2.02

4.2.2 Max-CUSUM Chart

Max-CUSUM was developed by Cheng and Thaga (2010). The statistics of equation (4.1) and (4.2) are integrated in CUSUM statistics to monitor the process mean and variance as given in equations 4.5-4.8 below:

$C_i^+ = max[0, Z_i - k + C_{i-1}^+]$ (4.7)

$C_i^- = max[0, -Z_i - k + C_{i-1}^-]$ (4.8)

$S_i^+ = max[0, Y_i - k + S_{i-1}^+]$ (4.9)

$S_i^- = max[0, -Y_i - k + S_{i-1}^-]$ (4.10)

where $C_0 = S_0 = 0$ are the starting points and k is the reference value. Since Z_i and Y_i are both normally distributed, then we can combine the statistics C_i^+, C_i^-, S_i^+ *and* S_i^-; and develop a new statistic that determine the highest value of the four. This new statistic is represented by N_i as it is in equation 4.11 below.

$N_i = max(C_i^+, C_i^-, S_i^+, S_i^-)$ (4.11)

Since the statistic N_i is always positive then it has only the upper control limit, h. Whenever N_i, exceeds the control limit h, then we say that the process is in an out-of-control state, otherwise, it is in a good state.

The ARL results for the Max- CUSUM control charting schemes are provided in Table 4.2.

Table4.2. ARL values for Max- CUSUM control charting schemes at $ARL_0 = 250$, $h = 5.05$ *and* $k = 0.50$

	a								
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	249.09	30.01	8.77	5.02	3.54	2.78	2.32	2.06	1.88
1.25	18.10	13.25	7.55	4.91	3.57	2.83	2.37	2.08	1.86
1.50	6.90	6.43	5.37	4.29	3.42	2.79	2.39	2.09	1.84
1.75	4.34	4.25	3.93	3.50	3.06	2.63	2.31	2.04	1.83
2.00	3.26	3.24	3.08	2.89	2.64	2.42	2.18	1.96	1.80

4.2.3 SS-EWMA Chart

Base on the statistics in equation 4.3 and 4.4, SS-EWMA statistic is constructed; it is given in Equation 4.12.

$SS1_i = U_i^2 + V_i^2 \;\; for\; i = 1,2,3,...$(4.12)

$SS1_i$ follows a chi-square distribution when, it is divided by $\sigma_{U_i}^2$, since $\frac{U_i}{\sigma_{U_i}}$ and $\frac{V_i}{\sigma_{V_i}}$ are independent and identical standard normal distribution.

$$\frac{SS1_i}{\sigma_{U_i}^2} = \frac{U_i^2}{\sigma_{U_i}^2} + \frac{V_i^2}{\sigma_{U_i}^2} \sim \chi_2^2 \qquad (4.13)$$

Based on the information of equation 4.11, the expected value and the variance of SS_i are given by

$$E\left(SS1_i\right) = 2\sigma_{U_i}^2 \qquad (4.14)$$

$$Var\left(SS1_i\right) = 4\sigma_{U_i}^4 \qquad (4.15)$$

Since $SS1_i$ is always positive, therefore, it has only the Upper Control Limit (UCL). Now, the UCL is given by

$$UCL = E\left(SS1_i\right) + L\sqrt{Var\left(SS1_i\right)} \qquad (4.16)$$

By the substitution of equation 4.14 and 4.15 in the UCL, then, we have

$$UCL = 2\sigma_{U_i}^2\left(1 + L\right) \qquad (4.17)$$

Since $\sigma_{U_i}^2 = \frac{\lambda}{2-\lambda}$ for the steady case, therefore the UCL approaches asymptotically to:

$$UCL = \frac{2\lambda}{2-\lambda}(1+L) \qquad (4.18)$$

The ARL results for the SS-EWMA control charting schemes are provided in Table 4.3.

Table4.3. ARL values for SS- EWMA control charting schemes at $ARL_0 = 250$, $L = 3.6$ and $\lambda = 0.10$

	a								
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	252.32	25.67	9.14	5.56	4.06	3.23	2.70	2.34	2.10
1.25	17.26	12.28	7.41	5.09	3.88	3.17	2.66	2.34	2.11
1.50	7.24	6.62	5.37	4.33	3.52	2.95	2.58	2.29	2.09
1.75	4.75	4.54	4.08	3.58	3.13	2.73	2.43	2.20	2.02
2.00	3.58	3.50	3.30	3.06	2.76	2.50	2.28	2.09	1.93

4.2.3 SS-CUSUM Chart

SS-CUSUM is developed by Thaga (2009) and it is based on equations 4.7-4.10 in Section 4.2.2 above. Let M_i and V_i be the maximum values of the CUSUM statistics that monitor process mean and standard deviation respectively.

$$M_i = max(C_i^+, C_i^-) \qquad (4.19)$$

$$V_i = max(S_i^+, S_i^-) \qquad (4.20)$$

The SS-CUSUM is the sum of square of the maximum values of the CUSUM statistics that monitor both process mean and standard deviation and it is given by:

$$SS2_i = M_i^2 + V_i^2 \qquad (4.21)$$

$SS2_i$ is plotted against the control limit, h.

The ARL results for the SS-CUSUM control charting schemes are provided in Table 4.4.

Table4.4. ARL values for SS- CUSUM control charting schemes at $ARL_0 = 250$, $h = 27.9$ and $k = 0.50$

	a								
b	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
1	250.53	26.011	8.16	4.9004	3.59	2.8908	2.45	2.1652	2.01
1.25	56.4846	17.4071	7.3958	4.6527	3.4824	2.8343	2.4167	2.1713	1.9977
1.5	22.96	11.8967	6.53	4.4188	3.35	2.752	2.40	2.1501	1.97
1.75	12.0418	8.5539	5.5789	4.0462	3.2229	2.6885	2.3386	2.0929	1.9219
2	7.44	6.1654	4.74	3.6821	3.00	2.5775	2.26	2.0267	1.87

4.3.0 THE PROPOSED CONTROL CHARTS

In this section, we propose seven new memory type univariate control charts that monitor both process mean and standard deviation simultaneously. Two of these charts are based on combining the effects of max-EWMA, max-CUSUM, SS-EWMA and SS-CUSUM charts. The inspiration of this approach is taken from Abbas et al. (2013 a&b) and Zaman et al. (2014). The remaining five proposed charts are the introduction of the three parameters logarithmic transformation to S^2 which was suggested by Castagliola (2005) to the statistic that monitors the process dispersion of Max-EWMA, Max-CUSUM, SS-EWMA, and SS-CUSUM. This section is divided into seven parts for the seven proposals separately.

4.3.1 MIXED MAX EWMA-CUSUM CONTROL CHART

Mixed max EWMA-CUSUM control chart (MMEC) is the integration of effects max-EWMA statistics in the max-CUSUM.

Assume $X_{ij} \sim N\left(\mu,\sigma^2\right)$, for $i=1,2,3,...$ *and* $j=1,2,3,..;\mathrm{n}_i$ where n_i is the sample size. Let $\bar{X}_i=(X_{i1}+...+\mathrm{X}_{in})/n$ and $S_i^2=\dfrac{\sum_{j=1}^{n}(X_{ij}-\bar{X})^2}{n-1}$ be the sample mean and variance of the distribution respectively. These two statistics are the minimum variance unbiased estimator of μ_0 and σ_0^2 respectively. These statistics are standardizing into normal distribution given by $Z_i=\sqrt{n}\dfrac{(\bar{X}-\mu_0)}{\sigma_0}$ and $W_i=\Phi^{-1}\left\{H\left[\dfrac{(n-1)S_i^2}{\sigma_0^2};n-1\right]\right\}$ respectively. Z_i and W_i are not affected by the size of the sample and they are transformed into EWMA statistics below:

$$U_i=(1-\lambda)U_{i-1}+\lambda Z_i \tag{4.22}$$

$$V_i=(1-\lambda)\mathrm{V}_{i-1}+\lambda W_i \tag{4.23}$$

where $0<\lambda\le 1$; V_0 *and* U_0 are the initial values of V_i *and* U_i respectively. The EWMA statistics are transformed into CUSUM statistics, as given in equation 4.24-4.27.

$$C_i^+=\max\left[0,\mathrm{U}_i-k^++C_{i-1}^+\right] \tag{4.24}$$

$$C_i^-=\max\left[0,-U_i-k^++C_{i-1}^-\right] \tag{4.25}$$

$$S_i^+=\max\left[0,\mathrm{V}_i-k^++S_{i-1}^+\right] \tag{4.26}$$

$$S_i^-=\max\left[0,-V_i-k^++S_{i-1}^-\right] \tag{4.27}$$

Since $\sigma_{U_i}^2 = \sigma_{V_i}^2 = \dfrac{\lambda}{2-\lambda}$ then the reference value k^+ is given by $k^+ = k\sigma_{U_i} = k\sigma_{V_i}$ where k can be any values greater than zero, it is usually taking to be 0.5.

$$k^+ = k\sqrt{\frac{\lambda}{2-\lambda}} \qquad (4.28)$$

Finally, we compute $MMEC_i$ which is the maximum value of the four statistics in equation 4.22-4.25 and compare it with the control limit H.

$$MMEC_i = \max(C_i^+, C_i^-, S_i^+, S_i^-) \qquad (4.27)$$

$$H = h\sqrt{Var(MMEC_i)} = 0.60281h\sqrt{\frac{\lambda}{2-\lambda}} \qquad (4.28)$$

The ARL results for the mixed max EWMA-CUSUM control chart schemes are provided in Table 4.5.

