A GENERAL STOCHASTIC MODEL FOR REFINERY PLANNING UNDER UNCERTAINTY

BY

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Dedication

This humble work is dedicated to my beloved parents, my wife, my brother and sister, my children; Khlood and Abdullah, and the soul of my uncle Abdullah.

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I really would like to give my deep gratitude and special thanks to my advisor, ***Dr. Mohammed Saleh Bashammakh***, for his priceless help, support, and confidence during the development of this work. It was my privilege to work with him and under his direct supervision. I also want to thank both of ***Dr. Naim Mohammed Faqir*** and ***Dr. Abdullah Abdulaziz Al-Shammari*** for being my committee members and for their constructive guidance, valuable advices and cooperation.

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ABSTRACT

Full Name **ABDULKHALEQ SALEM SAEED BAHAMOOD**

Title of Study **A GENERAL STOCHASTIC MODEL FOR REFINERY PLANNING UNDER UNCERTAINTY**

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The petroleum refining industry and its relative industries are considered for a major share in the worldwide energy and the consuming market. Usually, they are represented as the economic back-bone of the industrial countries.

In reality, refining industry faces daily serious changes in the products daily demand and properties and the crude oil cost which result in serious complexity in taking immediate decisions. Therefore, the use of optimization models becomes more effective and useful which make the process more realistically modeled.

The good planning and optimization considered as one of the most important problems for the process industries. The good planning helps to enhance the production and distribution process for the refined petroleum products based on the customer and market information.

In this research, we will develop a general stochastic optimization model for the refinery operational planning that might be used and applied in a refinery to help decision makers in making the appropriate immediate decisions which then will help raising the profitability value of the oil refinery.

ملخص الرسالة

الإسم	**عبدالخالق سالم سعيد باحمود**
عنوان الرسالة	**تطوير برنامج عشوائي عام لإدارة مصفاة في ظل حالة عدم الإستقرار**
التخصص	**هندسة كيميائية**
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تحتل صناعة تكرير النفط والصناعات المشتقة لها مركزاً رئيسياً ومهماً في صناعة الطاقة والسوق المستهلك في جميع أنحاء العالم. عادة يتم تمثيلها بالعمود الفقري الإقتصادي للدول الصناعية.

في الواقع، تواجه صناعة التكرير تغيرات يومية خطيرة في الطلب اليومي للمنتجات النفطية وأسعارها وخصائصها وفي تكلفة النفط الخام والتي بدورها تعقد إتخاذ قرارات فورية ومناسبة. ولذلك، أصبح إستخدام نماذج التحسين أكثر فعالية وفائدة حيث تجعل العملية ممثلة رياضياً وبصورة أكثر واقعية.

يعتبر التخطيط الجيد والأمثل واحداً من المشاكل الأكثر أهمية بالنسبة للصناعات العملية. والتخطيط الجيد يساعد على زيادة الإنتاج وعملية توزيع للمنتجات البترولية المكررة على أساس معلومات السوق والعملاء المستهلكين.

في هذا البحث، سوف نقوم بوضع موديل أو نموذج مثالي عام لتخطيط العمليات لمصفاة, والذي يمكن استخدامه وتطبيقه في مصفاة لمساعدة صانعي القرار في اتخاذ القرارات الفورية والمناسبة التي ستساعد ثم رفع القيمة الربحية للمصفاة النفط.

CHAPTER 1

INTRODUCTION

1.1 General Introduction

The simplest definition of an oil refinery is an industrial process in which the crude oil is refined and converted into useful products, such as gasoline, kerosene, diesel, asphalt, etc.

Oil refinery, which is simply shown in the process flow diagram in figure 1.1, usually consists of number of large and complex units which are combined to each other to produce more valuable products. Some of these units, that we are going to discuss later, are:

Crude distillation.
Catalytic reforming.
Vacuum distillation.
Fluid catalytic cracking.
Hydrocracker.

The petroleum refining industry and its relative industries are considered for a major share in the worldwide energy and the consuming market. Usually, they are represented as the economic back-bone of the industrial countries. These days, the environmental market and the prolonged change in consumer requirements result in constant pressure to seek opportunities that properly align and organize the incompatible elements of the industry. In particular, the coordination of the petroleum refining and its integration is having a great agreement of interest.

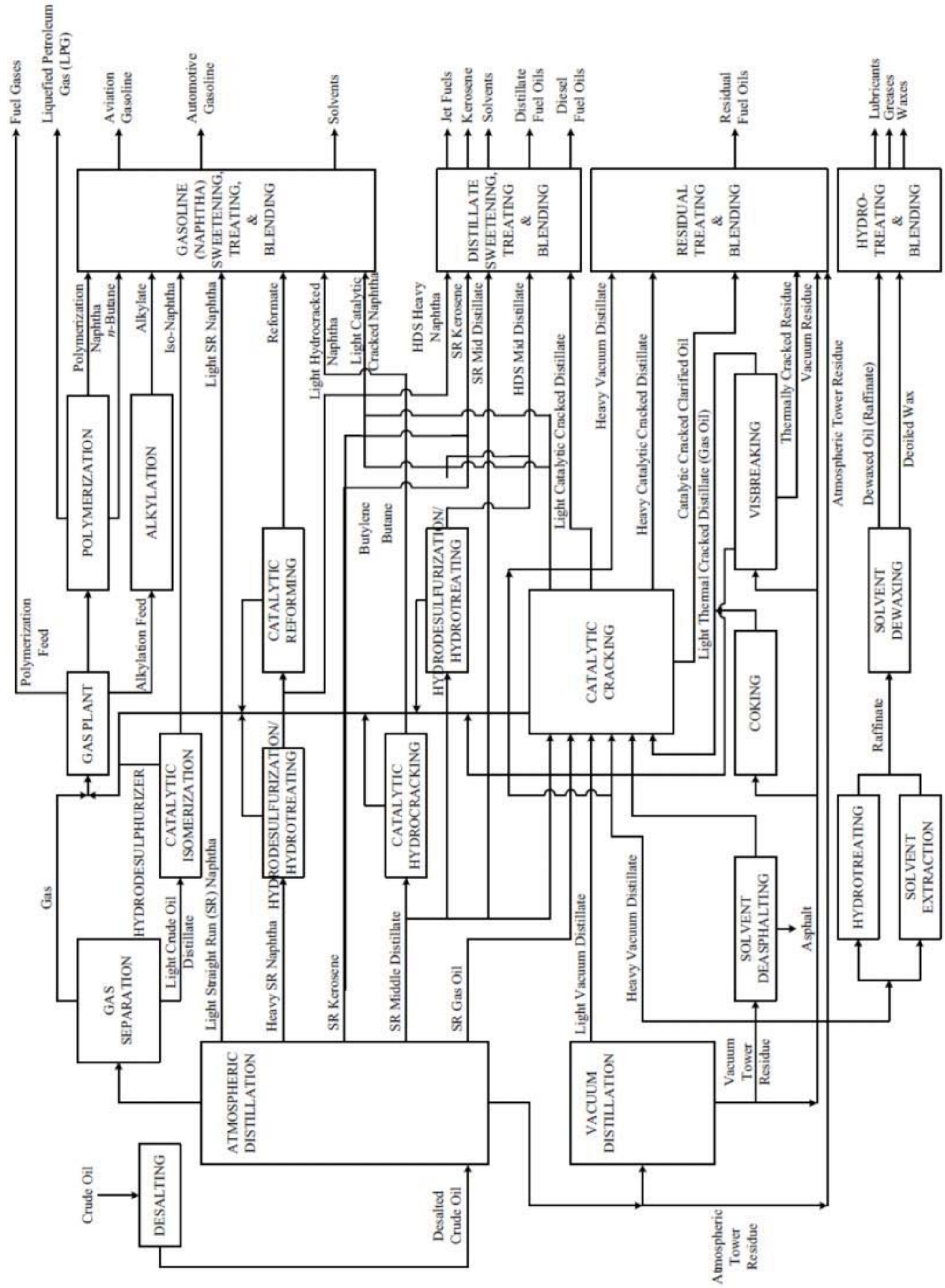

Figure 1.1 a simple Process Flow Diagram PFD of a modern refinery

Al-Qahtani & Elkamel (2010).

The first refinery was built in Titusville, Pennsylvania in 1860 and cost around $15,000. Other refineries and it only used batch distillation to part kerosene and heating oil from crude fractions. Earlier, refining separation was done using batch processing. Although with the sharp increase for oil products demand, refining became a continuous necessity. The first recognized continuous refinery plants came up around 1912. With the distinguishably and complexity of the oil products demand, the refining industry has been developed from few simple processing units to more complex production systems.

Usually, raw crude oil is not useful in industrial processes, although the lighter components of crude oil can be used as burning fuel in the refinery furnaces. On the other hand, hundreds of heavier hydrocarbon elements are separated in the refinery units into components which can be used as fuels, lubricants, and as feedstock in petrochemical industries which produce such products as plastics, detergents, solvents, and fibers such as nylon. Petroleum fuels are used in the internal combustion engines to provide power for ships, automobiles, aircraft engines, and such other machines.

Usually, a refinery is made up of several components that build up a complex production system, as in Figure 1.2. These components include:

1- Crude Supply and Blending: This part includes the receiving facilities and the tank area (tank farm) where all types of crude oil are received and gathered to be either blended or sent directly to the production system.

2- Production Units: they break crude oil up into other fractions or cuts, reform some of them, and convert heavy ones to light which are more useful. This zone also includes the refinery utilities which are a necessity for a safe refinery.

3- Product Blending and Transportation: In this part, the final products are processed according to either predetermined recipes and/or to certain product specifications. This area also includes the shipment of finished products to the customers.

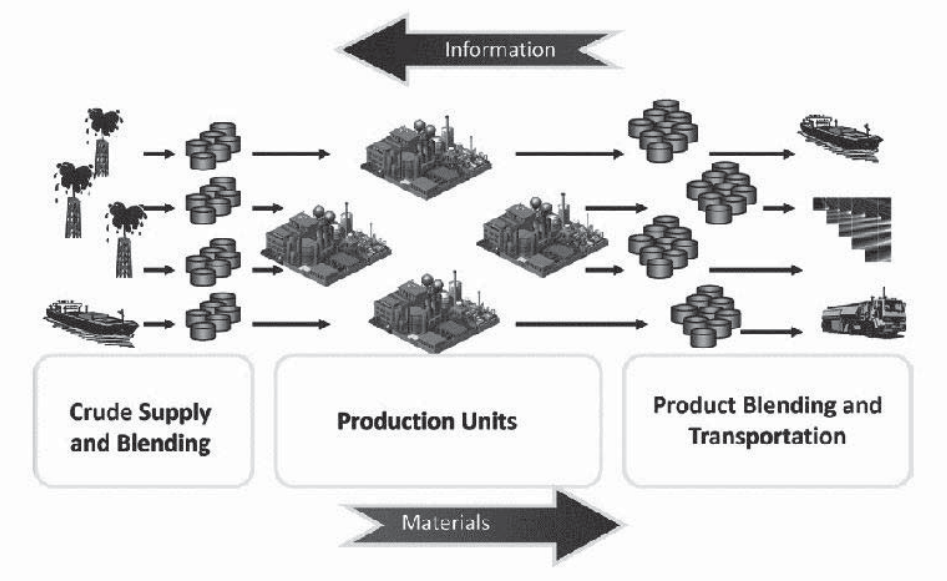

Figure 1.2 Schematic diagram of standard refinery configuration, Al-Qahtani & Elkamel(2010).

1.2 Configuration of Oil Refinery

1.2.1 Distillation Processes

The Crude distillation unit, which is considered to be the heart unit in the refinery, is used to force the crude oil to be separated into fractions by distillation according to their boiling points.

First of all, crude oil is treated to remove contained salt, if higher than 10 lb/ 1000 bbl, using single or multiple desalting units. This step is needed to minimize corrosion and fouling in the downstream heating trains and distillation columns. Distillation unit is usually divided into two steps, atmospheric and vacuum fractionation according to the applied pressure. This is done in order to achieve higher separation efficiencies at a lower cost. After heating the crude to near its boiling point, it is fed to the distillation column in which vapor rising through trays to help having direct contact with down-flowing liquid on the trays. During this process, higher boiling point fractions in the vapor phase are condensed and lighter fractions in the liquid are vaporized. This continuous process allows the various fractions of the crude oil with similar boiling points to achieve equilibrium and separate. Liquid can then be drawn off the column at different heights as product and sent for further treating or storage. Common products from the atmospheric distillation column include liquefied petroleum gas (LPG), naphtha, kerosene, gas oils and heavy residues.

The atmospheric bottom, also known as reduced oil, is then sent to the vacuum unit where it is further separated into vacuum gas oil and vacuum residues. Vacuum distillation improves the separation of gas oil distillates from the reduced oil at temperatures less than those at which thermal cracking would normally take place. The basic idea on which vacuum distillation operates is that, at low pressure, the boiling points of any material are reduced, allowing various hydrocarbon components in the reduced crude oil to vaporize or boil at a lower temperature. Vacuum distillation of the heavier product avoids thermal cracking and hence product loss and equipment fouling.

1.2.2 Coking and Thermal Processes

These processes are used to get higher quality products out of the heavy residues. They work together with other thermal processes to convert heavy fractions, usually from crude distillation processes, to more valuable and desirable products that are suitable for other refinery units.

One of the most famous and worldwide used coking processes is delayed coking. It involves severe thermal cracking of heavy residues such as vacuum oil, thermal tars, and sand bitumen. The actual reason behind its name is that the actual coking takes place in the heater effluent surge drum. The coke produced by this process is usually a hard and porous sponge-like material. This type of coke is called sponge coke and exists in a range of sizes and shapes. Many other types of coke are commercially available in the market and have a wide range of uses.

1.2.3 Catalytic Processes

There are two kinds of catalytic conversion units in the refining industry, cracking and reforming processes, and they are explained as follows:

1.2.3.1 Cracking Processes

This process converts heavy oils into lighter products that are able to be blended to give high value final products, such as gasoline, jet fuels and diesel. These processes mainly include catalytic cracking and hydrocracking. Catalytic cracking involves breaking down and rearranging heavy hydrocarbons into lighter ones with double bonds in order to increase the quality as well as the quantity of valuable products such as kerosene, gasoline, LPG, and petrochemical feedstocks which are the basis form of the petrochemical industry. The most commonly used process in the industry is fluid catalytic cracking (FCC) in which oil is cracked in a fluidized catalyst bed where it is continuously circulated between the reaction state and the regeneration state.

On the other side, hydrocracking is a process that combines catalytic cracking and hydrogenation where the feed is cracked in the presence of hydrogen to produce more desirable products. The other main importance role of hydrogen is to reduce tar formation and prevent the formation of coke on the catalyst.

1.2.3.2 Reforming Processes

Reforming processes convert feedstocks to higher quality streams by rearranging their structures. One of this kind's most famous and main processes is the catalytic reforming which is an important process used to convert low-octane feedstocks into high-octane gasoline components called reformate which can be produced with very high concentrations depending on the properties of the feedstock and the catalysts used. Hydrogen, which is a by-product of the reforming process, is separated from the products and reused as a feed in other refining processes. Some other examples of reforming processes are alkylation and isomerization processes.

1.2.4 Treatment Processes

They are applied to remove impurities, and other constituents that affect the properties of the finished products or reduce the efficiency of the conversion processes. A typical example of a treating process is hydrotreating which is a hydrogenation process used to remove about 90% of contaminants such as nitrogen, sulfur, oxygen, and metals from liquid petroleum fractions. These contaminants, if not removed from the petroleum fractions, can have a negative impact on the equipment, the catalysts, and the quality of the finished product. Hydrotreating is mainly used prior to catalytic reforming to reduce catalyst contamination and before catalytic cracking to reduce sulfur and improve product yields. It is also used to upgrade middle-distillate petroleum fractions into finished kerosene, diesel fuel, and heating fuel oils and converts olefins and aromatics to saturated compounds.

1.2.5 Product Blending

It is the process of mixing hydrocarbon fractions, additives, and other components to produce finished products with specific properties and desired characteristics. Products can be blended in-line through a manifold system, or batch blended in tanks and vessels. In-line blending of gasoline, distillates, jet fuel, and kerosene is accomplished by injecting proportionate amounts of each component into the main stream where turbulence promotes thorough mixing. Additives, including octane enhancers, metal deactivators, anti-oxidants, anti-knock agents, gum and rust inhibitors, detergents, and so on, are added during and/or after blending to provide specific properties not inherent in hydrocarbons.

1.3 The Research Contribution

Under the shadow of the open investment and changing economical and environmental conditions that the whole world witness these days, and as a result of fluctuations in the prices of raw materials needed for industry and the properties of the resulting products, industrial planning becomes much more important.

The good planning and optimization is considered as one of the most important problems for the process industries. This high importance is especially for petroleum refineries in which the planning suggestions are used to choose the best operational conditions, use the cheapest raw materials, use less amounts of energy, produce valuable materials, and meet the daily demand of the refined products in order to maximize the total profit and minimize the costs. The good planning helps to create production of high price materials, enhance their distribution process, good sales and future expanding and inventory plans based on the customer and market information.

1.4 Objective

In this humble research, we are aiming to present a detailed survey of existing literature in the stochastic refinery planning models, and based on that we will develop a general stochastic optimization model for the refinery operational planning that is applicable to be used and applied in refining industry as a tool which will serve in the oil refinery as the process consultant that helps decision makers in making the appropriate decisions which then will help raising the profitability of the oil refinery.

CHAPTER 2

LITERATURE REVIEW

The main objective of this chapter is to present a detailed survey of existing literature in the stochastic refinery planning models. The literature review will consider only journal papers since they are academically relevant. The review considers the published papers starting from the late nineties when the first work on stochastic optimization applied to the refinery planning, Liu & Sahinidis (1996) was published until now. First of all, overview of oil refinery planning has to be introduced to present a detailed idea about the thesis problem.

2.1 Overview of Oil Refinery Planning Under Uncertainty

The oil refinery planning can be defined as a development strategy for the allocation of equipment, utilities, or labor resources over a period of time in order to execute specific tasks to produce a single or more products, Leiras et al. (2011).

