

# KING FAHD UNIVERSITY OF PETROLEUM \& MINERALS <br> DHAHRAN 31261, SAUDI ARABIA 

## DEANSHIP OF GRADUATE STUDIES

This thesis, written by KHURRAM MASOOD under the direction of his thesis adviser and approved by his thesis committee, has been presented to and accepted by the Dean of Graduate Studies, in partial fulfillment of the requirements for the degree of DOCTOR OF PHILOSOPHY IN ELECTRICAL ENGINEERING.

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May 2013

Dedicated to my loving parents and family

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In the name of Allah, the Most Beneficent, the Most Merciful

All praise be to Allah for His boundless blessings. May Allah bestow peace and His choicest blessings on His last prophet, Hazrat Muhammad (Peace Be Upon Him), his family (May Allah be pleased with them), his companions (May Allah be pleased with them) and his followers.

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## Nomenclature

| Abbreviati |  |
| :---: | :---: |
| AC | Access Category |
| AIFS | Arbitration Inter-Frame Space |
| AIFSN | Arbitration Inter-Frame Spacing Number |
| AP | Access Point |
| BBR | Basic Bit Rate |
| BO | Back-off |
| BR | Bit Rate |
| CR | Cognitive Radio |
| CSMA | Carrier Sense Multiple Access |
| CSMA/CA | CSMA with Collision Avoidance |
| CSMA/CD | CSMA with Collision Detection |
| CT | Channel Type |
| CTB | Collapsed Transition onto Basis |
| CW | Contention Window |
| CWmax | Maximum Contention Window |
| CWmin | Minimum Contention Window |
| DCA | Dynamic Channel Assignment |
| DCF | Distributed Coordination Function |
| DFT | Defer First Transmission |


| DIFS | DCF Inter Frame Space |
| :---: | :---: |
| DPC | Dedicated Parallel Channel |
| EDCA | Enhanced Distributed Channel Access |
| EDCF | Enhanced Distributed Coordination Function |
| EPA | Equilibrium Point Analysis |
| ERP | Extended Rate PHY |
| FCA | Fixed Channel Assignment |
| FU-FB | Finite User Finite Buffer |
| IFT | Immediate First Transmission |
| LAN | Local Area Network |
| MA | Multiple Access |
| MAC | Medium Access Control |
| MH | MAC Header |
| MIMO | Multi-Input Multi-Output |
| MMAC | Multichannel MAC |
| P-ALOHA | Pure ALOHA |
| PCF | Point Coordination Function |
| PGF | Probability Generating Function |
| PH | Physical Header |
| PHY | Physical |
| QoS | Quality of Service |
| RBCS | Receiver Based Channel Selection |


| RFID | $:$ | Radio Frequency Identification |
| :--- | :--- | :--- |
| RTS/CTS | $:$ | Ready to Send/Clear to Send |
| S-ALOHA | $:$ | Slotted ALOHA |
| SOI | $:$ | Signal of Interest |
| SPC | $:$ | Shared Parallel Channel |
| SSC | $:$ | Shared Single Channel |
| SSCH | $:$ | Slotted Seeded Channel Hopping |
| STA | $:$ | Station |
| TC | $:$ | Traffic Category |
| TU | $:$ | Tagged User |
| TUA | $:$ | Tagged User Analysis |
| TXOP | $:$ | Transmission Opportunity |
| VPST | $:$ | Virtual Packet Service Time |
| WLAN | $:$ Wireless LAN |  |

## Notations

| $\beta$ | $:$ Path loss exponent |
| :--- | :--- |
| $\lambda$ | $:$ Packet arrival rate |
| $\lambda_{i, n}$ | $:$ Normalized offered load for $i^{\text {th }}$ group |
| $\lambda_{i}$ | $:$ Packet arrival rate for $i^{\text {th }}$ group |
| $\lambda_{n}$ | $:$ Normalized offered load |
| $\bar{P}_{k, m}$ | $:$ Average power for $m^{\text {th }}$ path of $k^{\text {th }}$ user |
| $\bar{P}_{1}$ | $:$ Average power of dominant path |
| $\bar{P}_{m}$ | $:$ Average power of $m^{\text {th }}$ path |
| $\bar{P}_{n}$ | $:$ Sum of average power of all the interfering signals |
| $\pi_{i, k}$ | $:$ Probability that leaving packets sees k packets in queue in $i^{\text {th }}$ group |
| $\pi_{k}$ | $:$ Probability that leaving packets sees k packets in queue |
| $\rho$ | $:$ Offered load |
| $\Theta$ | $:$ System throughput |
| $\theta$ | $:$ TU throughput |
| $\Theta_{i}$ | $:$ System throughput in $i^{\text {th }}$ group |
| $\theta_{i}$ | $:$ TU throughput in $i^{\text {th }}$ group |

$A(z) \quad: \quad$ PGF of $a_{k}$
$a_{i, k} \quad$ : Probability that k packets arrive during a service time in $i^{\text {th }}$ group
$A_{i}(z):$ PGF of $a_{k}$ in $i^{\text {th }}$ group
$a_{k} \quad: \quad$ Probability that k packets arrive during a service time
$b$ : Mean packet service time
$B(z)$ : PGF of VPST
$b_{i} \quad: \quad$ Mean packet service time in $i^{\text {th }}$ group
$B_{i}(z)$ : PGF of VPST in $i^{\text {th }}$ group
$C$ : Number of multiple channels
$C T_{1}$ : Channel of type1
$C T_{2}$ : Channel of type2
$D$ : Time for collision detection
$D_{i}^{\text {bas }}$ : Time for collision detection for a STA in group $i$
$D_{i}^{r t s} \quad$ : Time for collision detection for a STA in group $i$
$D_{i} \quad$ : Time for collision detection in $i^{\text {th }}$ group
$E\left[I_{q}\right]$ : Mean queue length
$E\left[t_{r}\right]$ : Mean packet response time
$E\left[t_{w}\right]$ : Mean waiting delay
$E_{I_{i}} \quad:$ Expected idle time for TU in $i^{\text {th }}$ group
$E_{i}\left[I_{q}\right]$ : Mean queue length in $i^{\text {th }}$ group
$E_{i}\left[t_{r}\right]$ : Mean packet response time in $i^{t h}$ group
$E_{i}\left[t_{w}\right]$ : Mean waiting delay in $i^{\text {th }}$ group
$f_{P_{m}}\left(p_{m}\right): \quad$ pdf for power of $m^{\text {th }}$ path
$f_{P_{n}}\left(p_{n}\right):$ pdf for power of interfering signals
$f_{P_{t}}\left(p_{t}\right) \quad: \quad$ pdf for power of SOI
$K_{i} \quad: \quad$ Maximum retries allowed by TU in $i^{\text {th }}$ group
$L \quad: \quad$ Buffer length
$L_{i} \quad: \quad$ Buffer length in $i^{\text {th }}$ group
$M \quad$ : Number of multipaths
$n \quad:$ Number of interferers
$N \quad: \quad$ Number of STAs
$N_{i} \quad: \quad$ Number of STAs in $i^{\text {th }}$ group
$p \quad:$ Channel access probability
$p_{c, c s} \quad: \quad$ Probability of selecting the $c^{\text {th }}$ channel
$p_{i, 0} \quad: \quad$ Idle probability in $i^{\text {th }}$ group
$p_{i, B} \quad: \quad$ Blocking probability in $i^{\text {th }}$ group
$p_{i, b} \quad: \quad$ Busy probability in $i^{\text {th }}$ group
$p_{i, c} \quad: \quad$ Probability that TU in group $i$ experiences a collision
$p_{i, c o g, j}$ : Probability, that during a busy duration in a cycle,
: there is a collision due to a STA in a group other than that of
: the TU group $i$
$p_{i, c o u, j}$ : Probability that, during a busy duration in a cycle,
: there is a collision by a STA (other than the TU) in group $i$ and : a STA in another group $j$ such that $D_{j}>D_{i}$
$p_{i, \text { cou }}$ : Probability that, during a busy duration in a cycle,
: there is a collision by a STA (other than the TU) in group $i$
: and a STA in group $j$ with $D_{j} \leq D_{i}$
$p_{i, c t u, j}$ : Probability that, during a busy duration in a cycle,
: there is a collision involving the TU in group $i$ and a STA in
: another group $j$ such that $D_{j}>D_{i}$
$p_{i, d} \quad: \quad$ Probability of packet drop by TU in $i^{\text {th }}$ group
$p_{i, I} \quad$ : Probability of sensing the channel idle by TU in group $i$
$p_{i, k} \quad:$ Probability that k packets are present in the queue in $i^{\text {th }}$ group
$p_{i, s} \quad$ : Probability of successful transmission by TU in $i^{\text {th }}$ group
$p_{i, s o g, j}:$ probability that, during a busy duration in a cycle,
: there is a successful transmission due to a STA in a group
: other than that of the TU group $i$
$p_{i, \text { sou }, j}$ : Probability that, during a busy duration in a cycle,
: the transmission by a STA (other than the TU) in group $i$ is successful
: in the presence of other transmitting STA from group $j$ such that $D_{j}>T_{i}$
$p_{i, \text { sou }}$ : probability that, during a busy duration in a cycle,
: the transmission by a STA in group $i$ (other than the TU) is a success
: and there is no STA transmitting from group $j$, such that $D_{j}>T_{i}$
$p_{i, s t u, j}$ : Probability that, during a busy duration in a cycle,
: the transmission by TU in group $i$ is a success with at least one
: transmitting STA in another group $j$ such that $D_{j}>T_{i}$
$P_{i}^{\text {bas }}$ : Time required for payload data transmission for a STA
$P_{i}^{r t s} \quad:$ Time required for payload data transmission for a STA
$p_{s \mid n} \quad: \quad$ Probability that TU transmission is successful in presence of $n$ interferers
$p_{T_{1}, c s} \quad: \quad$ Probability of selecting channel of type1
$p_{T_{2}, \text { cs }}$ : Probability of selecting channel of type2
$p_{0} \quad: \quad$ Idle probability
$p_{B} \quad:$ Blocking probability
$p_{b} \quad: \quad$ Busy probability
$p_{i} \quad$ : Channel access probability in $i^{\text {th }}$ group
$p_{k} \quad$ : Probability that k packets are present in the queue
$P_{n} \quad: \quad$ Power of interfering users
$p_{s} \quad: \quad$ Probability of successful transmission by TU
$P_{t} \quad: \quad$ Power of desired user
$S_{b o} \quad: \quad$ Back-off state
$S_{d} \quad: \quad$ Packet departure state
$S_{i, b o} \quad$ : Back-off state in $i^{\text {th }}$ group
$S_{i, d} \quad:$ Packet departure state in $i^{\text {th }}$ group
$S_{i, p r} \quad: \quad$ Packet ready state in $i^{\text {th }}$ group
$S_{i, t s} \quad: \quad$ Transmission start state in $i^{\text {th }}$ group
$S_{p r} \quad$ : Packet ready state
$S_{t s} \quad:$ Transmission start state
$T$ : Time for successful packet transmission
$T_{i}^{\text {bas }} \quad$ : Time required for successful transmission for a STA in
$T_{i}^{r t s} \quad$ : Time required for successful transmission for a STA
$T_{i} \quad: \quad$ Time for successful packet transmission in $i^{\text {th }}$ group
$W_{i, a v g}$ : Average backoff window size in group $i$
$W_{i} \quad$ : Minimum backoff window size in group $i$
$z_{0} \quad: \quad$ Capture ratio

## THESIS ABSTRACT

NAME:<br>Khurram Masood<br>TITLE OF STUDY: Analysis of Contention Based MAC Protocols under Multipath Fading and Resource Management<br>MAJOR FIELD: Electrical Engineering<br>DATE OF DEGREE: May 2013

The complexity of state space based methods for analysis of finite user finite buffer (FU-FB) random multiple access systems increases exponentially with buffer size and number of users. The presence of multipath frequency selective fading channel further adds to the complexity, making the analysis practically intractable. An approximate analysis technique called tagged user analysis (TUA) has been used to analyze the performance parameters of such systems over multipath and frequency selective fading channels for FU-FB systems. In TUA, the steady state system performance is evaluated from the analysis of a single user. Moreover, the state flow graph of TUA has just four states; thus reducing the complexity of the analysis. Simulation results confirm the validity of the TUA analysis.

The mathematical expressions for the pdf of the received signal impaired by
interference signal power are developed. These results are used to find the probability that the transmission by the tagged user will be successful in presence of interference. Random multiple access MAC protocols including S-ALOHA, the IEEE 802.11 DCF and EDCF based on CSMA/CA have been analysed. The analysis is presented for realistic scenarios with a) a multipath frequency selective channel between the users/stations (STAs) and the access point (AP), b) an unsaturated system having finite population of STAs, each having a finite buffer capacity, c) a ring/bell shaped spatial distribution for the STAs, d) the basic and ready to send/clear to send (RTS/CTS) access mechanisms for IEEE 802.11 distributed coordination function ( $D C F$ ) and enhanced $D C F(E D C F)$, e) homogeneous/heterogeneous groups of STAs.

Finally, the performance analysis of WLAN system using IEEE 802.11 EDCF as the access protocol with multiple parallel channels for transmission is presented. Different channel selection schemes and channel configurations have been used in the analysis and the results are presented.

## ملخص الرسالة

> الاسم الكامل: خرم مسعود

عنوان الرسالة: تحليل بروتوكولات التحكم بالوصول إلى الوسائط على أساس التزاحم تحت ظروف التلاشي وإدراة الموراد.

التخصص: هنسة كهربائية. تاريخ الارجة اللطمية: أَّار 2013.

تعقيد الطرق المعتمدة على فراغ الحالة في تحليل أنظمة الوصول المتعدد العشوائي محدودة الدستخدم-محدودة المخزّن يتز ايد أضعافاً مضاعفة مع حجم المخزّن و عدد المستخدمين. أيضاً، وجود ظاهرة التلاشي في القنوات تزيد من هذا التعقيد جاعلة من عملية تحليل الآداء صعبة التتفيذ. مؤخراً، تم إستخدام تقفيّة تقريبية لتحليل الآداء تسمى تحليل المستخدم الموسوم في تحليل آداء الأنظمة السابق ذكر ها تحت قنوات التلاشي إنتقائية النردد. في تقنيّة تحليل المستخلم الموسوم، آداء الحالة المستقرّة لللظام يتم حسابه من تحليل مستخدم واحد فقط. علاوة على ذلك، رسم تدفّق الحالة لتحليل المستخدم الموسوم له فقط 4 حالات؛ بالتاللي تقليل تعقيد التحليل. نتائج المحاكاة تؤكّد صحّة تقنيّة تحليل المستخدم الموسوم. تم إشتقاق التعبير ات الرياضية لدالّة الكثّافة الإحتمالية للإشارة المستقبلة التي تعاني من التداخل. ثم تم إستخدام النتائج السابقة في إيجاد إحتمالية نجاح إرسال المستخدم الموسوم بوجود تداخل. كما وتم تحليل بروتوكو لات الوصول الوتعدد العشو ائي مثل IEEE 802.11 DCF ،S-ALOHA و EDCF المعتمدة على CSMA/CA. تم تققيم التحليل لسيناريو هات و اقعية مع أ) قناة تلاشي إنتقائية التردد بين الدستخدمين|المحطات (STAs) ونقطة الوصول (AP)، ب) نظام غبر مشبع مع عدد محدد ممن ال ( كل واحد مع مخزّن محدود السعة. ج) توزيع فضائي حقياجرسي لل STAs. تفتيّات جاهز للجٕرسال|واضح للإرسال (RTS/CTS) لِ IEEE 802.11 دالة التتسيق المتوز عة (DCF) و ال DCF المطوّرة (EDCF)، د) مجموعات ال STAs المتجانسة|الغير متجانسة. أخيراً، تم تقديم تحليل آداء نظام ال WLAN بإستخدام IEEE 802.11 EDCF كبروتوكول وصول مع قنوات متوازية متعدّدة للإرسال. تم إستخدام مخططات إختيار وإعدادات قناة مختلفة في التحليل كما وتم تقديم النتائج.

## CHAPTER 1

## INTRODUCTION

The current trend in wireless communications to offer multimedia services has renewed interest in multimedia access methods applicable to wideband wireless systems. It has been found that the most efficient method of transmitting integrated voice, data, and image traffic is in the form of packets. This resulted in an upsurge in the analyses of access, transmission protocols and resource management for high-speed packet traffic. In the past, the random access methods were analyzed by making unrealistic assumptions like infinite user population, infinite or no buffer. These assumptions are not realistic. First, the number of users is finite and never infinite. Second, the data flow from the source necessitates a finite size buffer. These two realistic system parameters lead to the analysis of finite user - finite buffer systems and the analyses of these systems become quite involved. In the analysis, it is desirous to estimate, blocking probability, channel throughput, packet drop probability. These parameters are used in the design of packet networks.

The state of the channel, the number of users and their buffer states are needed for the exact analysis. Markov chain or state space methods are obviously applicable but the number of states increases exponentially and even for a modestly small system the analysis becomes tedious and sometimes intractable. The analysis if simplified by making certain assumptions and the results are now approximate and not exact. For example, equilibrium point analysis (EPA) is a fluid type of system with fixed number of users i.e. the number of users leaving the user pool are replaced by an equal number previously idle users and each user operates at an equilibrium point.

An analytical but approximate approach, proposed by Tao and Sheikh [1] known as Tagged User Analysis (TUA), uses existing results in queuing theory and can analyze many random access systems. The computational complexity of TUA is substantially lower than that of Markovian and EPA methods because TUA technique transforms a multi-dimensional system into a single dimension system with the help of several simplifying assumptions. The application of TUA to wireless communications requires the analysis for multiple access protocols in presence of frequency selective fading channels using TUA. Furthermore, application of TUA analysis in resource management where multiple channels are available to a heterogeneous group of users is an important topic, which is addressed in this research.

### 1.1 Background

Multiple access (MA) is needed wherever a resource is shared among a number of independent users. Due to their robust performance, random access protocols have been widely used in MA wireless communication systems such as packet satellite communications, wireless local area network (LAN), and random access channel in cellular mobile systems. Example of random access protocols include pure ALOHA (P-ALOHA), slotted ALOHA (S-ALOHA), carrier sense multiple access (CSMA), CSMA with collision detection (CSMA/CD) and CSMA with collision avoidance (CSMA/CA).

The future multimedia wireless communication systems will provide services in voice, data, and video. The current trend in networks is towards packet mode and it is very important that an analytical framework be developed to analyze multiuser FU-FB systems. The traditional analysis methods assume infinite user population and are not realistic. In practice the users of integrated multimedia systems have two distinct properties; they have a finite number of users each with a finite buffer length. For cognitive radio (CR) users that look for available white spaces in the spectrum, finite population of CR users each with a finite buffer size has been considered [2]-[4]. It has been shown that a buffered user significantly reduces the blocking and non-completion probabilities. These two realistic system parameters, FU-FB, lead to analytical difficulties when state space analysis is used. The number of states in a system grows exponentially with the number of users and the size of their buffer.

Multimedia applications of today require large amount of data to be transferred using wideband transmission methods. Wideband channels, which exhibit multipath and frequency selective fading, are required to handle such huge data. The presence of such a channel further adds to the complexity of the exact analysis. Analysis becomes prohibitively complex as the number of states increase from $N^{L}$ to $(N M)^{L}$ where $N$ is the number of users, $L$ is the buffer length and $M$ is the number of significant paths of the channel impulse response. The presence of multipaths adds $N \times(M-1)$ virtual users (the number becomes random and time variant). With this increase in the number of virtual users, the number of calculations of state transition probabilities becomes complex and practically intractable.

### 1.2 Literature Survey

The analysis of any wireless communication system entails evaluating a number of system parameters like, blocking probability, channel throughput, packet drop probability, packet response time, packet wait delay etc. The existing MA analysis methods fall in three classes - S-G analysis, Markov analysis, and equilibrium point analysis (EPA) [5]. S-G analysis assumes infinite population, which generates an aggregate traffic of S packets/slot, whereas the new transmissions and retransmissions combined together generate the channel traffic of $G$ packets/slot. Because of its simplicity S-G method has been applied to many multiple access protocols [6]-[9]. The S-G analysis assumes Poisson packet arrival and statistical
equilibrium. The analysis negates inclusion of buffers for each user as S-G analysis considers only the total offered traffic.

The system evolution in a random multiple access protocol is a stochastic process which is best analyzed by developing a Markov model of the system [10]. Markov chain or state-space method provides exact analysis [11]-[14]. Though Markovian analysis can be applied to all types of multiple access protocols, it has usually been applied to analyze homogeneous systems with unbuffered users, in which case the system state vector is a scalar and size of state space is just the total number of users in the system. In buffered homogeneous system, the state vector used to describe the system behavior becomes multi-dimensional, and size of state space is an exponential function of the user buffer capacity [15]. Taking channel fading into consideration further complicates the matter. The analysis becomes even more challenging when the channel not only fades but exhibits frequency selectivity. Due to the complex nature of the multipath frequency selective channels the analysis with Markov chain becomes practically impossible, where, in addition to the size of user population and the buffer, the number of paths in the channel increases the dimension of state space of the Markov analysis beyond tractability.

EPA is an approximate analysis technique that has also been used in the analysis of FU-FB systems [5], [16], [17]. EPA is a fluid-type approximate analysis, which is applied to analyze a system in steady state. EPA assumes a fixed number of users operating at their equilibrium points. For the analysis of dynamic systems,
where frequent arrivals and departures of users is common, EPA is difficult to apply. Though EPA can be applied to buffered systems, the size of its state space becomes prohibitively large with the increase of packet transmission time and buffered capacity.

Several other analytical techniques make use of ideas derived from the three methods [18]-[21]. The main focus is to reduce the state space dimensionality by using transition probabilities between a fewer number of states and exploiting the redundant information available in the systems. The most common assumption employed is that of channel symmetry where statistically each user has the same behavior. A two-dimensional Markov chain is used in [18]-[20], where one dimension relates to the number of busy users, $N$, and the other to the number of packets in user buffer, $L$. In [19], the system states are reduced to $(N+1)(2+(L-1) N) / 2$, whereas [20] uses the number of busy stations (STAs) in the system and the queue length at a particular station (STA) so as to derive an efficient, but approximate multiple access protocol with further reduced number of states, $N(L+1)$. A different approach for the analysis is taken in [21]. The statistical behavior of the system is obtained using the characteristics of the restricted urn model. The state is defined as the total number of packets in the whole system and transition probability is employed for the system analysis. Even using these approximations, the size of the state space in these approaches is still quite large, which presents a major hurdle in the analysis of buffered S-ALOHA and other multiple access protocols.

Over the past several years S-ALOHA has attracted the attention of researchers for studying random access protocols. Access network (infrastructure mode in WiFi) is an important application where multiple users compete for the opportunity to transmit data to an access point [22]. Recent work by [23] investigated the existing S-ALOHA medium access control (MAC) protocol and suggested some improvements for handling the large propagation delays involved in underwater sensor networks. Theoretical analysis of P-ALOHA and an intuitive explanation of S-ALOHA performance for underwater sensor networks were presented in [24]. In [25], researchers studied S-ALOHA based radio frequency identification system. The analytical results for finding the optimum retransmission probabilities considering S-ALOHA random access protocol with a Poisson arrival process more suitable for the traffic model in a cellular system for the users in the cells were presented in [26]. Cognitive MAC using S-ALOHA for cognitive radio networks was investigated in [27]. S-ALOHA has also been used in other applications including wireless relay networks [28], multi-input multi-output (MIMO) systems [29], wireless mesh networks [30], cooperative transmission [31], framed S-ALOHA based radio frequency identification (RFID) systems [32], and wireless sensor networks [33].

IEEE 802.11 [34] is widely used medium access control (MAC) protocol in wireless local area network (WLAN) systems. The medium access control (MAC) layer uses the mandatory distributed coordination function (DCF) and an optional point coordination function (PCF) to share a common radio channel among dif-
ferent STAs in WLAN system.
Bianchi [35], presented a saturated traffic model which is a simplification to the more practical finite traffic load. He used 2-D Markov chains to solve the system for saturation throughput. The model considers an ideal channel and there is no limit for retries after collision. The 2-D Markov chain method has been extensively used for the analysis of WLAN by other researchers as well [36]-[38]. These models either assume bufferless [36] or an infinite buffered [37],[38] user population. For infinite buffer case it is not possible to evaluate certain parameters like blocking probability. The work by Bianchi was extended by others to add more realistic scenarios for wireless networks. The limit on number of retries was introduced to Bianchi's model by [39] but the STAs were still assumed to be saturated. The first analysis for IEEE 802.11 DCF having users with finite buffer is presented in [40] under ideal channel conditions. The 3-D Markov chain was introduced as an extension to the existing 2-D Markov chain models. The $3^{r d}$ dimension is introduced to take into account the effect of buffer size. The proposed analysis uses collapsed transition onto basis (CTB) method to solve the 3-D Markov chains. This reduces the complexity but with the increase in the number of users and backoff stages the state space becomes too large and the system solution becomes intractable. A more practical work is presented in [41] where the same 3-D model has been applied under fading channel. In both ideal and fading channel cases the traffic is assumed to be homogeneous.

The IEEEE 802.11e standard was introduced to provide quality of service
(QoS) in WLANs. This standard uses an enhanced distributed coordination function (EDCF) to ensure QoS for real time applications like voice and video, which is backward compatible with DCF. Performance of IEEE 802.11e EDCF has been studied for WLANs during the past decade using simulations [42] - [45] while analyses have been done [46] - [51] using unrealistic assumptions like infinite user population, infinite or no buffer, saturation condition, fixed backoff, no retries limit, ideal channel conditions or flat fading channel condition. Most of the analyses are based on Markov chains analysis and are generally complicated. The effect of changing the QoS parameter, namely Arbitration inter-frame spacing number (AIFSN), for achieving priority for an access category (AC) is discussed in [42] using simulations whereas [43] shows the effectiveness of using other QoS parameters for achieving service differentiation. Performance under ideal channel conditions focusing on the backoff process is considered in [46] and in [47] throughput analysis under the assumption that there is no data loss due to bad channel is presented. The errors due to channel condition are considered by [48]. The performance of video streaming using EDCF considering the effect of channel noise on the transmission in addition to the packet loss due to collision is evaluated in [49]. An analysis is presented in [50], where the authors did not use Bianchi's decoupling assumption. The analysis is not able to fully implement the EDCA protocol for service differentiation as it considers all the STAs have same Arbitration inter frame space (AIFS), hence not capturing the benefits of QoS provided by IEEE 802.11e EDCF fully. These analyses assume the system to be saturated and hence
do not represent a practical scenario. In addition, the analysis involves rigorous Markovian framework and only provides saturation throughput. For unsaturated system [51] presents the analysis under ideal channel conditions.