Table4.5. ARL values for MMEC control charting schemes at $ARL_0 = 250$.

		a				
	b	0.00	0.50	1.00	1.50	2.00
H=59.51	1.00	247.93	17.22	10.68	8.26	6.97
λ=0.10	1.50	16.09	14.58	10.68	8.32	6.99
k=0.50	2.00	10.31	10.22	9.53	8.20	7.00
H=38.6	1.00	250.48	13.48	7.78	5.92	4.95
λ=0.20	1.50	12.37	10.92	7.77	5.96	4.95
k=0.50	2.00	7.54	7.41	6.81	5.82	4.96
H=28.51	1.00	249.40	11.72	6.40	4.78	3.98
λ=0.30	1.50	10.64	9.14	6.41	4.81	3.99
k=0.50	2.00	6.20	6.06	5.50	4.70	3.96
H=18.40	1.00	251.00	10.04	4.98	3.58	2.97
λ=0.50	1.50	8.86	7.35	4.97	3.63	2.98
k=0.50	2.00	4.81	4.65	4.15	3.50	2.94
H=11.17	1.00	249.20	8.99	3.94	2.69	2.10

λ=0.80	1.50	7.52	5.95	3.87	2.73	2.19
k=0.50	2.00	3.75	3.55	3.10	2.59	2.16

4.3.2 MIXED SUM of SQUARE EWMA-CUSUM CONTROL CHART

Follow from the equation 4.24-4.27 in Section 4.3.1 above, a new statistic is constructed and it is named as Mixed Sum of Square EWMA-CUSUM Control Chart (MSSEC). This is the sum of square of the maximum values of the CUSUM statistics that monitor both process mean and standard deviation.

Let M_i and V_i be the highest values of the CUSUM statistics that monitor process mean and standard deviation respectively. They are given as $M_i = \max\left(C_i^+, C_i^-\right)$ and $V_i = \max\left(S_i^+, S_i^-\right)$. Therefore,

$$MECSS_i = M_i^2 + V_i^2 \qquad (4.28)$$

$MECSS_i$ is plotted against the control limit h, and the value of h is derived through the simulation.The ARL results for the mixed sum of squareEWMA-CUSUM control chart schemes are provided in Table 4.6.

Table4.6. ARL values for MSSEC control charting schemes at $ARL_0 = 250$.

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
	1.00	252.05	31.87	17.84	13.35	11.04	9.60	8.55	7.82	7.19
K=0.50	1.25	27.78	22.47	16.36	12.95	10.89	9.50	8.52	7.76	7.19
H=77.5	1.50	16.27	15.47	13.64	11.79	10.32	9.20	8.34	7.66	7.11
λ=0.10	1.75	12.53	12.28	11.47	10.51	9.57	8.74	8.03	7.47	6.98
	2.00	10.50	10.37	9.98	9.45	8.83	8.24	7.68	7.18	6.78
	1.00	247.95	27.44	13.72	9.85	7.99	6.86	6.06	5.48	5.05
k=0.50	1.25	22.97	17.92	12.40	9.52	7.88	6.78	6.03	5.46	5.05
H=66.25	1.50	12.29	11.59	10.07	8.58	7.42	6.55	5.90	5.39	4.99
λ=0.20	1.75	9.15	8.96	8.30	7.55	6.84	6.20	5.67	5.25	4.89
	2.00	7.54	7.45	7.14	6.73	6.27	5.81	5.41	5.03	4.73

k=0.50 H=57.5 λ=0.30	1.00	250.70	25.91	11.91	8.25	6.58	5.59	4.90	4.40	4.06
	1.25	20.97	16.00	10.65	7.96	6.49	5.52	4.87	4.40	4.05
	1.50	10.51	9.85	8.45	7.10	6.08	5.33	4.77	4.34	4.01
	1.75	7.60	7.42	6.85	6.18	5.56	5.02	4.57	4.22	3.92
	2.00	6.17	6.09	5.81	5.47	5.08	4.69	4.35	4.03	3.79
k=0.50 H=44.5 λ=0.50	1.00	248.19	25.06	10.12	6.61	5.09	4.25	3.65	3.26	3.02
	1.25	18.88	14.05	8.86	6.34	5.02	4.19	3.65	3.27	3.01
	1.50	8.62	8.02	6.75	5.57	4.66	4.02	3.57	3.23	2.97
	1.75	5.96	5.80	5.31	4.74	4.23	3.78	3.42	3.14	2.90
	2.00	4.72	4.65	4.42	4.13	3.82	3.51	3.23	2.98	2.79
k=0.50 λ=0.80 H=33.1	1.00	249.66	26.75	9.06	5.48	4.03	3.26	2.75	2.40	2.14
	1.25	17.77	12.88	7.64	5.19	3.97	3.21	2.74	2.40	2.18
	1.50	7.23	6.64	5.53	4.45	3.62	3.07	2.68	2.38	2.18
	1.75	4.73	4.60	4.17	3.67	3.23	2.86	2.55	2.32	2.14
	2.00	3.61	3.56	3.39	3.14	2.88	2.61	2.40	2.21	2.07

4.3.3 A newsum of square EWMA control chart for monitoring process mean and variance simultaneously.

A new sum of square EWMA control chart for monitoring process mean and variance simultaneously (SS-EWMAVAR) is constructed by replacing the statistic that monitors the process dispersion in SS-EWMA control chart with the three parameters logarithmic transformation to S^2 which was suggested by Castagliola (2005).

Supposethat,$X_{ij} \sim N(\mu, \sigma^2)$, for $i = 1,2,3,...$ *and* $j = 1,2,3,..;\mathrm{n}_i$ where n_i is the sample size. Let $\bar{X}_i = (X_{i1} + ... + \mathrm{X}_{in}) / n$ and $S_i^2 = \frac{\sum_{j=1}^{n} (X_{ij} - \bar{X})^2}{n-1}$ be the sample mean and variance of the distribution respectively. These two statistics are the minimum variance unbiased estimator of μ_0 and σ_0^2 respectively. $\bar{X}_i$ is standardized into normal distribution and it is given by $G_i = \sqrt{n}\frac{\left(\bar{X}_i - \mu_0\right)}{\sigma_0}$

Castagliola (2005) use the three parameters logarithmic transformation in order to monitor the process variance which is given below by:

$$T_i = a + b ln\left(S_i^2 + c\right) \quad (4.29)$$

Where a, b and c are constants that are greater than zero and they are defined below by

$b = B(n),\ c = C(n)\sigma_0^2\ \ a = A(n) - 2B(n)1n\left(\sigma_0\right)$, where A(n), B(n) and C(n) are functions that depend on the value of the sample size (n). Table 4.7 reproduces the values of $A(n), B(n),\ C(n),\ \mu_T(n)\ and\ \ \sigma_T(n)$ for the sample size 3 up to 15.

Table 4.7: Values of $\boldsymbol{A(n), B(n), C(n), \mu_T(n)\ and\ \sigma_T(n)}$

n	A(n)	B(n)	C(n)	μ_T(n)	σ_T(n)
3	-0.6627	1.8136	0.6777	0.02472	0.9165
4	-0.7882	2.1089	0.6261	0.01266	0.9502
5	-0.8969	2.3647	0.5979	0.00748	0.967
6	-0.994	2.5941	0.5801	0.00485	0.9765
7	-1.0827	2.8042	0.5678	0.00335	0.9825
8	-1.1647	2.9992	0.5588	0.00243	0.9864
9	-1.2413	3.182	0.5519	0.00182	0.9892
10	-1.3135	3.3548	0.5465	0.00141	0.9912
11	-1.382	3.5189	0.5421	0.00112	0.9927
12	-1.4473	3.6757	0.5384	0.0009	0.9938
13	-1.5097	3.826	0.5354	0.00074	0.9947
14	-1.5697	3.9705	0.5327	0.00062	0.9955
15	-1.6275	4.11	0.5305	0.00052	0.996

In order to monitor the process variance, we will use the distribution T_i; and standardized it into normal so as to have the same distribution with G_i,

$$F_i = \frac{(T_i - \bar{\mu}_T(\text{n}))}{\sigma_T(n)} \sim N(0,1)$$

Now, G_i and F_i are transformed into EWMA statistics as below

$$U_i = (1-\lambda)\,\text{U}_{i-1} + \lambda G_i \qquad (4.30)$$

$$V_i = (1-\lambda)\,\text{V}_{i-1} + \lambda F_i \ (4.31)$$

$where \ \ 0 < \lambda \le 1$.

The initial values of U_i *and* V_i are $\text{U}_0 = 0$ *and* $V_0 = \text{A}(n) + \text{B}(n)\ln\{1 + \text{C}(n)\}$ respectively.

Base on the statistics in equation 4.30 and 4.31, SS-EWMA statistic is constructed; it is given in equation 4.32 below.

$$SSEW_i = U_i^2 + V_i^2 \, for \ i = 1,2,3, \ldots \qquad (4.32)$$

It has been proved in Section 4.2.3 that $SSEW_i$ follows a chi-square distribution and their expected value and the variance given by $E(SSEW_i) = 2\sigma_{U_i}^2$ and $Var(SSEW_i) = 4\sigma_{U_i}^4$ respectively.