According to this definition, oil refinery planning can be divided into three types due to its period of time frames:

- Long-term planning (strategic planning) which covers the time horizon from one to several years. This kind determines the supply chain's structure (e.g., capacity expansion, investment decisions, and the location of the production center).
- Mid-term planning (tactical planning) which covers the time horizon or period from three months up to one year. This kind deals with the production targets

assignment to the refineries and the transportation process from those refineries to the distribution centers. This one could be which type of crude oil (light or heavy) to be fed into the production process in a specific time period.

- Short-term planning (operational planning) which covers the daily and weekly assignments up to only three months. This one considers the units tasks in each refinery, putting resources as well as time constraints in mind. Its variables conclude the choice of operation modes, level of inventory, and the produced quantities in each unit demanded by the consuming market.

In reality, refining industry faces daily serious changes in the products daily demand and properties and the crude oil cost which result in serious complexity in taking immediate decisions. Therefore, the use of optimization models becomes more effective and useful which make the process more realistically modeled.

Due to the uncertain economical nature of the worldwide oil industry which includes the crude oil exploration and the market requirements of its valuable products that the whole world witness especially in the last five decades of the last century, and because of the extreme high economic incentives and their strategic importance, oil refinery planning has become extremely important. Therefore, oil refineries are interested in improving their operations planning, Leiras et al. (2011).

Uncertainty can be divided into short-term, mid-term, and long-term.

The short-term one refers to the unanticipated factors in the internal processes such as equipment failures and operational variations. On the other side, long-term uncertainties represent the external factors that affect the planning process for a long period. As an example of this one is supply and demand. The mid-term one includes both of short-term and long-term uncertainties.

Alternatively, it can be also classified as external (exogenous) and internal (endogenous) uncertainties, according to the point-of-view of process operations. The external uncertainty variable will be affected by an outside factor and the decisions are totally independent of those taken in the previous periods. The internal one arises from deficiencies in the complete knowledge of the process and so, the decisions at each stage depend on the previous one (Leiras et al., 2011). Some examples of uncertainty factors can be presented in table 2.1 according to the previous two criteria.

The integrated oil chain usually covers stages from oil exploration to the distribution of the final product which includes the complicated transformation processes which take place in the oil refinery. Leiras et al. (2011) has divided the activities that comprise the oil supply chain into three types:

1- Upstream segment; which usually includes the exploration and oil production in the oil field.
2- Midstream segment; usually known as the intermediate segment, consists of the refining activities that includes the oil transportation from the well site to the refinery.
3- Downstream segment; this one takes care about the logistical tasks necessary to move the final product from the refinery to the consuming market.

Table 2.1 classification of uncertainty factors, Leiras et al. (2011).

Time horizon	**Process Operations**	
	External (exogenous)	Internal (endogenous)
Long-term	Availability of sources of oil supply. Economic data on raw materials, finished products, utilities, etc. (prices, demands, and costs). Location. Budgets on capital investments for capacity expansion and new equipment purchases or replacements. Investment costs of processes. Regulatory issues concerning laws, regulations, and standards. chnology obsolescence. Political issues	
Mid-term	Economic data on raw materials, finished products, utilities, etc. (prices, demands, and costs). Type of oil available.	
Short-term		Type of oil available. Properties of components. Product/process yields. Blending options. Process variations (flow rates and temperatures). Machine availability.

Historically, oil refinery planning models were based mainly on linear programming LP and mixed-integer linear programming MILP. The development of non-linear programming NLP models and mixed-integer non-linear programming MINLP models has been restrained because of the extremely high complexity of the algorithms and the computation process, especially the last one "MINLP" which still considered as a challenge.

2.2 Optimization approaches under uncertainty

Dealing with uncertain refinery planning optimization problems can be done using some special techniques that can be shown in Figure (2.1) as follows:

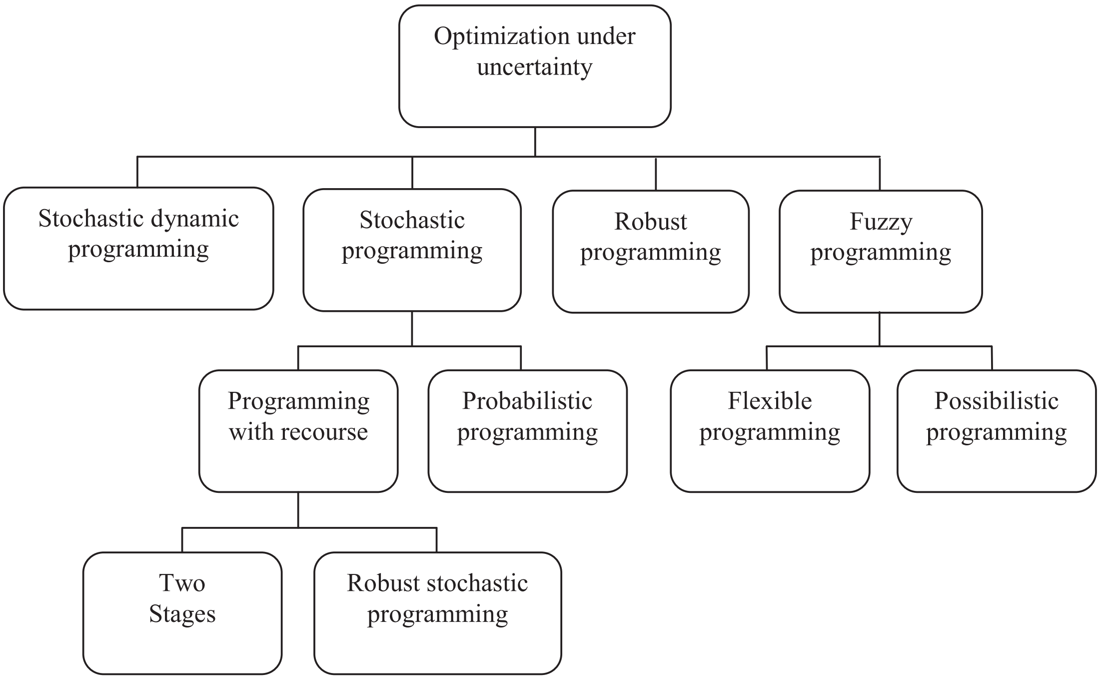

Figure 2.1 Approaches to optimization under uncertainty, Leiras et al. (2011).

Leiras et al., (2011) have studied the oil refinery planning under uncertainty over 40 articles, and concluded that each of LP and MILP represents 37.5% (15 articles), whereas NLP represents 22.5% (9 articles) and MINLP represents 20% (8 articles). They have also concluded that 55% (22 articles) of the published articles in this area has been accomplished using the stochastic programming techniques shown in figure 2.1.

In addition, very few articles (15% or 6 articles) address long-term (strategic) planning and most of the studied articles (65% or 26 articles) address short-term (operational) planning. The remaining (20% or 8 articles) address mid-term (tactical) planning.

2.3 Stochastic Optimization

Stochastic optimization plays a significant role in the analysis, design, and operation of modern systems. Methods for stochastic optimization provide a means of coping with inherent system noise and coping with models or systems that are highly nonlinear, high dimensional, or otherwise inappropriate for classical deterministic methods of optimization. For example, many modern data mining packages include methods such as simulated annealing and genetic algorithms as tools for extracting patterns in data. Specific applications include business, aerospace engineering, medicine, and traffic engineering. There are, of course, many other applications [Gentle et al., 2004].

Stochastic optimization algorithms have been growing rapidly in popularity over the last decade or two, with a number of methods now becoming "industry standard" approaches for solving challenging optimization problems [Gentle et al., 2004].

Therefore, our objective here is to give a detailed review on stochastic optimization as one of the most famous techniques used at refinery planning. But first we have to mention that we can write the uncertain refinery planning problem as a non-linear problem (NLP) as follows:

$$\underset{x \in X}{\mathrm{Min}} \{z(x)\} \text{ subject to } g_i(x) \leq 0, i = 1, \dots, m, x \in \Re^n \quad (2.1)$$

Where, the non-linearities arise from the final product specification constraints. Whereas NLP are usually difficult to be solved, the linear ones (LP) are perfectly posed because of the reason that they can be solved by applying the simplest method. Following Pongsakdi et al. (2006) and Lakkhanawat and Bagajewicz (2008), many non-linear parts in refinery planning models can be simplified in order to increase the computation speed. So, the planning problem (Equ. 2.1) can be rewritten as the following LP [Leiras et al., 2011]:

$$\underset{x \in X}{\mathrm{Min}}\{z(x) = c^T x\} \text{ subject to } Ax \leq b, x \in \Re^n, c \in \Re^n, b \in \Re^n, A \in \Re^n \qquad (2.2)$$

Furthermore, in the case of including the discrete decisions such as the operational mode choice and the minimum quantity of purchased oil in refinery planning models, the models were represented by (2.1) and (2.2) are formulated to include constraints of the form of $g(x) \leq b\rho$ and $Ax \leq b\rho$ $(\rho \in \{0,1\})$ to give mixed-integer non-linear models (MINLP) and mixed-integer linear models (MILP), respectively [Leiras et al., 2011].

Table (2.2) shows a collection of oil refinery planning articles which are classified due to the oil chain segment (upstream, midstream, and downstream), the decision planning level (long-term, mid-term, and short-term), the modeling technique used, and the uncertainty factor, as shown next.

Table 2.2 Literature review of stochastic refinery planning.

Author (year)	Segment			Decision level			Modeling tech.						Uncertainty factor					
	Upstream	**Midstream**	**Downstream**	**Strategic**	**Tactical**	**Operational**	**Two stages**	**Robust Stoc.**	**Probabilistic**	**Dynamic stoc**	**Robust**	**Fuzzy**	**Demand**	**Supply**	**Price**	**Cost**	**Yield**	**Other**
Al-Qahtani & Elkamel (2010)		×	×	×				×					×		×	×		
Park et al. (2010)		×				×		×							×			
Carneiro et al. (2010)	×	×	×	×				×					×	×	×			
Leiras et al. (2010)		×				×					×		×		×	×	×	
Ribas et al. (2010)	×	×	×	×			×				×		×	×	×			
Luo and Rong (2009)		×				×					×		×					×
Khor and Nguyen (2009)		×				×		×					×		×	×	×	
Al-Othman et al. (2008)	×	×	×		×		×						×		×			
Khor et al. (2008)		×			×		×	×					×		×		×	
Lakkhanawat & Bagajewicz (2008)		×				×		×					×		×	×		
Li et al. (2008)		×				×			×				×					
Pongsakdi et al. (2006)		×				×		×					×		×			
Neiro and Pinto (2006)		×				×				×			×		×			
Neiro and Pinto (2005)		×				×				×			×		×			
Li et al. (2004)		×			×				×				×	×				
Hsieh and Chiang (2001)		×	×		×							×	×			×		
Dempster et al. (2000)		×	×		×					×			×		×	×		
Escudero et al. (1999)		×	×		×		×						×		×	×		
Ahmed and Sahinidis (1998)		×		×				×					×	×	×	×		
Ravi and Reddy (1998)		×				×						×						×
Liu and Sahinidis (1997)		×		×								×	×	×	×	×	×	
Liu and Sahinidis (1996)		×			×		×					×	×	×	×			

As it has been shown in table 2.2; the development in the oil refinery planning has been increasing over the last two decades. The strategic models has been enhanced in the last few years as can be seen in the works by Al-Qahtani & Elkamel (2010), Carneiro et al. (2010), and Ribas et al. (2010).

The work of Al-Qahtani & Elkamel (2008) considered the multisite integration and coordination strategies in oil refinery network has been extended by Al-Qahtani & Elkamel (2010) in terms of uncertainty via robust optimization techniques. In the same way, Ribas et al. (2010) developed a strategic model for the oil chain under uncertainty with which they deal by developing two two-stage stochastic models which then be applied to a Brazilian oil chain. This work has been extended by Carneiro et al. (2010) including the risk management. The work of Liu and Sahinidis (1997) is considered to be the inspiration of applying strategic models to the oil chain. Ahmed and Sahinidis (1998) developed a robust stochastic programming model to solve the problem of the strategic planning.

Regarding the tactical models, Liu and Sahinidis (1996) developed a two-stage stochastic model and a fuzzy model for process planning under uncertainty. A method was proposed for comparing the two approaches. Overall, the comparison favored stochastic programming. Escudero et al. (1999) worked in the supply, transformation, and distribution planning problem that accounted for uncertainties in demands, supply costs, and product prices. As the deterministic treatment for the problem provided unsatisfactory results, they applied the two-stage scenario analysis based on a partial recourse approach. Dempster et al. (2000) formulated the tactical planning problem for an oil consortium as a dynamic recourse problem. A deterministic multi-period linear model was used as basis for implementing the stochastic programming formulation. Hsieh and Chiang (2001) developed a manufacturing-to-sale planning system and adopted fuzzy theory for dealing with demand and cost uncertainties. Li et al. (2004) proposed a probabilistic programming model to deal with demand and supply uncertainties in the tactical problem. Khor et al. (2008) treated the problem of medium-term planning of a refinery operation by using stochastic programming (a two-stage model) and stochastic robust programming. Al-Othman et al. (2008) have proposed a two-stage stochastic model for multiple time periods to optimize the supply chain of an oil company installed in a country that produces crude oil.

For the operational models, a non-linear integer programming application associated with uncertainty was investigated in the work by Neiro and Pinto (2005). They formulated a stochastic multi-period model for which the uncertainty is related to the prices of petroleum and product as well as to the product demand. Pongsakdi et al. (2006) treated the uncertainty and financial risk in the planning of operations for a refinery in Thailand using a two-stage linear stochastic model. The problem consists in determining how much of each crude oil had to be purchased and the anticipated production level of different products based on demand forecasts. The uncertainty was introduced by means of the demand and product price parameters. The first-stage decisions were represented by the amount of crude oil purchased for each period. Lakkhanawat and Bagajewicz (2008) extended the work of Pongsakdi et al. (2006) by incorporating the product pricing in their study. Luo and Rong (2009) also treated the integrated operational planning and scheduling of refineries but they dealt with uncertainty using the robust approach proposed by Janak et al (2007).

CHAPTER 3

MATHEMATICAL MODEL DEVELOPMENT

In this chapter a general deterministic model of oil refinery planning is represented in details based on several input and output parameters that affects the total profit of an oil refinery. In addition, a general stochastic model for oil refinery planning is developed based on the deterministic one taking into account the instability in the crude oil cost, products prices, and products daily demand. Both models consist of an objective function, to be maximized, restricted by a set of several types of constraints. The objective function to be maximized is the total profit.

3.1 Deterministic Model Formulation

As we know that the optimization algorithm determines the best solution for a given problem. The modeling and decision analysis process can be defined as shown in figure (3-1) where the situation represents the current or the real problem needed to be solved. By applying some suitable assumptions, the mathematical model can be obtained and coded into suitable software where then to give the optimal solution.

The challenging fault with this model is that the determined optimal solution or the decision is optimum for the model but not for the situation, and so the taken decision is quite different from the appropriate one.

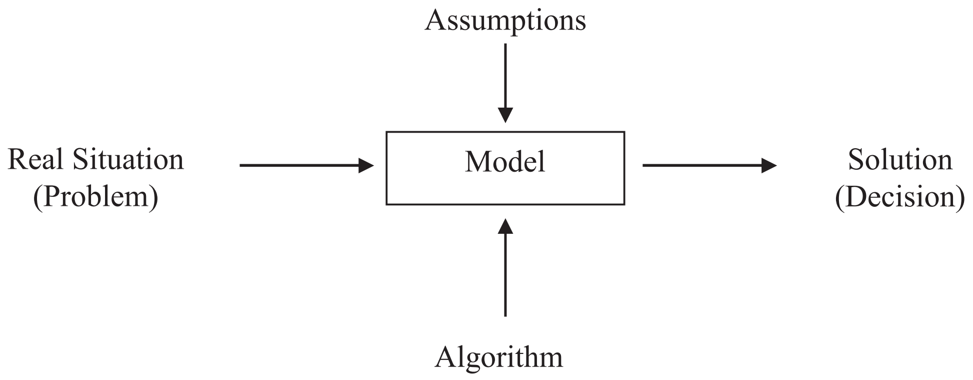

Fig. 3.1 the deterministic approach to make a decision.

The deterministic model represents the stable case under which the oil refinery works. It can be easily expressed in simple words as the total profit which is the difference between the total incomes and outcomes of the refinery.

In other words, the total incomes can be determined by multiplying the produced quantity of every main product which can be sold in the market, but the total outcomes can be determined by summarizing costs of raw materials, operation cost, inventory cost, etc., which have to be paid.

Therefore, in general,

Total profit = Products selling prices – costs (raw material, fixed, operating, Inventory, etc.)

Mathematically, the model can be written as:

$$\text{Maximize } Z = \sum_{s \in PRO_u} CPR_s . QOS_s - \sum_{s \in CRU_u} (CCR_s . QIS_s + CAD_s . QAD_s)$$

$$- \sum_{u \in CRU_u} CPM_u . a_u - \sum_{u \in U} (CFX_u + COP_u) - \sum_{u \in PRT} CINV_u . QINV_u \quad (3-1)$$

Subject to the following constraints:

1- Mass balance of components:

Each unit has number of input raw materials and output products which should be equalized. In addition, each component must be equalized around every unit as follows:

$$\sum_{u \in U} \sum_{s \in S} b_u . x_s = 0 \qquad (3-2)$$

$$\sum_{u \in U} \sum_{i \in N} b_u . x_i = D_j \quad (j \in M) \qquad (3-3)$$

2- Demand and supply:

It is common sense that the daily demand of a certain product must be lower than the supplied raw material of that product:

$$\sum_{i \in N} y_i \leq S_i \qquad (3-4)$$

$$\sum_{j \in M} y_j \geq D_j \qquad (3-5)$$

3- Quality constraints:

Basically, the quality of a different components blend (base stocks) is given by a rule called the blending rule as follows:

$$Q = \frac{\sum_{j \in M} q_j \cdot x_j}{\sum_{j \in M} x_j} \quad (3-6)$$

4- Unit Capacity constraints:

The summation of all quantities getting into a specific unit has not to exceed the capacity of that unit, as follows:

$$\sum_{s \in S} x_s \leq C_u \quad (3-7)$$

Where:

a_u	a binary variable (1 if there is another oil supply, 0 otherwise).
b_u	inlet material to a unit u.
CADs	cost of additional raw material.
CCR_s	cost of crude oil (raw material).
CFX_u	fixed cost of unit u.
$CINV_u$	cost of inventory of unit u.
COP_u	operating cost of unit u.
CPM_u	cost of pumping of additional amounts of raw materials.
CPR_s	price of outlet streams (product).
CRU_u	crude oil storage tanks.
C_u	unit capacity.
Dj	demand of product j.