It is important to note that the existing analytical techniques always start from basic principles and do not use the existing results in queuing theory except for two publications [52], [53]. As mentioned at the beginning Tagged User Analysis (TUA) is a relatively new analytical but approximate approach, proposed by Tao and Sheikh [1], that has been successfully applied to the analysis of FUFB systems for various random access protocols including S-ALOHA, R-ALOHA, CSMA/CD and DS/CDMA ALOHA for steady channels that are typical of computer networks [1],[54]-[58]. In these channels the only cause for unsuccessful packet transmission is the collision between packets. The application of TUA to wireless communications necessitates research into the behavior of system in the presence of fading. The application of TUA to flat fading channels [59]-[62] shows that it can be successfully applied in the presence of channel fading. In wireless communications, however, the presence of reflectors in the environment surrounding a transmitter and the receiver creates multiple paths that support propagation of transmitted signal. As a result, each multipath signal will experience differences in attenuation, delay and phase shift while traveling from the source to the receiver. The analysis becomes even more challenging when the channel not only fades but exhibits frequency selectivity. Due to this complex nature of the multipath frequency selective channels the analysis with Markov chain becomes
practically impossible, where, in addition to the size of user population and the buffer, the number of paths in the channel increases the dimension of state space of the Markov analysis beyond tractability.

TUA does not require the analysis of a system to be started from scratch but makes use of existing results in queuing theory when the type of the queue is recognized. Whereas in Markov chain based exact analysis methods, the number of states change with system parameters (queue size, number of users, number of channel taps) and necessitate an analysis from scratch. For each new random access method, one has to start from basics and develop a state space diagram enumerating all states and transition probabilities. This is a powerful and important result as it shortens considerably the effort and time required for system analysis.

There has been effort to increase the system throughput in order to meet the ever increasing demands in the communication systems. In order to fulfill the demand for higher throughput researchers have looked into using multiple channels that operate in parallel. In multiple channel systems an important feature is to appropriately select the channel suitable for transmission. Three types of channel scheduling schemes have been discussed in [63]. The $1^{\text {st }}$ approach uses homogeneous channels having same data rate, whereas the $2^{\text {nd }}$ approach assumes heterogeneous channels and a simple channel selection scheme where all the channels are randomly selected with an equal probability. The $3^{r d}$ and the most efficient technique uses heterogeneous channels and fastest channel first approach. This
means that the channels are selected according to the data rate they support. By doing so there is an increase in the system throughput and a corresponding decrease in the system delay. A comparison between single channel and multiplechannel CSMA/CD has been shown in [64]. They show that for equal capacity the multiple channel system shows greater throughput with a reduced response time.

In [65] multichannel S-ALOHA access systems have been shown to reduce collisions and accepts more packets by evenly distributing the system load over the available channels. The effect of using buffer and varying the retry limits alongwith the random backoff have been considered. It has been shown that multiple channels are useful in increasing the system throughput beyond the saturation point of the system. Random back-off algorithm alongwith multiple channels has been studied in [66] and it has been shown that such a system performs better than the normal CSMA/CA system. In another approach [67] have suggested to transmit a packet on more than one channel and randomly select one of these channels if there are more than one successful transmissions. Different channel allocation schemes have been discussed in [68]. Fixed channel assignment (FCA), where a number of STAs are allocated to a particular channel is a good approach to achieve higher system throughput under high load conditions but it is not able to handle changing traffic conditions and user distributions. This can result in some of the channels being overloaded while others having a lot of empty slots. At the cost of slightly more complexity, dynamic channel assignment (DCA) schemes
can be used. These schemes use a pool of STAs and the STA is assigned according to the need. As a fact in telephone traffic engineering single server is more efficient as compared to a group of small servers with equivalent aggregate capacity. FCA schemes behave as a group of small servers, while DCA combine all the STA in a large server. In [69] a receiver based channel selection (RBCS) mechanism has been presented. By reducing the collisions on the receiver side there is an increase in the throughput with a reduced delay.

The users experience a large number of collisions when the multiuser multiple access system is implemented using 802.11 MAC protocol. A quantitative analysis for the collision during transmission for 802.11a and 802.11b wireless LAN (WLAN) systems is given in [70]. The collision probability increases from $20 \%$ to $30 \%$ for 5 contending users when 802.11a is used as compared to 802.11b WLAN. For multihop wireless networks using 802.11 MAC protocol the problem is even worse. The throughput is reduced because of the interference from adjacent users. In order to solve this problem, multiple channels provided in 802.11 MAC protocol can be used in such a way that there is simultaneous transmission on these channels. This is achieved by allocating separate channels to different users in WLAN.

In order to utilize the multiple channels provided in 802.11 MAC protocol, researchers have applied different strategies. Some work has been done where a separate channel is dedicated for the access control. One such scheme is discussed in [71] where they use Dynamic Channel Assignment (DCA) for access control of
multichannel multiple access system. Using a separate channel for access control can reduce the system throughput and hence it has urged the researchers to use the same channel for data as well as access control. A multichannel MAC (MMAC) protocol has been proposed by [72], that resolves the problem of multichannel hidden terminals. The dynamic utilization of multiple channels hence results in performance improvement and higher channel utilization. Slotted Seeded Channel Hopping (SSCH) scheme is discussed in [73], in which each user is assigned a sequence for hopping. This sequence is determined by a seed which is assigned to each user. The communicating users meet in the same channel, whereas the disjoint users are assigned separate channels. In this way interference is avoided which results in improved system throughput. A simple scheme, Splash, for multiple channels in which the users switch to the next available channel once the channel of higher priority is busy is presented in [74]. In using this scheme the system throughput can be improved. Splash is a simple scheme which neither requires a separate access control channel nor does it require scheduled channel switching.

Computer networks have been migrating to wireless access network over the past decade. WiFi and WiMax systems have become popular methods to gain wireless access to internet respectively for short range and long range. In this context, analysis of the performance of access protocols requires a viable analytical framework. The TUA approach has been applied to the channels where collisions between packets are the only source of access failure or to the flat fading radio
channels. However, the wireless channels change dynamically and are impaired by signal fades, multipath propagation, interference, and noise. This channel peculiarity lead to the development of a generalized analytical model, which can be applied to frequency selective wireless fading channels. The research in this area is difficult and complex. The main challenge lies in the determination of the impact of activities of the users and the frequency fading effects on the common contention channels.

### 1.3 Dissertation Contributions

The main contributions of the dissertation can be summarized as follows:

1. The approximate TUA technique is established as a truly generalized analysis method for random multiple access protocols applicable under various channel conditions, i.e., ideal channel, Rayleigh fading and frequency selective. The technique is also applicable to heterogeneous networks and under realistic wireless network conditions like finite user finite buffer, unsaturated traffic. TUA results have been verified by matching with the simulations developed in MATLAB.
2. A generalized analytical framework for the analysis of WLAN systems using IEEE 802.11 under realistic conditions is developed for the first time using TUA. This analysis works for different variants of IEEE 802.11 and as an example results for IEEE 802.11 DCF and EDCF have been shown to match the simulations with reasonable accuracy.
3. Interference models have been developed and used for the evaluation of Quality of Service (QoS) parameters like system throughput, blocking probability, packet response time, waiting delay, average queue length and packet drop probability.
4. Determination of optimum operating range of re-transmission probability for these networks which delivers highest throughput while maintaining a unique stable operating point.
5. Application of TUA technique to multi-channel heterogeneous networks.

### 1.4 Dissertation Layout

This work investigates the application of TUA to the more realistic wireless channels which exhibit multipath frequency selective fading. The motivation is to establish TUA as a simple, approximate and generalized analysis technique for the random multiple access systems. In Chapter 1, background to the proposed research was presented and an in-depth review on the previous work was presented. The remainder of this report consists of 5 chapters. Chapter 2 , concentrates on mathematical model for interference signals and deriving the expressions for the probability of a successful transmission. The results of performance and stability analysis for S-ALOHA under multipath frequency selective fading appear in Chapter 3. In Chapter 4, a generalized model for analysis of IEEE 802.11 WLAN system with heterogeneous traffic is developed. System with multiple channels is
discussed in Chapter 5. Finally, Chapter 6, concludes the findings of the research and provides some future recommendations.

## CHAPTER 2

## SUCCESS PROBABILITY IN

## MULTIPATH FREQUENCY

## SELECTIVE FADING

## CHANNEL

In multiple access systems operating in infrastructure mode there are a number of users/stations (STAs) that share the same communication resource communicating with a central access point (AP). All the users send access or information data to a central base station. The base station receives data from these users and if more than one user transmit during the same slot collision occurs resulting in loss of data. In frequency selective fading channel the received signal of the desired user can still be accepted by the base station if its power is significantly higher compared to the interference signal power. This phenomena where the
packet from a user having the highest signal strength is successfully decoded by the base station, even in the presence of simultaneous transmissions due to other users, is known as capture. TUA uses the fact that at equilibrium, all users have statistically equivalent behavior. So each user can be assumed an independent queuing system operating with its own equilibrium probabilities. These equilibrium probabilities are affected by the other users sharing the same channel. In fact the probability that a transmission by any arbitrary user, referred as the tagged user (TU), is successful in presence of other users (the interferers) is used to compute the equilibrium probabilities of that TU. This makes it possible to find the system performance from the analysis of a single TU. In this chapter mathematical expressions for the pdf of the received signal impaired by interference signal power are developed and these results are used to find the probability that the transmission by the TU will be successful in presence of interference.

The analysis assumes a multipath frequency selective channel between the STAs and the AP. Each path of the channel is assumed to fade with independent Rayleigh statistics. The pdf of the power of a each multipath component is therefore exponentially distributed [75]. Each of $N$ users with $n$ interferers, $0 \leq n \leq(N-1)$, has $M$ multipath components. The signal through path $m(1 \leq m \leq M)$ of each STA has Rayleigh statistics. The corresponding signal power pdf for the $m^{\text {th }}$ path is given as

$$
\begin{equation*}
f_{P_{m}}\left(p_{m}\right)=\frac{1}{\bar{P}_{m}} \exp \left(-\frac{p_{m}}{\bar{P}_{m}}\right) \tag{2.1}
\end{equation*}
$$

where $\bar{P}_{m}$ is the mean power of the $m^{t h}$ path.
If $P_{t}$ and $P_{n}$ represent the powers of the TU and combined power of all the other interfering STAs, using the capture model [76], [77], the TU transmission is successful only if

$$
\begin{equation*}
P_{t}>z_{0} P_{n} \tag{2.2}
\end{equation*}
$$

where $z_{0}$ is the receiver signal capture ratio. Hence the conditional success probability of the tagged user, $p_{s \mid n}$, can be written as

$$
\begin{align*}
p_{s \mid n} & =p\left[P_{t}>z_{0} P_{n}\right] \\
& =1-p\left[\frac{P_{t}}{P_{n}}<z_{0}\right] \\
& =1-p\left[Z_{n}<z_{0}\right] \\
& =1-F_{Z_{n}}\left(z_{0}\right) \quad n=0, \ldots, N-1 \tag{2.3}
\end{align*}
$$

To find $F_{Z_{n}}\left(z_{0}\right)$, following procedure is adapted

$$
\begin{aligned}
Z_{n} & \triangleq \frac{P_{t}}{P_{n}} \\
W & \triangleq P_{n}
\end{aligned}
$$

The joint pdf of $Z_{n}$ and $W$ is given as

$$
\begin{aligned}
f_{Z_{n}, W}(z, w) & =f_{P_{t}, P_{n}}\left(p_{t}, p_{n}\right)|J| \\
& =f_{P_{t}}\left(p_{t}\right) f_{P_{n}}\left(p_{n}\right)\left|\begin{array}{cc}
\frac{\partial p_{t}}{\partial z} & \frac{\partial p_{t}}{\partial w} \\
\frac{\partial p_{n}}{\partial z} & \frac{\partial p_{n}}{\partial w}
\end{array}\right| \\
& =f_{P_{t}}\left(p_{t}\right) f_{P_{n}}\left(p_{n}\right) w
\end{aligned}
$$

Here it is assumed that $P_{t}$ and $P_{n}$ are independent. The marginal pdf for $Z_{n}$ can then be computed as

$$
f_{Z_{n}}(z)=\int_{0}^{\infty} f_{P_{t}}\left(p_{t}\right) f_{P_{n}}\left(p_{n}\right) w d w
$$

The CDF for $Z_{n}$ is then found by integrating w.r.t. $z$

$$
F_{Z_{n}}(z)=\int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w
$$

The success probability is then given by

$$
\begin{align*}
p_{s \mid n} & =1-F_{Z_{n}}\left(z_{0}\right) \\
& =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \tag{2.4}
\end{align*}
$$

Proceeding further the expressions for the power pdf of the signal of interest (SOI) and the interference signal are found. Two forms of user distribution around the AP are considered.

### 2.1 Ring Distribution of STAs

Assuming power control [78], the users will be distributed in an annular ring around the receiver; the width of the ring maybe related to the accuracy of the power control. The signals received by the receiver can be combined in two different ways [75], namely the coherent (Phasor) and non-coherent (Power) addition.

### 2.1.1 Phasor sum of interference signals

If the random phases of the signals do not vary significantly during a time interval larger than the bit interval, like in slow fading case, the phasors of signals arriving from different paths may be added. As each signal experiences Rayleigh fading, their phasor sum is also another Rayleigh phasor. The power is hence exponentially distributed with an average power equal to the sum of average powers of each path. Two scenarios are possible for the signal of interest (SOI), which are discussed below.

## Case A: SOI is dominant path of Tagged User (TU) transmission

For the TU signal there are $M$ multipath components, each fades with Rayleigh statistics. The dominant path of the TU signal is taken as the SOI and the remaining $M-1$ paths act as self interfering paths. The pdf for the power of the SOI, $f_{P_{t}}\left(p_{t}\right)$, is exponential as given by (2.1) with an average power $\bar{P}_{1}$ given as $\bar{P}_{1}=\max \bar{P}_{m}, \quad 1 \leq m \leq M$.

The pdf of the combined interference power, $f_{P_{n}}\left(p_{n}\right)$, is also exponential as given
in (2.1) with an average power, $\bar{P}_{n}$, which is the sum of average powers of all the interfering paths given as $\bar{P}_{n}=(n+1) \sum_{m=1}^{M} \bar{P}_{m}-\bar{P}_{1}, \quad 0 \leq n \leq(N-1)$. The conditional success probability in this case is obtained by solving (2.4)

$$
\begin{equation*}
p_{s \mid n}=\frac{1}{1+z_{0} \frac{\overline{P_{n}}}{\overline{P_{1}}}} \tag{2.5}
\end{equation*}
$$

There is a possibility that the power of more than one STA can be greater than the coherently added power of the interfering signals simultaneously. Therefore, the conditional success probability that transmission by a STA is successful in presence of $n$ interferers and there is no other STA whose power is greater than the coherently added power of the interferers is given as

$$
\begin{align*}
p_{s \mid n} & =\frac{1}{1+z_{0} \frac{\bar{P}_{n}}{\bar{P}_{1}}}\left(1-\frac{1}{1+z_{0} \frac{\bar{P}_{n}}{\bar{P}_{1}}}\right)^{n} \\
& =\frac{\left(z_{0} \frac{\bar{P}_{n}}{\bar{P}_{1}}\right)^{n}}{\left(1+z_{0} \frac{\bar{P}_{n}}{\bar{P}_{1}}\right)^{n+1}} \tag{2.6}
\end{align*}
$$

## Case B: SOI phasor sum of all paths of TU transmission

In this case, the TU transmitted signal is coherently added at the receiver by doing phasor sum of all the received signals. The power pdf, $f_{P_{t}}\left(p_{t}\right)$, is exponentially distributed as given by (2.1) with an average power $\bar{P}=\sum_{m=1}^{M} \bar{P}_{m}$ that is the sum average powers of all the multipth components. The pdf of the total interference power, $f_{P_{t}}\left(p_{t}\right)$, in this case is exponential as given by (2.1) with an average power $\bar{P}_{n}=n \sum_{m=1}^{M} \bar{P}_{m}, \quad 0 \leq n \leq N-1$.

The conditional success probability is computed by solving (2.4)

$$
\begin{equation*}
p_{s \mid n}=\frac{1}{1+n z_{0}} \tag{2.7}
\end{equation*}
$$

Similar to the previous case, the conditional success probability that transmission by a STA is successful in presence of $n$ interferers and there is no other STA whose power is greater than the coherently added power of the interferers is given as

$$
\begin{align*}
p_{s \mid n} & =\frac{1}{1+n z_{0}}\left(1-\frac{1}{1+n z_{0}}\right)^{n} \\
& =\frac{\left(n z_{0}\right)^{n}}{\left(1+n z_{0}\right)^{(n+1)}} \tag{2.8}
\end{align*}
$$

### 2.1.2 Power sum of interference signals

When the signal phase varies rapidly, the interference signals may be combined by using power sum at the base station. Under realistic channel conditions, the phases of the interference signals vary sufficiently fast, the result is a non-coherent addition of the phasors (power sum) at the base station. As the signal from each user is composed of $M$ multipaths, the SOI at the base station can have the following two cases.

## Case A: SOI is dominant path of TU transmission

The SOI is taken to be the strongest path of the TU and the remaining $M-1$ paths of the TU act as self interference terms. Any signal component due to users
other than the tagged user acts as interference. The effect of self interference term is to increase the strength of interference signal. The pdf for the power of the SOI, $f_{P_{t}}\left(p_{t}\right)$, is exponentially distributed as given by (2.1) with an average power $\bar{P}_{1}$ given as $\bar{P}_{1}=\max \bar{P}_{m}, \quad 1 \leq m \leq M$. There are $M$ number of multipaths for each STA. Each path is fading with certain statistics and hence some paths may be present or absent at the time of packet arrival. The average power of each corresponding path being the same for all the STAs, they can be combined as power sum. This results in Gamma distribution for power pdf of each path

$$
\begin{equation*}
f_{P_{p}}\left(p_{p}\right)=\frac{1}{\bar{P}_{m}} \frac{\left(p_{p} / \bar{P}_{m}\right)^{\alpha_{m}-1}}{\Gamma\left(\alpha_{m}\right)} \exp \left(-\frac{p_{p}}{\bar{P}_{m}}\right) \tag{2.9}
\end{equation*}
$$

where

$$
\alpha_{m}=\left\{\begin{array}{cc}
n, & \text { if } m=1  \tag{2.10}\\
n+1, & \text { if } m \geq 2
\end{array}\right.
$$

$P_{p}$ is the sum of power of interfering signals in path $m$ all having same mean power and $\Gamma$ (.) represents the Gamma function, $1 \leq m \leq M$ and $0 \leq n \leq N-1$. Finally, the sum of $M$ Gamma random variables gives the required pdf of the interference power sum [79] given as

$$
\begin{equation*}
f_{P_{n}}\left(p_{n}\right)=\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \times \sum_{k=0}^{\infty} \frac{\delta_{k} p_{n}^{M(n+1)+k-2} \exp \left(-p_{n} / \bar{P}_{M}\right)}{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)} \tag{2.11}
\end{equation*}
$$

where $\bar{P}_{M}=\min \bar{P}_{m}$ for $1 \leq m \leq M$.
The coefficient $\delta_{k}$ can be found by recursively solving the following equation

$$
\begin{equation*}
\delta_{k+1}=\frac{1}{k+1} \sum_{l=1}^{k+1}\left[\sum_{j=1}^{M} \alpha_{j}\left(1-\frac{\bar{P}_{M}}{\bar{P}_{j}}\right)^{l}\right] \delta_{k+1-l} \tag{2.12}
\end{equation*}
$$

with $\delta_{0}=1$. Substituting the expression for power pdf of SOI and (2.11) in (2.4), the conditional success probability of the tagged user, $p_{s \mid n}$, is given as

$$
\begin{equation*}
p_{s \mid n}=\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \frac{1}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right)^{M(n+1)-1}} \sum_{k=0}^{\infty} \frac{\delta_{k}}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right)^{k}} \tag{2.13}
\end{equation*}
$$

## Case B: SOI power sum of all paths of TU transmission

When all paths of the multipath frequency selective channel have distinct average powers, the $M$ multipaths can be combined to form the SOI as a power sum. Each path is assumed to be independently Rayleigh faded, hence it has an exponential power distribution. The pdf for the power of the SOI is the sum of exponential random variables with distinct average powers [79] as given below,

$$
\begin{equation*}
f_{P_{t}}\left(p_{t}\right)=\sum_{j=1}^{M} \bar{P}_{j}^{M-2} \exp \left(-\frac{p_{t}}{\bar{P}_{j}}\right) \prod_{k=1, k \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{k}} \tag{2.14}
\end{equation*}
$$

where all the $\bar{P}_{j}$ are distinct. The pdf of the total power of interference signal is computed by recognizing the fact that each interfering STA has $M$ independently fading paths. The average power of each path is assumed to be unique and corresponding path of each interfering STA has the same average power. Therefore,
taking the paths of different users with same average power and combining them as power sum results in a Gamma distribution given as

$$
\begin{equation*}
f_{P_{m}}\left(p_{m}\right)=\frac{1}{\bar{P}_{m}} \frac{\left(p_{m} / \bar{P}_{m}\right)^{n-1}}{\Gamma(n)} \exp \left(-\frac{p_{m}}{\bar{P}_{m}}\right) \tag{2.15}
\end{equation*}
$$

where $1 \leq m \leq M$ and $0 \leq n \leq N-1$.
Finally, sum of these $M$ Gamma random variables gives the required pdf of the interference power sum.

$$
\begin{equation*}
f_{P_{n}}\left(p_{n}\right)=\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \sum_{k=0}^{\infty} \frac{\delta_{k} p_{n}^{M n+k-1} \exp \left(-p_{n} / \bar{P}_{M}\right)}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)} \tag{2.16}
\end{equation*}
$$

where $\bar{P}_{M}=\min \bar{P}_{m}, 1 \leq m \leq M$

$$
\begin{equation*}
\delta_{k+1}=\frac{1}{k+1} \sum_{l=1}^{k+1}\left[\sum_{j=1}^{M}(n)\left(1-\frac{\bar{P}_{M}}{\bar{P}_{j}}\right)^{l}\right] \delta_{k+1-l} \tag{2.17}
\end{equation*}
$$

and $\delta_{0}=1$. Using (2.14) and (2.16) in (2.4) the expression for $p_{s \mid n}$ is as follows:

$$
\begin{equation*}
p_{s \mid n}=\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{i=1, i \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{i}\right)} \times \frac{1}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right)^{M n}} \sum_{k=0}^{\infty} \frac{\delta_{k}}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right)^{k}} \tag{2.18}
\end{equation*}
$$

It should be noted that (2.13) and (2.18) reduce to the case of flat fading with non-coherent addition of interference derived in [62], if the paths are reduced to one.

### 2.1.3 Power Phasor Sum of Interference Signals

The phasors of all the $M$ multipaths are added (phasor sum) to form a resulting Rayleigh phasor for each interferer. So, there are $n$ resultant interfering signals where $0 \leq n \leq N-1$, one for each interferer. The pdf for the power of each of these resultant signals is exponential function as given by (2.1) with average power $\bar{P}=\sum_{m=1}^{M} \bar{P}_{m}$, where $\bar{P}_{m}$ is the average power of the $m^{t h}$ path. The pdf of the total interference power in this case is taken as the power sum of these exponential random variables

$$
\begin{equation*}
f_{P_{n}}\left(p_{n}\right)=\frac{1}{\bar{P}} \frac{\left(p_{n} / \bar{P}\right)^{n-1}}{\Gamma(n)} \exp \left(-\frac{p_{n}}{\bar{P}}\right) \tag{2.19}
\end{equation*}
$$

## Case A: SOI phasor sum of M paths of TU transmission

All the received signals through $M$ multipath for the SOI are combined as a phasor sum. The pdf for the power of the SOI is again exponential as given by (2.1) having average power $\bar{P}=\sum_{m=1}^{M} \bar{P}_{m}$, where $\bar{P}_{m}$ is the average power of the $m^{\text {th }}$ path.

The conditional success probability is derived in this case using (2.4)

$$
\begin{equation*}
p_{s \mid n}=\frac{1}{\left(z_{0}+1\right)^{n}} \tag{2.20}
\end{equation*}
$$

## Case B: SOI power sum of M paths of TU transmission

All the received signals through $M$ multipath for the SOI are combined as a power sum. The pdf for the power sum of the SOI is given below,

$$
\begin{equation*}
f_{P_{t}}\left(p_{t}\right)=\sum_{j=1}^{M} \bar{P}_{j}^{M-2} \exp \left(-\frac{P_{t}}{\bar{P}_{j}}\right) \prod_{k=1, k \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{k}} \tag{2.21}
\end{equation*}
$$

where all the $\bar{P}_{j}$ are distinct.
The conditional success probability is derived in this case using (2.4) and is given below,

$$
\begin{equation*}
p_{s \mid n}=\sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} \times \frac{1}{\left(z_{0} \frac{\bar{P}}{\bar{P}_{j}}+1\right)^{n}} \tag{2.22}
\end{equation*}
$$

### 2.1.4 Summary of results for ring distributed STAs

In Section 2.1, the mathematical formulation for the evaluation of probability that a transmission by TU is successful in presence of interfering signals is presented. Different cases for combining the interference signal are discussed in the network setting where all the STAs are distributed around the AP in form of a ring. Interference signal is combined using coherent and in-coherent addition. Furthermore, cases are discussed where the SOI is the dominant path of the TU transmission or combination of $M$ multipaths.