Since $SSEW_i$ is always positive, therefore, it has only the Upper Control Limit (UCL). Now, the UCL is given by

$$UCL = E\left(SSEW_i\right) + L\sqrt{Var\left(SSEW_i\right)} \qquad (4.33)$$

By the substitution of $E(SSEW_i)$ and $V(SSEW_i)$in the UCL, then, we have $UCL = 2\sigma_{U_i}^2\left(1+L\right)$

Since $\sigma_{U_i}^2 = \dfrac{\lambda}{2-\lambda}$ for the steady case, therefore the UCL approaches asymptotically to:

$$UCL = \frac{2\lambda}{2-\lambda}\left(1+L\right) \qquad (4.34)$$

Table4.8. ARL values for SS-EWMAVARcontrol charting schemes at $ARL_0 = 250$.

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
L=3.55 λ=0.10	1.00	248.25	24.87	8.88	5.37	3.92	3.13	2.60	2.27	2.05
	1.25	13.85	10.13	6.39	4.54	3.54	2.89	2.48	2.20	2.00
	1.50	5.57	5.17	4.38	3.63	3.03	2.61	2.32	2.07	1.89
	1.75	3.67	3.57	3.27	2.91	2.62	2.35	2.13	1.95	1.78
	2.00	2.81	2.78	2.65	2.48	2.29	2.11	1.94	1.79	1.67
L=3.99 λ=0.20	1.00	250.83	31.02	8.70	4.77	3.34	2.62	2.19	1.95	1.74
	1.25	14.89	10.46	6.08	4.10	3.09	2.49	2.11	1.85	1.65
	1.50	5.36	4.93	4.05	3.27	2.67	2.26	1.98	1.75	1.57
	1.75	3.40	3.28	2.97	2.61	2.32	2.05	1.82	1.64	1.49
	2.00	2.52	2.50	2.37	2.20	2.01	1.83	1.67	1.53	1.41
L=4.15 λ=0.30	1.00	249.85	38.95	9.39	4.63	3.09	2.38	1.96	1.70	1.46
	1.25	15.54	10.92	6.05	3.91	2.87	2.27	1.89	1.62	1.43
	1.50	5.22	4.80	3.86	3.06	2.45	2.05	1.77	1.55	1.38
	1.75	3.19	3.07	2.76	2.40	2.10	1.85	1.64	1.47	1.33
	2.00	2.30	2.30	2.16	2.01	1.83	1.65	1.52	1.39	1.29
L=4.24 λ=0.50	1.00	248.19	57.15	12.42	5.05	2.99	2.15	1.68	1.40	1.19
	1.25	17.05	12.06	6.46	3.88	2.70	2.03	1.64	1.38	1.22
	1.50	5.18	4.71	3.71	2.87	2.22	1.83	1.56	1.36	1.22
	1.75	2.95	2.82	2.53	2.19	1.89	1.65	1.46	1.31	1.20
	2.00	2.08	2.06	1.94	1.80	1.64	1.48	1.37	1.26	1.18
L=4.267 λ=0.80	1.00	251.57	92.67	22.30	7.46	3.54	2.18	1.55	1.25	1.10
	1.25	20.34	15.12	8.16	4.48	2.87	2.00	1.53	1.28	1.14
	1.50	5.69	5.17	3.95	2.99	2.20	1.76	1.47	1.27	1.16
	1.75	2.98	2.85	2.52	2.14	1.83	1.57	1.38	1.24	1.15
	2.00	2.00	1.99	1.87	1.72	1.57	1.42	1.31	1.20	1.14

4.3.4 A new maximum EWMA control chart for monitoring process mean and variance simultaneously.

A new maximum EWMA control chart for monitoring process mean and variance simultaneously (Max-EWMAVAR) is constructed by replacing the statistic that monitors the process dispersion in Max-EWMA control chart with the three parameters logarithmic transformation to S^2.

Under the same assumption in Section 4.3.3 above, the statistics of equations 4.30 and 4.31 can be combined in another form as

$$MEW_i = \max\left(|U_i|, |V_i|\right)$$

MEW_i is plotted against the upper control limit (UCL) derived in Equation 4.6.

Table4.9: ARL values for Max-EWMAVARcontrol charting scheme at $ARL_0 =$ 250.

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
	1.00	250.29	24.54	8.73	5.26	3.84	3.07	2.55	2.24	2.03
L=2.77	1.25	14.06	10.99	7.08	4.98	3.78	3.06	2.58	2.25	2.05
λ=0.1	1.50	5.65	5.39	4.76	4.04	3.42	2.93	2.53	2.26	2.03
	1.75	3.69	3.62	3.42	3.22	2.94	2.63	2.38	2.17	2.00
	2.00	2.82	2.79	2.75	2.63	2.49	2.34	2.19	2.04	1.89
	1.00	251.18	30.54	8.43	4.62	3.23	2.53	2.13	1.89	1.67
L=2.99	1.25	15.51	11.40	6.59	4.35	3.22	2.56	2.14	1.86	1.65
λ=0.2	1.50	5.53	5.14	4.39	3.62	2.93	2.45	2.11	1.85	1.65
	1.75	3.46	3.40	3.15	2.83	2.54	2.25	1.99	1.80	1.61
	2.00	2.59	2.57	2.49	2.34	2.18	2.00	1.84	1.68	1.56
	1.00	251.4	38.3	9	4.46	2.98	2.29	1.89	1.63	1.38
L=3.06	1.25	16.22	11.9	6.51	4.12	2.96	2.31	1.9	1.61	1.41
λ=0.3	1.50	5.429	5	4.17	3.36	2.65	2.2	1.87	1.62	1.43
	1.75	3.276	3.19	2.94	2.58	2.29	2.01	1.76	1.58	1.42

	2.00	2.388	2.38	2.28	2.13	1.96	1.79	1.64	1.5	1.38
	1.00	250.5	89.6	21.2	7.01	3.34	2.06	1.49	1.22	1.08
L=3.12	1.25	21.88	16.7	9.11	4.83	3.01	2.04	1.54	1.28	1.13
λ=0.8	1.50	6.205	5.62	4.39	3.31	2.41	1.89	1.54	1.31	1.18
	1.75	3.175	3.08	2.74	2.34	1.98	1.68	1.47	1.3	1.19
	2.00	2.131	2.11	2	1.83	1.67	1.5	1.37	1.25	1.17

4.3.5 A new sum of square ofcumulative sum control chart for monitoring process mean and variance simultaneously.

A new sum of square of cumulative sum control chart for monitoring process mean and variance simultaneously (SS-CUSUMVAR) is constructed by replacing the statistic that monitors the process dispersion in SS-CUSUM control chart with the three parameters logarithmic transformation to S^2.

Following from the inspiration for section 4.3.3, the statistics G_i and F_i are defined in the same way.The statistics of equation (4.1) and (4.2) are integrated in CUSUM statistics to monitor the process mean and variance as given in equations 4.5-4.8 below:

$$SSC_i^+ = \max\left[0, G_i - k + SSC_{i-1}^+\right] \quad (4.5)$$

$$SSC_i^- = \max\left[0, -G_i - k + SSC_{i-1}^-\right] \quad (4.6)$$

$$SSS_i^+ = \max\left[0, F_i - k + SSS_{i-1}^+\right] \quad (4.7)$$

$$SSS_i^- = \max\left[0, -F_i - k + SSS_{i-1}^-\right] \quad (4.8)$$

where $SSC_0 = SSS_0 = 0$ are the starting points.

Let MC_i and VS_i be the maximum values of the CUSUM statistics that monitor process mean and standard deviation respectively.

$$MC_i = \max\left(C_i^+, C_i^-\right) \qquad (4.17)$$

$$VS_i = \max\left(S_i^+, S_i^-\right) \qquad (4.18)$$

The SS-CUSUM is the sum of square of the maximum values of the CUSUM statistics that monitor both process mean and standard deviation and it is given by:

$$SS3_i = MC_i^2 + VS_i^2 \qquad (4.19)$$

$SS3_i$ is plotted against the control limit h.

Table4.10: ARL values for SS-CUSUMVARcontrol charting schemes at $ARL_0 = 250$, $h = 27.66$, $and\ k = 0.5$.

		a								
		0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
b	1.00	249.90	29.13	8.85	5.12	3.62	2.84	2.37	2.09	1.92
	1.25	16.41	11.98	7.03	4.75	3.50	2.80	2.37	2.08	1.88
	1.50	6.44	6.05	4.92	3.93	3.20	2.65	2.30	2.04	1.84
	1.75	4.15	4.02	3.63	3.21	2.80	2.46	2.17	1.96	1.77
	2.00	3.14	3.08	2.91	2.70	2.48	2.23	2.03	1.85	1.69

4.3.6 A new maximumcumulative sum control chart for monitoring process mean and variance simultaneously.

A new maximum cumulative sum control chart for monitoring process mean and variance simultaneously (Max-CUSUMVAR) is developed based on the assumption in Section

4.3.5 above. The statistics of equations 4.30 and 4.31 can be combined in another form as:

$$MCVAR_i = \max(MC_i, VS_i) \qquad (4.9)$$

Since the statistic $MCVAR_i$ is always positive then it only has the upper control limit h. Anytime $MCVAR_i$, exceeds the control limit h, then we say that the process is in an out-of-control state otherwise, it is in a good state.