PRO_u	properties of the outlet stream of unit u.
PRT	production storage tanks.
Q	the product quality.
QAD_s	flow rate of additional amount of stream s.
$QINV_u$	amount of inventory of unit u.
QIS_s	inlet flow rate of stream s (crude oil).
q_j	the quality of a certain material.
QOS_s	the outlet flow rate of stream s (product).
s, S	stream.
S_i	supply of a certain raw material.
u, U	unit.
x_i	mole fraction of input stream.
x_j	mole fraction of output stream.
x_s	mole fraction of stream s.
y_i	the inlet quantities of all materials.
y_j	the outlet quantities of all products.
z	the objective function.

3.2 The Stochastic Model Formulation

3.2.1 Scenarios and Probabilities

Scenarios and probabilities are more accurate description for what really happens in the decision analysis process as can be shown in Fig (3-2), in which the future parameters are considered to be individually deterministic but they are different from each other. In scenarios, the model describes different possibilities for what will occur in supplies, demands, costs and prices in the near future and the solution gives advice for what should be done on the other side.

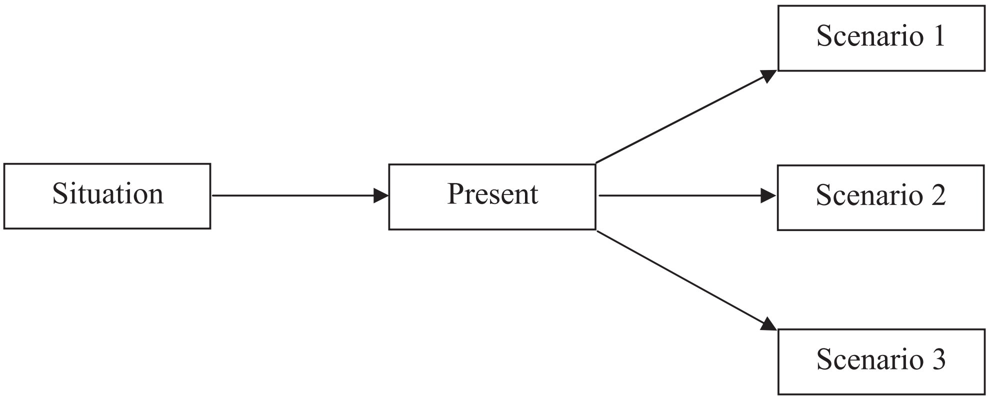

Fig. 3.2 decision problem with three possible scenarios.

This approach can be developed to be a multistage problem as shown in fig (3-3) where the problem can be divided into a number of periodic decisions. These periods can be represented as days, weeks, months or years based on the uncertainty factors.

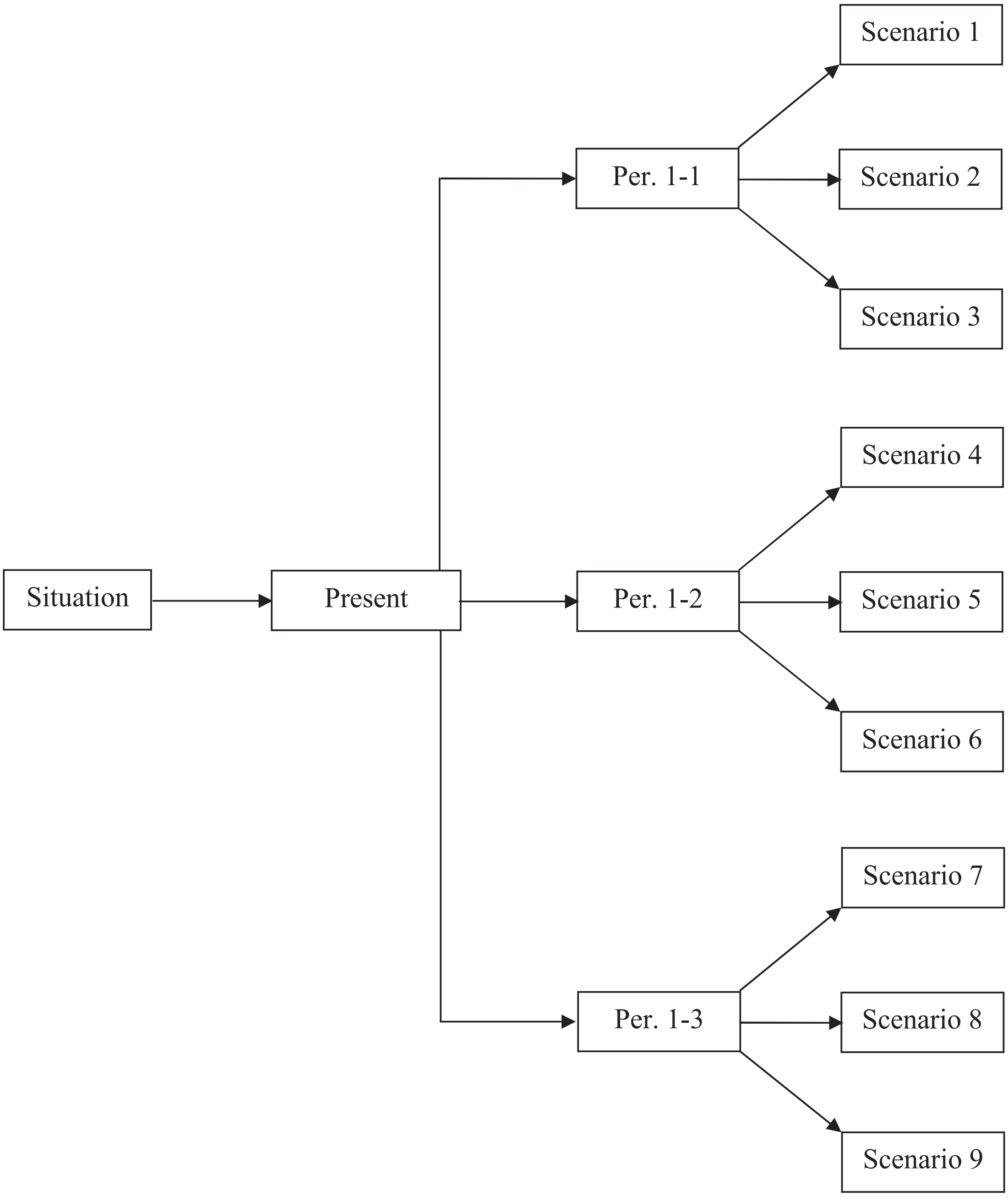

Fig. 3.3 multistage stochastic model.

The multistage stochastic programming model becomes much more difficult to be solved because of its size and the corresponding computational time needed to obtain the solution. The second difficulty is the structure of the structure of the mathematical programming model. However, the stochastic programming model is a general linear model.

3.2.2 The stochastic model

The mixed-integer linear programming MILP model for the refinery operational planning will be presented by putting scenarios and probabilities in mind. A two-stage stochastic linear model with fixed recourse is based on the work of the stochastic formulation presented by Neiro and Pinto (2005). It can be formulated as follows:

The objective function:

$$
\begin{aligned}
\text{Maximize } Z = & \sum_{se\in SE} \sum_{t\in T} \sum_{u\in PRU} \sum_{s\in PRO_u} Pr_{t,se} . CPR_{u,t,se} . QOS_{u,s,t,se} \\
& - \sum_{se\in SE} \sum_{t\in T} \sum_{u\in CRU} \sum_{s\in SRO_u} \big(Pr_{t,se} . CCR_{u,t,se} . QIS_{u,s,t,se} \\
& + CAD_{u,t,se} . QAD_{u,s,t,se} \big) - \sum_{se\in SE} \sum_{t\in T} \sum_{u\in CRU} CPM_u . a_{u,t,se} \\
& - \sum_{se\in SE} \sum_{t\in T} \sum_{u\in U} (CFX_u + COP_u) \\
& - \sum_{se\in SE} \sum_{t\in T} \sum_{u\in PRT} CINV_{u,t,se} . QINV_{u,t,se} \quad (3-8)
\end{aligned}
$$

Subject to the following constraints:

1- Balances of components:

$$\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{s\in S} b_{u,t,se}.x_{u,s,t,se} = 0 \qquad (3-9)$$

$$\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{i\in N} b_{u,t,se}.x_{i,u,t,se} = D_j \quad (j \in M) \qquad (3-10)$$

2- Demand and supply:

$$\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{i\in N} y_{i,u,t,se} \leq S_{i,t,se} \qquad (3-11)$$

$$\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{j\in M} y_{j,u,t,se} \geq D_{j,t,se} \qquad (3-12)$$

3- Quality constraints

$$Q = \frac{\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{j\in M} q_{j,t,se} \cdot x_{j,u,t,se}}{\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{j\in M} x_{j,u,t,se}} \qquad (3-13)$$

4- Unit Capacity constraints

$$\sum_{se\in SE}\sum_{t\in T}\sum_{u\in U}\sum_{s\in S} x_{s,u,t,se} \leq C_u \qquad (3-14)$$

Where:

$a_{u,t,se}$ a binary variable (1 if there is another oil supply, 0 otherwise).

$b_{u,t,se}$ inlet material to a unit u at time period t under scenario se.

$CAD_{u,t,se}$ cost of additional raw material that get into a unit u at time period t under scenario se.

$CCR_{u,t,se}$	cost of crude oil (raw material) of crude unit at time period t under scenario se.
CFX_u	fixed cost of unit u.
$CINV_{u,t,se}$	cost of inventory of unit u at time period t under scenario se.
COP_u	operating cost of unit u.
CPM_u	cost of pumping of additional amounts of raw materials.
$CPR_{u,t,se}$	price of outlet stream s (product) of unit u at time period t under scenario se.
CRU_u	crude oil storage tanks.
C_u	unit capacity.
D_j	total demand.
$D_{j,t,se}$	total demand at time period t under scenario se.
$Pr_{t,se}$	probability of scenario se in time period t.
PRO_u	properties of the outlet stream of unit u.
PRT	production storage tanks.
PRU	production units.
Q	product quality.
$QAD_{u,s,t,se}$	flow rate of additional amount of stream s of unit u at time period t under scenario se.
$QINV_{u,t,se}$	amount of inventory of unit u at time period t under scenario se.
$QIS_{u,s,t,se}$	inlet flow rate of stream s (crude oil) of crude unit u at time period t under scenario se.
$q_{j,t,se}$	the quality of a certain material at time period t under scenario se.
$QOS_{u,s,t,se}$	the outlet flow rate of stream s (product) of unit u at time period t under scenario se.

s, S	stream.
se, SE	scenario.
$S_{i,t,se}$	total supply at time period t under scenario se.
SRO_u	raw material tanks.
t, T	time period.
u, U	unit.
$x_{i,u,t,se}$	the inlet mole fraction of a certain material to a unit u at time period t under scenario se.
$x_{u,s,t,se}$	the inlet mole fraction of stream s to a unit u at time period t under scenario se.
$y_{i,u,t,se}$	the total inlet materials to a unit u at time period t under scenario se.
$y_{j,u,t,se}$	the total outlet materials from a unit u at time period t under scenario se.
z	the objective function.

Finally, these two approaches will be applied on a case study to determine the best optimal solution and compare the deterministic approach with the stochastic one.

First we have to discuss the case study on which we are going to apply our approaches, listed the needed data, and listed the suitable assumptions and basis to achieve what we are looking for. These all subjects will be covered in the next chapter.

CHAPTER 4

APPLICATION OF DEVELOPED MATHEMATICAL MODELS ON A CASE STUDY

4.1 Introduction

To obtain the best overall margin, refiners have to obtain a best combination of crude oil and feedstocks and examine the best ways and conditions to process them all.

In most industries operating, refineries must maximize their economic results. To do so, they must maximize their margins, i.e. the difference between their selling incomes from the products they manufacture, and their costs, Favennec J. (2001). There are some variables that control this process, for example:

- Raw materials (crude oil and imported feedstocks).
- Operating costs (maintenance, overheads, chemicals, catalysts, labor... etc.).
- Fixed and inventory costs.
- Any other excess costs.

In this work, the following assumptions will be considered:

- Both the monthly demand (±20%) of oil products and their prices (±15%) will be considered as uncertain variables.
- The probabilities of demand to remain normal and increase are 50% and 35% respectively.

- The probabilities of oil products prices to remain normal and increase are 40% and 30% respectively.
- Fixed cost will be neglected.
- The time period is one month.
- Inventory cost is 10 $/ton.

According to the second and the third assumptions, the scenarios probabilities are:

Table 4.1 scenarios probabilities.

	Demand	**Product Price**	**Probability**
Scenarios	Up (35%)	1- Up (30%)	0.105
		2- Normal (40%)	0.14
		3- Down (30%)	0.105
	Normal (50%)	4- Up (30%)	0.15
		5- Normal (40%)	0.2
		6- Down (30%)	0.15
	Down (15%)	7- Up (30%)	0.045
		8- Normal (40%)	0.06
		9- Down (30%)	0.045

Table 4.1 can be explained briefly as well as next.

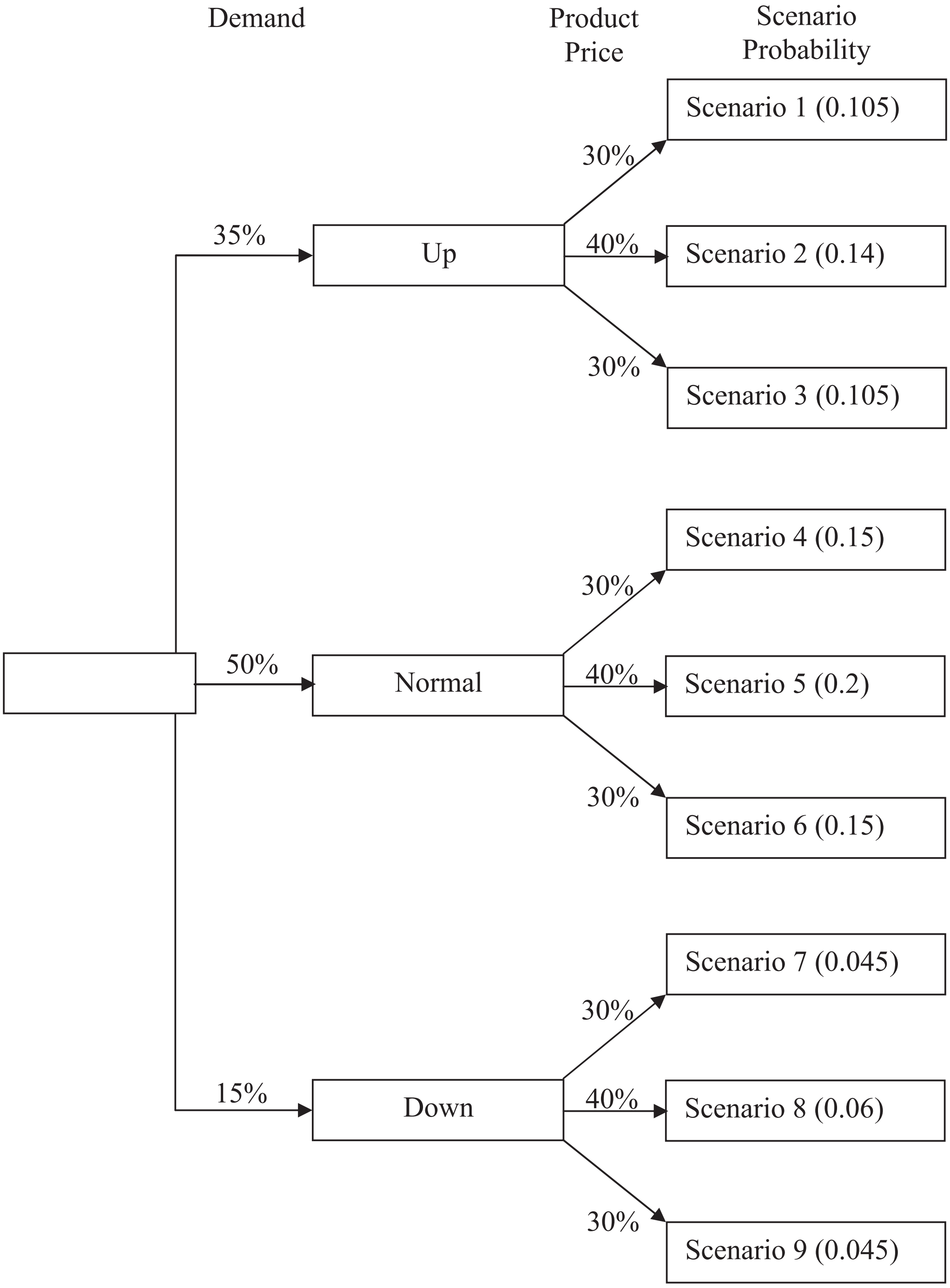

Fig. 4-1 suggested multistage stochastic model.