### 2.2 Spatial Distribution of STAs

The users are generally randomly distributed around the central AP. The average power received at the receiver is a function of the distance separating the transmitter and the receiver. Path loss models are available for different environments. The received power of the signal from the $m^{\text {th }}$ path of the $k^{\text {th }}$ STA transmitting at a distance $d$ from the receiver is given as

$$
\begin{equation*}
\bar{P}_{k, m}=K_{k} \bar{P}_{m} d^{-\beta} \tag{2.23}
\end{equation*}
$$

where $K_{k}$ is function of the receiver and transmitter antenna characteristics and $\beta$ is the path loss exponent. Assuming that the transmitter antennas are identical makes $K_{k}$ constant, which can be eliminated because the ratio of powers is considered in our capture model. So (2.23) can simply be written as

$$
\begin{equation*}
\bar{P}_{k, m}=\bar{P}_{m} d^{-\beta} \tag{2.24}
\end{equation*}
$$

The distance of the users can follow a certain distribution and some users are close to the receiver and some others can be located at a far away distance. If there is no power control available [80], then it is possible that the near/far effect can disturb the signal reception at the receiver. Also, if there is a concentration of STAs at a certain point, this can degrade the reception at the receiver. A significant number of STAs should not be present at large distances as it limits the frequency reuse and connectivity parameters of the network. Keeping these
points in view the bell-shaped user distribution is appropriate as used in [60]. The pdf for the distribution is given as in (2.25) and is shown in Figure 2.1.

$$
\begin{equation*}
f_{D}(d)=2 d \exp \left[-\frac{\pi}{4} d^{4}\right], \quad 0<d<\infty \tag{2.25}
\end{equation*}
$$

The average power pdf for $m^{t h}$ path of $k^{t h}$ STA can then be derived as

$$
\begin{align*}
f_{\bar{P}_{k, m}}\left(\bar{p}_{k, m}\right) & =f_{D}(d)\left|\frac{d}{d \bar{p}_{k, m}}(d)\right| \\
& =2\left(\frac{\bar{P}_{m}}{\bar{p}_{k, m}}\right)^{1 / \beta} \exp \left[-\frac{\pi}{4}\left(\frac{\bar{P}_{m}}{\bar{p}_{k, m}}\right)^{4 / \beta}\right] \frac{\left(\bar{P}_{m}\right)^{1 / \beta}}{\beta}\left(\bar{p}_{k, m}\right)^{-(\beta+1) / \beta} \\
& =\frac{2}{\beta}\left(\bar{P}_{m}\right)^{2 / \beta}\left(\bar{p}_{k, m}\right)^{-(\beta+2) / \beta} \exp \left[-\frac{\pi}{4}\left(\frac{\bar{P}_{m}}{\bar{p}_{k, m}}\right)^{4 / \beta}\right] \tag{2.26}
\end{align*}
$$

For $\beta=4$, which is a typical value used in land-mobile wireless channels and WiFi positioning system [81], the expression becomes

$$
\begin{equation*}
f_{\bar{P}_{k, m}}\left(\bar{p}_{k, m}\right)=\frac{1}{2}\left(\bar{P}_{m}\right)^{1 / 2}\left(\bar{p}_{k, m}\right)^{-3 / 2} \exp \left[-\frac{\pi}{4} \frac{\bar{P}_{m}}{\bar{p}_{k, m}}\right] \tag{2.27}
\end{equation*}
$$

Assuming that all the paths are Rayleigh faded, the conditional power pdf for $m^{\text {th }}$ path of $k^{\text {th }}$ STA is given as

$$
\begin{equation*}
f_{P_{k, m}}\left(p_{k, m} \mid \bar{p}_{k, m}\right)=\frac{1}{\bar{p}_{k, m}} \exp \left[-\frac{p_{k, m}}{\bar{p}_{k, m}}\right], \quad 1 \leq k \leq N, 1 \leq m \leq M \tag{2.28}
\end{equation*}
$$

The different cases for power of the SOI and the interference discussed for STAs distributed in a ring around the central base station are discussed in the following


Figure 2.1: Bell shaped user distribution pdf
sections with bell shaped spatial distribution. The pdf for the power of the SOI and interferers is as given below.

$$
\begin{gather*}
f_{P_{t}}\left(p_{t}\right)=\int_{0}^{\infty} f_{P_{t}}\left(p_{t} \mid \bar{p}_{t}\right) f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t}  \tag{2.29}\\
f_{P_{n}}\left(p_{n}\right)=\int_{0}^{\infty} f_{P_{n}}\left(p_{n} \mid \bar{p}_{n}\right) f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \tag{2.30}
\end{gather*}
$$

where $p_{t}$ and $p_{n}$ are the power of the SOI and the interferers.

### 2.2.1 Phasor sum of interference signals

The scenario discussed here is different from the one discussed in Section 2.1.1 as the average power for each path now also depends on the distance between the STA and the AP as given in (2.27). As all the signals experience Rayleigh fading, their phasor sum is also another Rayleigh phasor. The power is hence exponentially distributed with an average power equal to the sum of average powers of each path. Again two scenarios are possible for the signal of interest (SOI), which are discussed below.

## SOI is dominant path of TU transmission

This case assumes that the SOI consists of the dominant path of the TU transmitted signal. The conditional power pdf, $f_{P_{t}}\left(p_{t} \mid \bar{p}_{t}\right)$, is exponential as given by (2.1) with $\bar{p}_{t}$ as average power of SOI and the pdf of the average power is as given
in (2.27).

$$
\begin{equation*}
f_{\bar{P}_{t}}\left(\bar{p}_{t}\right)=\frac{1}{2}\left(\bar{P}_{1}\right)^{1 / 2}\left(\bar{p}_{t}\right)^{-3 / 2} \exp \left[-\frac{\pi}{4} \frac{\bar{P}_{1}}{\bar{p}_{t}}\right] \tag{2.31}
\end{equation*}
$$

The average power of the interference signal is the sum of the average power of $k$ users each with $M$ multipath signals. In addition the remaining paths of the TU also add up to the average interference signal power. The pdf of the total interference power is obtained by convolving the pdfs of all the individual interfering signals. Following Laplace transform pair is used

$$
\begin{equation*}
\frac{K t^{-3 / 2}}{2 \sqrt{\pi}} \exp \left[-\frac{K^{2}}{4 t}\right] \leftrightarrow \exp [-K \sqrt{s}] \tag{2.32}
\end{equation*}
$$

Comparing (2.32) with (2.27), the Laplace transform of the power pdf for $k^{\text {th }}$ user and $m^{\text {th }}$ path is given as

$$
\begin{equation*}
\mathcal{L}\left[f_{P_{k, m}}\left(p_{k, m}\right)\right]=\exp \left[-\sqrt{\pi \bar{P}_{m} s}\right] \tag{2.33}
\end{equation*}
$$

$M$ power pdfs for each of the $n$ interferers and another $M-1$ power pdfs for the TU transmission are convolved together. This gives us the following convolution for the interference power signal

$$
\begin{equation*}
\mathcal{L}\left[f_{\bar{P}_{n}}\left(\bar{p}_{n}\right)\right]=\exp \left[-(n+1) \sum_{m=1}^{M} \sqrt{\pi \bar{P}_{m} s}+\sqrt{\left.\pi \bar{P}_{1} s\right]}\right. \tag{2.34}
\end{equation*}
$$

Taking the inverse Laplace transform, the power pdf for the interference signal is
given as,

$$
\begin{equation*}
f_{\bar{P}_{n}}\left(\bar{p}_{n}\right)=\frac{1}{2} K_{t_{n}}\left(\bar{p}_{n}\right)^{-3 / 2} \exp \left[-\frac{\pi K_{t_{n}}^{2}}{4 \bar{p}_{n}}\right] \tag{2.35}
\end{equation*}
$$

where

$$
\begin{equation*}
K_{t_{n}}=(n+1) \sum_{m=1}^{M} \sqrt{\bar{P}_{m}}-\sqrt{\bar{P}_{1}} \tag{2.36}
\end{equation*}
$$

Equation (2.4) can be written as

$$
\begin{align*}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} d z \int_{0}^{\infty} d w \int_{0}^{\infty} w f_{P_{t}}(z w) \int_{0}^{\infty} f_{P_{n}}(w) \\
= & \int_{z_{0}}^{\infty} d z \int_{0}^{\infty} d w \int_{0}^{\infty} \frac{w}{\bar{p}_{t}} \exp \left(-\frac{z w}{\bar{p}_{t}}\right) f_{\bar{p}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
& \quad \int_{0}^{\infty} \frac{1}{\bar{p}_{n}} \exp \left(-\frac{w}{\bar{p}_{n}}\right) f_{\bar{p}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \tag{2.37}
\end{align*}
$$

Solving (2.37) gives the success probability given as

$$
\begin{equation*}
p_{s \mid n}=\frac{1}{1+K_{t_{n}} \sqrt{z_{0} / \overline{P_{1}}}} \tag{2.38}
\end{equation*}
$$

## SOI is phasor sum of M paths of TU transmission

In this case it is assumed that the TU transmitted signal can be coherently added at the receiver by doing phasor sum of the received signals. The conditional power pdf $f_{P_{t}}\left(p_{t} \mid \bar{p}_{t}\right)$ is exponential with $\bar{p}_{t}$ as average power of SOI and is equal to the
sum of average powers of all the multipaths of the TU transmission.

$$
\begin{equation*}
\bar{p}_{t}=\sum_{m=1}^{M} \bar{p}_{T U, m} \tag{2.39}
\end{equation*}
$$

The pdf of the total average power in (2.39) can be found by convolving the power pdf of each path of TU signal which is given in (2.27). For this the Laplace transform pair (2.32) is used.

$$
\begin{equation*}
f_{\bar{P}_{t}}\left(\bar{p}_{t}\right)=\frac{K_{t}\left(\bar{p}_{t}\right)^{-3 / 2}}{2 \sqrt{\pi}} \exp \left[-\frac{K_{t}^{2}}{4 \bar{p}_{t}}\right] \tag{2.40}
\end{equation*}
$$

where

$$
\begin{equation*}
K_{t}=\sum_{m=1}^{M} \sqrt{\pi \bar{P}_{m}} \tag{2.41}
\end{equation*}
$$

The interferers signals will also be combined as phasor sum and hence result in an exponential distribution of total power. The conditional power pdf for the interference signal, $f_{P_{n}}\left(p_{n} \mid \bar{p}_{n}\right)$, is exponential as given by (2.1) with average power $\bar{p}_{n}$. For each STA the pdf of the sum of average powers of its multipaths is same as in (2.40). The average powers of all the STAs is added to get the total average power of the interference signal by convolving the pdf of a user $n$ times.

$$
\begin{equation*}
f_{\bar{P}_{n}}\left(\bar{p}_{n}\right)=\frac{n K_{t}\left(\bar{p}_{n}\right)^{-3 / 2}}{2 \sqrt{\pi}} \exp \left[-\frac{n^{2} K_{t}^{2}}{4 \bar{p}_{n}}\right] \tag{2.42}
\end{equation*}
$$

Equation (2.4) can be written as

$$
\begin{align*}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} d z \int_{0}^{\infty} d w \int_{0}^{\infty} w f_{P_{t}}(z w) \int_{0}^{\infty} f_{P_{n}}(w) \\
= & \int_{z_{0}}^{\infty} d z \int_{0}^{\infty} d w \int_{0}^{\infty} \frac{w}{\bar{p}_{t}} \exp \left(-\frac{z w}{\bar{p}_{t}}\right) f_{\bar{p}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
& \quad \int_{0}^{\infty} \frac{1}{\bar{p}_{n}} \exp \left(-\frac{w}{\bar{p}_{n}}\right) f_{\bar{p}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \tag{2.43}
\end{align*}
$$

Solving (2.43) gives the success probability given as

$$
\begin{equation*}
p_{s \mid n}=\frac{1}{1+n \sqrt{z_{0}}} \tag{2.44}
\end{equation*}
$$

Here again there is a possibility of more than one STA having power more than the coherently added power of the interfering signals. Therefore, the conditional success probability is given as

$$
\begin{align*}
p_{s \mid n} & =\frac{1}{1+i \sqrt{z_{0}}}\left(1-\frac{1}{1+n \sqrt{z_{0}}}\right)^{n} \\
& =\frac{\left(n \sqrt{z_{0}}\right)^{n}}{\left(1+n \sqrt{z_{0}}\right)^{(n+1)}} \tag{2.45}
\end{align*}
$$

Table 2.1: Channel model B for NLOS mode

|  |  |  |  |  |  | , |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Tap index | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|  | Excess delay [ns] | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 |
| Cluster 1 | Power [dB] | 0 | -5.4 | -10.8 | -16.2 | -21.7 |  |  |  |  |
| AoA | $\begin{gathered} \text { AoA } \\ {\left[{ }^{\circ}\right]} \\ \hline \end{gathered}$ | 4.3 | 4.3 | 4.3 | 4.3 | 4.3 |  |  |  |  |
| $\begin{aligned} & \hline \text { AS } \\ & \text { (rx) } \\ & \hline \end{aligned}$ | $\begin{aligned} & \hline \text { AS } \\ & {\left[{ }^{\circ}\right]} \\ & \hline \end{aligned}$ | 14.4 | 14.4 | 14.4 | 14.4 | 14.4 |  |  |  |  |
| AoD | $\begin{gathered} \text { AoD } \\ {\left[{ }^{\circ}\right]} \\ \hline \end{gathered}$ | 225.1 | 225.1 | 225.1 | 225.1 | 225.1 |  |  |  |  |
| $\begin{aligned} & \hline \text { AS } \\ & (\mathrm{tx}) \end{aligned}$ | $\begin{aligned} & \hline \text { AS } \\ & {\left[{ }^{\circ}\right]} \\ & \hline \end{aligned}$ | 14.4 | 14.4 | 14.4 | 14.4 | 14.4 |  |  |  |  |
| Cluster 2 | Power [dB] |  |  | -3.2 | -6.3 | -9.4 | -12.5 | -15.6 | -18.7 | -21.8 |
| AoA | $\begin{gathered} \text { AoA } \\ {\left[{ }^{\circ}\right]} \\ \hline \end{gathered}$ |  |  | 118.4 | 118.4 | 118.4 | 118.4 | 118.4 | 118.4 | 118.4 |
| AS | $\begin{gathered} \text { AoA } \\ {\left[{ }^{\circ}\right]} \\ \hline \end{gathered}$ |  |  | 25.2 | 25.2 | 25.2 | 25.2 | 25.2 | 25.2 | 25.2 |
| AoD | $\begin{gathered} \mathrm{AoA} \\ {\left[{ }^{\circ}\right]} \\ \hline \end{gathered}$ |  |  | 106.5 | 106.5 | 106.5 | 106.5 | 106.5 | 106.5 | 106.5 |
| AS | AoA $\left[{ }^{\circ}\right]$ |  |  | 25.4 | 25.4 | 25.4 | 25.4 | 25.4 | 25.4 | 25.4 |

### 2.3 Results

The success probability for different scenarios discussed in this chapter are shown in Figures 2.2, 2.3 and 2.4. The success probability is dependent on the channel model used. In these plots channel model B recommended by IEEE 802.11 WLAN standard [82] is considered, in non-line-of-sight (NLOS) mode. This channel model has 9 Rayleigh-fading paths as shown in Table 2.1. The model comprises of two clusters, $1^{\text {st }}$ cluster has 5 paths with delays from 0 t0 40 nsec (in steps of 10 nsec ) and the $2^{\text {nd }}$ cluster contains 7 paths with delays of 20 to 80 nsec (in steps of 10 nsec). There are 3 overlapping paths between the two clusters. The results show


Figure 2.2: Success probability in presence of $n$ interferers


Figure 2.3: Success probability in presence of $n$ interferers with phasor sum that the success probability is very small for the all the cases where the SOI is only the dominant path of the TU transmission. This is because the signal is split into 9 paths and hence one path carries only a fraction of the total power


Figure 2.4: Success probability in presence of $n$ interferers with power sum resulting in a very small success probability even without any interferer present. The other case where SOI is due to the combined paths, the phasor sum gives a better success probability as compared to power sum. The success probability is 1 when there is no interfering signal in this case. When bell distributed users are considered, the average power of the STAs varies which results in a better success probability. So it is concluded that bell shaped STA distribution with phasor sum of interference signals results in the best chance of a successful transmission.

### 2.4 Summary

In this chapter the effect of frequency selective channel on the success probability of transmission by TU is discussed. Two different configurations for the placement of STAs around the AP, the ring distribution and the bell-shaped distribution,
are considered. Also different scenarios for combining the interference signals have been considered, including the phasor sum and the power sum. Finally, analysis has been done for the case when SOI is the dominant path of the TU transmission or is formed by the combination of all the paths of TU transmitted signal.

## CHAPTER 3

## APPLICATION OF TAGGED

## USER ANALYSIS TO SLOTTED

## ALOHA PERFORMANCE

Slotted ALOHA (S-ALOHA) is a simple and straight forward random multiple access technique, which has been used extensively in data and cellular networks as the protocol for random access. In analyzing finite user finite buffer (FU-FB) systems using Markov chains, the state space in the exponential order of buffer size and number of users is needed, hence making the analysis of FU-FB systems complex. An approximate analysis technique, tagged user analysis (TUA), has been used for evaluation of performance parameters of S-ALOHA over multipath and frequency selective fading channels for FU-FB systems. The analytical results are compared with simulation results.

### 3.1 Introduction

In this chapter, TUA method is applied to the analysis of more practical scenarios with FU-FB S-ALOHA system operating over multipath frequency selective fading radio channels and derive analytical expressions for system performance indices. It is shown that for a moderate number of active users, the simulation and analytical results fit closely demonstrating the accuracy of TUA method.

The chapter organization is as follows. The system model is explained in Section 3.2 and the TUA technique is elaborated in Section 3.3. The performance metrics and the iterative algorithm of TUA to compute these metrics are discussed in Section 3.4. Optimal selection for the channel access probability is discussed in Section 3.5. The numerical results from analysis (TUA) and simulation are compared in Section 3.6. The chapter is concluded in Section 3.7.

### 3.2 System Model

A homogeneous centralized S-ALOHA system in which a finite population of $N$ users is distributed randomly on a ring around the base station (equal pathloss) is considered. Each user has a finite buffer capacity of $L$ packets. A homogeneous system, i.e., system parameters like channel transmission probability, buffer size and packet arrival rate are same for all users, is considered. The packet arrival is assumed to be a Bernoulli process [26] with an average arrival rate of $\lambda$ packets per slot and the system traffic model is Poisson process. This model is widely used in literature, e.g., [40],[41],[83]-[86]. The channel access is assumed to have


Figure 3.1: State flow diagram of a tagged user
a geometric distribution with access probability, $p$. The channel is assumed to be frequency selective. If more than one user attempt to transmit packets during a given time slot, the user whose power exceeds the total power of all other contending users by the capture threshold, $z_{0}$, is deemed successful [87]. If none of the transmitting user fulfils the capture threshold, all users are deemed unsuccessful. Assuming a sufficiently high signal to noise ratio the effect of thermal noise is ignored.

Figure 3.1 shows the state flow graph for the tagged user (TU) under steady state condition. The TU can be modeled as a Geo/G/1/K queueing system [60]. The state flow graph of Figure 3.1 has four states; packet ready, $S_{p r}$, transmission start, $S_{t s}$, back off, $S_{b o}$, and packet departure, $S_{d}$. The state $S_{d}$ represents a successful transmission. A packet is in the virtual server when it is marked ready for transmission. The virtual packet service time (VPST) is defined as the time taken by the user in moving from state $S_{p r}$ to state $S_{d}$. The time required to move from one state to the other is indicated by the power of the parameter $z$. The user can be in any one of the above states at the start of a time slot. The late arrival
with delayed replacement rule [89] is followed. Thus, an arriving packet is dropped if the queue of the user is full, otherwise it enters the queue. A packet entering the queue has to wait until all the previous packets in the queue are successfully transmitted before it moves to the head-of-queue position. When a packet at the head-of-queue enters the virtual server, the user moves to state $S_{p r}$. From $S_{p r}$ the user moves to the state $S_{b o}$ without any delay. The user can also enter state $S_{b o}$ from state $S_{t s}$ if the previous transmission attempt was unsuccessful.

Defer first transmission (DFT) strategy has slight advantage over immediate first transmission (IFT) strategy for S-ALOHA system [90]. Therefore, the DFT principle is used. A user moves to the transmission state, $S_{t s}$, with a probability $p$ or remains in the same state for another slot with a probability $(1-p)$. A user in state $S_{t s}$ either moves to state $S_{b o}$, in case of unsuccessful transmission, or to state $S_{d}$, in case of successful transmission. If the user moves to state $S_{b o}$, it will attempt retransmission of its packet with probability $p$ while if it goes to state $S_{d}$, it removes the packet from the virtual server after $T$ slots. The user has to wait $D-1$ slots in state $S_{t s}$ for the acknowledgement of a successful transmission. The acknowledgements are assumed to be received error free. In case no acknowledgement is received by the user, the packet is marked for retransmission.

### 3.3 Tagged User Analysis

Under the assumptions that each user behaves as an independent queueing system operating at its equilibrium probabilities and the channel is symmetric, the system
performance is analyzed by the analysis of a single user. This single user is termed as the tagged user (TU).

TUA framework decouples the contention analysis from the queueing process analysis. This is done by modeling the coupling between users by a contention probability parameter. This contention probability is a function of parameters such as channel access probability of a user, the channel model and the capture ratio requirement for successful transmission. It also depends on the queue size of users, the packet arrival rate and the packet service policy. All these parameters in turn decide the packet success probability and hence the service time distribution of a packet undergoing multiple transmission attempts till it gets through. Nevertheless, the overall performance depends on the queueing process as much as on the channel access protocol. This coupling is taken care of in the iterative algorithm. The analysis can be divided into contention analysis and queueing analysis.

### 3.3.1 Channel Contention Analysis

The contention analysis deals with the determination of VPST distribution, which is dependent on the number of users and interference from these users, their busy probabilities and channel conditions. Using state flow graph techniques, the expression for probability generating function (PGF) of VPST, $B(z)$, is derived from the model described by the state transition diagram shown in Figure 3.1 under the assumption that the system is in steady state. Using Mason's formula
for state flow graphs, the final expression can be expressed as

$$
\begin{equation*}
B(z)=\frac{p p_{s} z^{T+1}}{1-(1-p) z-p\left(1-p_{s}\right) z^{D+1}} \tag{3.1}
\end{equation*}
$$

where $T$ is the packet transmission time and for S-ALOHA it is equal to duration of a slot i.e. $T=1$. After packet transmission, a busy user can be in any one of the two modes: CON (contending for the channel) or WAI (waiting for acknowledgement) that takes $D-1$ slots. Let $p_{c}$ be the probability that the user is in CON mode, then

$$
\begin{equation*}
p_{s}=\sum_{n=0}^{N-1}\binom{N-1}{n}\left(p p_{c}\right)^{n}\left(1-p p_{c}\right)^{N-1-n} p_{s \mid n} \tag{3.2}
\end{equation*}
$$

where, $p_{s \mid n}$ is the probability that the transmission is a success given there are $n$ interfering users. This probability is dependent on the channel conditions and the number of simultaneous transmissions from other users. From (3.1) the mean packet service time is given by

$$
\begin{equation*}
\dot{B}(1)=b=(T-D)+\frac{D}{p_{s}}+\frac{1}{p p_{s}} \tag{3.3}
\end{equation*}
$$

On average a user waits for acknowledgment for $\frac{D-1}{p_{s}}$ slots, hence

$$
\begin{equation*}
p_{c}=\frac{p_{b}}{b}\left(b-\frac{D-1}{p_{s}}\right) \tag{3.4}
\end{equation*}
$$

User busy probability, $p_{b}$ and user idle probability, $p_{0}$, satisfy the following relation

$$
\begin{equation*}
p_{b}+p_{0}=1 \tag{3.5}
\end{equation*}
$$

Using (3.1) to (3.4), $B(z)$ for the given $p_{c}$ can be computed.

### 3.3.2 Queueing Analysis

The queueing analysis is applied to determine the user busy probabilities, given the packet service time distribution and arrival process. As stated earlier, TUA allows us to utilize existing results of queueing theory. These results are available in standard queueing theory literature. These relations are non-linear but sufficient to determine VPST and the user busy probabilities. In order to find $p_{b}$ from $B(z)$, the algorithm for Geo/G/1/K system given in [89] is used. The related equations are reproduced here for easy reference. At most $L-1$ packets can arrive in one service time, so the PGF for $a_{k}$, which is the probability that $k$ packets arrive during a service time, is given as

$$
\begin{equation*}
A(z)=\sum_{k=0}^{L-1} a_{k} z^{k}=B(1-\lambda+\lambda z) \tag{3.6}
\end{equation*}
$$

Let $\pi_{k}, k=0,1, \cdots, L-1$, be the probability that a leaving packet sees $k$ packets in queue, from [88]

$$
\begin{align*}
& \pi_{k}=\pi_{0} a_{k}+\sum_{j=1}^{k+1} \pi_{j} a_{k-j+1}  \tag{3.7}\\
& \pi_{0}=\left(\sum_{k=0}^{L-1} \dot{\pi}_{k}\right)^{-1} \tag{3.8}
\end{align*}
$$

where

$$
\begin{equation*}
\pi_{k}^{\prime}=\frac{\pi_{k}}{\pi_{0}} \tag{3.9}
\end{equation*}
$$

and is computed using following recursive equations

$$
\begin{align*}
\pi_{0}^{\prime} & =1  \tag{3.10}\\
\pi_{k+1}^{\prime} & =\frac{1}{a_{0}}\left(\pi_{k}^{\prime}-\sum_{j=1}^{k} \pi_{j}^{\prime} a_{k-j+1}-a_{k}\right) \tag{3.11}
\end{align*}
$$

Defining $p_{k}$, as the probability that there are $k$ packets present in the system at any slot boundary, for $0 \leq k \leq L-1$,

$$
\begin{align*}
p_{k} & =\frac{\pi_{k}}{\pi_{0}+\rho}  \tag{3.12}\\
p_{L} & =1-\frac{1}{\pi_{0}+\rho} \tag{3.13}
\end{align*}
$$

where

$$
\begin{equation*}
b=\dot{B}(1) \quad \text { and } \quad \rho=\lambda b \tag{3.14}
\end{equation*}
$$

Equations (3.1) to (3.4) show us that $B(z)$ is a function of $p_{c}$ or equivalently $p_{b}$, while queueing theory results of (3.6) to (3.14) affirm the dependence of $p_{b}$ on $B(z)$. These relationships are independent and hence they can be solved simultaneously. However, due to non-linearity of the equations, numerical methods have to be used. The algorithm used to solve the problem is discussed in section 3.4.2. If the system has a unique operating (global equilibrium) point, different initial values for $p_{c}$ will converge to the same result. In the case of multiple operating (local equilibrium) points (e.g., bistable behavior), the results will be affected by the initial value used in the algorithm.