Table4.11. ARL values for Max-CUSUMVARcontrol charting scheme at $ARL_0 = 250,\ h = 5.035, and\ k = 0.5$.

		a								
		0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
b	1.00	250.21	29.25	8.77	5.04	3.56	2.78	2.32	2.05	1.88
	1.25	17.09	12.70	7.43	4.91	3.58	2.82	2.37	2.06	1.85
	1.50	6.70	6.38	5.30	4.23	3.39	2.78	2.37	2.08	1.86
	1.75	4.32	4.24	3.89	3.47	3.02	2.64	2.31	2.05	1.84
	2.00	3.28	3.22	3.09	2.90	2.69	2.42	2.19	1.99	1.80

4.4 COMPARATIVE ANALYSIS

In this section, we will compare the performance of the proposed with their counterparts includingmax-chart, Max-CUSUM, Max-EWMA, SS-CUSUM, and SS-EWMA control charts.For different amounts of shifts in process mean (a) and dispersion (b), the ARL values of the proposed charts and the other competing charts are provided in Tables 4.1-4.11. These results are based on 10^4Monte Carlo simulations, at each run, for our study purposes.For a comparative analysis of the proposed charts with their existing counterparts, we have listed the comparative results in Tables 4.12.

These results show that:

- SS-EWMAVAR and Max-EWMAVAR are better than Max-CUSUM, Max-EWMA, SS-CUSUM, and SS-EWMA at all shifts in the dispersion.
- SS-EWMAVAR is better than Max-CUSUM , SS-CUSUM and SS-EWMA at a very small shift ($a < 0.5$) of the process mean.
- Max-EWMAVAR is better than Max-CUSUM , SS-CUSUM at a very small shift ($a < 0.5$) of the process mean but slightly better and SS-EWMA all shifts of the mean.
- SS-EWMAVAR, Max-EWMAVAR and Max-EWMA have nearly equal values at all shifts in the process mean.
- SS-CUSUMVAR and Max-CUSUMVAR have almost the same ARL values with Max-CUSUM, Max-EWMA, SS-CUSUM, and SS-EWMA at all shifts in the process mean.
- SS-CUSUMVAR is slightly better than the performance of all its existing counterparts (except Max-Chart) at all shifts of the process dispersion.
- Max-CUSUMVAR is slightly better than the performance of all its existing counterparts in small and moderate shift of the process dispersion.
- SS-EWMAVAR, SS-CUSUMVAR, Max-CUSUMVARand Max-EWMAVAR are only performing better than Max-Chart at small and moderate shifts in both parameters.

Table4.12: Comparison Table Between the Proposed Charts and their Counterparts

SS-EWMAVAR									
	L=3.55	λ=0.10			a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	248.25	24.87	8.88	5.37	3.92	3.13	2.60	2.27	2.05
1.25	13.85	10.13	6.39	4.54	3.54	2.89	2.48	2.20	2.00
1.50	5.57	5.17	4.38	3.63	3.03	2.61	2.32	2.07	1.89

1.75	3.67	3.57	3.27	2.91	2.62	2.35	2.13	1.95	1.78
2.00	2.81	2.78	2.65	2.48	2.29	2.11	1.94	1.79	1.67
				SS-CUSUMVAR					
	H=27.66	k=0.5			a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	249.90	29.13	8.85	5.12	3.62	2.84	2.37	2.09	1.92
1.25	16.41	11.98	7.03	4.75	3.50	2.80	2.37	2.08	1.88
1.50	6.44	6.05	4.92	3.93	3.20	2.65	2.30	2.04	1.84
1.75	4.15	4.02	3.63	3.21	2.80	2.46	2.17	1.96	1.77
2.00	3.14	3.08	2.91	2.70	2.48	2.23	2.03	1.85	1.69
			MAX-CUSUMVAR						
				a	k=0.50	H=5.035			
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	250.21	29.25	8.77	5.04	3.56	2.78	2.32	2.05	1.88
1.25	17.09	12.70	7.43	4.91	3.58	2.82	2.37	2.06	1.85
1.50	6.70	6.38	5.30	4.23	3.39	2.78	2.37	2.08	1.86
1.75	4.32	4.24	3.89	3.47	3.02	2.64	2.31	2.05	1.84
2.00	3.28	3.22	3.09	2.90	2.69	2.42	2.19	1.99	1.80
				MAX-CUSUM					
		K=0.50	h=5.05		a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	249.09	30.01	8.77	5.02	3.54	2.78	2.32	2.06	1.88
1.25	18.10	13.25	7.55	4.91	3.57	2.83	2.37	2.08	1.86
1.50	6.90	6.43	5.37	4.29	3.42	2.79	2.39	2.09	1.84
1.75	4.34	4.25	3.93	3.50	3.06	2.63	2.31	2.04	1.83
2.00	3.26	3.24	3.08	2.89	2.64	2.42	2.18	1.96	1.80
				MAX-EWMA					
		L=2.785	λ=0.10		a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	250.68	24.88	8.82	5.32	3.85	3.07	2.57	2.23	2.04
1.25	17.70	13.20	7.88	5.27	3.86	3.10	2.60	2.27	2.06
1.50	7.35	6.94	5.87	4.66	3.76	3.08	2.62	2.30	2.06
1.75	4.80	4.66	4.32	3.87	3.38	2.95	2.57	2.27	2.05
2.00	3.61	3.59	3.45	3.23	2.96	2.69	2.44	2.21	2.02
				SS-CUSUM					
		k=0.5	h=27.9		a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	250.48	29.45	8.81	5.07	3.62	2.86	2.37	2.11	1.92
1.25	17.32	12.52	7.17	4.74	3.54	2.82	2.38	2.09	1.89
1.50	6.58	6.05	5.00	3.99	3.20	2.68	2.32	2.04	1.84
1.75	4.18	4.05	3.67	3.21	2.82	2.47	2.18	1.97	1.77
2.00	3.13	3.08	2.92	2.70	2.46	2.23	2.02	1.84	1.69
				MAX-CHART					
			h=3.09		a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	249.79	130.10	37.96	12.40	5.01	2.57	1.65	1.26	1.09
1.25	30.68	23.68	13.36	6.85	3.82	2.38	1.69	1.33	1.15

1.50	8.30	7.54	5.87	4.11	2.94	2.11	1.64	1.37	1.19
1.75	3.86	3.66	3.24	2.70	2.23	1.86	1.55	1.33	1.21
2.00	2.46	2.40	2.21	2.03	1.80	1.59	1.43	1.29	1.19
				SS-EWMA					
	L=2.785	λ=0.1			a				
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	252.32	25.67	9.14	5.56	4.06	3.23	2.70	2.34	2.10
1.25	17.26	12.28	7.41	5.09	3.88	3.17	2.66	2.34	2.11
1.50	7.24	6.62	5.37	4.33	3.52	2.95	2.58	2.29	2.09
1.75	4.75	4.54	4.08	3.58	3.13	2.73	2.43	2.20	2.02
2.00	3.58	3.50	3.30	3.06	2.76	2.50	2.28	2.09	1.93
				MAX-EWMAVAR					
		L=2.77	λ=0.1	a					
b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
1.00	250.29	24.54	8.73	5.26	3.84	3.07	2.55	2.24	2.03
1.25	14.06	10.99	7.08	4.98	3.78	3.06	2.58	2.25	2.05
1.50	5.65	5.39	4.76	4.04	3.42	2.93	2.53	2.26	2.03
1.75	3.69	3.62	3.42	3.22	2.94	2.63	2.38	2.17	2.00
2.00	2.82	2.79	2.75	2.63	2.49	2.34	2.19	2.04	1.89

4.5 ILLUSTRATIVE EXAMPLE

A practical example was taken from the book of Montgomery (2009), about the hard-bake process with photolithography in semiconductor manufacturing. Twenty-five samples were initially used in phase 1 with the subgroup each of 5 sample size of wafers and they are all in a statistical process. Another set of twenty samples of wafers are added to be monitored in the phase 2.The output of the proposed charts and their counterparts are shown in Table 4.13 and; their graphical displays are shown in figure 1-7. The sample mean and variance are calculated to be 1.53184 and 0.1360771 respectively.

The results show that:

- There is large increased values in the process at the sample 41 through 45.
- SS-CUSUMVAR and Max-CUSUMVAR detects 5 out-of-control signals at sample points 41-45.

- SS-EWMAVAR, Max-EWMAVAR, Max-EWMA, and SS-EWMAtriggers 3 out-of controlpoints, at sample points 43-45.
- Max-Chart gives only one out-of-control signal at sample point 45.

Table4.13: Output of the hard-bake process for the Proposed Charts and their Counterparts.