Given all the information above makes the objective function written as follows:

$$\text{Maximize } Z = \sum_{s \in PRO_u} CPR_s . QOS_s - \sum_{s \in CRU_u} (CCR_s . QIS_s + CAD_s . QAD_s) - \sum_{u \in U} COP_u - \sum_{u \in PRT} CINV_u . QINV_u \qquad (4-1)$$

and:

$$\text{Maximize } Z = \sum_{se \in SE} \sum_{t \in T} \sum_{u \in PRU} \sum_{s \in PRO_u} Pr_{t,se} . CPR_{u,t,se} . QOS_{u,s,t,se} - \sum_{se \in SE} \sum_{t \in T} \sum_{u \in CRU} \sum_{s \in SRO_u} \left(CCR_{u,t,se} . QIS_{u,s,t,se} + CAD_{u,t,se} . QAD_{u,s,t,se} \right) - \sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} COP_u - \sum_{se \in SE} \sum_{t \in T} \sum_{u \in PRT} CINV_{u,t,se} . QINV_{u,t,se} \qquad (4-2)$$

According to these constraints:

$$\sum_{u \in U} \sum_{s \in S} b_u . x_s = 0 \qquad (4-3)$$

$$\sum_{u \in U} \sum_{i \in N} b_u . x_i = D_j \quad (j \in M) \qquad (4-4)$$

$$\sum_{i \in N} y_i \leq S_i \qquad (4-5)$$

$$\sum_{j \in M} y_j \geq D_j \qquad (4-6)$$

$$Q = \frac{\sum_{j \in M} q_j \, . \, x_j}{\sum_{j \in M} x_j} \qquad (4-7)$$

$$\sum_{s \in S} x_s \leq C_u \qquad (4-8)$$

$$\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{s \in S} b_{u,t,se} . x_{u,s,t,se} = 0 \qquad (4-9)$$

$$\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{i \in N} b_{u,t,se} . x_{i,u,t,se} = D_j \quad (j \in M) \quad (4-10)$$

$$\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{i \in N} y_{i,u,t,se} \leq S_{i,t,se} \qquad (4-11)$$

$$\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{j \in M} y_{j,u,t,se} \geq D_{j,t,se} \qquad (4-12)$$

$$Q = \frac{\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{j \in M} q_{j,t,se} \, . \, x_{j,u,t,se}}{\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{j \in M} x_{j,u,t,se}} \qquad (4-13)$$

$$\sum_{se \in SE} \sum_{t \in T} \sum_{u \in U} \sum_{s \in S} x_{s,u,t,se} \leq C_u \qquad (4-14)$$

4.2 Case study

In this research, a practical refinery with complete data will be used as a case study. This refinery is the same one used by Al-shammari and Bashammakh (2010). The refinery consists of six main units; crude distillation unit CDU, reforming, fluidized catalytic cracking FCC, desulphurization, isomerization, and blending units. The refinery uses two types of crude oil (DC1 and DC2) to produce liquefied petroleum gas LPG, light naphtha LN, two types of gasoline (PG98 and ES95), jet fuel JF, gas oil GO, and heavy fuel oil HFO. All the refinery gas RG resulted from the CDU is blended with some amounts of other products to be used as a refinery fuel RF. All the above information and other details can be shown in the refinery PFD as in fig (4.1).

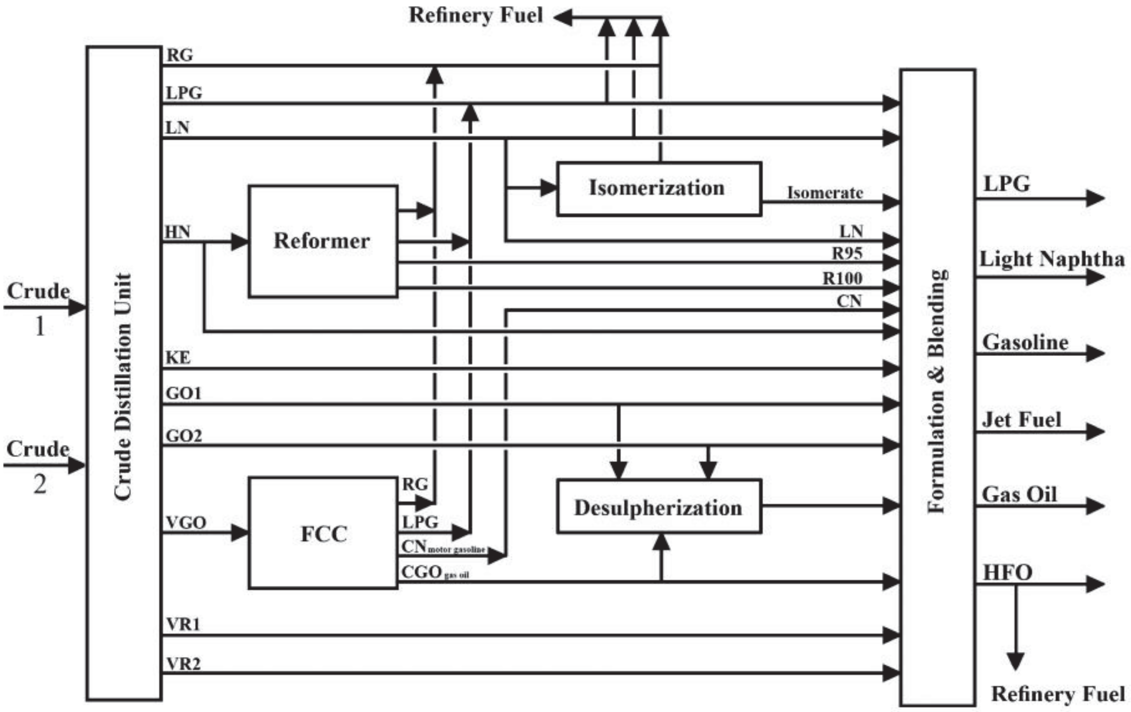

Figure 4.2 a simplified sketch for the oil refinery (case study).

All quantities of the refinery units are unknown, and so one of our objectives is to determine the resulted quantities of products is to determine those intermediate quantities, positive or zeros, that these values take in the optimum solution. To do so, some detailed data is needed.

The data includes:

1. The quantities of raw materials and intermediate products used as feedstock for the process units.
2. The quantities of intermediate products supplied to the blending facilities for the finished products production.
3. The quantities of products used as refinery fuel.
4. The quantities of finished products or intermediates (e.g. catalytic cracker feedstock) imported or stored.
5. The quantities of products manufactured according to predetermined formulations (if any).

The equations and inequalities provide the relationships between all these variables and are the means by which:

1- Quantities are controlled and any required restrictions are imposed;
2- The quality of the finished products is controlled;
3- The objective function is expressed.

Each unit has an output products, or base stocks, according to yields (% wt.) which are obtained from refinery process units and these are given next.

Table 4.2 yields obtained from distillation unit.

Atmospheric Distillation Unit (feedstock: crude)		
	Crude 1	Crude 2
Refinery gas RG	0.1	0.2
Liquefied petroleum gas LPG	1.2	1.5
Light naphtha LN	4.0	4.0
Heavy naphtha HN	14.5	7.5
Kerosene KE	15.0	9.0
Gas oil 1 GO1	31.0	--
Gas oil 2 GO2	--	20.3
Vacuum gas oil VGO	21.2	27.5
Vacuum residue 1 VR1	13.0	--
Vacuum residue 2 VR2	--	30.0
Total	100	100

Table 4.3 yields obtained from reforming unit.

Reforming unit (feedstock: heavy naphtha)		
	Severity	
	95	100
Refinery gas RG	8	9
Liquefied petroleum gas LPG	9	12
Reformate 95	83	--
Reformate 100	--	79
Total	100	100

Table 4.4 yields obtained from FCC.

Fluidized catalytic cracking unit (feedstock: VGO)		
Coke burnt in the unit = 5%	Maximizing	
	Mogas	AGO
Refinery gas RG	1.5	1.2
Liquefied petroleum gas LPG	5.3	4.6
Catalytic cracked spirit CN	43.6	38.1
Light cycle gas oil CGO	44.6	51.1
Total	95	95

Table 4.5 yields obtained from desulphurization unit.

Desulphurization Unit (to 97% of the sulphur)			
	Feedstock		
	G1	G2	CGO
Desulphurized raffinate	98	97	96
Refinery gas RG	2	3	4
Total	100	100	100

Table 4.6 yields obtained from isomerization unit.

Isomerization Unit (feedstock: light naphtha)	
Refinery gas RG	3
Isomerate ISO	97
Total	100

Jet fuel is produced according to two formulations as next.

Table 4.7 jet fuel formulations (% wt.).

Jet fuel formulation		
	F1	F2
Light naphtha LN	5	3.5
Heavy naphtha HN	10	7.5
Kerosene cut KE	85	89.9

The market demand for final products is given below:

Table 4.8 demand for sales products.

Demand (thousands of tons / month)	
Liquefied gas LPG	11
Light naphtha LN	6
Unleaded 98 mogas PG98	20
Unleaded 95 mogas ES95	80
Jet Fuel JF	70
AGO	160
HFO	148

The units capacities are given below.

Table 4.9 processing capacities.

Capacity limits (thousands of tons / month)	Min	Max
Distillation capacity	--	700
Reforming capacity	260	--
95 severity	2	--
Total	--	60
Total cracking capacity	--	135
Desulphurization capacity	--	150

The availability of crude oil is given below.

Table 4.10 raw material limits.

Crude availability (thousands of tons / month)	Min	Max
Crude 1	--	400
Crude 2	260	--

The unit operating costs and crude and products prices are given as follows.

Table 4.11 unit operating costs.

Unit operating costs ($ / ton)	
Distillation	6.28
Reforming	
95 severity	17.06
100 severity	20.22
Cracking	18.96
Isomerization	3.8
Desulphurization	
On GO1	6.28
On GO2	6.28
On CGO	8.84

Table 4.12 crude and imports costs.

Crude and imports costs ($ / ton)	
Crude 1	578
Crude 2	489.5
Catalytic reformer feedstock HN	682.3
Catalytic cracker feedstock VG	631.76
95 RON mogas	742.32
Jet fuel	710.73
AGO / HGO	694.94
HFO	347.47

Prices of the final products are listed as next.

Table 4.13 final products prices.

Final products prices ($ / ton)	
Liquefied petroleum gas LPG	655
Light naphtha LN	900
Unleaded 98 mogas PG98	995
Unleaded 95 mogas ES95	880
Jet fuel JF	1085
Gas oil GO	925
Heavy fuel oil HFO	550

Finally, the quality characteristics of intermediate and final products, and the specification requirements for finished products are given.

Table 4.14 quality characterstic of intermediate products.

Comp. or product	Relative density (g/cm^3)	Vapor pressure (bar)	Octane Number (clear)		Sulphur content (% wt.)	
			Research RON	Motor MON	Before HDS	After HDS
LPG	0.54	--	--	--	--	
C_4	0.58	4.300	94	90	--	
LN	0.65	0.800	71	68	--	
KE	0.77	--	--		0.1	
GO1	0.83	--	--		0.2	0.006
GO2	0.86	--	--		1.5	0.045
VGO	0.92	--	--		--	
VR1	0.98	--	--		--	
VR2	1.02	--	--		--	
R95	0.77	0.500	95	86	--	
R100	0.80	0.500	100	91	--	
ISO	0.665	0.400	91	86	--	
CN	0.75	0.650	93	82	--	
CGO	0.95	--	--		2	0.06

Where HDS is hydro-desulphurization.

The specification requirements for final products are:

- Butane content for both PG98 and ES95 is ≤ 5 % vol.
- Vapor pressure (in bars) for PG98 is in the range 0.500 ≤RVP≤ 0.860.
- Vapor pressure (in bars) for ES98 is in the range 0.450 ≤RVP≤ 0.800.
- Research octane number for PG98 and ES95 is ≥ 98 and ≥ 95 respectively.
- The sulphur content (% wt.) for AGO/HGO is ≤ 0.05.
- Sensitivity = RON – MON ≤ 10.

CHAPTER 5

RESULTS AND DISCUSSION

Both models that developed in chapter three were illustrated and coded into the General Algebraic Modeling System (GAMS) to maximize the total profit of the refinery mentioned in the case study.

The results will be discussed and compared by following three steps:

- The deterministic model is applied individually.
- The stochastic model is applied individually to have the results taking in mind the nine scenarios shown earlier in fig. 4.1 together and then compared to the deterministic one in terms of quantities and prices.
- The stochastic model is solved considering each scenario as well as deterministic model to compare the results.
- The sensitivity analysis is then illustrated to determine the accepted fluctuation in both price and demand.

5.1 Results

By applying both deterministic and stochastic models individually, the following results were achieved:

Table 5.1 results of deterministic and stochastic models.

Detailed composition		Deterministic (10^3 tons)	Stochastic (10^3 tons)
Crude 1 (DC1)		221.778	221.778
Crude 2 (DC2)		260	260
Light Naphtha LN		19.2711	19.2711
LPG	From DC1	2.661	2.661
	From DC2	3.9	3.9
	Rafinate 95 (R95)	0.18	0.18
	Rafinate 100 (R100)	6.204	6.204
	Vac. Distl. to FCC (Gasoline maximized) (FCCNA)	0	0
	Vac. Distl. to FCC (Gas oil maximized) (FCCGO)	5.452	5.452
TOTAL		18.397	18.397
PG98	Butane (C4)	0.746	0.746
	Light Naphtha (LN)	0	0
	Isomerate (ISO)	0	0
	Rafinate 95 (R95)	1.66	1.66
	Rafinate 100 (R100)	14.062	14.062
	Cat. Cracker Gasoline (CCG)	3.531	3.531
TOTAL		19.999	19.999
ES95	Butane (C4)	3.106	3.106
	Light Naphtha (LN)	0	0
	Isomerate (ISO)	8.487	8.487
	Rafinate 95 (R95)	0	0
	Rafinate 100 (R100)	26.784	26.784
	Cat. Cracker Gasoline (CCG)	41.624	41.624
TOTAL		80.001	80.001
JF	Jet Fuel (Formulation 1) (JF1)	66.667	66.667
	Jet Fuel (Formulation 2) (JF2)	0	0
	Light Naphtha (LN)	3.333	3.333
TOTAL		70	70
GO	Kerosine (KE)	0	0
	Non Desulpherized 1 (NGO1)	15.716	15.716
	Non Desulpherized 1 (NGO2)	0	0
	Non Desulpherized Cracked Gas Oil (NCGO)	0	0
	Desulpherized 1 (DGO1)	51.974	51.974
	Desulpherized 1 (DGO2)	51.197	51.197
	Desulpherized Cracked Gas Oil (DCGO)	18.541	18.541

Detailed composition		Deterministic (10^3 tons)	Stochastic (10^3 tons)
TOTAL		137.428	137.428
HFO	Desulpherized Cracked Gas Oil (NCGO)	41.248	41.248
	Vaccum residue from DC1 (VR1)	28.831	28.831
	Vaccum residue from DC2 (VR2)	78	78
TOTAL		148.079	148.079
RF	Refinery Gas	10.616	10.616
	LPG	3.546	3.546
	LN	1.188	1.188
	HFO	9.573	9.573
TOTAL		24.923	24.923
Imports	HN	8.713	8.713
	VG	0	0
	ES95	0	0
	JF	0	0
	GO	22.572	22.572
	HFO	9.493	9.493
TOTAL IMPORTS		40.778	40.778
INPUTS		522.556	522.556
OUTPUTS		495	495
INVENTORY		27.556	27.556
PROFIT (Millions of US Dollars)		38.861	51.936

According to table 5.1, both the deterministic and the stochastic models consume the same amounts of both crude oil DC1and DC2, imports of feedstocks (Reformer feedstock HN and Catalytic Cracker feedstock VG) and final products (ES95, JF, GO, and HFO) to meet the market demand.

A summation of 481.778×10^3 tons of two types of crude oil and total imports of 40.778×10^3 tons, which represent a total input of 522.556×10^3 tons, are fed to the refinery units to produce a total of 495.000×10^3 tons of final products and 27.556×10^3 tons of inventory.

The main difference between the deterministic model and the stochastic one is the objective function (Profit). The deterministic model gives a total profit of US$38.861million/month. However, the profit is increased in the stochastic model to US$51.936 million with an increment of US$13.075 million. This increment is because of the probabilities that mentioned in the stochastic model. To understand this, scenarios are divided into three segments:

1- The first segment consists of scenarios 1, 2, and 3.
2- The second segment consists of scenarios 4, 5, and 6.
3- The third segment consists of scenarios 7, 8, and 9.

This division is according to the market demand. For example, the first segment which consists of the first three scenarios have the same amounts of demand, so they definitely have the same amounts of crude oil, products, intra products, and imports as can be seen from table 5.1. The main difference is the objective function (Profit).

Referring to the scenarios represented earlier in Fig 4.1, the following paragraphs give the results obtained after running the deterministic model individually for each scenario. Every scenario is treated as a deterministic model and then compared to the deterministic results to see the effect of that scenario and its probability on the stochastic objective function. To do so, the applied scenario is 100% effectible and the other eight scenarios are neglected. But we have first to recall table 4.1 as given below.

Table 5.2 scenarios probabilities.

	Demand	Product Price	Probability
Scenarios	1st Segment Up (35%)	Scenario No.1 Up (30%)	0.105
		Scenario No.2 Normal (40%)	0.14
		Scenario No.3 Down (30%)	0.105
	2nd Segment Normal (50%)	Scenario No.4 Up (30%)	0.15
		Scenario No.5 Normal (40%)	0.2
		Scenario No.6 Down (30%)	0.15
	3rd Segment Down (15%)	Scenario No.7 Up (30%)	0.045
		Scenario No.8 Normal (40%)	0.06
		Scenario No.9 Down (30%)	0.045

By applying every scenario individually, the following results are obtained:

Table 5.3 results comparison.

Component (10^3 tons)	Deterministic (10^3 tons/month)	Segment (10^3 tons/month)								
		1			2			3		
		Scenario			Scenario			Scenario		
		1	2	3	4	5	6	7	8	9
DC1	221.778	298.623			221.778			105.496		
DC2	260	260			260			260		
LN	19.27112	22.34492			19.27112			14.61984		
LPG	18.397	20.825			18.397			14.618		
PG98	19.999	23.999			19.999			16.001		
ES95	80.001	87.336			80.001			64		
JF	70	84			70			48.454		
GO	137.428	166.845			137.428			93.207		
HFO	148.079	160.375			148.079			129.473		
RF	24.923	26.528			24.923			22.509		
Imports	40.778	64.686			40.778			55.439		
Inputs	522.556	623.308			522.556			420.935		
Outputs	495	594			495			396		
Inventory	27.556	29.308			27.556			24.935		
Profit (US$ million \month)	38.861	106.138	47.304	-11.53	87.89	38.861	-10.167	68.018	28.795	-10.427

Table 5.3 can be analyzed as follows.