### 3.4 Performance Measures and Iterative algorithm

In this section, the closed form analytical expressions for various performance measures of interest in a random access system using TUA are given [54] and the iterative algorithm used to derive the analytical results is listed.

### 3.4.1 Performance Measures

## Blocking Probability

Blocking probability, $p_{B}$, is the probability that an arriving packet finds the queue full and is given as

$$
\begin{equation*}
p_{B}=p_{L}=1-\frac{1}{\pi_{0}+\rho} \tag{3.15}
\end{equation*}
$$

## System Throughput

The system throughput, $\Theta$, is defined as the average number of packets transmitted per slot.

$$
\begin{equation*}
\Theta=N \theta \tag{3.16}
\end{equation*}
$$

where $\theta$ is the throughput of TU. For the system under study, any packet accepted into the queue of the TU will eventually be transmitted. The TU's throughput can be computed in terms of the average rate of packet acceptance in the queue of the TU and the mean packet transmission time. Mathematically,

$$
\begin{equation*}
\theta=\lambda\left(1-p_{B}\right) T \tag{3.17}
\end{equation*}
$$

where $\lambda\left(1-p_{B}\right)$ is the average packet acceptance rate. Recall that $b$ denotes the mean packet service time. The user throughput can be expressed as a ratio of the percentage of time when the tagged user is busy to mean packet service time, i.e.,

$$
\begin{equation*}
\theta=\frac{p_{b}}{b} \tag{3.18}
\end{equation*}
$$

## Mean Queue Length

Mean queue length is the average number of packets in the queue of a user of maximum queue size $L$. The mean queue length is given by

$$
\begin{equation*}
E\left[I_{q}\right]=\sum_{k=0}^{L} k p_{k} \tag{3.19}
\end{equation*}
$$



Figure 3.2: Time line of packet transmission

## Mean Packet Response time

Packet response time is defined only for those packets that are accepted into the system. It is the total time spent by the packet from its arrival into the queue to its successful departure from the virtual server. Figure 3.2 gives a pictorial description of the packet response time. The mean packet response time, $E\left[t_{r}\right]$, is given by

$$
\begin{equation*}
E\left[t_{r}\right]=\frac{E\left[I_{q}\right]}{\lambda\left(1-p_{B}\right)} \tag{3.20}
\end{equation*}
$$

## Mean Waiting delay

It is the time spent by a packet in the queue, i.e., from its arrival into the queue to its being ready for transmission. It is given by

$$
\begin{equation*}
E\left[t_{w}\right]=\frac{E\left[I_{q}\right]}{\lambda\left(1-p_{B}\right)}-b \tag{3.21}
\end{equation*}
$$

where $E\left[t_{w}\right]$ is the mean waiting delay.

### 3.4.2 The Algorithm

The expression of PGF of the VPST in (3.1) can be determined from (3.2) to (3.14). The numerical iterative algorithm to solve these equations is listed in Algorithm 1.

```
Algorithm 1 Tagged User Analysis
Require: Initial guess \(0 \leq p_{c}(0) \leq 1, \mathrm{n}=1, \delta=10^{-8}\).
Ensure: Contention probability \(p_{c}(n)\).
\(p_{s} \leftarrow(3.2)\)
\(b \leftarrow(3.3)\)
\(\rho \leftarrow(3.14)\)
\(\pi_{k}^{\prime} \leftarrow(3.10)\) and (4.23)
\(\pi_{k} \leftarrow(3.9)\)
\(p_{k} \leftarrow\) (4.24) and (4.25)
\(p_{b} \leftarrow 1-p_{0}\)
\(p_{c}(n) \leftarrow(3.4)\)
if \(\left|p_{c}(n)-p_{c}(n-1)\right| \leq \delta\) then
        STOP
    else
        \(\mathrm{n}=\mathrm{n}+1\)
        GOTO 1
    end if
```


### 3.5 Selection of Optimum system operating

## point

Although there is no stability issue for finite buffer systems [1], it is important to analyze the optimal selection of channel access probability, in order to maximize the system throughput and achieve a globally stable equilibrium point. The packet transmission time, $T$, and the acknowledgement delay, $D$, are assumed to be equal
to 1 . This is true for S-ALOHA where the duration of a slot is equal to packet transmission time. This simplifies the expressions for the mean packet service time in (3.3), user contention probability $p_{c}$ in (3.4) and user success probability $p_{s}$ in (3.2) as follows,

$$
\begin{gather*}
b=\frac{p+1}{p p_{s}}  \tag{3.22}\\
p_{c}=p_{b}=1-p_{0}  \tag{3.23}\\
p_{s}=\sum_{n=0}^{N-1}\binom{N-1}{n}\left[p\left(1-p_{0}\right)\right]^{n}\left[1-p\left(1-p_{0}\right)\right]^{N-1-n} p_{s \mid n} \tag{3.24}
\end{gather*}
$$

Using (3.18) the following relation can be derived,

$$
\begin{equation*}
p_{0}=1-\theta b \tag{3.25}
\end{equation*}
$$

From (3.25) and (3.22),

$$
\begin{equation*}
p_{0}=1-\frac{\theta(p+1)}{p p_{s}} \tag{3.26}
\end{equation*}
$$

Equation (3.26) has $N$ solutions for $p_{0}$, out of which only those solutions are sensible where $p_{0} \in[0,1]$. Let us assume $y \in[0,1]$ and is given by

$$
\begin{equation*}
y=p\left(1-p_{0}\right) \tag{3.27}
\end{equation*}
$$

Using (3.24), (3.26), and (3.27)

$$
\begin{equation*}
\theta=\frac{1}{(p+1)} y \sum_{n=0}^{N-1}\binom{N-1}{n} y^{n}(1-y)^{N-1-n} p_{s \mid n} \tag{3.28}
\end{equation*}
$$

The value of $y$ for which the throughput is maximized can be found using (3.28) by letting $\frac{d \theta}{d y}=0$. One solution is $y=0$, that implies that either $p=0$ suggesting that there is no permission to access the channel, or $p_{0}=1$ which suggests that the user is always idle. In both cases $\theta$ is zero. The second possible solution is $y=1$, in which case $p=1$ and $p_{0}=0$. This yields $\theta=0$ which means that although the user is always busy yet there is no successful packet transmitted. The only solution that is reasonable that maximizes the throughput, $y=y_{\max }$, is given by

$$
\begin{equation*}
y_{\max }=\frac{1}{N} \frac{\sum_{n=0}^{N-1}\binom{N-1}{n}(n+1) p_{s \mid n}}{\sum_{n=0}^{N-1}\binom{N-1}{n} p_{s \mid n}} \tag{3.29}
\end{equation*}
$$

where $y_{\max }$ is the value of $y$ for which the throughput is maximized. Using (3.27) at $y=y_{\text {max }}$, the following relation is achieved

$$
\begin{equation*}
p_{0}=1-\frac{y_{\max }}{p} \tag{3.30}
\end{equation*}
$$

The value of $p$ should be greater than or equal to $y_{\max }$ in order to obtain $p_{0} \in$ $[0,1]$. This provides the lower bound on $p$ as $y_{\max }$ such that $p \in\left[y_{\max }, 1\right]$. The
lower bound of $p$ for the case of ideal channel is reported in [1] as $\frac{1}{N}$. Here for the frequency selective case with SOI as one dominant path, it is $\frac{1.28}{N}$, which is higher than the ideal channel case. This is expected because of the capture phenomena in frequency selective fading channel. In order to find the upper limit for channel access probability, the fluid approximation trajectories for S-ALOHA with finite buffer capacity as shown in Figure 3.3 are used. The analysis is done using throughput and load lines following an idea similar to [1]. The curves obtained using (3.17) are called the load lines, each corresponding to a packet arrival rate of the tagged user. The throughput line describes the packet output rate or user throughput, which is given by (3.28) (derived using (3.18)). This curve describes the relation between the user throughput and its idle probability $p_{0}$ for a given channel access probability $p$. The intersection of throughput line with the load line defines the equilibrium point for the system. The solid and the dashed lines in Figure 3.3 represent the throughput and the load lines respectively.

Figure 3.3 shows load lines for different values of $\lambda$ with $p>y_{\max }$. There is only one equilibrium point for the system if the value of $\lambda$ is either relatively small like $\lambda=0.002$ and 0.0025 , or large like $\lambda=0.004$. In the former case the channel idle probability is large, whereas the later case exhibits a smaller channel idle probability and hence a longer packet response time. For the in between values of $\lambda$ there is a region where three different equilibrium points exist. Among these points the one with the largest value of channel idle probability is the desired equilibrium point. For $\lambda=0.00326$ and $\lambda=0.0036$ the two equilibrium points


Figure 3.3: Fluid approximation trajectories for $p=0.03>y_{\text {max }}$
overlap and the system behaves as it has only one global equilibrium point. For example when $\lambda=0.00326$ point $B$ is the globally stable point and for $\lambda=0.0036$ point E behaves as the globally stable equilibrium point. But point E corresponds to a very large packet response time.

Hence, from Figure 3.3 it can be seen that for the given $p$, the upper limit for $\lambda$ is 0.00326 so that the system has one globally stable equilibrium point. Also, for a given $\lambda$ the line passing through B and D gives the maximum possible value for $p$. Note that the lower limit for $p$ is $y_{\text {max }}$.

### 3.6 Numerical Results and Discussion

The simulation scenario considers a wireless slotted ALOHA system where a homogeneous group of users attempts to communicate with a base station. The number of users in the system, $N$, is taken as 100 . For simplicity, the users are assumed to be distributed uniformly on a ring around the base station, which is equivalent to some form of power control implementation, to counter shadowing. The packet arrival is assumed to be a Bernoulli process. Each user has a length $L$ queue. If the queue of a particular user is full, the incoming packet is dropped, otherwise it enters the queue. Once a packet enters the queue, it will stay in the queue until it is successfully transmitted and an acknowledgement is received by the user.

Based on the availability of a packet in the queue and channel access probability, $p$, each user will attempt to transmit its packets. It is assumed that all users
are contending for a single channel for transmission. In case of multiple users trying to access the channel during the same time slot, the user with power exceeding the sum of total power of other active users by a certain threshold (capture ratio $\left.z_{0}\right)$ is deemed successful. The simulations are carried out for a capture ratio of $z_{0}=4 \mathrm{~dB}$, but other capture ratios may also be used. For a successfully transmitted packet, the user receives the acknowledgement $D-1$ slot later. It is assumed that the acknowledgment is received error free. While waiting for acknowledgement, the user is in idle state, i.e., it will not attempt to capture the channel. If a user does not receive an acknowledgement within $D-1$ slots, it assumes the packet transmission was unsuccessful and marks the packet for retransmission.

The channel is assumed to be frequency selective and the interference signals are combined using power sum as discussed in Section 2.1.2. The ITU channel model for outdoor to indoor and pedestrian test environment as given in Table 3.1 is used. The system is simulated for 100,000 time slots and averaged over 10 runs. Note that TUA is applied to a single user and its results are extrapolated to the whole system to get the analytical system performance while simulation is carried out for the entire system of $N$ users to get the simulated system performance.

Table 3.1: ITU channel model for outdoor to indoor and pedestrian test environment

| Taps | Relative delay <br> $(\mathrm{ns})$ | Average power <br> $(\mathrm{dB})$ | Doppler <br> spectrum |
| :---: | :---: | :---: | :---: |
| 1 | 0 | 0 | Classic |
| 2 | 110 | -9.7 | Classic |
| 3 | 190 | -19.2 | Classic |
| 4 | 410 | -22.8 | Classic |

### 3.6.1 SOI is dominant path of TU transmission

Figure 3.4(a) -3.4(d) show a set of curves for the case when SOI comprises of only the strongest path of the dominant user. The markers are TUA results while the lines depict the simulation results. These figures show the throughput, packet response time, blocking probability and waiting delay for a range of channel access probabilities with packet arrival rate $\lambda=0.0035,100$ users $(N=100)$ and two different buffer sizes ( $L=1$ and $L=8$ ). Buffer size $L=1$ corresponds to a system with no queue. So if user $k$ has a packet to serve, it will not accept any more packets until the current packet is successfully transmitted and its acknowledgement is received.

In the analysis, the algorithm starts with an assumed initial condition for user busy probability, $p_{b}$. Simulations were carried out with initial condition that the user queue is either empty or full at start. The empty markers are for initial condition that the user is idle and filled markers for the initial condition that the user is busy. Dashed lines correspond to the simulation results of initially empty queue while solid lines correspond to those of initially full queue.

From Figure 3.4(a), it can be seen that for $L=8$, throughput is reasonably high (more than 0.3355 packets/slots) for $p$ between 0.012 and 0.0226 . The throughput decreases dramatically outside this range. The explanation of this behavior is as follows: The channel access probability, $p$, corresponds to the likelihood of a user making a transmission attempt. Low $p$ means that most users will be in idle state while high $p$ means most users will attempt to transmit in a given


Figure 3.4: Analytic and simulation results for an S-ALOHA system with immediate acknowledgement, non-coherent addition, SOI dominant path only. $N=100$, $\lambda=0.0035$
time slot, provided they have a packet in their queue. As shown in Figure 3.4(c), a low value of $p$ reduces the chances of collision but it will cause the user queue to fill up. This results in a large waiting delay and packet response time as shown in Figures 3.4(b) and 3.4(d). A high $p$ results in more active users per time slot which increases the interference from other users as well as self interference. This leads to reduced throughput, and an increase in both waiting delay and packet response time. The limits on practical values of $p$ is set by packet response time based on acceptable latency in packet transmission as shown in Figure 3.4(d). For a system with $L=8$, the minimum packet response time occurs for $p=0.0226$. There is a good match between the TUA and simulation results except for the mismatch at $p=0.065$ for the $L=8$ case with initial queue full conditions. This is explained by the fact that at this value of $p$, the system is bistable and oscillates between the two equilibrium points. So the simulation either converges to one equilibrium point or the other and on the average result lies between the two TUA results, one for the empty queue and the other for full queue initial condition.

Using the technique discussed in Section 3.5, the optimal value for channel access probability can be achieved, which maximizes the throughput while maintaining a global equilibrium point. The lower limit for $p$ has already been shown to be $\frac{1.28}{N}$ while the upper limit comes out to be $p=0.06$ for $L=1$ and $p=0.021$ for $L=8$, as shown in Figure 3.5. The global operating point is marked as A and B for the two cases, respectively. So $\frac{1.28}{N} \leq p \leq 0.06$ and $\frac{1.28}{N} \leq p \leq 0.021$ defines the operating range for channel access probability to achieve maximum


Figure 3.5: Throughput and load lines for $(\mathrm{L}=1, \mathrm{p}=0.06)$ and $(\mathrm{L}=8, \mathrm{p}=0.021)$, non-coherent addition, SOI dominant path only. $N=100, \lambda=0.0035$
throughput and a unique operating point, with a buffer length of $L=1$ and $L=8$ respectively. These results are in agreement with the simulation results as shown in Figure 3.4(a).

### 3.6.2 SOI power sum of all paths of TU transmission

Figure 3.6(a) -3.6(d) show a set of curves for the case when SOI comprises of the power sum of all the multipath components of the dominant user. The figures show the throughput, packet response time, blocking probability and waiting delay for a range of channel access probabilities with packet arrival rate $\lambda=0.0035, N=100$ and two different buffer sizes ( $L=1$ and $L=8$ ).


Figure 3.6: Analytic and simulation results for an S-ALOHA system with immediate acknowledgement, non-coherent addition, SOI sum of multipaths. $N=100$, $\lambda=0.0035$

There is a marked difference in the results of empty and full queue initial conditions. A high value of $p$ will allow more users to be active in a given time slot provided they have a packet to transmit. The number of users contending for the channel during every time slot will be large if the queues of all or most of the users are always full. A user's queue will start to fill up and subsequent arriving packets dropped if the rate at which the packets leave the queue is less than the rate at which packets arrive at the queue. As a result, a system with the initial condition of full queue will experience critical failure due to congestion at a lower value of $p$ as compared to a system which start with an initially empty queue.

Comparing the results of buffer sizes $L=1$ and $L=8$ in Figure 3.6(a), reveals that a larger buffer size results in better throughput and much lower blocking probability. This is expected as larger buffer size means that an arriving packet is less likely to find the queue full and thus less likely to be dropped. A similar trend is observed in Figure 3.6(c) where the blocking probability increases for $p=0.0227$. Figure 3.6(a) also shows that for $N=100, \lambda=0.0035$ and $L=8$, the maximum throughput is achieved for $p \geq 0.0123$ (empty queue initial condition) and for $0.0123 \leq p \leq 0.0277$ (full queue initial condition).

From Figure 3.6(d), it can be seen that the minimum packet response time, for case of full queue initial condition, occurs at $p=0.0277$ while for the empty queue initial condition, the packet response time follows the same shape as that of full queue up to $p=0.0277$. Afterwards it keeps on decreasing for larger values of $p$. The minimum of the common portion of the two curves would give the optimum
value of $p$, i.e., $p=0.0277$. Note that the optimum operating point of the system can be found accurately using TUA.

Similar to the case where SOI is only the dominant path, the optimal range for channel access probability can also be obtained for the current case. As shown in Figure 3.7, the upper limit of channel access probability comes out to be $p=0.068$ and $p=0.032$ for $L=1$ and $L=8$ respectively. The global operating points are marked as A and B for the two cases. Thus, $\frac{1.28}{N} \leq p \leq 0.068$ and $\frac{1.28}{N} \leq p \leq 0.032$ defines the operating range for channel access probability to achieve maximum throughput and a unique operating point, with a buffer size of $L=1$ and $L=8$ respectively. These results again match to the simulation results as shown in Figure 3.6(a).

### 3.6.3 Comparison between SOI dominant path and SOI sum of multipaths

A comparison of Figures 3.4 and 3.6, reveals that the optimum channel access probability is different for the two cases. Furthermore, for the system with $L=$ 8, the packet response time of the scheme that performs a power sum of the multipaths of the dominant user to form the SOI is seven times less than the scheme which takes only the strongest path as the SOI (from 481.8 time slots to 66.03 time slots). This is a significant reduction in packet response time and provides an argument for combining the multipaths of the dominant user to form the SOI.


Figure 3.7: Throughput and load lines for $(\mathrm{L}=1, \mathrm{p}=0.068)$ and $(\mathrm{L}=8, \mathrm{p}=0.032)$, non-coherent addition, SOI sum of multipaths. $N=100, \lambda=0.0035$

### 3.6.4 Effect of the buffer length on system performance

The advantage of increased buffer size is that it ensures lesser number of packets being blocked and hence there is an improvement in the system throughput. From Figure 3.4(a), it can be seen that the throughput increases from 0.31 to 0.35 , an improvement of about $15 \%$, when the queue size is increased from $L=1$ to $L=8$. There is a corresponding increase in the response time with increase in the buffer size. This is mainly due to the waiting delay. The waiting delay is larger if more packets are already present in the buffer, as they need to be serviced before the present packet can be served. Although the response time is less in case of $L=1$,
the packet blocking probability is increased and hence user throughput is reduced.
The effect of increasing the buffer size has been studied and the results show that there is no significant performance improvement beyond a buffer length of $L=8$. It has been found that a buffer of length $L=8$ is sufficient to guarantee large throughput with an acceptable response time.

Finally, Figure 3.8 shows that the presented analysis is a general formulation that encompasses the flat fading analysis of [62] and ideal channel analysis of [1] as special cases. For all cases discussed in the paper, it is observed that TUA is able to accurately analyze the system behavior, regardless of the environment over which it is operating.

### 3.7 Summary

An approximate analysis for FU-FB S-ALOHA system operating in frequency selective fading channel using TUA is presented. System performance parameters which include throughput, packet response time, packet blocking probability and waiting delay have been analyzed for different buffer sizes. For the SOI, two cases have been discussed and it is found that the system throughput is better for the case when SOI is the power sum of multipath signals of the desired user. It is concluded that increasing the buffer size does not improve the system performance beyond a certain value of buffer size. A good match is obtained between simulation and analytical results at much lower computational cost as compared to Markov based methods.


Figure 3.8: Throughput vs channel access probability for a system with 100 users in ideal and fading scenario, $\mathrm{D}=1$, and capture ratio $=4 \mathrm{~dB}$ (fading)

In this chapter it is shown that the TUA technique is a truly generalized analysis method for random multiple access protocols applicable under various channel conditions, i.e., ideal channel, Rayleigh fading and frequency selective. The TUA framework presented for the frequency selective fading case encapsulates the Rayleigh fading and ideal channel scenarios as special cases which shows the versatility of TUA. Quality of Service (QoS) parameters like system throughput, blocking probability, packet response time and waiting delay have been evaluated. Finally, a method for determining the optimum operating range of re-transmission probability for the system, which delivers highest throughput while maintaining a unique stable operating point is presented.

## CHAPTER 4

## PERFORMANCE ANALYSIS

## OF IEEE 802.11 EDCF

This chapter presents a case study on the application of Tagged User analysis to evaluate the performance of IEEE 802.11 enhanced distributed coordination function (EDCF), that uses carrier sense multiple access with collision avoidance (CSMA/CA) as the access mechanism, for finite-user finite-buffer (FU-FB) system. The users/stations (STA) are spatially distributed around the access point (AP) and the radio channels between STAs and AP are independent multipath frequency selective. The analysis considers a number of heterogeneous groups where the STAs within each group have same parameters. Both the basic and ready-to-send/clear-to-send (RTS/CTS) access schemes in the protocol are considered. The performance parameters are evaluated using tagged user analysis (TUA) approach. The simulation results are compared with those obtained analytically and a good match is obtained.

### 4.1 Introduction

The analysis using a realistic scenario for the WLAN system is presented in this chapter. In the analysis, an unsaturated system with a finite population of STAs, each having a finite buffer capacity is considered. Under these conditions certain QoS parameters like blocking probability, average queue length, wait delay are evaluated. The channel between a STA and the AP is a realistic multipath frequency selective channel. A bell shaped spatial distribution for the STAs is considered. The analysis is applicable to both the basic and RTS/CTS access mechanisms for EDCF. In addition, heterogeneous groups of STAs are considered; each group represents a system with one of these data types: voice, video and data traffic. This allows to study the effect of changes in various system parameters while ensuring QoS for each traffic category (TC) as is explained in section 4.2. An approximate technique TUA [54] is used for the analyses as it makes it possible to analyze the FU-FB system under multipath frequency selective fading with reasonable accuracy. Extensive simulations have been done to validate the results.

### 4.2 Overview of IEEE 802.11

### 4.2.1 IEEE 802.11 DCF

The IEEE 802.11 DCF is based on carrier sense multiple access with collision avoidance (CSMA/CA) MAC protocol. Two mechanisms are used to access the
channel using DCF, which are the basic access and a 4 -way handshake procedure called the RTS/CTS access mechanism. In both the access schemes each STA must sense the channel idle for at least a time interval called the DCF inter frame space (DIFS) before it can start transmission. After sensing the channel idle for required interval of time, the STA waits for a random time interval called the backoff (BO) time. The BO time is determined by the contention window (CW) size, whose initial value is set to a minimum CW size (CWmin) and could increase upto a maximum CW size (CWmax). The BO counter is decremented by a slot time $\sigma$ if the channel is found idle for DIFS interval, whereas if the channel is busy then the BO counter is frozen. The BO counter starts decrementing from the same value when the channel is sensed idle for DIFS. If the transmission results in a collision the STAs follow binary exponential backoff by doubling the size of CW and enter the next BO stage. The maximum number of BO stages is specified using the parameter $m$. The doubling of the CW continues for $m$ BO stages and after that the CW size remains fixed as CWmax until the maximum number of retries $K$ are made. After $K$ retries if the packet has not been successfully transmitted, it is dropped from the system. The CW size is reset to CWmin after a successful transmission. Further details for IEEE 802.11 DCF can be found in [34].

### 4.2.2 IEEE 802.11 QoS EDCF

IEEE 802.11 EDCF was introduced to guarantee QoS for different traffic categories in WLAN. The enhanced distributed channel access (EDCA) defines four access categories, namely, voice, video, best effort and background access categories. Each access category has its own set of EDCF parameters to set the priority. These parameters include the CWmin, CWmax, arbitration inter frame space (AIFS), transmission opportunity (TXOP) limit. AIFS is used to sense the channel idle just like DIFS in IEEE 802.11 DCF. The traffic category with higher priority is given a smaller value of AIFS. This ensures that whenever the channel is idle the STA with higher priority gets higher chance to decrement its BO counter because it has to sense the channel idle for a smaller interval of time. Also, having a smaller value for the CWmin and CWmax, the random time that the STA with higher priority has to wait is lesser than the STA with larger values of these parameters. This again helps in setting the priority for different groups of users.