S/no	N_i	M_i	$SS1_i$	$SSEW_i$	$SS3_i$	$MCVAR_i$	MaxChat	$SS2_i$	MEW_i
1	0.28	0.08	0.01	0.01	0.10	0.31	0.78	0.08	0.081
2	0.10	0.09	0.01	0.01	0.01	0.10	0.60	0.01	0.09
3	1.17	0.16	0.04	0.04	1.33	1.07	1.67	1.55	0.163
4	0.92	0.25	0.07	0.07	0.94	0.92	1.00	0.99	0.247
5	1.14	0.29	0.09	0.09	1.30	1.14	0.72	1.30	0.294
6	2.00	0.40	0.16	0.16	4.00	2.00	1.36	4.00	0.4
7	0.70	0.28	0.08	0.08	0.56	0.70	0.80	0.55	0.28
8	0.16	0.25	0.06	0.06	0.03	0.16	0.37	0.03	0.248
9	0.06	0.26	0.07	0.07	0.00	0.06	0.40	0.00	0.263
10	0.07	0.27	0.07	0.07	0.02	0.15	0.57	0.00	0.267
11	0.11	0.25	0.06	0.06	0.02	0.12	0.61	0.01	0.253
12	0.49	0.23	0.07	0.07	0.28	0.53	0.87	0.24	0.233
13	1.75	0.44	0.23	0.23	4.03	1.75	2.25	3.88	0.435
14	1.35	0.40	0.17	0.17	1.88	1.35	0.69	1.85	0.401
15	2.88	0.56	0.34	0.34	8.30	2.88	2.03	8.30	0.564
16	2.33	0.50	0.39	0.39	9.36	2.33	2.23	8.99	0.503
17	2.57	0.53	0.41	0.41	9.99	2.57	0.73	9.69	0.526
18	3.29	0.60	0.47	0.47	12.77	3.29	1.22	12.66	0.596
19	2.04	0.46	0.30	0.29	4.68	2.04	0.75	4.70	0.461
20	2.13	0.46	0.42	0.42	8.69	2.20	1.89	8.39	0.457
21	2.49	0.51	0.36	0.36	6.65	2.49	1.03	6.63	0.515
22	1.87	0.45	0.24	0.23	4.59	1.87	0.94	4.36	0.451
23	1.15	0.36	0.14	0.14	2.65	1.34	0.72	2.17	0.361
24	2.75	0.26	0.09	0.08	7.05	2.65	2.10	7.57	0.264
25	1.77	0.25	0.07	0.06	2.80	1.67	0.48	3.12	0.247
26	2.26	0.27	0.10	0.10	4.95	2.23	1.00	5.13	0.275
27	1.46	0.28	0.09	0.09	2.09	1.45	0.30	2.14	0.276
28	0.71	0.25	0.07	0.07	0.52	0.72	0.25	0.51	0.247
29	1.42	0.41	0.17	0.17	2.66	1.42	1.92	2.56	0.414
30	1.28	0.41	0.17	0.17	2.22	1.28	1.24	2.19	0.409
31	1.76	0.47	0.22	0.22	3.16	1.76	0.98	3.13	0.465
32	1.30	0.42	0.21	0.22	2.88	1.30	1.32	2.74	0.423
33	0.80	0.38	0.16	0.17	0.74	0.80	0.31	0.69	0.381
34	0.28	0.26	0.08	0.08	0.08	0.28	0.78	0.08	0.265
35	1.21	0.41	0.20	0.20	1.84	1.21	1.71	1.78	0.409
36	1.52	0.45	0.21	0.21	2.40	1.52	0.81	2.39	0.45
37	1.90	0.47	0.32	0.30	4.91	1.69	2.40	6.46	0.472

38	2.02	0.35	0.21	0.19	3.34	1.63	1.33	4.76	0.32
39	2.37	0.23	0.06	0.04	5.75	2.37	2.04	6.09	0.202
40	3.68	0.17	0.04	0.04	13.61	3.68	1.81	13.60	0.143
41	5.48	0.35	0.13	0.12	30.17	5.48	2.30	30.14	0.345
42	6.51	0.46	0.22	0.22	42.38	6.51	1.53	42.38	0.464
43	8.72	0.69	0.51	0.50	76.55	8.72	2.71	76.50	0.689
44	9.87	0.78	0.62	0.62	98.73	9.87	1.65	98.56	0.785
45	13.28	1.10	1.21	1.21	176.86	13.28	3.91	176.83	1.098
Control limits	5.05	0.644	0.484	0.479	27.6	5.035	3.09	27.9	0.642