5.1.1 Profit Analysis

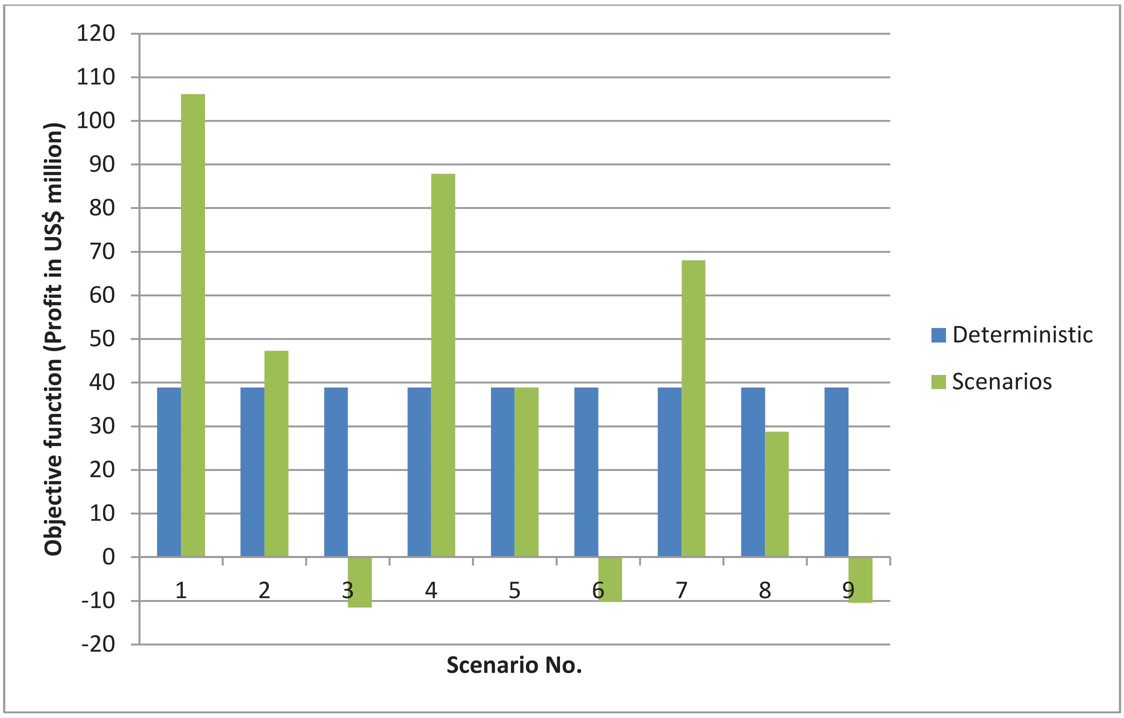

Fig. 5.1 Objective function (Profit) analysis.

Fig. 5.1 shows that the deterministic profit is constant at (US$ 38.861 million), the profit of all scenarios vary from the highest value to the lowest one according to final products price. The gap between the highest value and the lowest one is getting smaller within the same segment according to difference in the market demand.

As shown in table 5.3 and fig. 5.1, there are three negative points of profit which means that the refinery has to pay extra money to keep the demand which is not acceptable. This problem is explained at the sensitivity part.

5.1.2 Crude Oil Analysis

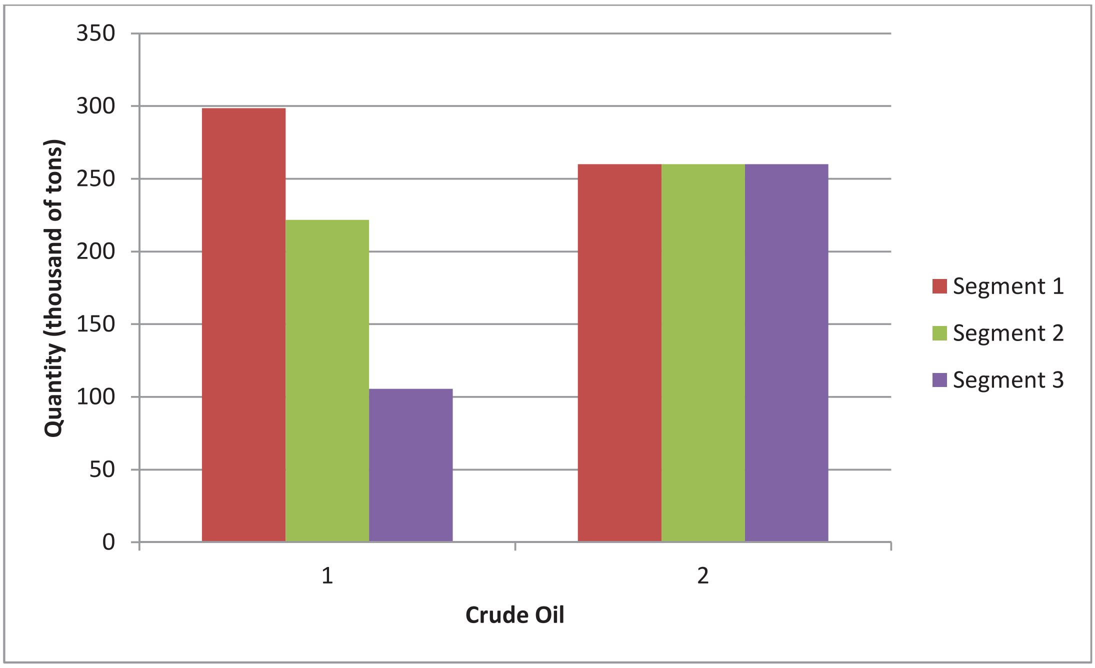

Fig. 5.2. Crude Oil Analysis.

As shown above in fig. 5.2, the consumption of crude oil 2 is constant not because of its low cost compared to crude oil 1 but because of constraint that we must use at least 260 thousand of tons of DC2 and the high quality of DC1.

The consumption of DC1, which gives high quality final products, is the main difference between segments. The highest consumption of DC1 (298.623×10^3 tons) is in the first segment and the lowest one (105.496×10^3 tons) is in the third segment according to the high and low market demand respectively. This means that DC1 is the main controller of raw materials fed to the refinery.

5.1.3 Imports Analysis

Table 5.4 Detailed Imported feedstocks.

Imported Feedstocks	Segment		
	1	2	3
HN	5.2	8.713	13.099
VG	0	0	0
ES95	8.665	0	0
JF	0	0	7.546
GO	25.155	22.572	34.794
HFO	25.666	9.493	0
Total (thousands of tons)	64.686	40.778	55.439

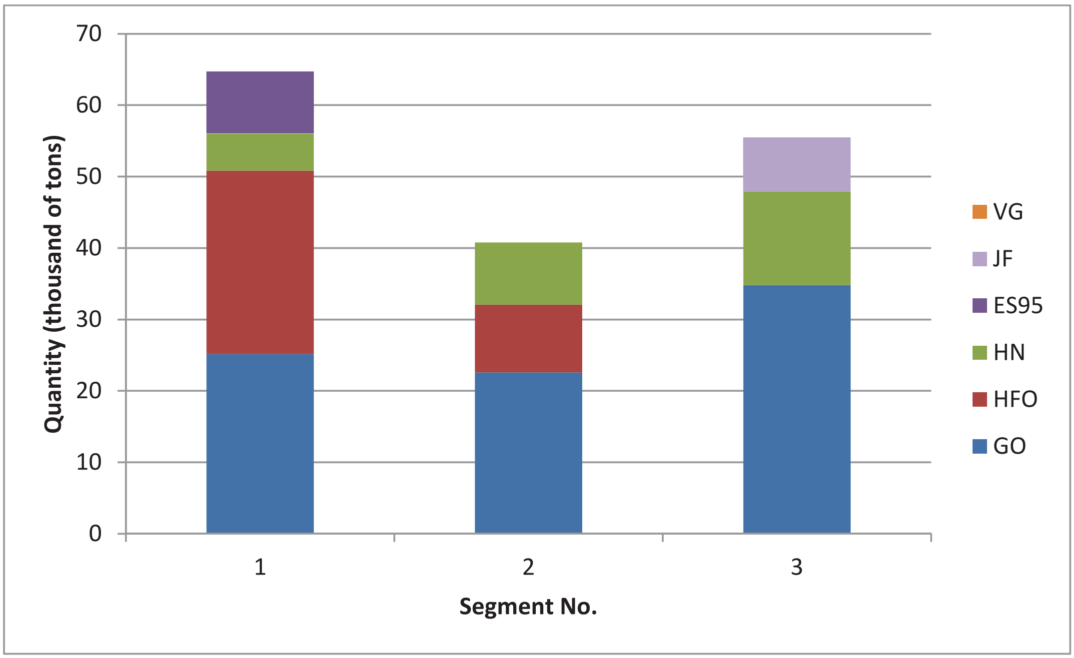

Fig. 5.3. Imports analysis.

GO and HFO are the highest two imported feedstocks among the three segments because of the low cost of them and demand constraint. They both represent 79%, 79%, and 63% of the total imports of segments 1, 2, and 3 respectively. HN is imported in the three segments. However the largest amount is imported in the last segment to be fed to the reformer to produce and meet the demand of gasoline; PG98 and ES95. There are small quantities of both ES95 and JF in the first and the last segment respectively, but there is no VG imported.

Generally, imports are huge in the first and the last segment compared to the second one and that might be because of the huge fluctuation in the market demand.

5.1.4 Inventory Analysis

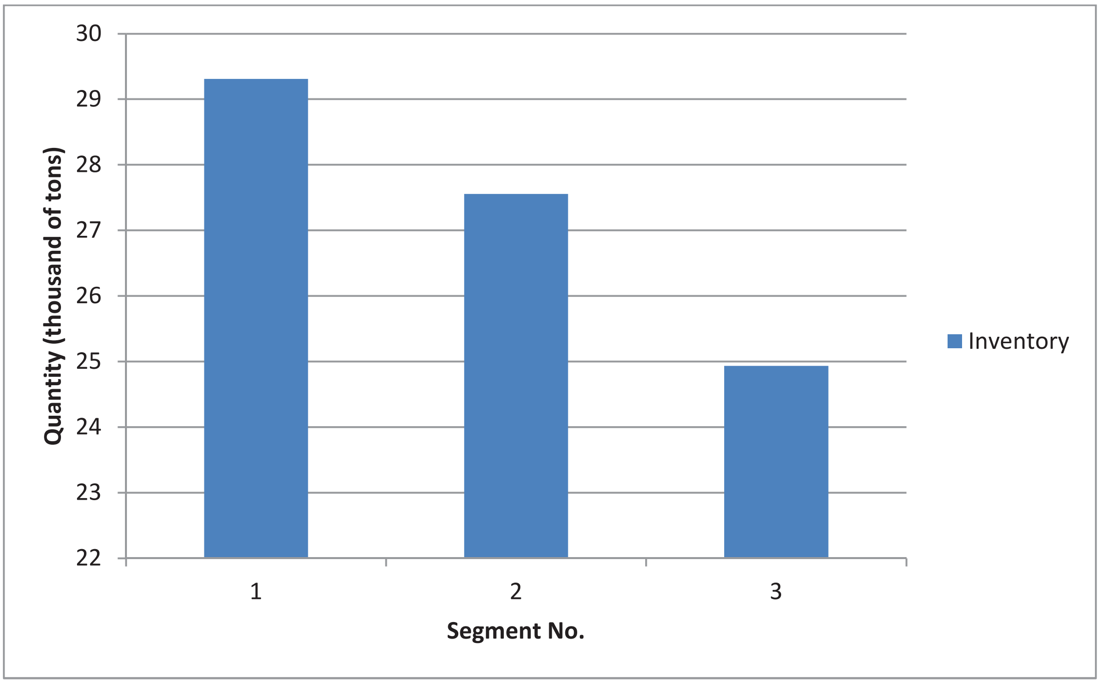

Fig. 5.4 Inventory analysis.

Because of the high demand of final products and so the high consumption of crude oil and imported feedstocks, inventory will be huge axiomatically. The effect of demand and crude oils is clearer than the effect of imports.

5.1.5 Product Composition Analysis

Table 5.5 LPG composition analysis.

		Segment		
LPG	Component	1	2	3
	From DC1	3.584	2.661	1.266
	From DC2	3.9	3.9	3.9
	R95	0.18	0.18	0.18
	R100	6.96	6.204	4.954
	FCCNA	0	0	0
	FCCGO	6.201	5.452	4.318
TOTAL		20.825	18.397	14.618

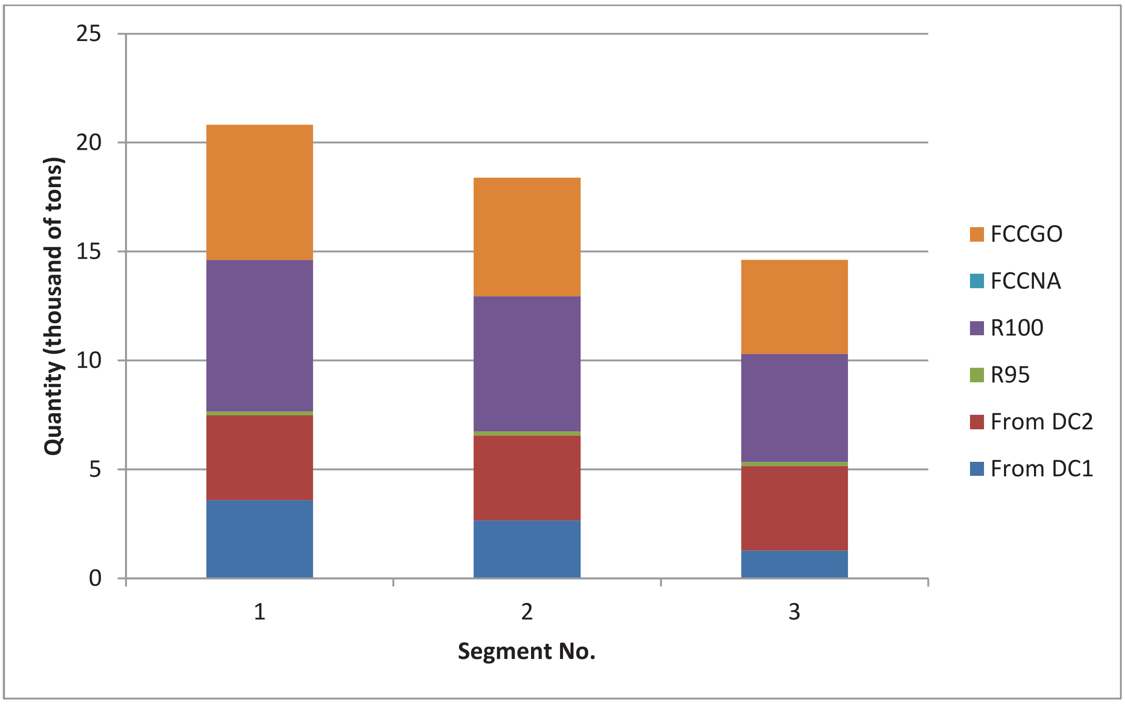

Fig. 5.5 LPG composition analysis.

As shown in table 5.5 and fig. 5.5, R100 is the biggest component of LPG which consists mainly from the produced LPG from DC1, DC2 and FCCGO. There is low amount of R95 (1800 tons) contained. The demand effect is very clear.

Table 5.6 PG98 composition analysis.

		Segment		
PG98	Component	1	2	3
	C4	0.895	0.746	0.597
	LN	0	0	0
	ISO	0	0	0
	R95	1.66	1.66	1.66
	R100	16.975	14.062	11.15
	CCG	4.469	3.531	2.594
TOTAL		23.999	19.999	16.001

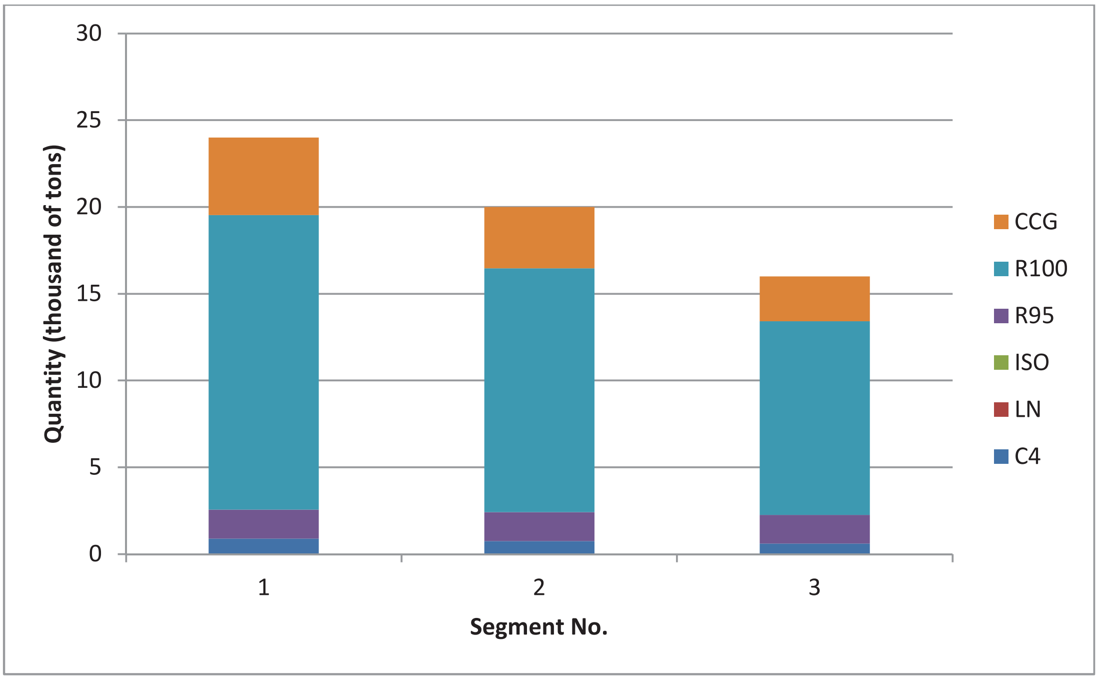

Fig. 5.6 PG98 composition analysis.