### 4.3 System Model

The system comprises of an infrastructure network with a central access point (AP) communicating with a heterogeneous group of $N$ STAs. IEEE 802.11 EDCF is used for the channel access which uses CSMA/CA as the access mechanism. The STAs with same statistical properties like arrival rate, buffer size and other set of parameters, as described in Section 4.2, are grouped together. These STAs are distributed around the AP following a bell shaped spatial distribution discussed
in Section 2.2.1. The transmission by a STA is successful if

$$
\begin{equation*}
P_{t}>z_{0} P_{n}, \quad 0 \leq n \leq N-1 \tag{4.1}
\end{equation*}
$$

where $P_{t}$ and $P_{n}$ are the powers of the signal transmitted by the desired STA and all the other transmissions acting as interferers, $z_{0}$ is the receiver capture ratio and $n$ is the number of interferers. Each group behaves as a group of homogeneous STAs with a set of parameters which are same for all the STAs within the group. The STA in group $i$ have a finite buffer length $L_{i}$ with $N_{i}$ STAs. Packets arrive in the buffer following Bernoulli process with an average arrival rate of $\lambda_{i}$ arrivals per second. The normalized offered load is defined as the number of packets that arrive in the buffer of each STA per time for successful transmission. The time for successful packet transmission $T_{i}$ is different for different groups. The longest value of $T_{i}$ is designated as $T_{\max }$, i.e. $T_{\max }=\max \left(T_{i}\right)$, and is used for the normalized offered load.

$$
\begin{equation*}
\lambda_{i, n}=N_{i} \lambda_{i} T_{\max } \tag{4.2}
\end{equation*}
$$

where $T_{\max }$ is the time for successful transmission such that $T_{\max }=\max \left(T_{i}\right)$ for all the groups $i$.

Late arrival model with delayed replacement rule and deferred first transmission (DFT) rule are used [89]. When a packet arrives at a STA, it observes the channel for any activity and follows the IEEE 802.11 EDCF as explained in section 4.2.

### 4.4 Analytical Model for TUA

TUA uses the fact that at equilibrium, all STAs have statistically equivalent behavior [54]. This makes it possible to find the system performance from the analysis of any one STA. The contention analysis is decoupled from the queueing process analysis using TUA.

### 4.4.1 Channel Contention Analysis

The contention analysis deals with the determination of virtual packet service time (VPST) distribution, which is dependent on the number of STAs, interference from these STAs and their busy probabilities $p_{b}$. The probability generating function (PGF) of VPST is derived from the model described by the state transition diagram shown in Figure 4.1 under the assumption that the system is in steady state.


Figure 4.1: State Flow Graph for CSMA/CA for a certain group

This state transition diagram is different from the one presented in Chapter 3 and includes a new state which takes care of the packets being dropped from the
system. The packets are dropped if not successfully transmitted after a number of unsuccessful tries. The maximum number of retries are specified in the parameters for the protocol. The tagged user (TU) in the $i^{t h}$ group has a queue of size $L_{i}$ and the packets arrive in the queue following Bernoulli arrival process. The packet service time has a general distribution. Hence the queue discipline is described as $G e o / G / 1 / L_{i}$ in the state flow diagram. The TU can be in any one of the five states as shown in Figure 4.1. The packet at the head of the queue enters the back-off state, $S_{i, b o}$, once the previous packet in the system has been served. In $S_{i, b o}$ state the the TU senses the channel and if it is busy it remains in the same state for another slot. Once the channel becomes idle for an interval equal to the AIFS, the TU then waits for a random backoff time determined by the backoff counter and after the expiry of the backoff counter the TU enters the transmission start state, $S_{i, t s}$. Otherwise it remains in the same state for an idle slot time. Once in the $S_{i, t s}$ state the TU transmits the packet and can enter any one of the three states namely the departure state, $S_{i, d e}$, the drop state, $S_{i, d r}$, or the backoff state. The TU can enter the $S_{i, d e}$ state if the transmission results in a success or enters the $S_{i, d r}$ state if packet has to be dropped from the system after $K_{i}$ unsuccessful retransmissions. If on the other hand the transmission results in a collision the TU goes back and enters the $S_{i, b o}$ state. From the $S_{i, d e}$ state the packet departs the system and the next packet in the queue enters the $S_{i, b o}$ state.

The probability that transmission of TU in group $i$ is a success is given as

$$
\begin{align*}
p_{i, s} & =\sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times \prod_{j \neq i} \sum_{u_{j}=0}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} p_{s \mid\left(u_{i}+\sum_{j \neq i} u_{j}\right)} \tag{4.3}
\end{align*}
$$

where $p_{i, b}$ is the probability that a STA in group $i$ is busy and $p_{i}$ is the transmission permission probability of any STA in $S_{i, b o}$ state. $p_{s \mid n}$ is the probability that the transmission is successful in presence of $n$ interferers. In (4.3) all possible combinations for STAs from all groups transmitting simultaneously with the TU in group $i$ are considered.

The probability $p_{i, d}$ that a packet is involved in $K_{i}+1$ collisions and gets dropped from the system is given by,

$$
\begin{equation*}
p_{i, d}=\left(1-p_{i, s}\right)^{K_{i}+1} \tag{4.4}
\end{equation*}
$$

The expression for PGF of VPST using signal flow graph in Figure 4.1 is given in (4.5),

$$
\begin{equation*}
B_{i}(z)=\frac{p_{i} p_{i, I} p_{i, s} z^{T_{i}+1}}{1-\left\{\left(1-p_{i, I}\right) z+p_{i, I}\left(1-p_{i}\right) z^{\sigma_{i}}\right\}-p_{i} p_{i, I}\left(1-p_{i, s}-p_{i, d}\right) z^{D_{i}+1}} \tag{4.5}
\end{equation*}
$$

where $p_{i, I}$ is the probability of sensing the channel idle, $T_{i}$ and $D_{i}$ are the times
for successful packet transmission and collision detection in group $i . p_{i}$ depends on the backoff procedure used and is derived from the overall average backoff window. $B_{i}(z)$ is a function of $p_{i, b}$ and $p_{i, I}$, so the relationship between $p_{i, I}$ and $p_{i, b}$ is derived in order to solve the system.

## Probability of Sensing the Channel Idle by TU

$p_{i, I}$ is determined by analyzing the behavior of the channel. The channel time is divided into a sequence of cycles. Each cycle consists of an idle duration and a busy duration. The busy status may be due to a collision or a successful transmission. The TU in group $i$ is busy with a probability 1 , since it has packets available for transmission at a certain time. All the other users are busy with a probability $p_{i, b}$, since they operate at their equilibrium probability by assumption.

To find the $p_{i, I}$, the time when the channel is idle and also the time when the TU in group $i$ will sense the channel are needed. The expected idle time for the TU in $i^{\text {th }}$ group in each cycle is given as

$$
\begin{equation*}
E\left(I_{i}\right)=\frac{\left(1-p_{i}\right)\left(1-p_{i} p_{i, b}\right)^{\left(N_{i}-1\right)} \prod_{j \neq i}\left(1-p_{j} p_{j, b}\right)^{N_{j}}}{1-\left(1-p_{i}\right)\left(1-p_{i} p_{i, b}\right)^{\left(N_{i}-1\right)} \prod_{j \neq i}\left(1-p_{j} p_{j, b}\right)^{N_{j}}} \tag{4.6}
\end{equation*}
$$

A TU does not sense the channel while it is busy transmitting a packet, resulting either in a success or a collision. There is a possibility that the transmission by TU in group $i$ is successful while there is another STA in group $j$ transmitting simultaneously, such that $D_{j}>T_{i}$. In such a case the TU will not sense the channel for the initial period of $T_{j}$ sec during which it is transmitting, but it will
sense the channel for the remaining $\left(D_{j}-T_{i}\right)$ sec during which the STA in group $j$ will be transmitting. Thus, we need to find the probability that, during a busy duration in a cycle, the transmission by TU in group $i$ is a success with at least one transmitting STA in another group $j$ such that $D_{j}>T_{i}$. We denote this probability as $p_{i, s t u, j}$ and it is given by (4.7).

$$
\begin{align*}
p_{i, s t u, j} & =\left[E\left(I_{i}\right)+1\right] p_{i} \sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}}>D_{j}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \times \sum_{\substack{u_{j}=1 \\
D_{j}>T_{i}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod _ { \substack { j _ { l } \neq i , j \\
D _ { j _ { l } } < D _ { j } } } \sum _ { u _ { j _ { l } } = 0 } ^ { N _ { j _ { l } } } ( \begin{array} { c } 
{ N _ { j _ { l } } } \\
{ u _ { j _ { l } } }
\end{array} ) \left(p_{j_{l}} p_{\left.\left.\left.j_{l, b}, b\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \times p_{s \mid\left(u_{i}+u_{j}+\right.} \sum_{j_{l} \neq i, j} u_{j_{l}}\right)}\right.\right. \tag{4.7}
\end{align*}
$$

To explain (4.7) we note that the TU in group $i$ is transmitting with $p_{i}$, there are other STAs from group $i$ transmitting with probability given by $\sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}}$. STAs from group $j$ such that $D_{j}>T_{i}$ are transmitting with probability given by $\sum_{\substack{u_{j}=1 \\ D_{j}>T_{i}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}}$. The probability $\prod_{\substack{j_{h} \neq i, j, j \\ D_{j_{h}}>D_{j}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}$ takes care of the fact that there is no STA transmitting form a group $j_{h}$ such that $D_{j_{h}}>D_{j}$. The term
$\prod_{\substack{j_{l} \neq i, j \\ D_{j_{l}} D_{j}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}$ takes into account the transmission by STAs from groups whose time for collision detection is less than the time for collision detection of a STA in group $j$, i.e. $D_{j_{l}}<D_{j}$. Finally, $p_{s \mid\left(u_{i}+u_{j}+\right.} \sum_{j_{l} \neq i_{l}, j} u_{\left.j_{l}\right)}$ is used to compute the probability of successful transmission by Tu in group $i$ in the presence of $\left(u_{i}+u_{j}+\sum_{j_{l} \neq i, j} u_{j_{l}}\right)$ simultaneous transmissions.

Similarly, the transmission by another STA in group $i$ (same group as of the TU) can be successful and the corresponding success probability $p_{i, s o u, j}$ is given by (4.8). It is the probability that, during a busy duration in a cycle, the transmission by a STA (other than the TU) in group $i$ is successful in the presence of other transmitting STA from group $j$.
$p_{i, \text { sou }}$ given by (4.9) is the probability that, during a busy duration in a cycle, the transmission by a STA in group $i$ (other than the TU) is a success and there is no STA transmitting from group $j$, such that $D_{j}>T_{i}$.

$$
\begin{align*}
p_{i, s o u, j} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}}>D_{j}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \times \sum_{\substack{u_{j}=1 \\
D_{j}>T_{i}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l}}<D_{j}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l, b}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \times p_{s \mid\left(u_{i}-1+u_{j}+\sum_{j_{l} \neq i, j} u_{\left.j_{l}\right)}\right)} \tag{4.8}
\end{align*}
$$

$$
\begin{align*}
p_{i, s o u} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times \prod_{\substack{j_{h} \neq i \\
D_{j_{h}>T_{i}}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}} \times\left\{\prod_{\substack{j_{l} \neq i \\
D_{j_{l}<T_{i}}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\left(\begin{array}{c}
N_{j_{l}} \\
\\
u_{j_{l}}
\end{array}\right)\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l} l} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \times p_{s \mid\left(u_{i}-1+\sum_{j_{l} \neq i} u_{j_{l}}\right)} \tag{4.9}
\end{align*}
$$

The probability that, during a busy duration in a cycle, there is a successful transmission due to a STA in a group other than that of the TU group $i$, is denoted by $p_{i, s o g, j}$ and is given by (4.10).

$$
\begin{align*}
p_{i, s o g, j} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right)\left(1-p_{i} p_{i, b}\right)^{N_{i}-1} \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}}, T_{j}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\}\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l}}<T_{j}}} \sum_{j_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{\left.\left.j_{l} p_{j_{l}, b}\right)^{u_{j l}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\}}\right\}\right. \\
& \times p_{s \mid\left(u_{j}-1+\sum_{j_{l} \neq i, j} u_{j_{l}}\right)} \tag{4.10}
\end{align*}
$$

STAs in each group experience a different time for collision detection and hence the collision probabilities for each collision scenario needs to be computed. The probability, $p_{i, c t u, j}$, that during a busy duration in a cycle, there is a collision involving the TU in group $i$ and a STA in another group $j$ such that $D_{j}>D_{i}$ is
given by (4.11).

$$
\begin{align*}
p_{i, c t u, j} & =\left[E\left(I_{i}\right)+1\right] p_{i} \sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}}>D_{j}, D_{i}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l}}<D_{j}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \times\left(1-p_{s \mid\left(u_{i}+u_{j}+\sum_{j_{l}} \sum_{i, j} u_{\left.j_{l}\right)}\right)}\right. \tag{4.11}
\end{align*}
$$

Similarly, the transmission by another STA in group $i$ (same group as of the TU) can result in a collision and the corresponding collision probabilities, during a busy duration in a cycle, are denoted by $p_{i, c o u, j}$ when the collision is with another STA in group $j$ with $D_{j}>D_{i}$, and $p_{i, \text { cou }}$ when the collision involves a STA in group $j$ with $D_{j} \leq D_{i}$.

$$
\begin{align*}
p_{i, c o u, j} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}}>D_{j}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \sum_{\substack{u_{j}=1 \\
D_{j}>D_{i}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{j} \neq i, j \\
D_{j_{l}}<D_{j}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l} l}-u_{j_{l}}}\right\} \\
& \times\left(1-p_{s \mid\left(u_{i}-1+u_{j}+\sum_{j_{l}} u_{i, j}\right)} u_{\left.j_{l}\right)}\right) \tag{4.12}
\end{align*}
$$

$$
\begin{align*}
p_{i, c o u} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i} p_{i, b}\right)^{u_{i}}\left(1-p_{i} p_{i, b}\right)^{N_{i}-1-u_{i}} \\
& \times \prod_{\substack{j_{h} \neq i \\
D_{j_{h}}>D_{i}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\left\{\prod_{\substack{j_{l} \neq i}} \sum_{D_{j_{l}<D_{i}}}^{N_{j_{l}}}\left(\begin{array}{c}
N_{j_{l}} \\
\\
u_{j_{l}}
\end{array}\right)\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \times\left(1-p_{\left.s \mid\left(u_{i}-1+\sum_{j_{l} \neq i, j} u_{j_{l} l}\right)\right)}\right. \tag{4.13}
\end{align*}
$$

The probability, that during a busy duration in a cycle, there is a collision due to a STA in a group other than that of the TU group $i, p_{i, \text { cog }, j}$ is given by (4.14).

$$
\begin{align*}
p_{i, \text { cog }, j} & =\left[E\left(I_{i}\right)+1\right]\left(1-p_{i}\right)\left(1-p_{i} p_{i, b}\right)^{N_{i}-1}\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h}>D_{j}}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \\
& \times \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j} p_{j, b}\right)^{u_{j}}\left(1-p_{j} p_{j, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j}} \sum_{\substack{u_{j_{l}}=0}}^{D_{j_{l}<D_{j}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l}} p_{j_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l}} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \left.\times\left(1-p_{s \mid\left(u_{i}+u_{j}-1+\sum_{j_{l}} \sum_{i, j}\right.} u_{j_{l} l}\right)\right) \tag{4.14}
\end{align*}
$$

The TU will find the channel idle for $\left(E\left(I_{i}\right)+1\right)$ slots and the slots during which the TU can sense the channel are given as

$$
E\left(I_{i}\right)+1+s_{i, s u}+s_{i, c o l}
$$

where $s_{i, s u}$ are the slots in which the TU senses during a successful transmission and is given as

$$
\begin{aligned}
& s_{i, s u}=\sum_{\substack{j \neq i \\
D_{j}>T_{i}}} p_{i, s t u, j}\left(D_{j}-T_{i}\right)+\sum_{\substack{j \neq i \\
D_{j}>T_{i}}} p_{i, s o u, j}\left(D_{j}-1\right)+p_{i, s o u}\left(T_{i}-1\right) \\
&+\sum_{j \neq i} p_{i, s o g, j}\left(T_{j}-1\right)
\end{aligned}
$$

and $s_{i, \text { col }}$ are the slots in which the TU senses the channel during a collision, given as

$$
\begin{array}{r}
s_{i, \text { col }}=\sum_{\substack{j \neq i \\
D_{j}>D_{i}}} p_{i, \text { ctu }, j}\left(D_{j}-D_{i}\right)+\sum_{\substack{j \neq i \\
D_{j}>D_{i}}} p_{i, \text { cou }, j}\left(D_{j}-1\right)+p_{i, \text { cou }}\left(D_{i}-1\right) \\
\\
+\sum_{j \neq i} p_{i, \text { cog }, j}\left(D_{j}-1\right)
\end{array}
$$

Therefore, the probability $p_{i, I}$ is obtained as

$$
\begin{equation*}
p_{i, I}=\frac{E\left(I_{i}\right)+1}{E\left(I_{i}\right)+1+s_{i, s u}+s_{i, c o l}} \tag{4.15}
\end{equation*}
$$

Equations (4.7)-(4.15) establish the relation between $p_{i, I}$ and $p_{i, b}$. Let $p_{i, 0}$ be the probability that a STA in group $i$ has an empty buffer, then the following relation holds

$$
\begin{equation*}
p_{i, b}+p_{i, 0}=1 \tag{4.16}
\end{equation*}
$$

Using the relations developed above, the distribution for packet service time $B_{i}(z)$ given the user idle probability $p_{i, 0}$ can be determined.

## Average backoff window and the permission probability

The permission probability $p_{i}$ for a STA in group $i$ used in the previous section depends on the contention window size used for the protocol. The average size of the contention window can be derived as discussed in [39], given as

$$
\begin{array}{r}
W_{i, a v g}=\frac{1}{1-p_{i, c}^{K_{i}}} \times\left(\frac{W_{i}\left(1-p_{i, c}\right)\left(1-\left(2 p_{i, c}\right)^{m_{i}}\right)}{2\left(1-2 p_{i, c}\right)}-\frac{1-p_{i, c}^{m_{i}}}{2}\right. \\
\left.+\frac{\left(2^{m_{i}} W_{i}-1\right)\left(p_{i, c}^{m_{i}}-p_{i, c}^{K_{i}}\right)}{2}\right) \tag{4.17}
\end{array}
$$

where $p_{i, c}$ is the probability that the transmission by TU in group $i$ experiences a collision, and is given as

$$
\begin{equation*}
p_{i, c}=1-p_{i, s} \tag{4.18}
\end{equation*}
$$

Based on the overall average backoff window, the probability $p_{i}$ that the TU in group $i$ (when it is busy) attempts to transmit in any arbitrary slot is given by

$$
\begin{equation*}
p_{i}=\frac{1}{W_{i, a v g}+1} \tag{4.19}
\end{equation*}
$$

Using (4.3), (4.17) and (4.18), $p_{i, c}$ and $W_{i, a v g}$ can be computed. The permission probability is then computed using (4.19).

### 4.4.2 Queueing Analysis

The queueing analysis is used to determine the user busy probabilities, given the packet service time distribution and arrival process.

The PGF for $a_{i, k}$, which is the probability that $k$ packets arrive during a service time, is given as

$$
\begin{equation*}
A_{i}(z)=\sum_{k=0}^{L_{i}-1} a_{i, k} z^{k}=B_{i}\left(1-\lambda_{i}+\lambda_{i} z\right) \tag{4.20}
\end{equation*}
$$

since at most $L_{i}-1$ packets arrive in one service time. Denoting $\pi_{i, k}$, where $k=0,1, \ldots, L_{i}-1$, as the probability that leaving packet sees $k$ packets in queue, it follows (from [89])

$$
\begin{align*}
& \pi_{i, k}=\pi_{i, 0} a_{i, k}+\sum_{j=1}^{k+1} \pi_{i, j} a_{i, k-j+1}  \tag{4.21}\\
& \pi_{i, 0}=\left(\sum_{k=0}^{L_{i}-1} \hat{\pi}_{i, k}\right)^{-1} \tag{4.22}
\end{align*}
$$

where $\pi_{i, k}^{\prime}=\frac{\pi_{i, k}}{\pi_{i, 0}}$ which can be computed from the recursion

$$
\begin{equation*}
\dot{\pi}_{i, k+1}=\frac{1}{a_{i, 0}}\left(\pi_{i, k}^{\prime}-\sum_{j=1}^{k} \pi_{i, j}^{\prime} a_{i, k-j+1}-a_{i, k}\right) \tag{4.23}
\end{equation*}
$$

with $\pi_{i, 0}^{\prime}=1$, and where $p_{i, k}$ is the probability that there are $k$ packets present in the system at any slot boundary. For $0 \leq k \leq L_{i}-1$,

$$
\begin{align*}
p_{i, k} & =\frac{\pi_{i, k}}{\pi_{i, 0}+\rho_{i}}  \tag{4.24}\\
p_{i, L} & =1-\frac{1}{\pi_{i, 0}+\rho_{i}} \tag{4.25}
\end{align*}
$$

where

$$
\begin{equation*}
\rho_{i}=\lambda_{i} b_{i} \tag{4.26}
\end{equation*}
$$

and $b$ is defined as

$$
\begin{equation*}
b_{i}=\dot{B}_{i}(1) \tag{4.27}
\end{equation*}
$$

Equations (4.20) to (4.24) relate $p_{b}$ and $B_{i}(z)$. These equations along with those presented in the contention analysis can be solved using iterative algorithm discussed in 4.5.2.

### 4.5 Performance parameters and iterative Algorithm

### 4.5.1 Performance parameters

Following performance measures are used in the analysis of the IEEE 802.11 EDCF.

## Packet Blocking Probability

If the arriving packet finds the queue full, it is not accepted into the system and is blocked. The probability that a packet is blocked in group $i$ is given as

$$
\begin{equation*}
p_{i, B}=1-\frac{1}{\pi_{i, 0}+\rho_{i}} \tag{4.28}
\end{equation*}
$$

where $\pi_{i, 0}$ is the probability that the queue of TU in group $i$ is empty when a packet leaves the system, $\rho_{i}$ is the offered load given by $\rho_{i}=\lambda_{i} b_{i}$ and $b_{i}$ is average virtual packet service time for a STA in group $i$.

## Packet Drop Probability

A packet is dropped from the $i^{\text {th }}$ group if it is not successfully transmitted even after $K_{i}$ retries. The probability that a packet will be dropped from the system is given as

$$
\begin{equation*}
p_{i, d}=\left(1-P_{i, s}\right)^{K_{i}+1} \tag{4.29}
\end{equation*}
$$

## System Throughput

The system throughput is defined as the average number of data bits transmitted per second. The system throughput is calculated as follows

$$
\begin{equation*}
\Theta=\sum_{i=1}^{G} N_{i} \theta_{i} \tag{4.30}
\end{equation*}
$$

where $\theta_{i}$ is the throughput of the TU in group $i$ which is given as

$$
\begin{equation*}
\theta_{i}=\lambda_{i} N_{i} P_{i}\left(1-p_{i, B}\right)\left(1-p_{i, d}\right) \tag{4.31}
\end{equation*}
$$

where $P_{i}$ is the packet payload in bits per frame and $\lambda_{i}$ is the packet arrival rate in frames per second per STA.

## Expected queue length

The STA's in group $i$ have a finite buffer capacity of $L_{i}$. The expected queue length for a STA in group $i$ is given as

$$
\begin{equation*}
E_{i}\left[I_{q}\right]=\sum_{k=0}^{L_{i}} k p_{i, k} \tag{4.32}
\end{equation*}
$$

## Mean Packet Response time, wait time and VPST

Packet response time is the time that a packet spends in the virtual server from its arrival in the queue to its successful departure. The mean packet response time, $E_{i}\left[t_{r}\right]$, is given by

$$
\begin{equation*}
E_{i}\left[t_{r}\right]=\frac{E_{i}\left[I_{q}\right]}{\lambda_{i}\left(1-p_{i, B}\right)} \tag{4.33}
\end{equation*}
$$

Packet response time is the sum of wait time and the virtual packet service time (VPST). Wait time is the time spent by the packet in the queue before being ready for transmission and the VPST is the time that a packet spends in the system from being ready for transmission to the successful departure of the packet from the system.

$$
\begin{equation*}
E_{i}[w]=E_{i}\left[t_{r}\right]-b \tag{4.34}
\end{equation*}
$$

where $b$ is the VPST.

### 4.5.2 Iterative Algorithm

The numerical iterative algorithm to complete the analysis is listed in Algorithm
2.

```
Algorithm 2 Tagged User Analysis
Require: Initial guess \(0 \leq p_{i, 0}(0) \leq 1, \mathrm{n}=1, \delta=10^{-8}\).
Ensure: Buffer occupancy probabilities \(p_{i, k}\).
    \(p_{i, b} \leftarrow(4.16)\)
    \(p_{i} \leftarrow\) Section 4.4.1
    \(p_{i, s} \leftarrow(4.3)\)
    \(E\left(I_{i}\right) \leftarrow(4.6)\)
    \(p_{i, I} \leftarrow\) Section 4.4.1
    \(p_{i, k} \leftarrow\) Section 4.4.2
    if \(\left|p_{i, 0}(n)-p_{i, 0}(n-1)\right| \leq \delta\) then
        STOP
    else
        \(\mathrm{n}=\mathrm{n}+1\)
        GOTO 1
    end if
```


### 4.6 Results and discussion

### 4.6.1 An example of WLAN network with heterogeneous

 trafficTo validate the analytical results shown, an example of a heterogeneous system with three traffic categories (TC) is presented. These three TCs represent voice, video and data applications with $N_{i}=3$ STAs in each TC. Extensive simulations developed using Matlab are conducted to validate the proposed analysis using TUA. The IEEE 802.11 EDCF protocol with parameters shown in Table 4.1 is

Table 4.1: IEEE 802.11 parameters common to all traffic categories

| Parameter |  | Parameter |  |
| :---: | :---: | :---: | :---: |
| Bit rate $(\mathrm{BR})$ | 11 Mbps | Basic bit rate $(\mathrm{BBR})$ | 1 Mbps |
| SIFS | $10 \mu \mathrm{sec}$ | Slot time | $20 \mu \mathrm{sec}$ |
| Physical header $(\mathrm{PH})$ | $(144+48) / \mathrm{BBR}$ | MAC header $(\mathrm{MH})$ | $272 / \mathrm{BR}$ |
| RTS | $(160 / \mathrm{BR}+\mathrm{PH})$ | CTS | $112 / \mathrm{BR}+\mathrm{PH}$ |
| ACK | $112 / \mathrm{BR}+\mathrm{PH}$ |  |  |

used to simulate the system. As discussed, the TCs can have their own set of parameters that ensure the QoS as given in Table 4.2. The results presented are for RTS/CTS access mechanism, with STAs distributed around the AP following a bell shaped user distribution. A multipath frequency selective fading channel model, IEEE 802.11 Channel Model B [82], has been used to simulate the channel between the STAs and the AP. The normalized offered load has been varied to evaluate the system performance parameters including the system throughput, packet blocking probability, packet response and wait times and average buffer length.