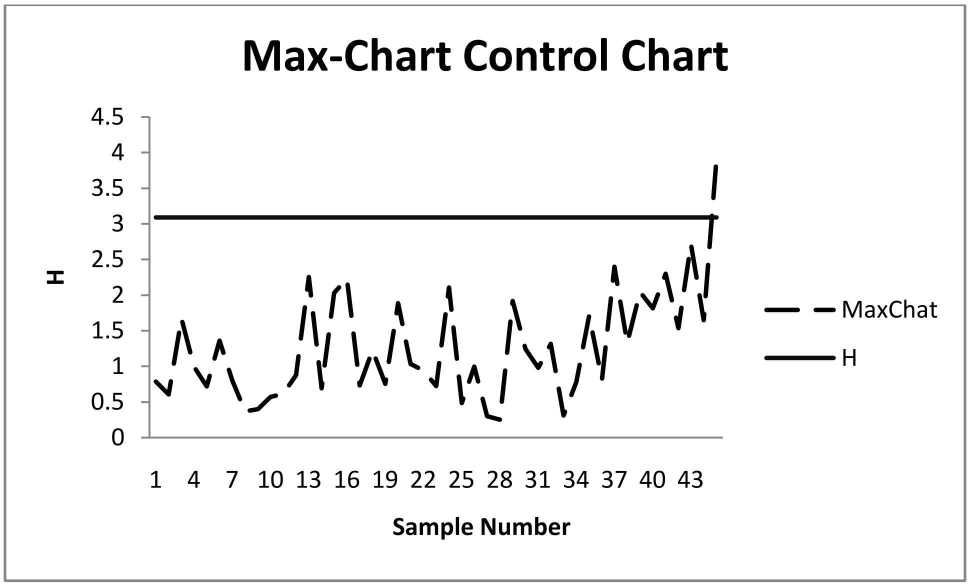

Figure 4.1. Max-Chart when h=3.09 at ARL_0 =250

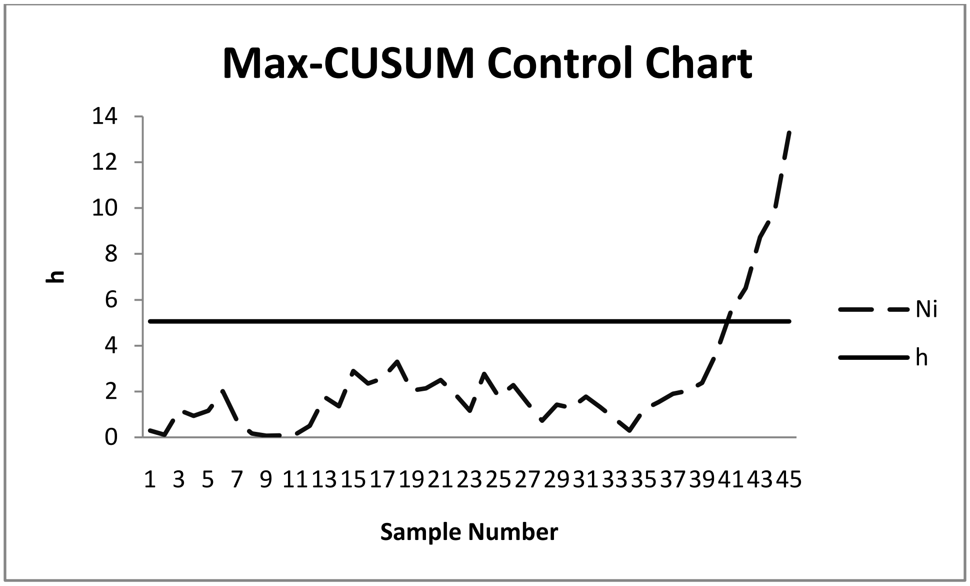

Figure 4.2. Max-CUSUM when h=5.05 and k=0.5 at ARL_0 =250

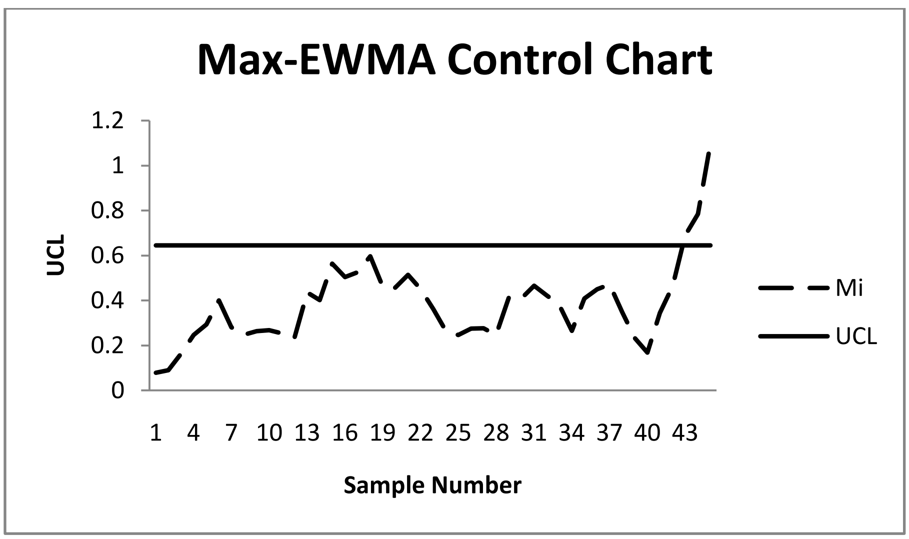

Figure 4.3. Max-EWMA when L=2.785 and λ=0.1 at ARL_0 =250

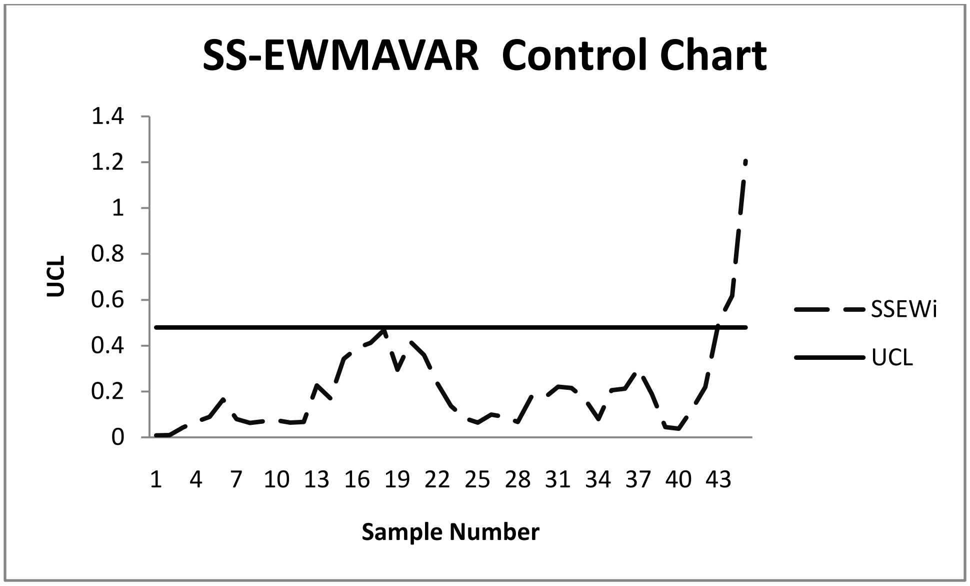

Figure 4.4. SS-EWMAVAR when L=3.55and λ=0.1 at ARL_0 =250

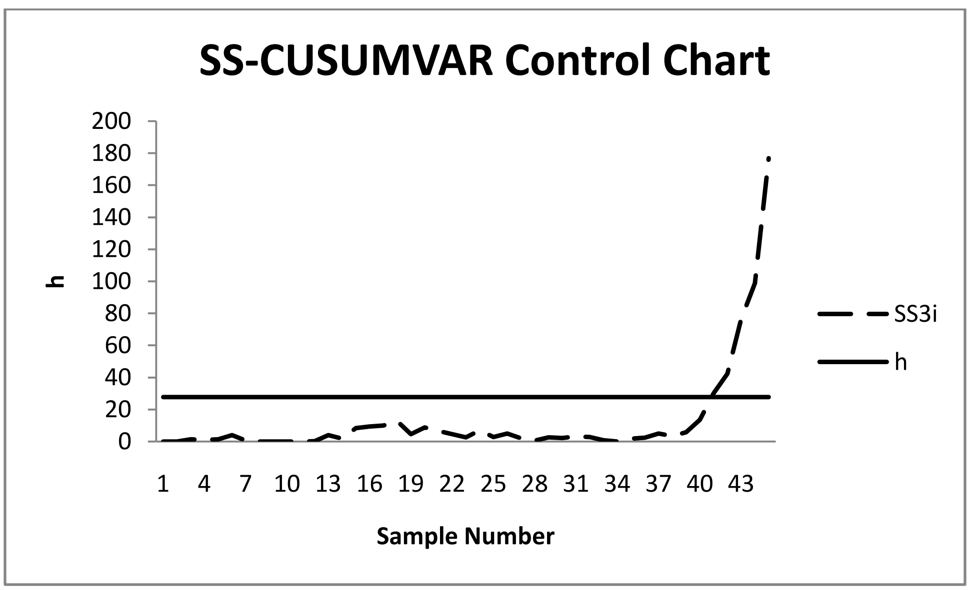

Figure 4.5. SS-CUSUMVAR when h=27.66and k=0.5 at ARL_0 =250

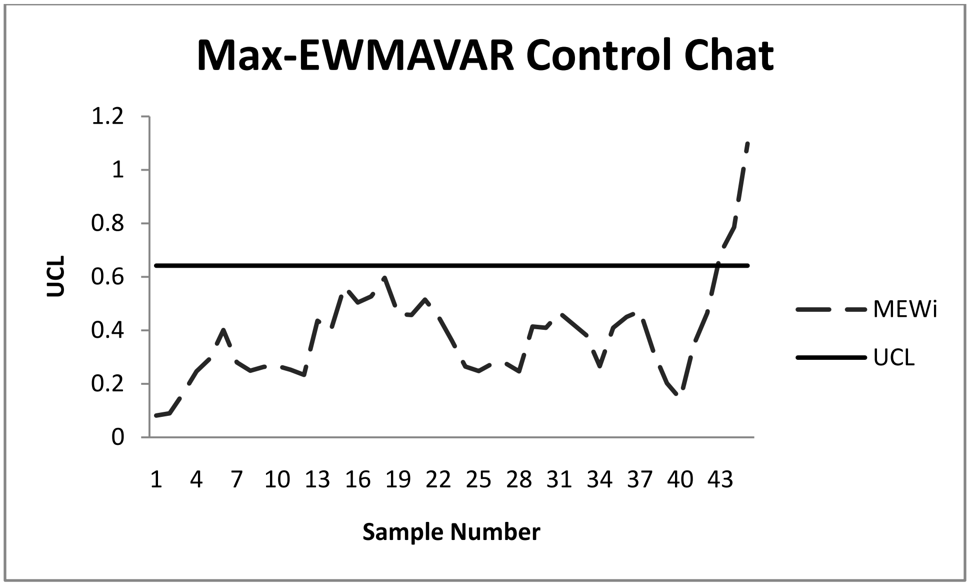

Figure 4.6. Max-EWMAVAR when L=2.77and λ=0.1 at ARL_0 =250

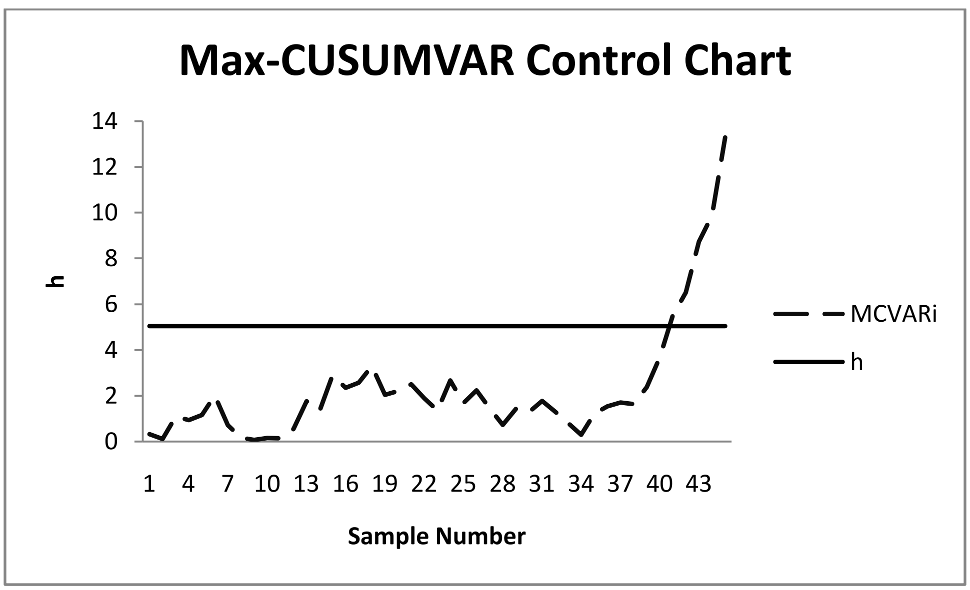

Figure 4.7. Max-CUSUMVAR when h=5.035and k=0.5at ARL_0 =250

4.6 SUMMARY, CONCLUSIONS AND RECOMMENDATIONS

In this study, we have proposed seven new control charts that monitor the process mean and dispersion simultaneously. We have investigated ARL properties of the proposed schemes and compared them with the existing counterparts including Max-EWMA, Max-CUSUM, $SS-EWMA$ and $SS-CUSUM$. The comparisons showed that the proposed schemes are really perform better for the shifts in the process dispersion than the other existing schemes covered in this study. They have slightly the same ARL values in the process mean with their counterparts. The scope of this study may be extended for monitoring process mean and dispersion simultaneously in the multivariate setups for an improved and efficient monitoring of process parameters.

CHAPTER 5

MIXED MULTIVARIATE EWMA-CUSUM CONTROL CHART FOR MONITORING PROCESS MEAN AND STANDARD DEVIATION.

Memory type control is very effective in detecting small and moderate shift in process mean and/or dispersion. Many charts have been developed to monitor the process mean and dispersion simultaneously in the univariate and multivariate control charts.

In this chapter, we propose a new multivariate chart that mixes the effect of multivariate max-EWMA and multivariate max-CUSUM. The proposed chart will be compared with its existing counterparts by their run length properties.

5.1 INTRODUCTION

Shewhart control chart is a memoryless chart which is effective in detecting a large shift in a process. It uses only the current information. Cumulative sum and exponential weighted moving average developed by Page (1954) and Robert (1959) respectively are the most memory type control charts that detect small and moderate disturbances in a process parameter.

Sometimes, we are interested in monitoring more than one correlated quality variables like the hardness and tensile strength of steel; multivariate control charts are employed. Hotelling's (1947) introduce a chart that monitors two or more correlated quality characteristics and named it Chi-squared control chart. It is an analog of Shewhart control chart in the univariate set-up. Lowry et al. (1992), Pignatiello and Runger (1990), Healy (1987) and Crosier (1988) also developed some memory type multivariate control charts that use both previous and current information. These charts are good at detecting small and moderate changes in the process parameters.