R100 is the biggest component of PG98 as shown in both table 5.6 and fig. 5.6. PG98 also consists of medium amounts of R95 and CCG. A low amount of C4 is contained also. PG98 in the first and the second segment meets the market demand, but in the last one no and that is why HV is imported widely in the last segment.

Table 5.7 ES95 composition analysis

		Segment		
	Component	1	2	3
ES95	C4	3.386	3.106	2.485
	LN	0	0	0
	ISO	8.212	8.487	6.884
	R95	0	0	0
	R100	28.845	26.784	21.462
	CCG	46.893	41.624	33.169
TOTAL		87.336	80.001	64

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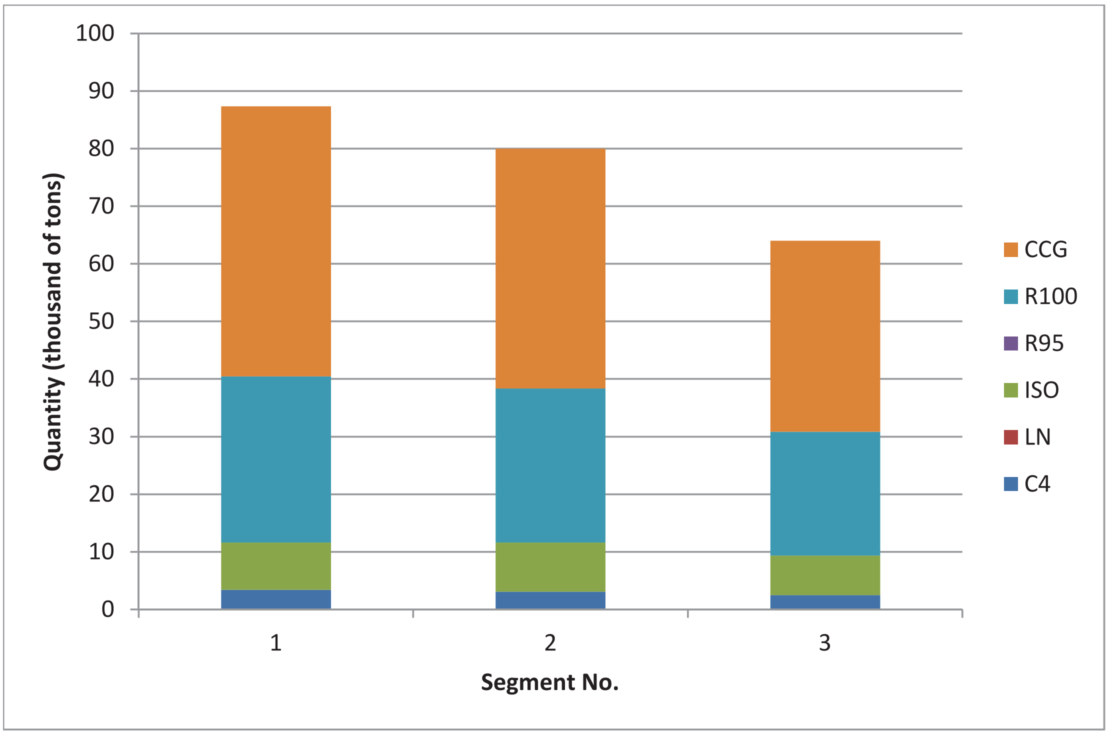

Fig. 5.7 ES95 composition analysis

As shown above, R100 and CCG are the biggest components of ES95. A medium amount of ISO and low amount of C4 are existence. There is no LN or R95. In the first and the second segment ES95 meets the demand, but in the last an extra amount of ES95 is imported.

Table 5.8 JF composition analysis

		Segment		
	Component	1	2	3
JF	JF1	80	66.667	46.146
	JF2	0	0	0
	LN	4	3.333	2.308
TOTAL		84	70	48.454

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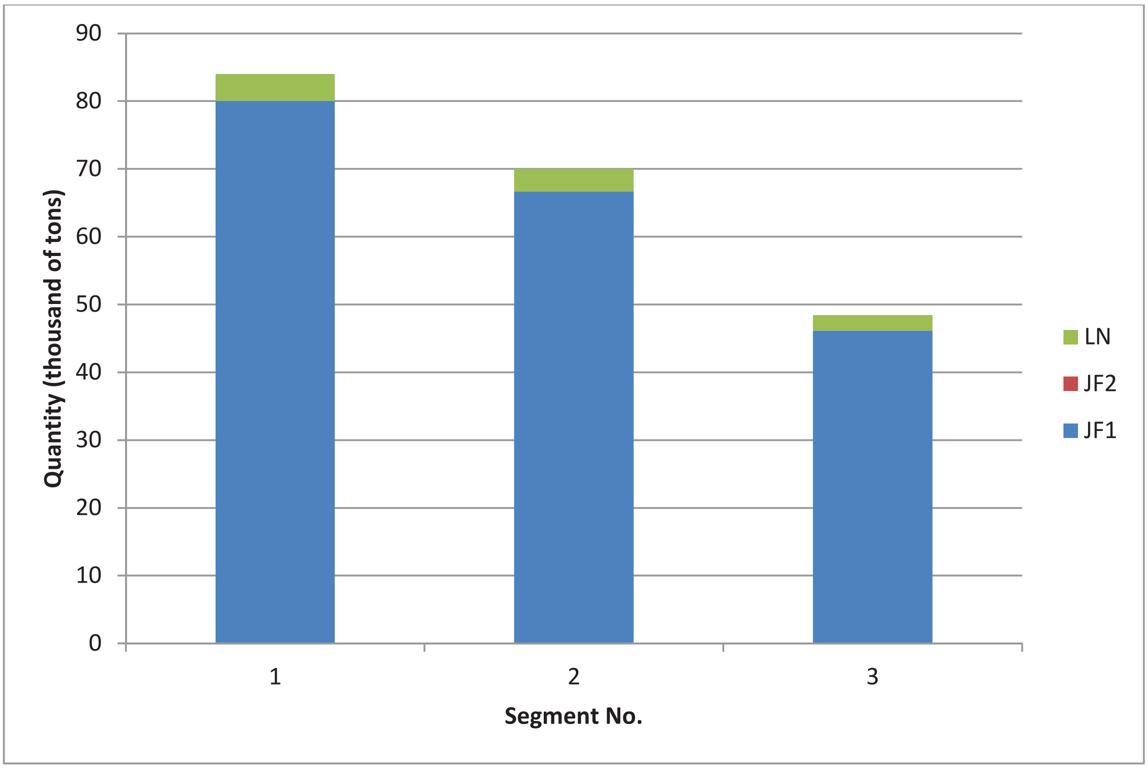

Fig. 5.8 JF composition analysis.

JF is mainly consists of JF1, which is the result of jet fuel formulated using the first formula, and small amount of LN. JF1 represents more than 95%. On the other hand, JF2 does not exist.

Table 5.9 GO composition analysis

		Segment		
	Component	1	2	3
GO	KE	0.193	0	0
	NGO1	20.686	15.716	8.12
	NGO2	0	0	0
	NCGO	0	0	0
	DGO1	70.449	51.974	24.092
	DGO2	51.197	51.197	51.197
	DCGO	24.32	18.541	9.798
TOTAL		166.845	137.428	93.207

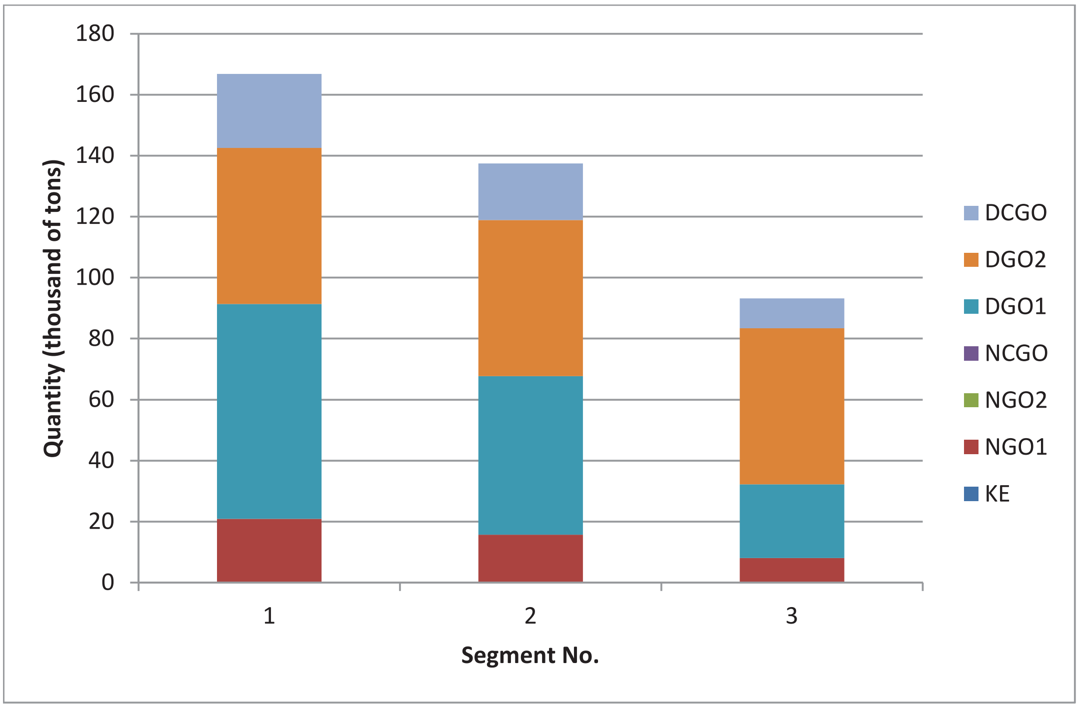

Fig. 5.9 GO composition analysis.

From table 5.9 and fig. 5.9, we can see that NGO2 and NCGO are totally excluded from the GO component in all segments. KE is included in the first segment to complete the GO quantity needed and totally excluded from the second and the third one. However, the quantity of DGO2 is constant because of the constant quantity of DC2 used.

Table 5.10 HFO composition analysis.

		Segment		
	Component	1	2	3
HFO	NCGO	43.554	41.248	37.759
	VR1	38.821	28.831	13.714
	VR2	78	78	78
TOTAL		160.375	148.079	129.473

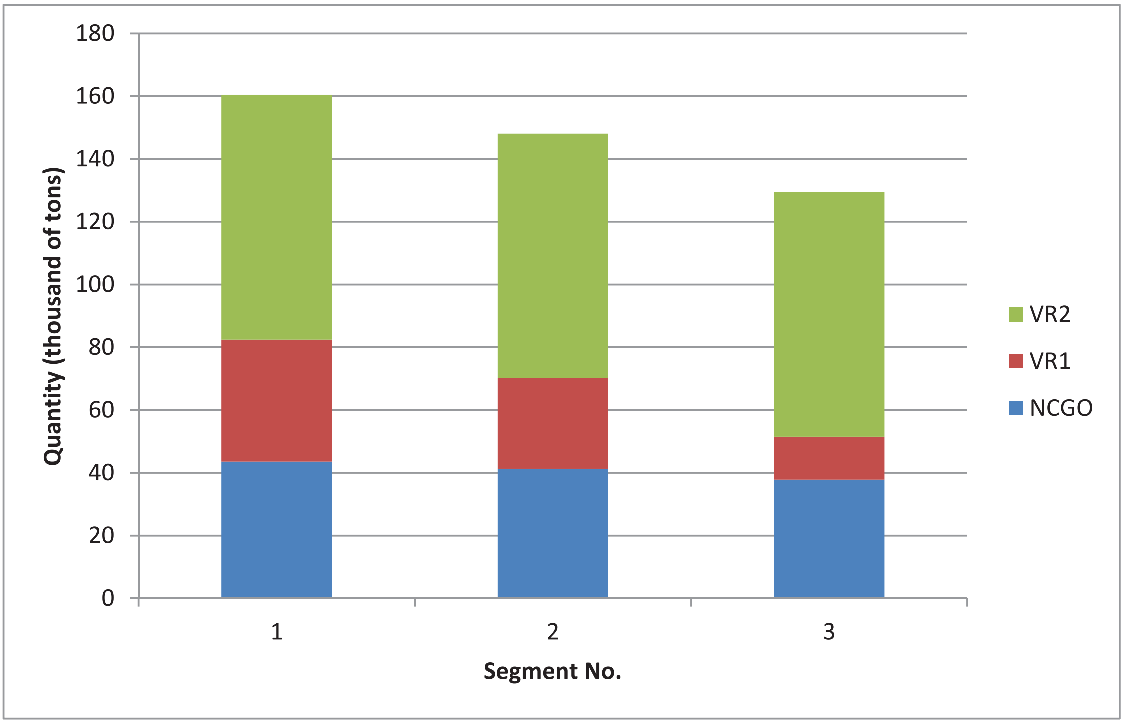

Fig. 5.10 HFO composition analysis.

Mainly, HFO consists of varying quantities of VR1 and NCGO according to the HFO daily demand and the consumption of the used quantity of the used raw material. It also consists of constant quantity of VR2 according to the constant quantity of DC2 used.

Table 5.11 RF composition analysis.

		Segment		
	Component	1	2	3
RF	RG	12.065	10.616	8.283
	LPG	3.343	3.546	2.736
	LN	2.679	1.188	0.416
	HFO	8.441	9.573	11.074
TOTAL		26.528	24.923	22.509

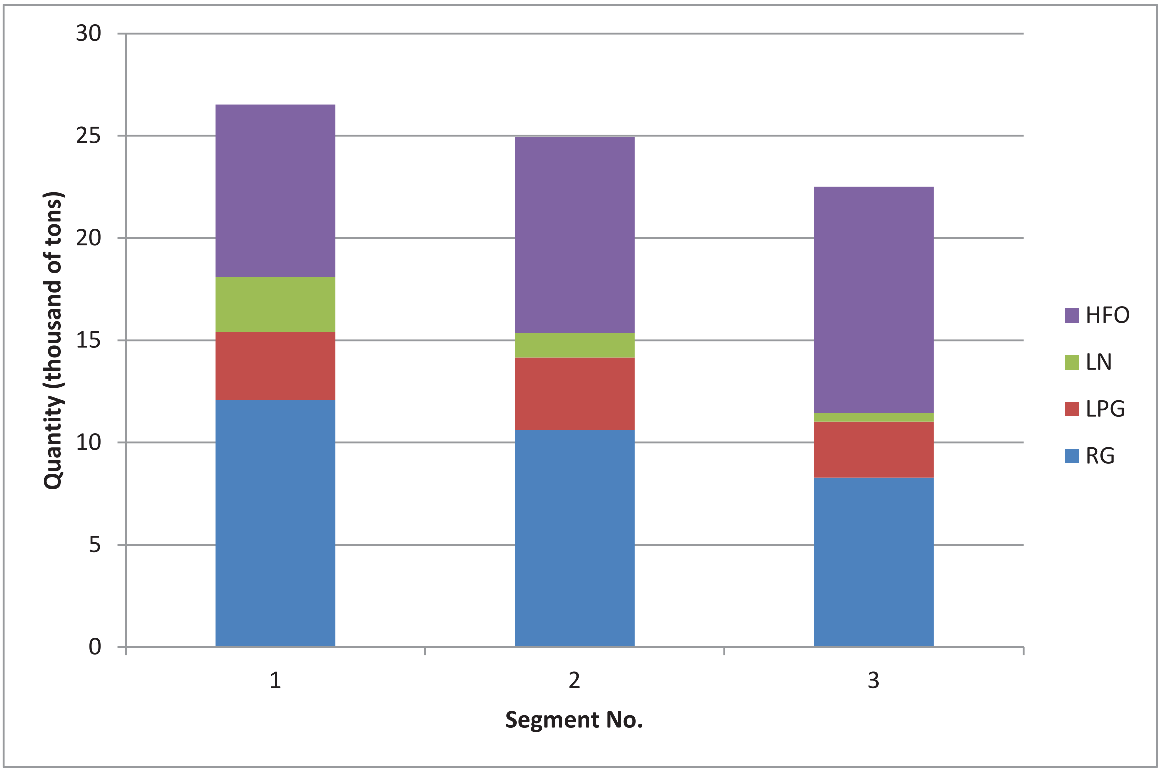

Fig. 5.11 RF composition analysis.

RF consists of RG, LPG, HFO, and small amount of LN. Amounts of all components decrease except HFO which increases when the demand decreases. LPG increases in the second segment and then decreases sharply in the third one.

5.2 Sensitivity Results

As it is previously shown in table 5.3, and for the given assumed probabilities and scenarios shown in table 4.1, the total profit of the case study is decreasing several millions US dollars per month. The profit values with the negative sign mean that the refinery has to pay extra money, which means loss, in order to balance the market and that is unacceptable for the oil industry.

This loss is because of the fluctuation in products demand and price or in the assumed probabilities and scenarios. Since the probabilities and scenarios are assumed to be correct, the main reason behind this is the assumed fluctuation percentages for demand, price, or both. So to examine that and see the effect of demand, price must be constant and vice versa and so the following results are obtained:

Table 5.12 effect of demand fluctuation.