Table 4.2: Parameters related to the traffic categories

|  | Voice | Video | Data |
| :---: | :---: | :---: | :---: |
| AIFS | $50 \mu$ sec | $50 \mu \mathrm{sec}$ | $50 \mu \mathrm{sec}$ |
| Retry limit, K | 8 | 8 | 15 |
| $C W_{\min }$ | 8 | 16 | 32 |
| $C W_{\max }$ | 16 | 64 | 1024 |
| Persistence factor | 2 | 2 | 2 |
| Data transmitted $[\mathrm{Mbps}]$ | $24.4-244.2$ | $122.1-1221$ | $97.7-976.8$ |
| Frames per sec | $12.21-122.1$ | $12.21-122.1$ | $12.21-122.1$ |
| Payload $[\mathrm{bits} / \mathrm{frame}]$ | 2000 | 10000 | 8000 |
| $T_{i}^{\text {bas }}[\mu \mathrm{sec}]$ | 612 | 1340 | 1158 |
| $D_{i}^{\text {bas }}[\mu \mathrm{sec}]$ | 399 | 1127 | 945 |
| $P_{i}^{\text {bas }}[\mu \mathrm{sec}]$ | 189 | 910 | 728 |
| $T_{i}^{\text {rts }}[\mu \mathrm{sec}]$ | 1042 | 1770 | 1588 |
| $D_{i}^{\text {rts }}[\mu \mathrm{sec}]$ | 207 | 207 | 207 |
| $P_{i}^{\text {rts }}[\mu \mathrm{sec}]$ | 189 | 910 | 728 |

The times for successful transmission or a collision are given by the following relations.

$$
\begin{aligned}
T_{i}^{\text {bas }} & =A I F S+H+P+S I F S+A C K \\
D_{i}^{\text {bas }} & =A I F S+H+P \\
T_{i}^{r t s} & =A I F S+R T S+S I F S+C T S+S I F S+H+P+S I F S+A C K \\
D_{i}^{\text {rts }} & =A I F S+R T S
\end{aligned}
$$

Here bas and $r$ ts stand for basic access and access using RTS/CTS in IEEE 802.11 respectively, $H$ is the header which is the sum of physical and MAC header. Voice traffic has been assigned the highest priority by selecting the minimum size for $C W$ min and a single $B O$ stage. The video traffic is next in priority with a $C W \min$ greater than the voice traffic but less than data. Video traffic also has a single $B O$ stage. Data traffic is the least priority traffic with highest value of $C W \min$ and five $B O$ stages.

Figure 4.2 shows the throughput of each TC and the aggregate throughput plotted against the normalized offered load. It can be seen that as the offered load increases the throughput increases upto a value of $\lambda_{n}=1$. After further increase in $\lambda_{n}$ the least priority data traffic throughput does not further increase, whereas for other TC's of voice and video the throughput increases with the offered load. After $\lambda_{n}=1.4$ the video traffic also does not see improvement in throughput. The voice traffic with highest priority has a linearly increasing throughput whereas the other TC's with lower priority suffer in terms of throughput with increase in the


Figure 4.2: Throughput: Aggregate and each TC
normalized offered load. This shows that the TC with least priority is the first one to suffer in terms of throughput, whereas the TC with highest priority will have an improved throughput for a larger offered load.

The blocking probability is shown in Figure 4.3 for the three TCs. The data packets have the highest blocking probability, which is due to its lowest priority and high value for retry limit. So the data packets remain in the system for a longer time with a higher packet response time and wait delay as shown in Figures 4.4 and 4.5 but are not dropped from the system. This results in more rapid filling of the queue as shown in Figure 4.6. The highest priority traffic with less number of retries has a lower blocking probability. Also, the response time for the highest priority time critical TC of voice is less than 10 msec . The VPST is shown in

TUA: Lines, Simulation: Markers


Figure 4.3: Blocking probability for each TC


Figure 4.4: Packet response time for each TC


Figure 4.5: Waiting time for each TC


Figure 4.6: Average queue length of each TC

Figure 4.7.
In voice-oriented continuous traffic wireless networks there is more voice traffic. Throughput results for such a network configuration are shown in Figure 4.8.

### 4.6.2 Comparison of phasor and power sum for interference signal in WLAN network with homogeneous traffic

A scenario where a homogeneous group of STAs attempt to communicate with an access point (AP) using IEEE 802.11 DCF in infrastructure mode is presented. The normalized offered load is varied to evaluate the system parameters. QoS parameters including system throughput, packet blocking and dropping proba-


Figure 4.7: Virtual packet service time for each TC


Figure 4.8: Throughput for voice oriented continuous traffic wireless networks: Aggregate and for each TC
bilities, packet response time, waiting delay and average queue length have been analyzed. The system parameters used for simulation are listed in Table 4.3. IEEE 802.11 Channel Model B as described in [82] has been used for the simulation. The simulation is developed in MATLAB and the sampling time is taken as $1 \mu s e c$, so that all the times are multiple of this sampling time. The results show a comparison for the coherent (Phasor) and in-coherent (Power) addition of

Table 4.3: IEEE 802.11 DCF simulation parameters

| Basic bit rate (BBR) | 1 Mbps | $C W_{\min }$ | 31 |
| :--- | :--- | :--- | :--- |
| Bit rate (BR) | 11 Mbps | $C W_{\max }$ | 1023 |
| Physical Header (PH) | $(144+48) / \mathrm{BBR}$ | DIFS | $50 \mu \mathrm{sec}$ |
| MAC Header (MH) | $272 / \mathrm{BR}$ | SIFS | $10 \mu \mathrm{sec}$ |
| ACK | $112 / \mathrm{BR}+\mathrm{PH}$ | Slot time | $20 \mu \mathrm{sec}$ |
| T | $1225 \mu \mathrm{sec}$ | D | $1012 \mu \mathrm{sec}$ |

the interference signals where STAs are distributed in form of a ring, as discussed in Section 2.1. It can be seen that the performance is better in case of phasor addition of signals when the normalized traffic is greater than one.

Figure 4.9 shows the system throughput with varying normalized load. It is seen that the throughput increases linearly with increase in the normalized offered load upto a certain value. After that the throughput is not affected by any further increase in the load. The analytic results match well with the simulation results. Figure 4.10 shows the corresponding packet blocking probability. This is an important QoS parameter that could not be measured using the 2-D Markov chains models. The blocking probability as expected increases with the increase in the normalized offered load, $\lambda_{n}>1$. The packets that are not successfully transmitted even after $K$ collisions are dropped from the system and the corresponding packet dropping probability is shown in Figure 4.11. The simulation results are not shown as the number of packets dropped during network simulation were insufficient to give any meaningful statistics even when the network was simulated for 125 sec .

Packet response time shown in Figure 4.12 increases with the increase in the load with a corresponding increase in the packet blocking probability and a decrease in the system throughput. Figure 4.13 gives the waiting delay for the packets and average queue length for the TU is given in Figure 4.14. The larger queue will result in more waiting delay for large $\lambda_{n}$. This is because the packet will have to wait in the queue before the other packets already in the queue get


Figure 4.9: System throughput for IEEE 802.11 DCF


Figure 4.10: Blocking probability for IEEE 802.11 DCF


Figure 4.11: Packet dropping probability for IEEE 802.11 DCF


Figure 4.12: Response time for IEEE 802.11 DCF


Figure 4.13: Waiting delay for IEEE 802.11 DCF


Figure 4.14: Average queue length for IEEE 802.11 DCF
served, which will take longer time. So increasing the buffer size will help in handling increase in load for a shorter time but with a corresponding increase in wait time and packet response time. From Figure 4.14 the value of the offered load after which the queue will get full and hence packets will be blocked, resulting in congestion, can also be estimated.

### 4.7 Summary

This chapter presented an approximate technique for the analysis of IEEE 802.11 EDCF system in a nonsaturated heterogeneous scenario with finite STAs each having a finite buffer size. The contention and queueing processes are decoupled using an iterative algorithm. Important QoS parameters have been evaluated using the proposed technique. The analysis is done for realistic channel conditions with STAs spatially distributed around the AP. The analysis is also valid for IEEE 802.11 DCF, homogeneous users, saturated conditions, different channel conditions like ideal channel or fading channel as special cases. Hence, it is concluded that TUA is a general analysis method which can be used for IEEE 802.11 WLAN systems under different network scenarios.

## CHAPTER 5

## APPLICATION OF TUA TO

## RESOURCE MANAGEMENT

The performance analysis of WLAN system using IEEE 802.11 EDCF as the access protocol with multiple parallel channels for transmission is presented in this chapter. In order to achieve better system performance in terms of system throughput and delay, multiple channels provided in IEEE 802.11 can be utilized. Different channel selection schemes and channel configurations have been used in the analysis and the results are presented. The analysis is done using Tagged User Analysis (TUA) and MATLAB simulations have been used to validate the results obtained from the analysis.

The chapter organization is as follows. Section 5.1 introduces the multiple channel system used throughout the chapter. The system model is explained in Section 5.2 and the TUA technique for analysis of multiple channel system is elaborated in Section 5.3. The performance metrics and the iterative algorithm of

TUA to compute these metrics are discussed in Section 5.4. The numerical results from TUA analysis and simulation are compared in Section 5.5. The chapter is concluded in Section 5.6.

### 5.1 Introduction

Multiple channels available in IEEE 802.11 WLAN systems are utilized in the analysis. Heterogeneous traffic is considered in the analysis and different configurations for the multiple channels have been discussed. First, the case is considered when there are a number of channels available and the traffic is distributed among these channels in different ways. The effect of increased arrival rate of a certain traffic category (TC) and the TC priority is also discussed. Then, comparison is presented for the case when the total capacity considering all the channels is same, but each individual channel can have different capacity. In this comparison different channel selection scenarios are discussed.

### 5.2 System Model

IEEE 802.11 EDCF [34] infrastructure BSS with a centralized AP is used for a multiple channel system. The STAs are distributed around the AP under multipath frequency selective fading environment. The Extended Rate PHY (ERP) specifications provide for data rates of $1,2,5.5,11,22$, and 33 Mbps using different modulation schemes. The $2.4-2.4835 \mathrm{GHz}$ frequency band is used by IEEE

Table 5.1: High rate PHY frequency channel plan for IEEE 802.11

| Channel ID | Frequency <br> $[\mathrm{MHz}]$ | Channel ID | Frequency <br> $[\mathrm{MHz}]$ |
| :---: | :---: | :---: | :---: |
| 1 | 2412 | 8 | 2447 |
| 2 | 2417 | 9 | 2452 |
| 3 | 2422 | 10 | 2457 |
| 4 | 2427 | 11 | 2462 |
| 5 | 2432 | 12 | 2467 |
| 6 | 2437 | 13 | 2472 |
| 7 | 2442 | 14 | 2484 |

802.11 WLAN system and the channels with the ID and the frequency range are as given in Table 5.1. In order to keep a frequency separation of 20 MHz , the channels with ID 1, 4 and 11 are used in parallel without interference.

The flow chart for multiple channel protocol is shown in Fig 5.1. When a STA has a packet in the head of the queue to be transmitted, it selects a channel. After appropriate channel selection it waits for a time equal to AIFS. If the channel is idle for AIFS interval of time the STA then waits for a random interval of time depending on the BO counter. Once the BO counter reaches zero the STA transmits the packet and this transmission can either be successful or a failure due to collision. If the transmission is successful the packet departs the system. If a collision occurs the STA increments the retry limit counter $C_{r e t}$ and checks if the counter has reached the maximum number of retries. If maximum retries have been made the packet is dropped from the system, otherwise the STA again looks for the channel being to be idle for AIFS interval of time and then waits for another random BO interval following binary exponential backoff.

The scheme discussed so far for multiple channel systems assumes that the al-


Figure 5.1: Flow diagram for multi-channel IEEE 802.11
gorithm used for the channel selection will select a channel for packet transmission of a STA. This channel will then not be changed until the packet is successfully transmitted or has been dropped after maximum retries. There are other possible scenarios as listed below.

If in the selected channel, the STA experiences a collision, it can leave the channel and look for another suitable channel. Maximum number of tries is decremented along with the update of the BO counter. The next channel selection can avoid the selection of the previously used channel.

In the case discussed above, when a STA is assigned a channel it can be busy. If the STA finds the channel busy, it has to stay in the same channel and wait for it to become idle. One alternate can be to leave the channel if it is busy and look for some other channel which is idle.

### 5.2.1 Channel Selection Strategies

In order to analyse the multiple channel system three different schemes for channel selection have been considered.

- Like channels with random channel selection
- Unlike channels with equal probability of channel selection
- Unlike channels with priority channel selection


## Like channels with random channel selection probability

In this strategy, it is assumed that all the channels are alike, i.e. the data rate of all the channels is same. This is the simplest scenario where the STAs select any
channel with equal probability,

$$
\begin{equation*}
p_{c, c s}=\frac{1}{C} ; \text { for } 1 \leq c \leq C \tag{5.1}
\end{equation*}
$$

## Unlike channels with equal channel selection probability

The Unlike channels, where different channels have different data rates, are considered here. The STAs select any of the available channels with equal probability.

## Unlike channels with priority channel selection

In this scheme, where again Unlike channels are considered, the channel with the highest data rate is selected with a higher probability. This ensures that the fastest channel gets more STAs and delivers more throughput to the system. The channel selection probability for different channels available can be selected according to the data rate supported by that channel or the time required for successful packet transmission on that channel.

### 5.3 Tagged User Analysis Model for Multiple Channel System

### 5.3.1 Channel Contention Analysis

The contention analysis deals with the determination of the distribution of VPST and for that the state flow graph in Figure 5.2 is used. For simplicity it is assumed


Figure 5.2: State flow graph for IEEE 802.11 EDCF using 2-parallel channels that there are two types of channels, channel type $1\left(C T_{1}\right)$ and channel type 2 $\left(C T_{2}\right)$. In case of Unlike channels, $C T_{1}$ contains only one channel with higher data rate and all other $(C-1)$ channels have same data rate that is less than the data rate of $C T_{1}$. The idea can be extended to more channel types and more than one channel in each type. The probability of selecting $C T_{1}$ and $C T_{2}$ is denoted by $p_{T_{1}, \text { cs }}$ and $p_{T_{2}, \text { cs }}$ respectively. Following relation exists for a system with two types of channels

$$
\begin{equation*}
p_{T_{2}, c s}=1-p_{T_{1}, c s} \tag{5.2}
\end{equation*}
$$

Also, the probability of selecting $c^{\text {th }}$ channel is given as

$$
p_{c, c s}= \begin{cases}p_{T_{1}, c s}, & c=1  \tag{5.3}\\ \frac{p_{T_{2}, c s}}{C-1}, & c=2,3, \ldots, C\end{cases}
$$

where it is assumed that channel 1 is the only channel in $C T_{1}$, and $C T_{2}$ has $(C-1)$ channels.

The sequence of operations is explained using the state flow graph shown in Figure 5.2. This is an extended version of the SFG used for single channel case Figure 4.1, but with two parallel channels. The SFG can be extended in a similar fashion for more channels. Packets arrive in the queue of TU in the $i^{\text {th }}$ group and head of the queue packet enters the packet ready state $S_{i, p r}$. It selects one of the $C$ channels, with a probability $p_{c, c s}$, where

$$
\begin{equation*}
\sum_{c=1}^{C} p_{c, c s}=1 \tag{5.4}
\end{equation*}
$$

After selecting a channel, TU enters the back-off state $S_{i c, b o}$. The selection of the channel is an important feature in the analysis and the system performance depends on this selection. The system performance can be degraded if all the STAs try to select the same channel, or there is an uneven distribution of busy users among the different channels. For this reason different approaches for the channel selection are discussed in Section 5.2.1.

When a channel is selected, following the procedure discussed in Section 5.2.1, the TU senses that channel and if idle, TU waits for AIFS before proceeding. If on the other hand the channel is busy, the TU remains in the same state and looks for the channel activity in the next time slot. Once the channel is idle for AIFS, the TU waits for the BO counter to decrement to zero before entering the transmission start state $S_{i c, t s}$. In $S_{i c, t s}$ the packet is transmitted successfully with
a probability of $p_{i c, s}$ and enters the departure state $S_{i, d}$. The packet can experience a collision with probability $\left(1-p_{i c, s}-p_{i c, d}\right)$ and goes to the backoff state $S_{i c, b o}$. If the packet transmission is not successful even after $K_{i}$ retries it is dropped from the system with a probability $p_{i c, d}$. The BO counter is updated according to the binary exponential backoff algorithm.

The probability that transmission of TU in group $i$ and channel $c$ is a success is given as

$$
\begin{align*}
p_{i c, s} & =\sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times \prod_{j \neq i} \sum_{u_{j}=0}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} p_{s \mid\left(u_{i}+\sum_{j \neq i} u_{j}\right)} \tag{5.5}
\end{align*}
$$

where $p_{i c, b}$ is the probability that a STA in group $i$ is busy in channel $c$ and $p_{i} c$ is the transmission permission probability of any STA in $S_{i c, b o}$ state. $p_{s \mid n}$ is the probability that the transmission is successful in presence of $n$ interferers.

The probability $p_{i c, d}$ that a packet of a STA in group $i$ and $c^{t h}$ channel is involved in $K_{i}+1$ collisions and gets dropped from the system is given by,

$$
\begin{equation*}
p_{i c, d}=\left(1-p_{i c, s}\right)^{K_{i}+1} \tag{5.6}
\end{equation*}
$$

The expression for PGF of VPST using signal flow graph of Figure 5.2 is given
in (5.7),

$$
\begin{equation*}
B_{i}(z)=\sum_{c=1}^{C} \frac{p_{i c} p_{i c, I} p_{i c, s} z^{T_{i c}+1}}{1-\left\{\left(1-p_{i c, I}\right) z+p_{i c, I}\left(1-p_{i c}\right) z^{\sigma_{i c}}\right\}-p_{i c} p_{i c, I}\left(1-p_{i c, s}-p_{i c, d}\right) z^{D_{i c}+1}} \tag{5.7}
\end{equation*}
$$

where $p_{i c, I}$ is the probability of sensing the $c^{\text {th }}$ channel idle, $T_{i c}$ and $D_{i c}$ are the times for successful packet transmission and collision detection in the $c^{\text {th }}$ channel in group $i$. $p_{i c}$ depends on the backoff procedure used and is derived from the overall average backoff window. $B_{i}(z)$ is a function of $p_{i c, b}$ and $p_{i c, I}$, so the relationship between $p_{i c, I}$ and $p_{i c, b}$ is derived in order to solve the system.

## Probability of Sensing the $c^{\text {th }}$ Channel Idle by TU $i$

$p_{i c, I}$ is determined by analyzing the behavior of the $c^{\text {th }}$ channel. The channel has a time duration that is divided into a sequence of cycles. Each cycle has an idle duration and a busy duration. In the busy duration there can be a collision or a successful transmission. The TU in group $i$ is busy with a probability 1 , since it has packets available for transmission at a certain time. All the other users are busy with a probability $p_{i, b}$, since they operate at their equilibrium probability by assumption. The probability that a STA in group $i$ is busy in $c^{t h}$ channel is given as

$$
\begin{equation*}
p_{i c, b}=p_{c, c s} \times p_{i, b} \tag{5.8}
\end{equation*}
$$

To find the $p_{i c, I}$, the time when the $c^{\text {th }}$ channel is idle and also the time when the TU in group $i$ will sense the $c^{\text {th }}$ channel are needed. The expected idle time
for the TU in $i^{\text {th }}$ group and $c^{\text {th }}$ channel in each cycle is given as

$$
\begin{equation*}
E\left(I_{i c}\right)=\frac{\left(1-p_{i c}\right)\left(1-p_{i c} p_{i c, b}\right)^{\left(N_{i}-1\right)} \prod_{j \neq i}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}}}{1-\left(1-p_{i c}\right)\left(1-p_{i c} p_{i c, b}\right)^{\left(N_{i}-1\right)} \prod_{j \neq i}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}}} \tag{5.9}
\end{equation*}
$$

The TU will not sense the $c^{t h}$ channel for the time during which it is transmitting a packet. The transmission can result in either a success or a collision. In both cases if another STA is transmitting simultaneously in $c^{\text {th }}$ channel with the TU, and the time for collision detection for the other STA in group $j$ is more than the time for successful transmission of the TU in group $i$, then the TU will sense the $c^{\text {th }}$ channel for the time during which the other STA in group $j$ is transmitting after the completion of transmission by TU in group $i$. So, the probability that during a busy duration in a cycle the transmission by TU in group $i$ and $c^{t h}$ channel is a success with at least one transmitting STA in another group $j$ whose time for collision detection is more than the time for successful transmission of TU in group $i, D_{j c}>T_{i c}$ is calculated. This probability is denoted by $p_{i c, s t u, j}$ and is given by (5.10).

$$
\begin{align*}
& p_{i c, s t u, j}=\left[E\left(I_{i c}\right)+1\right] p_{i c} \sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c}>D_{j c}}}\left(1-p_{j_{h}} p_{j_{h}, b}\right)^{N_{j_{h}}}\right\} \times \sum_{\substack{u_{j}=1 \\
D_{j c}>T_{i c}}}^{N_{j}}\left(\begin{array}{c}
N_{j} \\
\\
u_{j}
\end{array}\right)\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod _ { \substack { j _ { l } \neq i , j \\
D _ { j _ { l } c } < D _ { j c } } } \sum _ { u _ { j _ { l } } = 0 } ^ { N _ { j _ { l } } } ( \begin{array} { c } 
{ N _ { j _ { l } } } \\
{ u _ { j _ { l } } }
\end{array} ) \left(p_{\left.\left.j c_{l} p_{j c_{l}, b}\right)^{u_{j_{l}}}\left(1-p_{j c_{l}} p_{j_{c}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\}, ~\left(p_{0}\right)}\right.\right. \\
& \left.\times p_{s \mid\left(u_{i}+u_{j}+\right.} \sum_{j_{l} \neq i, j} u_{j_{l}}\right) \tag{5.10}
\end{align*}
$$

Similarly, the transmission by another STA in group $i$ (same group as of the TU) can be successful and the corresponding success probabilities are denoted as $p_{i c, s o u, j}$ and $p_{i c, s o u} . p_{i c, s o u, j}$ given by (5.11) is the probability that during a busy duration in a cycle the transmission by a STA (other than the TU) in group $i$ and $c^{\text {th }}$ channel is successful in the presence of other STA from group $j$, such that $D_{j c}>T_{i c}$ and $p_{i c, s o u}$ given by (5.12) is the probability that during a busy duration in a cycle the transmission by a STA in group $i$ and $c^{t h}$ channel (other than the TU) is a success and there is no STA transmitting from group $j$, such
that $D_{j c}>T_{i c}$.

$$
\begin{align*}
& p_{i c, s o u, j}=\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c>D_{j c}}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \times \sum_{\substack{u_{j}=1 \\
D_{j c}>T_{i c}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l} c}<D_{j c}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l c} c} p_{j l c, b}\right)^{u_{j_{l}}}\left(1-p_{j l c} p_{j_{l c}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \left.\times p_{s \mid\left(u_{i}-1+u_{j}+\right.} \sum_{j_{l} \neq i, j} u_{j_{l}}\right) \\
& p_{i c, \text { sou }}=\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i \\
D_{j_{h}} \gg T_{i c}}}\left(1-p_{j_{h} c} c p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \\
& \times\left\{\prod_{\substack{j_{l} \neq i \\
D_{j_{c} c}<T_{i c}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l} c} p_{j_{l}, b, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l} c} p_{j_{l} c, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \times p_{s \mid\left(u_{i}-1+\sum_{j_{l} \neq i} u_{j_{l}}\right)} \tag{5.12}
\end{align*}
$$

The probability that during a busy duration in a cycle there is a successful transmission due to a STA in a group other than that of the TU group $i$ in $c^{\text {th }}$ channel,
is denoted by $p_{i c, s o g, j}$ and is given by (5.13).

$$
\begin{align*}
p_{i c, s o g, j} & =\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right)\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1} \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c}>T_{j_{j c}}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \\
& \times\left\{\prod _ { \substack { j _ { j } \neq \neq j , j \\
D _ { j _ { l } < T _ { j c } } } } \sum _ { u _ { j _ { l } } = 0 } ^ { N _ { j _ { l } } } ( \begin{array} { c } 
{ N _ { j _ { l } } } \\
{ u _ { j _ { l } } }
\end{array} ) \left(p_{\left.\left.j_{l c} p_{j l} p_{j c, b}\right)^{u_{j l}}\left(1-p_{j_{l c} c} p_{j_{l c}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\}}\right.\right. \\
& \times p_{s \mid\left(u_{j}-1+\underset{j_{l} \neq i, j}{ } u_{j_{l}}\right)} \tag{5.13}
\end{align*}
$$

STAs in each group experience a different time for collision detection and hence the collision probabilities for each collision scenario needs to be computed. The probability, $p_{i c, c t u, j}$, that during a busy duration in a cycle there is a collision involving the TU in group $i$ and $c^{\text {th }}$ channel and a STA in another group $j$ such that $D_{j c}>D_{i c}$ is given by (5.14).

$$
\begin{align*}
& p_{i c, c t u, j}=\left[E\left(I_{i c}\right)+1\right] p_{i c} \sum_{u_{i}=0}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c} c D_{j c}, D_{i c}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l} c}<D_{j c}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l} c} p_{j_{l} c, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l} c} p_{j_{l}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \left.\times\left(1-p_{s \mid\left(u_{i}+u_{j}+\right.} \sum_{j_{l} \neq i, j} u_{j_{l}}\right)\right) \tag{5.14}
\end{align*}
$$