Many control charts in the literature monitor a single parameter (location or dispersion). Some other control charts monitor both parameters on different charts. Max-chart introduced by Chen and Cheng (1998) combines the statistics of both parameters and plot it against a control limit. Following the inspiration from Max-chart which is a memoryless control chart, Xie (1999), Thaga (2009) and Cheng and Thaga (2010) developed new control charts that use both present and past information. The above charts are also developed in the multivariate set-up; Multivariate Max-CUSUM control chart by Cheng and by Thaga (2005); and Multivariate Max-EWMA chart by Xie (1999).

Abbaset al. (2013a&b)usedtheideaofmergingthestructuresofEWMAandCUSUMchartsforlocationand dispersionparameters.Later,Zamanet al. (2014)extendedthisideainareversemixingpattern. In this chapter, we introduced a new chart that combines the effect Multivariate Max-CUSUM control chart and Multivariate Max-EWMA.In our current study, we have used Average run length (ARL) as a performance measure that is an effective measure of comparing the performance of the control charts.

Therestofthisstudyisorganizedas:Section5.2givesthebackgroundoftheexistingmulti variatecontrolcharts monitor both parameters on a single chart.Section5.3presentsthedetailsofourproposedmixedmultivariate MaximumEWMA-CUSUMcontrolcharts.Section5.4providescomparativeanalysisoftheproposalswiththeexistingcontrolcharts.Finally, Section5.5containstheconclusionsfromthisstudy.

5.2 Memory-Type Multivariate Control Charts that Monitor both Process Vector Mean and Dispersion.

Background of some memory type multivariate control charts that monitor both mean vector and dispersion will be given in this section.

5.2.1 A Multivariate Max-EWMA Control Chart

Let $X \sim N_p(\mu, \Sigma)$, where p is the number of correlated quality characteristics to be monitored simultaneously. Suppose that there are n sample size drawn from the process, that is, $X_{i1}, X_{i2}, X_{i3}, \ldots, X_{in}$ for i=1,2,…, and $\bar{X}_i$ represent the sample mean vector of the distribution. The in-control mean vector and the standard covariance matrix are denoted with μ_0 *and* Σ_0 respectively.

In order to monitor the process vector, $\bar{X}_i$ will be transformed into the EWMA statistic as below:

$$Z_i = (1-\lambda) Z_{i-1} + \lambda \left(\bar{X}_i - \mu_0 \right) \qquad (5.1)$$

The statistic that monitors the process mean vector is given as

$$U_i = \phi^{-1}\left[H_p \left\{ \frac{n(2-\lambda)}{\lambda(1-\lambda)^{2i}} Z_i^{'}\Sigma_0^{-1}Z_i \right\} \right] \quad (5.2)$$

We can calculate the following statistics, given below, in order to monitor the process standard deviation.

$$W_i = \sum_{i=1}^{n}\left(X_{ij} - \bar{X}_i \right)^{'}\Sigma_0^{-1}\left(X_{ij} - \bar{X}_i \right) \quad (5.3)$$

$Y_i = (1-\lambda)Y_{i-1} + \lambda\phi^{-1}\left[H_{p(n-1)}(W_i) \right]$ and $V_i = \sqrt{\frac{2-\lambda}{\lambda(1-\lambda)^{2i}}}Y_i$

Since U_i *and* V_i are independent and normally distributed then we combine them together to form a new statistic given by

$$M_i = \max\left(|U_i|,|V_i|\right)$$

M_i is compared with the upper control limit (UCL). The process will be in a control state in as much as M_i is below the control limit. The control limit is computed below as:

$$UCL = E(M_i) + L\sqrt{Var(M_i)}$$

The expected value and variance of M_i are derive through the numerical computation and they are given to be $E(M_i)$=1.12379 and $Var(M_i)$=0.363381 respectively.

Therefore, UCL=1.128379+0.602811L.

The average run length (ARL) table of this scheme will be given in table 5.1.

5.2.2 Multivariate Max-CUSUM Control Chart

Cheng and Thaga (2005) developed the Multivariate Max-CUSUM Control Chart by combining the statistics of both process mean and dispersion which were introduced by Healy (1987). They evaluated the maximum values of the statistics and plotted them against the control limit.

The following upper, C_i^+ and lower, C_i^- , multivariate CUSUM statistics are calculated in order to monitor the process vector mean.

$C_i^+ = \max\left(0,\ Z_i - 0.5D + C_{i-1}^+\right)$ and $C_i^- = \max\left(0,\ -0.5D - Z_i + C_{i-1}^-\right)$

Where $Z_i = a'\left(\bar{X}_i - \mu_0\right)$ and D is a non-centrality parameter which is defined as

$D = \sqrt{\left(\mu_1 - \mu_0\right)' \Sigma^{-1} \left(\mu_1 - \mu_0\right)}$

For monitoring the process variability, Healy defined the following statistics

$S_i^+ = \max\left(0,\ Y_i - k + S_{i-1}^+\right)$ and $S_i^- = \max\left(0, -k - Y_i + S_{i-1}^-\right)$

Where $Y_i = \phi^{-1}\left\{H\left[\left(X_i - \mu\right)' \Sigma^{-1}\left(X_i - \mu\right); p\right]\right\}$; H(.;p) represents the Chi-squared distribution with p degree of freedom and $\Phi^{-1}(.)$ represents inverse standard normal cumulative distribution function.

Healy (1987) defined a' and k as below

$$a' = \frac{(\mu_1 - \mu_0)\Sigma^{-1}}{\left[(\mu_1 - \mu_0)' \Sigma^{-1} (\mu_1 - \mu_0)\right]^{1/2}} \quad \text{and} \quad k = 0.5 \frac{(\mu_1 + \mu_0)' \Sigma^{-1} (\mu_1 - \mu_0)}{\left[(\mu_1 - \mu_0)' \Sigma^{-1} (\mu_1 - \mu_0)\right]^{1/2}}$$

Cheng and Thaga (2005) combined the four statistics that monitor the process vector mean and dispersion and evaluated their maximum value, M_i as given below and it is plotted against the control limit, h.

$M_i = \max(C_i^+, C_i^-, \mathrm{S}_i^+, \mathrm{S}_i^-)$. The ARL values of this chart is given in Table 5.2.

5.3 THE PROPOSED CHART

Let $X \sim N_p(\mu, \Sigma)$, where p is the number of correlated quality characteristics to be monitored simultaneously.

In order to monitor the process vector, $\bar{X}_i$ will be transformed into the EWMA statistic as below:

$$Z_i = (1-\lambda) Z_{i-1} + \lambda \left(\bar{X}_i - \mu_0 \right) \tag{5.4}$$

The max-EWMA statistic that monitors the process mean vector is given as

$$U_i = \phi^{-1} \left[H_p \left\{ \frac{n(2-\lambda)}{\lambda(1-\lambda)^{2i}} Z_i' \Sigma_0^{-1} Z_i \right\} \right] \tag{5.5}$$

We can calculate the following statistics, given below, in order to monitor the process standard deviation.

$$W_i = \sum_{i=1}^{n} \left(X_{ij} - \bar{X}_i \right)' \Sigma_0^{-1} \left(X_{ij} - \bar{X}_i \right) \qquad (5.6)$$

$$Y_i = (1-\lambda) Y_{i-1} + \lambda \phi^{-1} \left[H_{p(n-1)} (W_i) \right] \text{and } V_i = \sqrt{\frac{2-\lambda}{\lambda (1-\lambda)^{2i}}} Y_i$$

We can integrate both U_i *and* V_i into multivariate CUSUM statistics to monitor process mean vector and standard deviation respectively.

$$MMC_i^{+} = \max\left(0, U_i - 0.5D + MMC_{i-1}^{+}\right) \text{ and } MMC_i^{-} = \max\left(0, -0.5D - U_i + MMC_{i-1}^{-}\right)$$

$$MMS_i^{+} = \max\left(0, V_i - k + MMS_{i-1}^{+}\right) \text{ and } MMS_i^{-} = \max\left(0, -k - V_i + MMS_{i-1}^{-}\right)$$

Where $Z_i = a' \left(\bar{X}_i - \mu_0 \right)$ and D is a non-centrality parameter which is defined as $D = \sqrt{(\mu_1 - \mu_0)' \Sigma^{-1} (\mu_1 - \mu_0)}$ and k is a reference value.

We combined the four statistics that monitor the process vector mean and dispersion; and evaluated their maximum value, MMM_i as given below. It is plotted against the control limit, h. The ARL values of this chart are given in Table 5.3.

$$MMM_i = \max(\mathrm{MM}C_i^{+}, \mathrm{MM}C_i^{-}, \mathrm{MMS}_i^{+}, \mathrm{MMS}_i^{-}).$$

5.4 Performance Measures

In this section, we will compare the performance of the proposed with their counterparts includingmultivariate Max-CUSUM and multivariate Max-EWMA control charts.For different amounts of shifts in process mean (a) and dispersion (b), the ARL values of the proposed charts and the other competing charts are provided in Tables 1-9. These results are based on 10^3Monte Carlo simulations, at each run, for our study purposes.These results show that:

- The proposed chart is better than Multivariate Max-CUSUM in the mean vector at a small shift in the process (a<1).
- It is better than Multivariate Max-CUSUM chart at all shifts of the process dispersion when the shift in the process mean is less that 1(i.e. a<1).
- The proposed chart is performing poorer when compared to the changes in process mean vector and/or dispersion of the Multivariate Max-CUSUM chart.