Scenarios	Profit (US $ Millions/month)			
	5%	15%	20%	25%
1	92.779	102.004	106.138	110.049
2	41.3	45.621	47.304	48.764
3	-10.18	-10.761	-11.53	-12.522
4	87.89	87.89	87.89	87.89
5	38.861	38.861	38.861	38.861
6	-10.167	-10.167	-10.167	-10.167
7	83	73.054	68.018	62.705
8	36.423	31.38	28.795	25.934
9	-10.154	-10.294	-10.427	-10.837

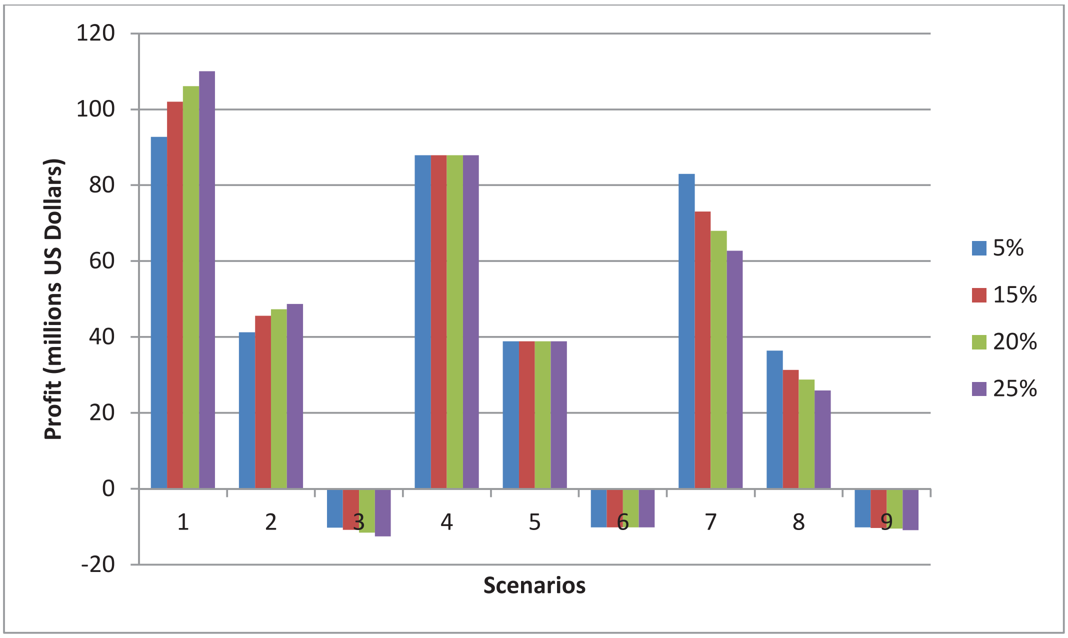

Fig. 5.12 effect of demand fluctuation.

As shown in table 5.12 and Fig. 5.12, the demand fluctuation has no effect on the objective function.

To examine the effect of price, demand should be constant and results can be shown in the following table:

Table 5.13 Effect of price fluctuation.

Scenarios	Profit (US $ Millions/month)		
	10%	15%	20%
1	86.527	106.138	125.749
2	47.307	47.304	47.304
3	8.082	-11.53	-31.141
4	71.547	87.89	104.232
5	38.861	38.861	38.861
6	6.176	-10.167	-26.51
7	54.944	68.018	81.092
8	28.795	28.795	28.795
9	2.647	-10.427	-23.501

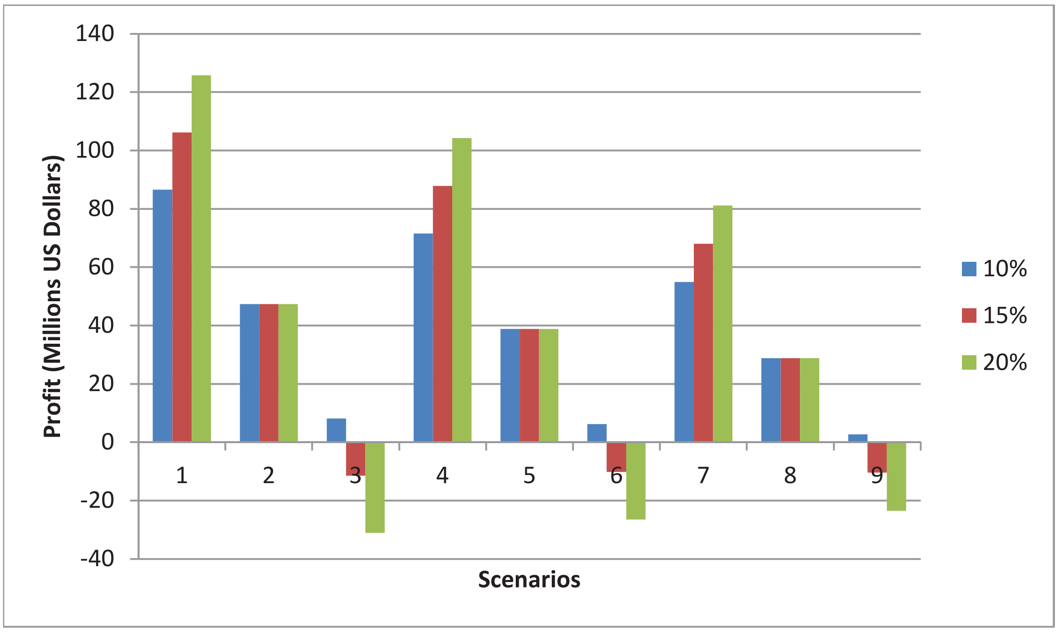

Fig. 5.13 effect of price fluctuation.

As shown in table 5.13 and fig. 5.13, the fluctuation in the product price ±15% and ±20% still affect the total price as the negative sign appears. However, there is no negative sign in the ±10% fluctuation.

Therefore, price fluctuation should not exceed ±10%.

CHAPTER 6

CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

A general stochastic model for refinery planning under uncertainty was formulated and applied into a real case study. Two approaches were implemented:

- Deterministic – Stochastic approach
- Deterministic – Segments approach

First, both the deterministic and the stochastic models were applied on the case study and the results showed that 481.778×10^3 ton/m of both crude oil 1 and crude oil 2 and a total imports of 40.778×10^3 ton/m were used to produce total production of 495×10^3 ton/m of final products and an inventory of 27.556×10^3 ton/m. On the other side, a comparison between the deterministic results and the stochastic one showed that the stochastic model has increased the total profit to 51.936 million US dollars instead of 38.861 million US dollars. The extra profit might be a little high because of the neglected extra costs such as fixed cost, maintenance, labor, and safety cost. This means that the stochastic model can be used not only to enhance the profit of the refinery under tough fluctuation in demand and price, but also to economize the use of raw material and imported feedstocks.

For the deterministic – segments approach; the stochastic results were divided into three segments according to the demand fluctuation. For every scenario, it was treated as a deterministic one to find the total profit if that scenario was the controlling one. Results showed that for every segment the used raw material, imported feedstock, total production, inventory, and the composition of the resulted products were all the same and that is because of the fixed demand for every segment. The objective function was going up and down according to the fluctuation in price.

Some results with a negative sign were not acceptable, so a sensitivity determination analysis was done to suggest the maximum allowed price and demand fluctuation. The sensitivity analysis showed that the demand has no effect on the negative sign and it can be up to ±25%.

By implementing the sensitivity determination analysis on product price, the analysis showed that price should not exceed ±10%.

6.2 Recommendation

According to the results above, the following recommendations can be made to extend this research:

- Increasing the uncertain factors instead of two.
- Study the same case study under smoother circumstances.
- Study the same case study under equal fluctuation.
- Study the same case study including air emission.

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APPENDIX

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*STOCHASTIC MODEL

*This is a model for a complex refinery comprised of:

*1) crude unit, 2) vacuum unit, 3)reformer unit, 4)catalytic cracker unit,

*5)isomerization unit, 6)desulfurization unit and 7)refinery fuel and blending units.

*Products are:

* LPG  =1

* LN   =2

* PG98 =3

* ES95 =4

* JF   =5

* GO   =6

* HFO  =7

*-----------------------------------------------------------------------------------

Sets

i "product type" /1*7/

s "scenarios" /1*9/

Table pc(i,s) "price of product type i per realization s"

          1         2         3        4         5         6         7         8         9

1     753.25      655         556.75     753.25       655          556.75       753.25        655

556.75

2       1035       900        765      1035       900        765       1035       900        765
```

```
3       1144.25     995         845.75     1144.25     995         845.75      1144.25     995
845.75
4      1012       880        748       1012       880        748       1012       880        748
5      1247.75     1085        922.25     1247.75     1085        922.25      1247.75     1085
922.25
6       1063.75     925         786.25     1063.75     925         786.25      1063.75     925
786.25
7        632.5       550         467.5      632.5       550         467.5       632.5       550
467.5;

Table d(i,s) demand of product type i per realization s
     1      2      3      4      5      6      7      8      9
1    13.2    13.2    13.2    11     11     11     8.8     8.8     8.8
2    7.2     7.2     7.2     6      6      6      4.8     4.8     4.8
3    24      24      24      20     20     20     16      16      16
4    96      96      96      80     80     80     64      64      64
5    84      84      84      70     70     70     56      56      56
6    192     192     192     160    160    160    128     128     128
7    177.6   177.6   177.6   148    148    148    118.4   118.4   118.4;

parameters
p(s)  probability of the realization of scenario  /1 0.105, 2 0.14, 3 0.105, 4 0.15, 5 0.2, 6
0.15, 7 0.045, 8 0.06, 9 0.045/
```

* desnsities

DNSTYLG	Light gas density	/0.54/
DNSTYC4	C4 density	/0.58/
DNSTYLN	Light naphtha density	/0.65/
DNSTYHN	Heavy naphtha density	/0.74/
DNSTYKE	Kerosene density	/0.77/
DNSTYGO1	Gasoil of crude 1 density	/0.83/
DNSTYGO2	Gasoil of crude 2 density	/0.86/
DNSTYVGO	Vacuum gasoil density	/0.92/
DNSTYVR1	Vacuum residue of crude 1 density	/0.98/
DNSTYVR2	Vacuum residue of crude 2 density	/1.02/
DNSTYR95	Gasoline R95 density	/0.77/
DNSTYR100	Gasoline R100 density	/0.80/
DNSTYISO	Isomerate density	/0.665/
DNSTYCrN	Cracked naphtha density	/0.75/
DNSTYCGO	Cycle gasoil density	/0.95/

*Ried Vapor Pressures

RVPC4	C4 RVP	/4.3/
RVPLN	Light naphtha RVP	/0.8/
RVPR95	R95 RVP	/0.5/
RVPR100	R100 RVP	/0.5/

```
          RVPISO          Isomerate RVP                 /0.4/

          RVPCrN          Cracked naphtha RVP           /0.65/


*RESEARCH OCTANE NUMBER

          RONC4           C4 RON                        /94/

          RONLN           Light naphtha RON             /71/

          RONR95          R95 RON                       /95/

          RONR100         R100 RON                      /100/

          RONISO          Isomerate RON                 /91/

          RONCrN          Cracked naphtha RON           /93/


*MOTOR OCTANE NUMBER

          MONC4           C4 MON                        /90/

          MONLN           Light naphtha MON             /68/

          MONR95          R95 MON                       /86/

          MONR100         R100 MON                      /91/

          MONISO          Isomerate MON                 /86/

          MONCrN          Cracked naphtha MON           /82/


*SULFUR CONTENT BEFORE HDS

          SCBKE           Kerosene sulfur before HDS            /0.1/

          SCBGO1          Gasoil from crude 1 before HDS        /0.2/

          SCBGO2          Gasoil from crude 2 before HDS        /1.5/
```

```
          SCBCGO       Cycle gasoil before HDS            /2/


*SULFUR CONTENT AFTER HDS

          SCAGO1       Gasoil from crude 1 after HDS      /0.006/

          SCAGO2       Gasoil from crude 2 after HDS      /0.045/

          SCACGO       Cycle gasoil after HDS             /0.06/


*VISCOSITY BLENDING INDEX

          VBIVR1       Vacuum residue from crude 1 viscosity            /38/

          VBIVR2       Vacuum residue from crude 2 viscosity            /43/

          VBICGO       Cycle gasoil viscosity                           /12/;


Positive variables

          DC1          Quantity of crude 1 distilled

          DC2          Quantity of crude 2 distilled

          QTYR95       Quantity of heavy naphtha fed to catalytic reformer (95 severity)

          QTYR100      Quantity of heavy naphtha fed to catalytic reformer (100 severity)

          QTYFCCNA     Quantity of vacuum distillate fed to cracker (gasoline maximized)

          QTYFCCGO     Quantity of vacuum distillate fed to cracker (gas oil maximized)

          QTYISO       Quantity of light naphtha fed to the isomerization unit

          QTYDESGO1    Quantity of gas oil 1 fed to the desulfurization uit

          QTYDESGO2    Quantity of gas oil 2 fed to the desulfurization unit

          QTYDESGO     Quantity of LCO fed to the desulfurization unit
```

QTYJFF1	Quantity of jet fuel manufactured according to formulation 1
QTYJFF2	Quantity of jet fuel manufactured according to formulation 2
QTYC4S98	Quantity of butane as a component of Super 98
QTYLNS98	Quantity of light naphtha as a component of Super 98
QTYISOS98	Quantity of isomerate as a component of Super 98
QTYR95S98	Quantity of 95 reformate as a component of Super 98
QTYR100S98	Quantity of 100 reformate as a component of Super 98
QTYCCGS98	Quantity of cat cracker gasoline as a component of Super 98
QTYVS98	Volume of Super 98 manufactured
QTYC4S95	Quantity of butane as a component of Eurosuper 95
QTYLNS95	Quantity of light naphtha as a component of Eurosuper 95
QTYISOS95	Quantity of isomerate as a component of Eurosuper 95
QTYR95S95	Quantity of 95 reformate as a component of Eurosuper 95
QTYR100S95	Quantity of 100 reformate as a component of Eurosuper 95
QTYCCGS95	Quantity of cat cracked gasoline a component of Eurosuper 95
QTYVS95	Volume of Eurosuper 95 manufactured
QTYKEGO	Quantity of kerosene as a gas oil component
QTYNGO1GO	Qantity of non-desulphurised gas oil 1 as a gas oil component
QTYNGO2GO	Quantity of non-desulphurised gas oil 2 as a gas oil component
QTYNCGOGO	Quantity of non-desulphurised CGO as a gas oil component
QTYDGO1GO	Quantity of desulphurized gas oil 1 as a gas oil component
QTYDGO2GO	Quantity of desulpherized gas oil 2 as a gas oil component
QTYDCGOGO	Quantity of desulphurized CGO as a gas oil component

QTYGO	Weight of gas oil manufactured
QTYNCGOHFO	Quantity of non-desulphurised CGO as a HFO component
QTYVR1HFO	Quantity of vacuum residue from crude oil 1 as a HFO component
QTYVR2HFO	Quantity of vacuum residue from crude oil 2 as a HFO component
QTYVHGO	Volume of HGO manufactured
QTYRGRF	Quantity of refinery gas used a refinery fuel
QTYLPGRF	Quantity of LPG used as refinery fuel
QTYLNRF	Quantity of light naphtha used as a refinery fuel
QTYHFORF	Quantity of HFO used a refinery fuel
QTYIMHN	Imports of catalytic reformer feedstock (heavy naphtha)
QTYIMVG	Imports of cracker feedstock (vacuum distillate)
QTYIMES95	Imports of Eurosuper 95
QTYEXES95	Exports of Eurosuper 95
QTYIMJF	Imports of jet fuel
QTYEXJF	Exports of jet fuel
QTYIMGO	Imports of gas oil
QTYEXGO	Exports of gas oil
QTYIMHFO	Imports of HFO
QTYEXHFO	Exports of HFO
RG	Quantity of Refinery Gas produced

```
          LPG             Quantity of LPG produced

          LN              Quantity of Light Naphtha produced

          PG98            Quantity of Super 98 produced

          ES95            Quantity of Euro Super 95 produced

          JF              Quantity of Jet Fuel produced

          GO              Quantity of Gas Oil produced

          HFO             Quantity of Heavy Fuel Oil produced

          MONPG98         Motor Octane Number of PG98

          RONPG98         Research Octane Number of PG98

          MONES95         Motor Octane Number of ES95

          RONES95         Research Octane Number of ES95

          INPUTS          Total Input Quantities

          OUTPUTS         Total Output Quantities;


free variable z;


equations

          R1BARG

          R1BALG

          R1BALN

          R1BAHN

          R1BAKE

          R1BAGO1
```

R1BAGO2

R1BAVGO

R1BAVR1

R1BAVR2

R1BAR95

R1BAR100

R1BAISO

R1BACN

R1BACGO

R1BADG1

R1BADG2

R1BADCG

R1DPG98

R1DES95

R1DJP

R1DGO

R1DHF

R1VOPG98

R1LIC4PG98

R1VPMXPG98

R1VPMNPG98

R1ORMNPG98

R1SENSPG98

R1VOES95

R1LIC4ES95

R1VPMXES95

R1VPMNES95

R1ORMNES95

R1SENSES95

R1WTGO

R1SUMXGO

R1VOHF

R1VMXHF

R1VMNHF

R1BARF

R1C1MAX

R1C2MIN

R1CAPADMX

R1REFMAX

R1RF95MN

R1FCCMX

R1CAPDSMX

QutyRG

QutyLPG

QutyLN

QutyPG98

```
          QutyES95

          QutyJF

          QutyGO

          QutyHFO

          R1MONPG98

          R1RONPG98

          R1MONES95

          R1RONES95

          QTYIN

          QTYOUT

          COST;


*Balance of refinery gas

R1BARG..                    0.001*DC1+0.002*DC2+0.06*QTYR95+0.09*QTYR100   +

0.015*QTYFCCNA+0.012*QTYFCCGO+0.03*QTYISO+0.02*QTYDESGO1+0.03*Q

TYDESGO2+0.04*QTYDESGO-QTYRGRF=e=0;


*Balance of liquefied gas

R1BALG..