Similarly, during a busy duration in a cycle the transmission by another STA in group $i$ (same group as of the TU) and $c^{\text {th }}$ channel can result in a collision and the corresponding collision probabilities are denoted by $p_{i c, c o u, j}$ when the collision is with another STA in group $j$ with $D_{j c}>D_{i c}$, and $p_{i c, c o u}$ when the collision involves a STA in group $j$ with $D_{j c} \leq D_{i c}$.

$$
\begin{align*}
& p_{i c, c o u, j}=\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c}>D_{j c}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \sum_{\substack{u_{j}=1 \\
D_{j c}>D_{c i}}}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l} c}<D_{j c}}} \sum_{u_{j_{l}}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l} c} p_{j_{l} c, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l} c} p_{j_{l} c, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \left.\times\left(1-p_{s \mid\left(u_{i}-1+u_{j}+\right.}^{j_{l} \neq i, j} u_{j_{l}}\right)\right) \tag{5.15}
\end{align*}
$$

$$
\begin{align*}
p_{i c, c o u} & =\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right) \sum_{u_{i}=1}^{N_{i}-1}\binom{N_{i}-1}{u_{i}}\left(p_{i c} p_{i c, b}\right)^{u_{i}}\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1-u_{i}} \\
& \times\left\{\prod_{\substack{j_{h} \neq i \\
D_{j_{h} c}>D_{i_{i c}}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \\
& \times\left\{\prod_{\substack{j_{l} \neq i \\
D_{j_{l} c}<D_{i c}}} \sum_{u_{j_{l}=0}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l} c} p_{j_{l} c, b}\right)^{u_{j_{l}}}\left(1-p_{j_{l} c} p_{j_{l c}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\} \\
& \times\left(1-p_{s \mid\left(u_{i}-1+\sum_{j_{l}} z_{i, j}\right.} u_{\left.j_{l}\right)}\right) \tag{5.16}
\end{align*}
$$

The probability that during a busy duration in a cycle there is a collision due to a STA in $c^{\text {th }}$ channel and in a group other than that of the TU group $i, p_{i c, \text { cog }, j}$ is given by (5.17).

$$
\begin{align*}
p_{i c, c o g, j} & =\left[E\left(I_{i c}\right)+1\right]\left(1-p_{i c}\right)\left(1-p_{i c} p_{i c, b}\right)^{N_{i}-1}\left\{\prod_{\substack{j_{h} \neq i, j \\
D_{j_{h} c}>D_{j c}}}\left(1-p_{j_{h} c} p_{j_{h} c, b}\right)^{N_{j_{h}}}\right\} \\
& \times \sum_{u_{j}=1}^{N_{j}}\binom{N_{j}}{u_{j}}\left(p_{j c} p_{j c, b}\right)^{u_{j}}\left(1-p_{j c} p_{j c, b}\right)^{N_{j}-u_{j}} \\
& \times\left\{\prod_{\substack{j_{l} \neq i, j \\
D_{j_{l}}<D_{j c}}} \sum_{u_{j_{l}=0}=0}^{N_{j_{l}}}\binom{N_{j_{l}}}{u_{j_{l}}}\left(p_{j_{l} c} p_{\left.j_{l} c, b\right)^{u_{j l}}}\left(1-p_{j_{l} c} p_{j_{l c}, b}\right)^{N_{j_{l}}-u_{j_{l}}}\right\}\right. \\
& \times\left(1-p_{\left.s \mid\left(u_{i}+u_{j}-1+\underset{j_{l}}{ } \sum_{j_{l i, j}} u_{j_{l}}\right)\right)}\right. \tag{5.17}
\end{align*}
$$

The TU will find the $c^{t h}$ channel idle for $\left(E\left(I_{i c}\right)+1\right)$ slots and the slots during which the TU can sense the channel are given as

$$
E\left(I_{i c}\right)+1+s_{i c, s u}+s_{i c, c o l}
$$

where $s_{i c, s u}$ are the slots in which the TU senses $c^{\text {th }}$ channel during a successful transmission and is given as

$$
\begin{array}{r}
s_{i c, s u}=\sum_{\substack{j \neq i \\
D_{j c}>T_{i c}}} p_{i c, s t u, j}\left(D_{j c}-T_{i c}\right)+\sum_{\substack{j \neq i \\
D_{j c}>T_{i c}}} p_{i c, s o u, j}\left(D_{j c}-1\right)+p_{i c, s o u}\left(T_{i c}-1\right) \\
+\sum_{j \neq i} p_{i c, s o g, j}\left(T_{j c}-1\right)
\end{array}
$$

and $s_{i c, \text { col }}$ are the slots in which the TU senses the $c^{\text {th }}$ channel during a collision, given as

$$
\begin{array}{r}
s_{i c, c o l}=\sum_{\substack{j \neq i \\
D_{j c}>D_{i c}}} p_{i c, c t u, j}\left(D_{j c}-D_{i c}\right)+\sum_{\substack{j \neq i \\
D_{j c}>D_{i c}}} p_{i c, c o u, j}\left(D_{j c}-1\right)+p_{i c, c o u}\left(D_{i c}-1\right) \\
\\
+\sum_{j \neq i} p_{i c, c o g, j}\left(D_{j c}-1\right)
\end{array}
$$

Therefore, the probability $p_{i c, I}$ is obtained as

$$
\begin{equation*}
p_{i c, I}=\frac{E\left(I_{i c}\right)+1}{E\left(I_{i c}\right)+1+s_{i c, s u}+s_{i c, c o l}} \tag{5.18}
\end{equation*}
$$

Equations (5.10)-(5.18) establish the relation between $p_{i c, I}$ and $p_{i c, b}$. Let $p_{i, 0}$ be the probability that a STA in group $i$ has an empty buffer, then the following relation holds

$$
\begin{equation*}
p_{i, b}+p_{i, 0}=1 \tag{5.19}
\end{equation*}
$$

Using the relations developed above, the distribution for packet service time $B_{i}(z)$ given the user idle probability $p_{i, 0}$ can be determined.

Average backoff window and the permission probability for a STA in group $i$ and $c^{\text {th }}$ channel

The permission probability $p_{i} c$ for a STA in group $i$ used in the previous section depends on the contention window size used for the protocol. The average size of
the contention window can be derived as discussed in [39], given as

$$
\begin{array}{r}
W_{i c, a v g}=\frac{1}{1-p_{i c, c}^{K_{i}}} \times\left(\frac{W_{i}\left(1-p_{i c, c}\right)\left(1-\left(2 p_{i c, c}\right)^{m_{i}}\right)}{2\left(1-2 p_{i c, c}\right)}-\frac{1-p_{i c, c}^{m_{i}}}{2}\right. \\
\left.+\frac{\left(2^{m_{i}} W_{i}-1\right)\left(p_{i c, c}^{m_{i}}-p_{i c, c}^{K_{i}}\right)}{2}\right) \tag{5.20}
\end{array}
$$

where $p_{i c, c}$ is the probability that the transmission by TU in group $i$ experiences a collision in $c^{t h}$ channel, and is given as

$$
\begin{equation*}
p_{i c, c}=1-p_{i c, s} \tag{5.21}
\end{equation*}
$$

Based on the overall average backoff window, the probability $p_{i c}$ that the TU in group $i$ (when it is busy) attempts to transmit in any arbitrary slot in $c^{\text {th }}$ channel is given by

$$
\begin{equation*}
p_{i c}=\frac{1}{W_{i c, a v g}+1} \tag{5.22}
\end{equation*}
$$

Using (5.5), (5.20) and (5.21), $p_{i c, c}$ and $W_{i c, a v g}$ can be computed. The permission probability is then computed using (5.22).

### 5.4 Performance Measures

The performance parameters are as described in 4.5.1.

Table 5.2: IEEE 802.11 parameters common to all traffic categories

| Parameter |  | Parameter |  |
| :---: | :---: | :---: | :---: |
| Bit rate $(\mathrm{BR})$ | $5.5,11,22,33 \mathrm{Mbps}$ | Basic bit rate $(\mathrm{BBR})$ | 1 Mbps |
| SIFS | $10 \mu \mathrm{sec}$ | Slot time | $20 \mu$ sec |
| Physical header $(\mathrm{PH})$ | $(144+48) / \mathrm{BBR}$ | MAC header $(\mathrm{MH})$ | $272 / \mathrm{BR}$ |
| RTS | $(160 / \mathrm{BR}+\mathrm{PH})$ | CTS | $112 / \mathrm{BR}+\mathrm{PH}$ |
| ACK | $112 / \mathrm{BR}+\mathrm{PH}$ |  |  |

### 5.5 Results and discussion

To validate the analytical results shown, an example of a heterogeneous system with three traffic categories (TC) is presented. These three TCs represent voice, video and data applications with $N_{i}=6$ STAs in each TC. Extensive simulations developed using Matlab are conducted to validate the proposed analysis using TUA. The IEEE 802.11 EDCF protocol, with parameters shown in Table 5.2, is used to simulate the system. As discussed, each TC can have its own set of parameters that ensure the QoS as given in Table 5.3.

The results presented are for RTS/CTS access mechanism, with STAs distributed around the AP following a bell shaped user distribution. A multipath

Table 5.3: Parameters related to the traffic categories

|  | Voice | Video | Data |
| :---: | :---: | :---: | :---: |
| AIFS <br> $\mu$ sec $]$ | 50 | 50 | 50 |
| Retry <br> limit, K | 8 | 8 | 15 |
| $C W_{\min }$ | 8 | 16 | 32 |
| $C W_{\max }$ | 16 | 64 | 1024 |
| Persistence <br> factor | 2 | 2 | 2 |
| Payload <br> $[$ bits/frame $]$ | 2000 | 10000 | 8000 |

frequency selective fading channel model, IEEE 802.11 Channel Model B [82], has been used to simulate the channel between the STAs and the AP. The normalized offered load has been varied to evaluate the system performance parameters including the system throughput, packet blocking probability, packet response and wait times and average buffer length.

The times for successful transmission or a collision are given by the following relations.

$$
\begin{aligned}
T_{i}^{\text {bas }} & =A I F S+H+P+S I F S+A C K \\
D_{i}^{\text {bas }} & =A I F S+H+P \\
T_{i}^{r t s} & =A I F S+R T S+S I F S+C T S+S I F S+H+P+S I F S+A C K \\
D_{i}^{r t s} & =A I F S+R T S
\end{aligned}
$$

Here bas and $r$ ts stand for basic access and access using RTS/CTS in IEEE 802.11 respectively, $H$ is the header which is the sum of physical and MAC header. The values for $T, D$ and $P$ at different data rates for different TCs are given in Table 5.4.

Normalized offered load is defined as the number of frame arrivals per time for successful transmission per all users and varies in the range, $\lambda_{n}=0.5: 0.5: 6$. The time for successful transmission is taken as the time required by video traffic (with maximum T ) to successfully transmit a packet @33 Mbps and is equal to $T 33 \mathrm{Mbps}=1174 \mu \mathrm{sec}$.

Table 5.4: Times (T, D and P) at different data rates for the traffic categories

|  | Voice |  |  |  | Video |  |  |  | Data |  |  |  |
| :---: | :---: | :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| Data rate <br> $[\mathrm{Mbps}]$ | 5.5 | 11 | 22 | 33 | 5.5 | 11 | 22 | 33 | 5.5 | 11 | 22 | 33 |
| $T_{i}^{\text {bas }}$ <br> $\mu \mathrm{sec}]$ | 879 | 662 | 554 | 518 | 2334 | 1390 | 918 | 761 | 1970 | 1208 | 827 | 700 |
| $D_{i}^{\text {bas }}$ <br> $[\mu \mathrm{sec}]$ | 656 | 449 | 346 | 312 | 2111 | 1177 | 710 | 555 | 1747 | 995 | 619 | 494 |
| $P_{i}^{\text {bas }}$ <br> $[\mu \mathrm{sec}]$ | 364 | 189 | 91 | 61 | 1819 | 910 | 455 | 304 | 1455 | 728 | 364 | 243 |
| $T_{i}^{\text {rts }}$ <br> $\left[\mu_{\mathrm{sec}}\right]$ | 1334 | 1092 | 972 | 931 | 2789 | 1820 | 1336 | 1174 | 2425 | 1638 | 1245 | 1113 |
| $D_{i}^{\text {rts }}$ <br> $[\mu \mathrm{sec}]$ | 272 | 257 | 250 | 247 | 272 | 257 | 250 | 247 | 272 | 257 | 250 | 247 |
| $P_{i}^{\text {rs }}$ <br> $[\mu \mathrm{sec}]$ | 364 | 189 | 91 | 61 | 1819 | 910 | 455 | 304 | 1455 | 728 | 364 | 243 |

### 5.5.1 Like channels

In order to analyse the MC system with Like channels, a system with 3 channels is selected. All the 3 channels operate with the same data rate of 11 Mbps . Following three cases are discussed

1. CASE1: The 3 TCs, each with $N_{i}=6$ STAs, can transmit on any of the parallel channels. This scenario is referred to as Shared Parallel Channel (SPC) and each channel has a data rate of 11 Mbps . The normalized offered load $\lambda_{n}=\lambda * N * T 33 M b p s$ where $N=N 1+N 2+N 3$.
2. CASE2: In this case each TC is assigned to specific channels and is termed as Dedicated Parallel Channel (DPC) scheme. In case of 3 channels each TC is assigned to 1 channel.
3. CASE3: For comparison purposes a case is discussed with a single channel
operating at the same data rate, i.e. 33 Mbps . All the TCs share the single channel for data transmission and the case is termed as Shared Single Channel (SSC). Results for SSC are for data rate of 33 Mbps , to compare the results with same total data rate.

For all the three cases discussed above the frames arrive in the queue of a STA for each TC with the same rate. The throughput for all the 3 cases is shown in Figure 5.3. From this figure it is seen that when the $\lambda_{n}$ is low which corresponds to light traffic load all the three cases perform in a similar way. For heavy traffic conditions Case3 for SSC gives the worst aggregate throughput as well as for each TC the throughput is the minimum. This is because at $\lambda_{n}>1$ the rate of collision increases and hence the performance in this case deteriorates. The best aggregate throughput is in Case2 for DPC, where each TC is assigned a separate channel. This reduces the collision among users from different TCs and hence improves the system throughput. The Case1 for SPC, when there are 3 channels available to all the TCs shows reasonable results. Especially the throughput for the TC with highest priority is the best in this case. The TC with least priority suffers in this case but with the advantage of providing QoS for the TC with highest priority. So it is concluded that with same frame arrival rate for each TC the QoS can be guaranteed by using multiple parallel channels.

An interesting feature is seen in case where the frames for different TCs arrive at different rate. When the voice traffic arrives at a faster rate as compared to video and data traffic, the results are shown in Figure 5.4. Similar results are


Figure 5.3: Throughput comparison for 3 cases


Figure 5.4: Throughput comparison with different offered load per TC. Vo:Vi:Da::4:1:1
shown for higher arrival rates of video and data frames in Figures 5.5 and 5.6 respectively. When the video traffic has a higher frame arrival rate SPC performs better than DPC for $\lambda_{n}<3$. This is explained by the fact that when the video frames arrive at a higher rate a single channel will not be able to process the input data, whereas 3 channels will accomodate the traffic with good throughput. From Figure 5.5 it can be seen that video traffic has a better throughput for this range of $\lambda_{n}$.

The effect of changing $C W$ min for improving the throughput for data traffic is shown in Figure 5.7, when $C W \min =16$ for both video and data traffic. This improved data throughput is accompanied by a slight decrease in the video traffic throughput. The voice traffic being the one with highest priority has almost the same throughput. In order to further improve the throughput $C W \min$ can be made even smaller and $C W \min =10$ gives results as shown in Figure 5.8. It can be seen that there is further improvement in the aggregate throughput but with slightly reduced throughput for the voice traffic.

### 5.5.2 Unlike channels

Channels with different data rates are considered to see the effect on system throughput. Following cases are considered:

1. CASE1: There are 2 channels with a fast channel @ 22 Mbps and the other @ 11 Mbps .
2. CASE2: There are 3 channels with a fast channel @ 22 Mbps and the other


Figure 5.5: Throughput comparison with different offered load per TC. Vo:Vi:Da::1:4:1


Figure 5.6: Throughput comparison with different offered load per TC. Vo:Vi:Da::1:1:4


Figure 5.7: Throughput for C=3 SPC each @11Mbps and same CWmin=16 for video and data traffic


Figure 5.8: Throughput for $\mathrm{C}=3 \mathrm{SPC}$ each @11Mbps and same CWmin=10 for video and data traffic

2 channels @ 5.5 Mbps each.
3. CASE3: There are 5 channels with a fast channel @ 11 Mbps and the other 4 channels @ 5.5 Mbps each.

The total data rate supported by the parallel channels in all cases is the same. The channel selection is first considered to be random and then the channel selection probability depending on the time required for successful packet transmission is used. For random selection each channel is selected with equal probability and $p_{c, c s}=\frac{1}{C}$. The channel selection is then considered where the channel with faster time for a successful packet transmission is selected with a higher probability, $p_{T_{1}, c s}$. The other channels are selected with probability $p_{T_{2}, c s}$ such that

$$
\begin{equation*}
p_{T_{2}, c s}=1-p_{T_{1}, c s} \tag{5.23}
\end{equation*}
$$

If $p_{c, c s}$ is the probability of selecting a channel then

$$
p_{c, c s}= \begin{cases}p_{T_{1}, c s}, & c=1  \tag{5.24}\\ \frac{p_{T_{2}, c s}}{C-1}, & c=2,3, \ldots, C\end{cases}
$$

The $p_{T_{1}, c s}$ is related to $p_{c, c s} ; c=2,3, \ldots, C$ by the following relation

$$
\begin{equation*}
p_{T_{1}, c s}=\frac{T_{2}}{T_{1}} \times p_{c, c s} \tag{5.25}
\end{equation*}
$$

Therefore, using (5.23)-(5.25) it can be shown that

$$
\begin{aligned}
p_{T_{1}, c s} & =\frac{\left(T_{2} / T_{1}\right)}{C-1+\left(T_{2} / T_{1}\right)} \\
p_{T_{2}, c s} & =\frac{C-1}{C-1+\left(T_{2} / T_{1}\right)}
\end{aligned}
$$

The results for CASE1 are shown in Figure 5.9. Similar results for CASE2 and CASE3 are shown in Figure 5.10 and 5.11 respectively. It can be seen that for the CASE2 where 3 channels with the fast channel @22Mbps and the other 2 channels @ 5.5 Mbps are used, the gain in throughput is more significant when the probability of channel selection based on the ratio of time for successful packet transmission between the 2 channels operating at different data rates is used. This is because for CASE2 this ratio is significant as compared to other cases. It is around 2 for CASE2 whereas it is maximum 1.5 for CASE3 and 1.3 for CASE1. So, for CASE1 and CASE3 the random selection of channels is simple approach and gives almost the same results as with priority channel selection scheme. It is also noted that although in CASE2 the fast channel has a data rate that is 4 times higher than the other channels, the time for successful transmission is only half and not one fourth. This is because of the overhead in terms of the PHY header that is transmitted at basic data rate of 1 Mbps and RTS, CTS frames that include the PHY header. So it is not advisable to select the channel only on the basis of its high data rate.


Figure 5.9: Throughput for C=2 Unlike channels @22,11 Mbps


Figure 5.10: Throughput for $\mathrm{C}=3$ Unlike channels @22,2x5.5 Mbps


Figure 5.11: Throughput for $\mathrm{C}=5$ Unlike channels @11,4x5.5 Mbps

### 5.6 Summary

Analysis for multiple channel systems, to evaluate the QoS parameters that had not been analysed due to the difficulty in analysing FU-FB systems, is presented using TUA. Channels of different types SPC, DPC and SSP have been considered for the heterogeneous traffic in the analysis. Scenarios for different network conditions have been considered. Performance under different channel selection schemes have been compared. Results show that by having multiple parallel channels instead of a single channel the system performance can be improved. Higher system throughput is achieved with increase in the number of channels. When the packet arrival rate is different for each TC, SPC performs better than the other schemes. Also, the priority channel selection gives good results only if the ratio of
time required for successful packet transmission between the channels operating at different data rates is significant i.e. more than 2 .

## CHAPTER 6

## CONCLUSIONS AND FUTURE

## RECOMMENDATIONS

### 6.1 Achievements of the work

In this thesis, the approximate but simple technique of TUA has been extended to the analysis of FU-FB systems under multipath frequency selective fading channels. The technique has been applied to different multiple access protocols including the S-ALOHA and IEEE 802.11 (CSMA/CA) protocols (DCF and EDCF). The results show that TUA can be successfully applied to the analysis of random multiple access protocols under various multipath channels. The stability analysis under such channel conditions has also been done. The TUA method has been applied to determine the range of channel transmission probability for the system where we can achieve maximum throughput while maintaining a unique system operating point. It has been shown that if channel transmission probability is
selected beyond this determined range, then the system operates with more than one equilibrium point and system can oscillate between these equilibrium points. The TUA has been established as a generalized analysis technique, that can be applied to various multiple access protocols under different channel conditions like ideal, flat fading and multipath frequency selective fading channels.

### 6.2 Summary of main contributions

The main contributions to the work carried out in this project can be summarised as follows:

- Mathematical model for the interference analysis under multipath frequency selective fading channels.
- Performance analysis under multipath frequency selective fading channels using TUA.
- Stability analysis for multiple access system under multipath frequency selective fading channels.
- Analysis of IEEE 802.11 DCF WLAN systems using TUA.
- Analysis of IEEE 802.11 EDCF WLAN systems with heterogeneous traffic using TUA.
- Applicability of TUA to multiple channel systems.


### 6.3 Future recommendations

In this thesis, TUA has been applied to the analysis of S-ALOHA and CSMA/CA based systems under multipath frequency selective fading channels. Different mathematical models, developed for computing the interference signal power, have been used to analyse homogeneous as well as heterogeneous traffic networks with ring and bell shaped distributed users. Finally, a framework has been presented for the analysis of CSMA/CA based IEEE 802.11 EDCF with multiple parallel channels available for transmission. Some recommendations for future work based on the work done are listed below.

- The analysis has been done for Rayleigh fading channels. These results can be extended to a more general fading channel setup, which includes Rayleigh, Rician, Nakagami-m, Weibull and Nakagami-q channels as special cases.
- The effect of AWGN and shadowing can be incorporated to make the analysis complete.
- The Poisson arrival process has been considered for analysis. Different arrival processes like Markov arrival process, which is a generalization of Poisson process, and the batch Markov arrival process can be used to simulate different types of traffics.
- The analysis can be extended to other MAC protocols like Mobile Slotted ALOHA (MS-ALOHA), Reservation ALOHA (R-ALOHA) etc.


## APPENDIX A

## PGF FOR VPST USING

## SIGNAL FLOW GRAPH

The expression for PGF of VPST in Figure 4.1 is evaluated by using signal flow graph (SFG). The signal flow graph is simplified as shown in Figure A.1. The same SFG can be used to fine the PGF for VPST by properly selecting the path gains. The path gains represented by (a-g) for IEEE 802.11 EDCF and S-ALOHA are given in Table A.1.

Table A.1: Gains for IEEE 802.11 EDCF and S-ALOHA

| Gain | IEEE802.11 EDCF | S-ALOHA |
| :---: | :---: | :---: |
| a | 1 | 1 |
| b | $\left(1-p_{i, I}\right) z$ | 0 |
| c | $p_{i, I}\left(1-p_{i}\right) z^{\sigma_{i}}$ | $(1-p) z$ |
| d | $p_{i} p_{i, I} z$ | $p z$ |
| e | $\left(1-p_{i, s}-p_{i, d}\right) z^{D_{i}}$ | $\left(1-p_{s}\right) z^{D}$ |
| f | $p_{i, s} z^{T_{i}}$ | $p_{s} z^{T}$ |
| g | $p_{i, d} z^{D_{i}}$ | 0 |

The PGF for VPST is finally obtained as





Figure A.1: Evaluation of PGF using state flow graph

$$
B(z)=\frac{a d f}{1-(b+c)-e d}
$$

Using above equation and gains in Table A.1, PGF for VPST of IEEE 802.11 EDCF (4.5) and S-ALOHA system (3.1) are obtained.