Table5.1: Multivariate Max-CUSUM Chart

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
	1.00	201.73	98.19	33.60	16.08	9.89	7.12	5.53	4.54	3.86
H=4.8	1.25	75.01	50.97	25.73	14.51	9.50	6.99	5.51	4.54	3.88
K=0.5	1.50	34.32	28.73	18.98	12.48	8.87	6.76	5.44	4.50	3.87
	2.00	14.40	13.53	11.46	9.28	7.44	6.12	5.10	4.36	3.78

Table 5.2: Multivariate Max-EWMA Chart

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
	1.00	199.97	37.44	11.36	6.66	4.69	3.75	3.14	2.67	2.35
L=2.8659	1.25	10.95	10.12	8.21	6.06	4.63	3.73	3.08	2.70	2.39
λ=0.10	1.50	4.88	4.88	4.80	4.28	3.87	3.37	3.00	2.61	2.38
	2.00	2.57	2.61	2.56	2.54	2.47	2.38	2.33	2.20	2.11

Table 5.3: Multivariate Maximum Mixed EWMA-CUSUM

		a								
	b	0.00	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
k=0.5	1.00	204.54	36.35	17.24	12.30	9.90	8.51	7.50	6.73	5.98
λ=0.10	1.25	16.80	16.29	14.49	11.85	9.81	8.48	7.53	6.66	5.90
h=26.5	1.50	10.49	10.36	10.30	9.88	9.19	8.25	7.45	6.61	5.86
	2.00	6.88	6.89	6.89	6.86	6.83	6.68	6.50	6.05	5.64

5.5 Conclusions

We have proposed a new multivariate mixed maximum EWMA-CUSUM control chart, to monitor changes in the process mean vector and dispersion on a single chart. The performance of the proposed schemes is evaluated in terms of ARL and compared with other competing charts like multivariate max-CUSUM and multivariate max-EWMA control charts. The comparisons showed that the proposed schemeis really better than multivariate max-CUSUM for detecting the small shifts in the process mean vector and/or dispersion.

CHAPTER6

CONCLUSIONS

In this study, we have proposed some improvements on the mixed EWMA- CUSUM control charts with varying FIR features in the form of MECHS, MECFIR and MECFIRHS control charting schemes. We have investigatedARL properties of the proposed schemes and compared them with the existing counterparts including classical CUSUM, classical EWMA, FIR EWMA and FIR CUSUM. We have observed that the proposals of the studyimprove the detection ability of the mixed EWMA-CUSUM chart for the processes that are off-target at the start-up.The comparisons showed that the proposed schemes are really good at detecting shifts (especially of smaller magnitude) in the process relative tom the other existing schemes covered in this study.

We have also proposed two multivariate mixed EWMA-CUSUM control charts,in the form of MEC1 and MEC2 control charts, to monitor changes in the process mean vector. The performance of the proposed schemes is evaluated in terms of ARL and compared with other competing charts like MCUSUM, MEWMA, MC1 and Hotelling T^2/χ^2 control charts. The comparisons showed that the proposed schemes are really good at detecting the small shifts in the process as compared with the other schemes under study.

We also proposed six new control charts that monitor the process mean and standarddeviation simultaneously in the univariate set-up. We have investigatedARL properties of the proposed schemes and compared them with the existing counterparts including Max-EWMA, Max-CUSUM, $SS-EWMA$ and $SS-CUSUM$ .The comparisons showed that the proposed schemes are really good at detecting shifts (especially of smaller magnitude) in the process relative tom the other existing schemes covered in this study.

Finally, we proposed a new multivariate mixed maximum EWMA-CUSUM control chart, to monitor changes in the process mean vector and dispersion on a single chart. The comparisons showed that the proposed schemeis really better than multivariate max-CUSUM for detecting the small shifts in the process mean vector and/or dispersion.

References

1. Abbas, N., Riaz, M. and Does, R. J. M. M (2013 a). CS-EWMA Chart for Monitoring Process Dispersion. *Quality and Reliability Engineering International*, 29(5), 653-663.
2. Abbas, N., Riaz, M., and Does, R.J.M.M. (2013 b). Mixed Exponentially Weighted Moving Average – Cumulative Sum charts for Process Monitoring. *Quality and Reliability Engineering International*, 29(3), 345 – 356.
3. ASTM D6299 (2013). Applying Statistical Quality Control Assurance and Control Charting Techniques to Evaluate Analytical Measurement System Performance: *American Society for Testing and Materials*.
4. ASTM E2587 (2012). Use of Control Charts in Statistical Process Control: USA: American Society for Testing and Materials.
5. Castagliola P. (2005). A New S^2-EWMA Control Chart for Monitoring the Process Variance. *Quality and Reliability Engineering International*,21(8), 781-794.
6. Castagliola P, Celano G, Fichera S. (2009). A New CUSUM-S^2 Control Chart for Monitoring the Process Variance. *Journal of Quality in Maintenance Engineering,* 15(4), 344–357.
7. Castagliola P, Celano G, Fichera S. (2010). A Johnson's Type Transformation EWMA-S^2 Control Chart. *International Journal of Quality Engineering and Technology*, 1(3), 253–275.
8. Chen G and Cheng S.W. (1998). Max-Chart: Combining X-Bar Chart and S Chart. *Statistica Sinica*, 8, 263-271.

9. Cheng S.W and Thaga K. (2005). Multivariate Max-CUSUM Chart. *Quality Technology and Quantitative Management*, 2(2), 221-235.

10. Cheng S.W and Thaga K. (2010). The Max-CUSUM Chart. *Frontiers in Statistical Quality Control*, 9.

11. Crosier, R.B. (1988). Multivariate Generalizations of Cumulative Sum Quality Control Scheme. *Technometrics*, 30, 291-303.

12. Hotelling, H. (1947). "Multivariate Quality Control-Illustrated by the Air Testing of Sample Bombsights," in Techniques of Statistical Analysis, eds. C. Eisenhart, M. W. Hastay, and W. A. Wallis, New York: McGraw-Hill, pp. 111-184.

13. Haq, A; Brown, J; and Moltchanova, E. (2014). Improved Fast Initial Response Features for Exponentially Weighted Moving Average and Cumulative Sum Control Charts. *Quality and Reliability Engineering International*, 30(5), 697 -710.

14. ISO7870-1 (2014). Control Charts- Part 1: General Guidelines. Geneva, Switzerland: ISO.

15. ISO7870-2 (2013). Control Charts-Part 2: Shewhart Control Charts. Geneva, Switzerland: ISO.

16. ISO7870-3 (2012). Process Acceptance Control Charts-Part 3: Geneva, Switzerland: ISO.

17. ISO7870-4 (2011). Cumulative Sum Charts-Part 4: Geneva, Switzerland: ISO.

18. ISO7870-5 (2014). Specialized Control Charts-Part 5: Geneva, Switzerland: ISO.

19. ISO7870-6 (2014). EWMA Control Charts-Part 6 (under development): Geneva, Switzerland: ISO.

20. Kim, M. J. (2005). *Number of replications required in control chart Monte Carlo simulation studies*, Ph. D. dissertation, University of Northern Colorado.

21. Lowry, C.A., Woodall, W.H., Champ, C.W., Rigdon, S.E. (1992). A multivariate exponentially weighted moving average control chart. Technometrics. 34(1), 46–53.

22. Lucas, J.M; Crosier, R.B. (1982) Fast Initial Response for CUSUM Quality-Control Scheme. *Technometrics*, 24, 199–205.

23. Montgomery D.C. (2009) *Introduction to Statistical Quality Control*. 6th ed. John Wiley & Sons, New York.

24. Mundform, D. J., Schaffer, J., Kim, M. J., Shaw, D., Thongteeraparp, A., & Supawan, P. (2011). Number of Replications Required in Monte Carlo Simulation Studies: A Synthesis of Four Studies. *Journal of Modern Applied Statistical Methods*, *10*(1), article#4.

25. Page, E.S. (1954) Continuous Inspection Schemes. *Biometrika*, 41, 100–115.

26. Pignatiello, J. J., Jr., and Runger, G. C. (1990). Comparisons of Multivariate CUSUM Charts. *Journal of Quality Technology*, 22, 173-186.

27. Riaz M, Abbas N, Does RJMM (2011). Improving the Performance of CUSUM Charts. *Quality and Reliability Engineering International*, 27(4), 415-424

28. Roberts, S.W. (1959) Control Chart Tests Based on Geometric Moving Averages. *Technometrics*, 1, 239–250.

29. Schaffer, J. R. and Kim, M. J. (2007), Number of replications in control chart Monte Carlo simulation studies, *Communications in Statistics-Simulation & Computation* **36**, 1075–1087.

30. Shewhart W. (1931). *Economic Control of Quality Manufactured Product*, D. Van Nostrand, New York; reprinted by the American Society for Quality Control in 1980, Milwauker, WI.

31. Steiner, S.H. (1999) EWMA control charts with time varying control limits and fast initial response. *Journal of Quality Technology*, 31(1), 75–86.

32. Thaga K. (2009). SS-CUSUM Chart. *Economic Quality Control*, 24(1), 117-128.

33. Xie, H. (1999). Contribution to Qualimetry. PhD thesis, University of Manitoba, Winnipeg, Canada.

34. Zaman, B., Riaz, M., Abbas, N. and Does, R. J. M. M. (2014). Mixed CUSUM-EWMA Control Charts: An Efficient Way of Monitoring Process Location. *Quality and Reliability Engineering International*, DOI: 10.1002/qre.1678.

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