0.012*DC1+0.015*DC2+0.09*QTYR95+0.12*QTYR100+0.053*QTYFCCNA+0.046*

QTYFCCGO-QTYC4S98-QTYC4S95-QTYLPGRF=e=d('1','4');


*Balance of light naphtha
```

```
R1BALN..        0.04*DC1+0.04*DC2-QTYISO-0.05*QTYJFF1-0.035*QTYJFF2-
QTYLNS98-QTYLNS95-QTYLNRF=e=d('2','4');

*Balance of heavy naphtha
R1BAHN..        0.145*DC1+0.075*DC2-QTYR95-QTYR100-0.1*QTYJFF1-
0.075*QTYJFF2+QTYIMHN=e=0;

*Balance of kerosene
R1BAKE..        0.15*DC1+0.09*DC2-0.85*QTYJFF1-0.899*QTYJFF2-
QTYKEGO=e=0;

*Balance of gas oil crude 1
R1BAGO1..       0.31*DC1-QTYDESGO1-QTYNGO1GO=e=0;

*Balance of gas oil crude 2
R1BAGO2..       0.203*DC2-QTYDESGO2-QTYNGO2GO=e=0;

*Balance of vacuum distillate
R1BAVGO..       0.212*DC1+0.275*DC2-QTYFCCNA-
QTYFCCGO+QTYIMVG=e=0;

*Balance of vacuum residue crude 1
R1BAVR1..       0.13*DC1-QTYVR1HFO=e=0;
```

```
*Balance of vacuum residue crude 2

R1BAVR2..       0.3*DC2-QTYVR2HFO=e=0;


*Balance of reformate 95

R1BAR95..       0.83*QTYR95-QTYR95S98-QTYR95S95=e=0;


*Balance of reformate 100

R1BAR100..       0.79*QTYR100-QTYR100S98-QTYR100S95=e=0;


*Balance of isomerate

R1BAISO..       0.97*QTYISO-QTYISOS98-QTYISOS95=e=0;


*Balance of cat cracked gasoline

R1BACN..                         0.436*QTYFCCNA+0.381*QTYFCCGO-QTYCCGS98-
QTYCCGS95=e=0;


*Balance of CGO

R1BACGO..                         0.446*QTYFCCNA+0.511*QTYFCCGO-QTYDESGO-
QTYNCGOGO-QTYNCGOHFO=e=0;


*Balance of desulfurization gas oil crude 1

R1BADG1..       0.98*QTYDESGO1-QTYDGO1GO=e=0;
```

```
*Balance of desulfurization gas oil crude 2
R1BADG2..       0.97*QTYDESGO2-QTYDGO2GO=e=0;

*Balance of desulfurization CGO
R1BADCG..       0.96*QTYDESGO-QTYDCGOGO=e=0;

*Demand for Super 98
R1DPG98..
QTYC4S98+QTYLNS98+QTYISOS98+QTYR95S98+QTYR100S98+QTYCCGS98=e
=d('3','4');

*Demand for Eurosuper 95
R1DES95..
QTYC4S95+QTYLNS95+QTYISOS95+QTYR95S95+QTYR100S95+QTYCCGS95+Q
TYIMES95-QTYEXES95=e=d('4','4');

*Demand for jet fuel
R1DJP..                 QTYJFF1+QTYJFF2+0.05*QTYJFF1+0.035*QTYJFF2+QTYIMJF-
QTYEXJF=e=d('5','4');

*Demand for gas oil
```

```
R1DGO..
QTYKEGO+QTYNGO1GO+QTYNGO2GO+QTYNCGOGO+QTYDGO1GO+QTYDG
O2GO+QTYDCGOGO+QTYIMGO-QTYEXGO=e=d('6','4');

*Demand for HFO
R1DHF..                              QTYNCGOHFO+QTYVR1HFO+QTYVR2HFO-
QTYHFORF+QTYIMHFO-QTYEXHFO=e=d('7','4');

*Calculation of volume of Super 98 manufactured
R1VOPG98..
(1/DNSTYC4)*QTYC4S98+(1/DNSTYLN)*QTYLNS98+(1/DNSTYISO)*QTYISOS9
8+(1/DNSTYR95)*QTYR95S98
          +(1/DNSTYR100)*QTYR100S98+(1/DNSTYCrN)*QTYCCGS98-
QTYVS98=e=0;

*Maximum (5% volume) butane content of Super 98
R1LIC4PG98..     (1/DNSTYC4)*QTYC4S98-0.05*QTYVS98=l=0;

*Maximum vapor pressure of unleaded Super 98
R1VPMXPG98..
(RVPC4/DNSTYC4)*QTYC4S98+(RVPLN/DNSTYLN)*QTYLNS98+(RVPISO/DNS
TYISO)*QTYISOS98+
```

```
(RVPR95/DNSTYR95)*QTYR95S98+(RVPR100/DNSTYR100)*QTYR100S98+(RVP
CrN/DNSTYCrN)*QTYCCGS98-0.86*QTYVS98=l=0;

*Minimum vapor pressure of unleaded Super 98
R1VPMNPG98..
(RVPC4/DNSTYC4)*QTYC4S98+(RVPLN/DNSTYLN)*QTYLNS98+(RVPISO/DNS
TYISO)*QTYISOS98+

(RVPR95/DNSTYR95)*QTYR95S98+(RVPR100/DNSTYR100)*QTYR100S98+(RVP
CrN/DNSTYCrN)*QTYCCGS98-0.5*QTYVS98=g=0;

*Minimum RON of unleaded Super 98
R1ORMNPG98..
(RONC4/DNSTYC4)*QTYC4S98+(RONLN/DNSTYLN)*QTYLNS98+(RONISO/DNS
TYISO)*QTYISOS98+

(RONR95/DNSTYR95)*QTYR95S98+(RONR100/DNSTYR100)*QTYR100S98+(RO
NCrN/DNSTYCrN)*QTYCCGS98-98*QTYVS98=g=0;

*Maximum sensitivity of unleaded Super 98
R1SENSPG98..                  ((RONC4-MONC4)/DNSTYC4)*QTYC4S98+((RONLN-
MONLN)/DNSTYLN)*QTYLNS98+
```

```
                ((RONISO-MONISO)/DNSTYISO)*QTYISOS98+((RONR95-
MONR95)/DNSTYR95)*QTYR95S98+
                ((RONR100-MONR100)/DNSTYR100)*QTYR100S98+((RONCrN-
MONCrN)/DNSTYCrN)*QTYCCGS98-10*QTYVS98=l=0;

*Calculation of volume of Eurosuper 95 manufactured
R1VOES95..
(1/DNSTYC4)*QTYC4S95+(1/DNSTYLN)*QTYLNS95+(1/DNSTYISO)*QTYISOS9
5+(1/DNSTYR95)*QTYR95S95+
               (1/DNSTYR100)*QTYR100S95+(1/DNSTYCrN)*QTYCCGS95-
QTYVS95=e=0;

*Maximum (5% volume) butane content of Eurosuper 95
R1LIC4ES95..       (1/DNSTYC4)*QTYC4S95-0.05*QTYVS95=l=0;

*Maximum vapor pressure of unleaded Eurosuper 95
R1VPMXES95..
(RVPC4/DNSTYC4)*QTYC4S95+(RVPLN/DNSTYLN)*QTYLNS95+(RVPISO/DNS
TYISO)*QTYISOS95+

(RVPR95/DNSTYR95)*QTYR95S95+(RVPR100/DNSTYR100)*QTYR100S95+(RVP
CrN/DNSTYCrN)*QTYCCGS95-0.8*QTYVS95=l=0;
```

```
*Minimum vapor pressure of unleaded Eurosuper 95
R1VPMNES95..
(RVPC4/DNSTYC4)*QTYC4S95+(RVPLN/DNSTYLN)*QTYLNS95+(RVPISO/DNS
TYISO)*QTYISOS95+

(RVPR95/DNSTYR95)*QTYR95S95+(RVPR100/DNSTYR100)*QTYR100S95+(RVP
CrN/DNSTYCrN)*QTYCCGS95-0.45*QTYVS95=g=0;

*Minimum RON of unleaded Eurosuper 95
R1ORMNES95..
(RONC4/DNSTYC4)*QTYC4S95+(RONLN/DNSTYLN)*QTYLNS95+(RONISO/DNS
TYISO)*QTYISOS95+

(RONR95/DNSTYR95)*QTYR95S95+(RONR100/DNSTYR100)*QTYR100S95+(RO
NCrN/DNSTYCrN)*QTYCCGS95-95*QTYVS95=g=0;

*Maximum sensitivity of unleaded Eurosuper 95
R1SENSES95..                  ((RONC4-MONC4)/DNSTYC4)*QTYC4S95+((RONLN-
MONLN)/DNSTYLN)*QTYLNS95+
          ((RONISO-MONISO)/DNSTYISO)*QTYISOS95+((RONR95-
MONR95)/DNSTYR95)*QTYR95S95+
          ((RONR100-MONR100)/DNSTYR100)*QTYR100S95+((RONCrN-
MONCrN)/DNSTYCrN)*QTYCCGS95-10*QTYVS95=l=0;
```

```
*Calculation of weight of gas oil manufactured
R1WTGO..
QTYKEGO+QTYNGO1GO+QTYNGO2GO+QTYNCGOGO+QTYDGO1GO+QTYDG
O2GO+QTYDCGOGO-QTYGO=e=0;

*Maximum sulfur content of gas oil
R1SUMXGO..
SCBKE*QTYKEGO+SCBGO1*QTYNGO1GO+SCBGO2*QTYNGO2GO+SCBCGO*
QTYNCGOGO+SCAGO1*QTYDGO1GO+
          SCAGO2*QTYDGO2GO+SCACGO*QTYDCGOGO-0.05*QTYGO=l=0;

*Calculation of volume of HFO manufactured
R1VOHF..
(1/DNSTYCGO)*QTYNCGOHFO+(1/DNSTYVR1)*QTYVR1HFO+(1/DNSTYVR2)*
QTYVR2HFO-QTYVHGO=e=0;

*Maximum viscosity of HFO
R1VMXHF..
(VBICGO/DNSTYCGO)*QTYNCGOHFO+(VBIVR1/DNSTYVR1)*QTYVR1HFO+(
VBIVR2/DNSTYVR2)*QTYVR2HFO-33*QTYVHGO=l=0;

*Minimum viscosity of HFO
```

```
R1VMNHF..
(VBICGO/DNSTYCGO)*QTYNCGOHFO+(VBIVR1/DNSTYVR1)*QTYVR1HFO+(
VBIVR2/DNSTYVR2)*QTYVR2HFO-30*QTYVHGO=g=0;

*Balance of refinery fuel
R1BARF..                -0.018*DC1-0.018*DC2-0.019*QTYR95-0.026*QTYR100-
0.007*QTYFCCNA-0.007*QTYFCCGO-
        0.04*QTYISO-0.02*QTYDESGO1-0.02*QTYDESGO2-
0.02*QTYDESGO+1.3*QTYRGRF+1.2*QTYLPGRF+1.1*QTYLNRF+QTYHFORF=e
=15.2;

*Maximum availability of crude 1
R1C1MAX..    DC1=l=400;

*Minimum treatment of crude 2
R1C2MIN..    DC2=g=260;

*Maximum crude oil distillation capacity
R1CAPADMX..    DC1+DC2=l=700;

*Maximum catalytic reformer capacity
R1REFMAX..    QTYR95+QTYR100=l=60;
```

```
*Minimum quantity of catalytic reformer feedstock at 95 severity

R1RF95MN..      QTYR95=g=2;


*Maximum cracking capacity

R1FCCMX..       QTYFCCNA+QTYFCCGO=l=135;


*Maximum gas oil desulfurization capacity

R1CAPDSMX..       QTYDESGO1+QTYDESGO2+QTYDESGO=l=150;


*Quantity of Refinery Gas

QutyRG..        [0.001*DC1+0.002*DC2+0.06*QTYR95+0.09*QTYR100


+0.015*QTYFCCNA+0.012*QTYFCCGO+0.03*QTYISO+0.02*QTYDESGO1+0.03*

QTYDESGO2+0.04*QTYDESGO-QTYRGRF]-RG=e=0;

*Quantity of liquefied gas

QutyLPG..

[0.012*DC1+0.015*DC2+0.09*QTYR95+0.12*QTYR100+0.053*QTYFCCNA+0.046*

QTYFCCGO-QTYC4S98-QTYC4S95-QTYLPGRF]-LPG=e=0;


*Quantity of light naphtha

QutyLN..                [0.04*DC1+0.04*DC2-QTYISO-0.05*QTYJFF1-0.035*QTYJFF2-

QTYLNS98-QTYLNS95-QTYLNRF]-LN=e=0;
```

```
*Quantity of Super 98

QutyPG98..

[QTYC4S98+QTYLNS98+QTYISOS98+QTYR95S98+QTYR100S98+QTYCCGS98]-

PG98=e=0;


*Quantity of Eurosuper 95

QutyES95..

[QTYC4S95+QTYLNS95+QTYISOS95+QTYR95S95+QTYR100S95+QTYCCGS95+

QTYIMES95-QTYEXES95]-ES95=e=0;


*Quantity of jet fuel

QutyJF..          [QTYJFF1+QTYJFF2+0.05*QTYJFF1+0.035*QTYJFF2+QTYIMJF-

QTYEXJF]-JF=e=0;


*Quantity of gas oil

QutyGO..

[QTYKEGO+QTYNGO1GO+QTYNGO2GO+QTYNCGOGO+QTYDGO1GO+QTYD

GO2GO+QTYDCGOGO+QTYIMGO-QTYEXGO]-GO=e=0;


*Quantity of HFO

QutyHFO..                       [QTYNCGOHFO+QTYVR1HFO+QTYVR2HFO-

QTYHFORF+QTYIMHFO-QTYEXHFO]-HFO=e=0;
```

```
*Calculation of Super 98 MON
R1MONPG98..
[(MONC4/DNSTYC4)*QTYC4S98+(MONLN/DNSTYLN)*QTYLNS98+(MONISO/D
NSTYISO)*QTYISOS98+

(MONR95/DNSTYR95)*QTYR95S98+(MONR100/DNSTYR100)*QTYR100S98+(M
ONCrN/DNSTYCrN)*QTYCCGS98]=g=MONPG98*QTYVS98;

*Calculation of Super 98 RON
R1RONPG98..
[(RONC4/DNSTYC4)*QTYC4S98+(RONLN/DNSTYLN)*QTYLNS98+(RONISO/DN
STYISO)*QTYISOS98+

(RONR95/DNSTYR95)*QTYR95S98+(RONR100/DNSTYR100)*QTYR100S98+(RO
NCrN/DNSTYCrN)*QTYCCGS98]=g=RONPG98*QTYVS98;

*Calculation of Eurosuper 95 MON
R1MONES95..
[(MONC4/DNSTYC4)*QTYC4S95+(MONLN/DNSTYLN)*QTYLNS95+(MONISO/D
NSTYISO)*QTYISOS95+

(MONR95/DNSTYR95)*QTYR95S95+(MONR100/DNSTYR100)*QTYR100S95+(M
ONCrN/DNSTYCrN)*QTYCCGS95]=g=MONES95*QTYVS95;
```

```
*Calculation of Eurosuper 95 RON
R1RONES95..
[(RONC4/DNSTYC4)*QTYC4S95+(RONLN/DNSTYLN)*QTYLNS95+(RONISO/DN
STYISO)*QTYISOS95+


(RONR95/DNSTYR95)*QTYR95S95+(RONR100/DNSTYR100)*QTYR100S95+(RO
NCrN/DNSTYCrN)*QTYCCGS95]=g=RONES95*QTYVS95;

*Total Input Quantity
QTYIN..
DC1+DC2+QTYIMHN+QTYIMVG+QTYIMES95+QTYIMJF+QTYIMGO+QTYIMH
FO=e=INPUTS;

*Total Output Quantity
QTYOUT..
RG+LPG+LN+PG98+ES95+JF+GO+HFO+QTYEXES95+QTYEXJF+QTYEXGO+QT
YEXHFO=e=OUTPUTS;


*Economic function (cost to maximize)
COST..          [p('1')*[pc('1','1')*d('1','1')  +  pc('2','1')*d('2','1')  +  pc('3','1')*d('3','1')  +
pc('4','1')*d('4','1') + pc('5','1')*d('5','1') + pc('6','1')*d('6','1')]
```

+p('2')*[pc('1','2')*d('1','2') + pc('2','2')*d('2','2') + pc('3','2')*d('3','2') + pc('4','2')*d('4','2') + pc('5','2')*d('5','2') + pc('6','2')*d('6','2')]

+p('3')*[pc('1','3')*d('1','3') + pc('2','3')*d('2','3') + pc('3','3')*d('3','3') + pc('4','3')*d('4','3') + pc('5','3')*d('5','3') + pc('6','3')*d('6','3')]

+p('4')*[pc('1','4')*d('1','4') + pc('2','4')*d('2','4') + pc('3','4')*d('3','4') + pc('4','4')*d('4','4') + pc('5','4')*d('5','4') + pc('6','4')*d('6','4')]

+p('5')*[pc('1','5')*d('1','5') + pc('2','5')*d('2','5') + pc('3','5')*d('3','5') + pc('4','5')*d('4','5') + pc('5','5')*d('5','5') + pc('6','5')*d('6','5')]

+p('6')*[pc('1','6')*d('1','6') + pc('2','6')*d('2','6') + pc('3','6')*d('3','6') + pc('4','6')*d('4','6') + pc('5','6')*d('5','6') + pc('6','6')*d('6','6')]

+p('7')*[pc('1','7')*d('1','7') + pc('2','7')*d('2','7') + pc('3','7')*d('3','7') + pc('4','7')*d('4','7') + pc('5','7')*d('5','7') + pc('6','7')*d('6','7')]

+p('8')*[pc('1','8')*d('1','8') + pc('2','8')*d('2','8') + pc('3','8')*d('3','8') + pc('4','8')*d('4','8') + pc('5','8')*d('5','8') + pc('6','8')*d('6','8')]

+p('9')*[pc('1','9')*d('1','9') + pc('2','9')*d('2','9') + pc('3','9')*d('3','9') + pc('4','9')*d('4','9') + pc('5','9')*d('5','9') + pc('6','9')*d('6','9')]

-[(578*DC1+489.5*DC2)]

-[(682.3*QTYIMHN+631.76*QTYIMVG+742.32*QTYIMES95+710.73*QTYIMJF+694.94*QTYIMGO+347.47*QTYIMHFO)]

-2*[3.14*(DC1+DC2)+(8.53*QTYR95+10.11*QTYR100+6.32*QTYIMHN)+9.48*(QT

```
YFCCNA+QTYFCCGO+QTYIMVG)+1.9*(QTYISO)+3.14*(QTYDESGO1+QTYDES
GO2)+4.42*QTYDESGO] -10*[(INPUTS-OUTPUTS)]]/1000=e=z;
model refineryplan /all/;
* added bu AAA to control tolerance
option optcr = 0.0001;

solve refineryplan using lp maximizing z;
```