## APPENDIX B

## DERIVATION OF SELECTED

## EQUATIONS IN CHAPTER 2

## B. 1 Derivation of Equation 2.5

The conditional success probability is obtained by solving (2.4) using the expressions for the exponential power pdf of the SOI and the interfering signals

$$
\begin{aligned}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} \int_{0}^{\infty} \frac{1}{\bar{P}_{1}} \exp \left(-\frac{z w}{\bar{P}_{1}}\right) \frac{1}{\bar{P}_{n}} \exp \left(-\frac{w}{\bar{P}_{n}}\right) w d w d z \\
& =\frac{1}{\bar{P}_{1} \bar{P}_{n}} \int_{0}^{\infty} \exp \left(-\frac{w}{\bar{P}_{n}}\right) w \int_{z_{0}}^{\infty} \exp \left(-\frac{z w}{\bar{P}_{1}}\right) d z d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
\begin{aligned}
p_{s \mid n} & =\frac{1}{\bar{P}_{n}} \int_{0}^{\infty} \exp \left(-\frac{w}{\bar{P}_{n}}\right) \exp \left(-\frac{z_{0} w}{\bar{P}_{1}}\right) d w \\
& =\frac{1}{\bar{P}_{n}} \int_{0}^{\infty} \exp \left[-\frac{1}{\bar{P}_{n}}\left(1+\frac{\bar{P}_{n}}{\bar{P}_{1}}\right) w\right] d w \\
& =\frac{1}{1+z_{0} \frac{\bar{P}_{n}}{\overline{P_{1}}}}
\end{aligned}
$$

## B. 2 Derivation of Equation 2.7

The conditional success probability is computed by solving (2.4) using the expressions for the exponential power pdf of the SOI and the interfering signals.

$$
\begin{aligned}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} \int_{0}^{\infty} \frac{1}{\bar{P}} \exp \left(-\frac{z w}{\bar{P}}\right) \frac{1}{\bar{P}_{n}} \exp \left(-\frac{w}{\bar{P}_{n}}\right) w d w d z \\
& =\frac{1}{\overline{P P}_{n}} \int_{0}^{\infty} \exp \left(-\frac{w}{\bar{P}_{n}}\right) w \int_{z_{0}}^{\infty} \exp \left(-\frac{z w}{\bar{P}}\right) d z d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
\begin{aligned}
p_{s \mid n} & =\frac{1}{\bar{P}_{n}} \int_{0}^{\infty} \exp \left(-\frac{w}{\bar{P}_{n}}\right) \exp \left(-\frac{z_{0} w}{\bar{P}}\right) d w \\
& =\frac{1}{\bar{P}_{n}} \int_{0}^{\infty} \exp \left[-\frac{1}{\bar{P}_{n}}\left(1+\frac{\bar{P}_{n}}{\bar{P}}\right) w\right] d w \\
& =\frac{1}{1+z_{0} \frac{\bar{P}_{n}}{\bar{P}}}
\end{aligned}
$$

where $\bar{P}=\sum_{m=1}^{M} \bar{P}_{m}$ and $\bar{P}_{n}=n \sum_{m=1}^{M} \bar{P}_{m}$, hence

$$
p_{s \mid n}=\frac{1}{1+n z_{0}}
$$

## B. 3 Derivation of Equation 2.13

Substituting the expression of power pdf for the SOI and (2.11) in (2.4), the expression for the conditional success probability of the tagged user, $p_{s \mid n}$, is calculated as given below:

$$
\begin{aligned}
p_{s \mid n}= & \int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
= & \int_{z_{0}}^{\infty} \int_{0}^{\infty} \frac{1}{\bar{P}_{1}} \exp \left(-\frac{w z}{\bar{P}_{1}}\right) \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \\
& \quad \times \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M(n+1)+k-2} \exp \left(-w / \bar{P}_{M}\right)}{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)} w d w d z \\
= & \frac{1}{\bar{P}_{1}} \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \int_{0}^{\infty} \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M(n+1)+k-2} \exp \left(-w / \bar{P}_{M}\right)}{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)} w \\
& \quad \times \int_{z_{0}}^{\infty} \exp \left(-\frac{w z}{\bar{P}_{1}}\right) d z d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
\begin{aligned}
p_{s \mid n}= & \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \int_{0}^{\infty} \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M(n+1)+k-2} \exp \left[-\frac{1}{\bar{P}_{M}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right) w\right]}{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)} d w \\
= & \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \sum_{k=0}^{\infty} \frac{\delta_{k}}{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)} \\
& \quad \times \int_{0}^{\infty} w^{M(n+1)+k-2} \exp \left[-\frac{1}{\bar{P}_{M}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right) w\right] d w
\end{aligned}
$$

Using the following definite integral relation and integrating w.r.t. w

$$
\begin{gather*}
\int_{0}^{\infty} x^{a} \exp (-b x) d x=\frac{\Gamma(a+1)}{b^{a+1}}  \tag{B.1}\\
p_{s \mid n}=\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \sum_{k=0}^{\infty} \frac{\bar{P}_{M}^{M(n+1)-1+k} \Gamma(M(n+1)-1+k)}{\delta_{k}} \\
\times \frac{\Gamma(M(n+1)-1+k)}{\left[\frac{1}{\bar{P}_{M}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right)\right]^{M(n+1)-1+k}} \\
= \\
\prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{\alpha_{m}} \frac{1}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right)^{M(n+1)-1}} \sum_{k=0}^{\infty} \frac{\delta_{k}}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{1}}+1\right)^{k}}
\end{gather*}
$$

## B. 4 Derivation of Equation 2.18

Using (2.14) and (2.16) in (2.4) the expression for the conditional success probability $p_{s \mid n}$ is derived as follows:

$$
\begin{aligned}
p_{s \mid n}= & \int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
= & \int_{z_{0}}^{\infty} \int_{0}^{\infty} \sum_{j=1}^{M} \bar{P}_{j}^{M-2} \exp \left(-\frac{w z}{\bar{P}_{j}}\right) \prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}} \\
& \times \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\overline{\bar{P}}_{m}}\right)^{n} \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M n+k-1} \exp \left(-w / \bar{P}_{M}\right)}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)} w d w d z \\
= & \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\overline{\bar{P}}_{m}}\right)^{n} \int_{0}^{\infty} \sum_{j=1}^{M} \bar{P}_{j}^{M-2} \int_{z_{0}}^{\infty} \exp \left(-\frac{w z}{\bar{P}_{j}}\right) d z \prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}} \\
& \times \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M n+k-1} \exp \left(-w / \bar{P}_{M}\right)}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)} w d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
\begin{aligned}
& p_{s \mid n}= \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \int_{0}^{\infty} \sum_{j=1}^{M} \bar{P}_{j}^{M-1} \exp \left(-z_{0} \frac{w}{\bar{P}_{j}}\right) \prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}} \\
& \times \sum_{k=0}^{\infty} \frac{\delta_{k} w^{M n+k-1} \exp \left(-w / \bar{P}_{M}\right)}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)} d w \\
&= \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \int_{0}^{\infty} \sum_{j=1}^{M} \bar{P}_{j}^{M-1} \prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}} \\
&= \times \sum_{m=1}^{\infty} \frac{\delta_{k} w^{M n+k-1} \exp \left[-\frac{1}{\bar{P}_{M}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right) w\right]}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)} d w \\
&\left.\bar{P}_{m}\right)^{n} \sum_{j=1}^{M} \bar{P}_{j}^{M-1} \prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}} \\
& \times \sum_{k=0}^{\infty} \frac{\delta_{k} \int_{0}^{\infty} w^{M n+k-1} \exp \left[-\frac{1}{\bar{P}_{M}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right) w\right] d w}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)}
\end{aligned}
$$

Using (B.1) and integrating w.r.t. w

$$
\begin{aligned}
p_{s \mid n}= & \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{i=1, i \neq j}^{M} \frac{1}{\bar{P}_{j}-\bar{P}_{i}}} \\
& \times \sum_{k=0}^{\infty} \frac{\delta_{k} \Gamma(M n+k)}{\bar{P}_{M}^{M n+k} \Gamma(M n+k)\left[\frac{1}{\overline{P_{M}}}\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right)\right]^{M n+k}} \\
= & \prod_{m=1}^{M}\left(\frac{\bar{P}_{M}}{\bar{P}_{m}}\right)^{n} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{i=1, i \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{i}\right)} \times \frac{1}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right)^{M n}} \sum_{k=0}^{\infty} \frac{\delta_{k}}{\left(z_{0} \frac{\bar{P}_{M}}{\bar{P}_{j}}+1\right)^{k}}
\end{aligned}
$$

## B. 5 Derivation of Equation 2.20

Using (2.19) and the exponential pdf for the power of SOI in (2.4) the expression for the conditional success probability $p_{s \mid n}$ is derived as follows:

$$
\begin{aligned}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} \int_{0}^{\infty} \frac{1}{\bar{P}} \exp \left(-\frac{z w}{\bar{P}}\right) \frac{1}{\bar{P}} \frac{(w / \bar{P})^{n-1}}{\Gamma(n)} \exp \left(-\frac{w}{\bar{P}}\right) w d w d z \\
& =\frac{1}{\bar{P}^{n+1}} \int_{0}^{\infty} \frac{w^{n-1}}{\Gamma(n)} \exp \left(-\frac{w}{\bar{P}}\right) w \int_{z_{0}}^{\infty} \exp \left(-\frac{z w}{\bar{P}}\right) d z d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
p_{s \mid n}=\frac{1}{\bar{P}^{n} \Gamma(n)} \int_{0}^{\infty} w^{n-1} \exp \left[-\left(\frac{z_{0}+1}{\bar{P}}\right) w\right] d w
$$

Using (B.1) and integrating w.r.t. w

$$
\begin{aligned}
p_{s \mid n} & =\frac{1}{\bar{P}^{n} \Gamma(n)} \times \frac{\Gamma(n)}{\left(\frac{z_{0}+1}{\bar{P}}\right)^{n}} \\
& =\frac{1}{\left(z_{0}+1\right)^{n}}
\end{aligned}
$$

## B. 6 Derivation of Equation 2.22

Using (2.19) and (2.21) in (2.4) the expression for the conditional success probability $p_{s \mid n}$ is derived as follows:

$$
\begin{aligned}
p_{s \mid n} & =\int_{z_{0}}^{\infty} \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w d z \\
& =\int_{z_{0}}^{\infty} \int_{0}^{\infty} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-2} \exp \left(-\frac{z w}{\bar{P}_{j}}\right)}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} \times \frac{1}{\bar{P}} \frac{(w / \bar{P})^{n-1}}{\Gamma(n)} \exp \left(-\frac{w}{\bar{P}}\right) w d w d z \\
& =\frac{1}{\bar{P}^{n} \Gamma(n)} \int_{0}^{\infty} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-2}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} w^{n-1} \exp \left(-\frac{w}{\bar{P}}\right) w \int_{z_{0}}^{\infty} \exp \left(-\frac{z w}{\bar{P}_{j}}\right) d z d w
\end{aligned}
$$

Integrating w.r.t. z,

$$
\begin{aligned}
p_{s \mid n} & =\frac{1}{\bar{P}^{n} \Gamma(n)} \int_{0}^{\infty} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} w^{n-1} \exp \left[-\frac{1}{\bar{P}}\left(z_{0} \frac{\bar{P}}{\bar{P}_{j}}+1\right) w\right] d w \\
& =\frac{1}{\bar{P}^{n} \Gamma(n)} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} \int_{0}^{\infty} w^{n-1} \exp \left[-\frac{1}{\bar{P}}\left(z_{0} \frac{\bar{P}}{\bar{P}_{j}}+1\right) w\right] d w
\end{aligned}
$$

Using (B.1) and integrating w.r.t. w

$$
\begin{aligned}
p_{s \mid n} & =\frac{1}{\bar{P}^{n} \Gamma(n)} \sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} \times \frac{\Gamma(n)}{\left[\frac{1}{\bar{P}}\left(z_{0} \frac{\bar{P}}{\bar{P}_{j}}+1\right)\right]^{n}} \\
& =\sum_{j=1}^{M} \frac{\bar{P}_{j}^{M-1}}{\prod_{k=1, k \neq j}^{M}\left(\bar{P}_{j}-\bar{P}_{k}\right)} \times \frac{1}{\left(z_{0} \frac{\bar{P}}{\bar{P}_{j}}+1\right)^{n}}
\end{aligned}
$$

## B. 7 Derivation of Equation 2.38

The expression for success probability is derived using (2.4), given below,

$$
\begin{equation*}
p_{s \mid n}=1-F_{Z_{n}}\left(z_{0}\right) \tag{B.2}
\end{equation*}
$$

where the CDF for $Z_{n}$ is given as

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w
$$

Substituting the values from (2.29) and (2.30), we have

$$
\begin{align*}
& F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{z_{0}} d z \int_{0}^{\infty} w d w \int_{0}^{\infty} f_{P_{t}}\left(z w \mid \bar{p}_{t}\right) f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
& \int_{0}^{\infty} f_{P_{n}}\left(w \mid \bar{p}_{n}\right) f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \tag{B.3}
\end{align*}
$$

The conditional power pdf for the interfering signals is exponentially distributed and is given as below.

$$
\begin{equation*}
f_{P_{n}}\left(p_{n} \mid \bar{p}_{n}\right)=\frac{1}{\bar{p}_{n}} \exp \left(-\frac{p_{n}}{\bar{p}_{n}}\right) \tag{B.4}
\end{equation*}
$$

where $\bar{p}_{n}$ is the average power of the interference signals and the pdf of the average power is as given in (2.35). Using (B.4) and exponential conditional power pdf
for the case when SOI is dominant path of the TU transmitted signal in (B.3)

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right)= & \int_{0}^{z_{0}} d z \int_{0}^{\infty} d w \int_{0}^{\infty} \frac{w}{\bar{p}_{t}} \exp \left(-\frac{w z}{\bar{p}_{t}}\right) f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
& \quad \int_{0}^{\infty} \frac{1}{\bar{p}_{n}} \exp \left(-\frac{w}{\bar{p}_{n}}\right) f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \\
= & \int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \\
& \quad \int_{0}^{\infty} \frac{w}{\bar{p}_{t} \bar{p}_{n}} \exp \left[-\left(\frac{z}{\bar{p}_{t}}+\frac{1}{\bar{p}_{n}}\right) w\right] d w
\end{aligned}
$$

Integrating w.r.t. $w$,

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right) & \left.=\int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \frac{\bar{p}_{n} / \bar{p}_{t}}{\left(z \overline{\bar{p}}_{n}\right.}+1\right)^{2} \\
& =\int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \int_{0}^{z_{0}} \frac{\bar{p}_{n} / \bar{p}_{t}}{\left(z \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)^{2}} d z
\end{aligned}
$$

Now integrating w.r.t. $z$,

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) \frac{z_{0} \bar{p}_{n} / \bar{p}_{t}}{\left(z_{0} \overline{\bar{p}}_{n}+1\right)} d \bar{p}_{n}
$$

Substituting the values from (2.31) and (2.35) in above equation,

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right) & =\int_{0}^{\infty} \frac{\bar{P}_{1}^{1 / 2}}{2} \bar{p}_{t}^{-3 / 2} \exp \left(-\frac{\pi}{4} \frac{\bar{P}_{1}}{\bar{p}_{t}}\right) \\
& \times\left[\int_{0}^{\infty} \frac{K_{t_{n}}}{2} \bar{p}_{n}^{-3 / 2} \exp \left(-\frac{\pi K_{t_{n}}^{2}}{4 \bar{p}_{n}}\right) \frac{z_{0} \bar{p}_{n} / \bar{p}_{t}}{\left(z_{0} \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)} d \bar{p}_{n}\right] d \bar{p}_{t}
\end{aligned}
$$

First solving the inner integral by defining $t=\bar{p}_{n}^{-1 / 2}$, so

$$
\left.\begin{array}{rl}
F_{Z_{n}}\left(z_{0}\right) & =\int_{0}^{\infty} \frac{\bar{P}_{1}^{1 / 2}}{2} \bar{p}_{t}^{-3 / 2} \exp \left(-\frac{\pi}{4} \bar{P}_{1}\right. \\
\bar{p}_{t}
\end{array}\right)
$$

Using the definite integral relation

$$
\begin{gather*}
\int_{0}^{\infty} \frac{\exp \left(-a t^{2}\right)}{t^{2}+x^{2}} d t=\frac{\pi}{2 x} \exp \left(a x^{2}\right) \operatorname{erfc} \sqrt{a} x  \tag{B.5}\\
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} \frac{\bar{P}_{1}^{1 / 2}}{2} \bar{p}_{t}^{-3 / 2} \exp \left(-\frac{\pi}{4} \frac{\bar{P}_{1}}{\bar{p}_{t}}\right) \\
\quad \times \sqrt{\pi} \sqrt{\frac{\pi K_{t_{n}}^{2} z_{0}}{4 \bar{p}_{t}}} \exp \left(\frac{\pi K_{t_{n}}^{2} z_{0}}{4 \bar{p}_{t}}\right) \operatorname{erfc}\left(\sqrt{\frac{\pi K_{t_{n}}^{2} z_{0}}{4 \bar{p}_{t}}}\right) d \bar{p}_{t}
\end{gather*}
$$

Now solving the outer integral by defining $x=\frac{\pi}{4 \bar{p}_{t}}$, so

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} \bar{P}_{1}^{1 / 2} K_{t_{n}} \sqrt{z_{0}} \exp \left[-\left(\bar{P}_{1}-K_{t_{n}}^{2} z_{0}\right) x\right] \operatorname{erfc}\left(\sqrt{K_{t_{n}}^{2} z_{0} x}\right) d x
$$

Recognizing that $\operatorname{erfc}(t)=1-\operatorname{erf}(t)$ and using the definite integral relation

$$
\begin{gather*}
\int_{0}^{\infty} \exp (-a t) \operatorname{erf}(\sqrt{b t}) d t=\frac{1}{a} \sqrt{\frac{b}{a+b}}  \tag{B.6}\\
F_{Z_{n}}\left(z_{0}\right)=\frac{K_{t_{n}} \sqrt{z_{0}}}{\sqrt{\bar{P}_{1}}+K_{t_{n}} \sqrt{z_{0}}}
\end{gather*}
$$

Substituting value of $F_{Z_{n}}\left(z_{0}\right)$ in (B.2),

$$
p_{s \mid n}=\frac{1}{1+K_{t_{n}} \sqrt{\frac{z_{0}}{P_{1}}}}
$$

## B. 8 Derivation of Equation 2.44

The expression for success probability is derived using (B.2).

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{P_{t}}(z w) f_{P_{n}}(w) w d w
$$

Substituting the values from (2.29) and (2.30), we have

$$
\begin{gather*}
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{z_{0}} d z \int_{0}^{\infty} w d w \int_{0}^{\infty} f_{P_{t}}\left(z w \mid \bar{p}_{t}\right) f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
\int_{0}^{\infty} f_{P_{n}}\left(w \mid \bar{p}_{n}\right) f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \tag{B.7}
\end{gather*}
$$

The conditional power pdf for the interfering signals is exponentially distributed and is given as below.

$$
\begin{equation*}
f_{P_{n}}\left(p_{n} \mid \bar{p}_{n}\right)=\frac{1}{\bar{p}_{n}} \exp \left(-\frac{p_{n}}{\bar{p}_{n}}\right) \tag{B.8}
\end{equation*}
$$

where $\bar{p}_{n}$ is the average power of the interference signals and the pdf of the average power is as given in (2.35). Using (B.8) and the conditional power pdf for the SOI
with all M paths of TU combined as phasor sum in (B.7)

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right)= & \int_{0}^{z_{0}} d z \int_{0}^{\infty} d w \int_{0}^{\infty} \frac{w}{\bar{p}_{t}} \exp \left(-\frac{w z}{\bar{p}_{t}}\right) f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \\
& \quad \int_{0}^{\infty} \frac{1}{\bar{p}_{n}} \exp \left(-\frac{w}{\bar{p}_{n}}\right) f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \\
= & \int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \\
& \quad \int_{0}^{\infty} \frac{w}{\bar{p}_{t} \bar{p}_{n}} \exp \left[-\left(\frac{z}{\bar{p}_{t}}+\frac{1}{\bar{p}_{n}}\right) w\right] d w
\end{aligned}
$$

Integrating w.r.t. $w$,

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right) & =\int_{0}^{z_{0}} d z \int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \frac{\bar{p}_{n} / \bar{p}_{t}}{\left(z \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)^{2}} \\
& =\int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) d \bar{p}_{n} \int_{0}^{z_{0}} \frac{\bar{p}_{n} \bar{p}_{t}}{\left(z \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)^{2}} d z
\end{aligned}
$$

Now integrating w.r.t. $z$,

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} f_{\bar{P}_{t}}\left(\bar{p}_{t}\right) d \bar{p}_{t} \int_{0}^{\infty} f_{\bar{P}_{n}}\left(\bar{p}_{n}\right) \frac{z_{0} \bar{p}_{n} / \bar{p}_{t}}{\left(z_{0} \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)} d \bar{p}_{n}
$$

Substituting the values from (2.40) and (2.42) in above equation,

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right) & =\int_{0}^{\infty} \frac{K_{t}\left(\bar{p}_{t}\right)^{-3 / 2}}{2 \sqrt{\pi}} \exp \left(-\frac{K_{t}^{2}}{4 \bar{p}_{t}}\right) \\
& \times\left[\int_{0}^{\infty} \frac{n K_{t}\left(\bar{p}_{n}\right)^{-3 / 2}}{2 \sqrt{\pi}} \exp \left(-\frac{n^{2} K_{t}^{2}}{4 \bar{p}_{n}}\right) \frac{z_{0} \bar{p}_{n} / \bar{p}_{t}}{\left(z_{0} \frac{\bar{p}_{n}}{\bar{p}_{t}}+1\right)} d \bar{p}_{n}\right] d \bar{p}_{t}
\end{aligned}
$$

First solving the inner integral by defining $t=\bar{p}_{n}^{-1 / 2}$, so

$$
\begin{aligned}
F_{Z_{n}}\left(z_{0}\right) & =\int_{0}^{\infty} \frac{K_{t}\left(\bar{p}_{t}\right)^{-3 / 2}}{2 \sqrt{\pi}} \exp \left(-\frac{K_{t}^{2}}{4 \bar{p}_{t}}\right) \\
& \times\left[\int_{0}^{\infty} \frac{n K_{t} z_{0} / \bar{p}_{t}}{\sqrt{\pi}\left(z_{0} / \bar{p}_{t}+t^{2}\right)} \exp \left(-\frac{n^{2} K_{t}^{2} t^{2}}{4}\right) d t\right] d \bar{p}_{t}
\end{aligned}
$$

Using the definite integral relation (B.5),

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} \frac{K_{t}^{2} n \sqrt{z_{0}} \bar{p}_{t}^{-2}}{4} \exp \left[-\frac{K_{t}^{2}}{4 \bar{p}_{t}}\left(1-n^{2} z_{0}\right)\right] \operatorname{erfc}\left(\sqrt{\frac{K_{t_{n}}^{2} n^{2} z_{0}}{4 \bar{p}_{t}}}\right) d \bar{p}_{t}
$$

Now solving the outer integral by defining $x=\frac{K_{t}^{2}}{4 \bar{p}_{t}}$, so

$$
F_{Z_{n}}\left(z_{0}\right)=\int_{0}^{\infty} n \sqrt{z_{0}} \exp \left[-\left(1-n^{2} z_{0}\right) x\right] \operatorname{erfc}\left(\sqrt{n^{2} z_{0} x}\right) d x
$$

Recognizing that $\operatorname{erfc}(t)=1-\operatorname{erf}(t)$ and using the definite integral relation (B.6),

$$
F_{Z_{n}}\left(z_{0}\right)=\frac{n \sqrt{z_{0}}}{1+n \sqrt{z_{0}}}
$$

Substituting value of $F_{Z_{n}}\left(z_{0}\right)$ in (B.2),

$$
p_{s \mid n}=\frac{1}{1+n \sqrt{z_{0}}}
$$

## APPENDIX C

## DERIVATION OF SELECTED

## EQUATION IN CHAPTER 4

## C. 1 Derivation of Equation 4.17

As discussed in Section 4.2 the backoff (BO) procedure follows a binary exponential backoff. The contention window size is initially set to $W_{i}$ (=CWmin) and is doubled if there is a collision. The average BO window size, if there is no collision, is given as $\left(W_{i}-1\right) / 2$. If there is a collision with probability $p_{i, c}$, the backoff window is doubled and the average BO window size becomes $\left(2 W_{i}-1\right) / 2$. This continues for $m_{i}$ collisions and after that the BO window size is fixed with an average BO window size of $\left(2_{i}^{m} W_{i}-1\right) / 2$ until the $K_{i}^{t h}$ retry.

The overall BO window can hence be written as

$$
\begin{align*}
W_{i, a v g} & =\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left(\frac{W-1}{2}\right)+\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right) p_{i, c}\left(\frac{2 W-1}{2}\right)+\ldots \\
& +\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right) p_{i, c}^{m_{i}-1}\left(\frac{2^{m_{i}-1} W-1}{2}\right)+\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right) p_{i, c}^{m_{i}}\left(\frac{2^{m_{i}} W-1}{2}\right)+ \\
& +\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right) p_{i, c}^{m_{i}+1}\left(\frac{2^{m_{i}} W-1}{2}\right)+\ldots+\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right) p_{i, c}^{K_{i, c}-1}\left(\frac{2^{m_{i}} W-1}{2}\right) \\
& =\underbrace{\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left\{\left(\frac{W-1}{2}\right)+p_{i, c}\left(\frac{2 W-1}{2}\right)+\ldots+p_{i, c}^{m_{i}-1}\left(\frac{2^{m_{i}-1} W-1}{2}\right)\right\}}_{\text {SUM1 }}+ \\
& +\underbrace{\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left(\frac{2^{m_{i}} W-1}{2}\right)\left\{p_{i, c}^{m_{i}}+p_{i, c}^{m_{i}+1}+\ldots+p_{i, c}^{K_{i}-1}\right\}}_{\text {SUM2 }} \tag{C.1}
\end{align*}
$$

where $\left(1-p_{i, c}\right)$ is the probability of successful transmission and $\left(1-p_{i, c}^{K}\right)$ is the normalization term. The expressions for SUM1 and SUM2 are simplified as follows.

$$
\begin{aligned}
S U M 1 & =\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left\{\left(\frac{W-1}{2}\right)+p_{i, c}\left(\frac{2 W-1}{2}\right)+\ldots+p_{i, c}^{m_{i}-1}\left(\frac{2^{m_{i}-1} W-1}{2}\right)\right\} \\
& =\frac{1}{2}\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left\{(W-1)+p_{i, c}(2 W-1)+\ldots+p_{i, c}^{m_{i}-1}\left(2^{m_{i}-1} W-1\right)\right\} \\
& =\frac{1}{2}\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left\{W\left(1+2 p_{i, c}+\ldots+2^{m_{i}-1} p_{i, c}^{m_{i}-1}\right)-\left(1+p_{i, c}+\ldots+p_{i, c}^{m_{i}-1}\right)\right\}
\end{aligned}
$$

The following theorem is used for calculating geometric series,

$$
\begin{equation*}
\sum_{k=0}^{n-1} a r^{k}=a\left(\frac{r^{n}-1}{r-1}\right) \tag{C.2}
\end{equation*}
$$

So,

$$
S U M 1=\frac{1}{2}\left(\frac{1}{1-p_{i, c}^{K}}\right)\left\{\frac{W_{i}\left(1-p_{i, c}\right)\left(1-\left(2 p_{i, c}\right)^{m_{i}}\right)}{2\left(1-2 p_{i, c}\right)}-\frac{1-p_{i, c}^{m_{i}}}{2}\right\}
$$

Also, SUM2 is simplified as given below,

$$
\begin{aligned}
S U M 2 & =\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left(\frac{2^{m_{i}} W-1}{2}\right)\left\{p_{i, c}^{m_{i}}+p_{i, c}^{m_{i}+1}+\ldots+p_{i, c}^{K_{i}-1}\right\} \\
& =\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left(\frac{2^{m_{i}} W-1}{2}\right) p_{i, c}^{m_{i}}\left\{1+p_{i, c}+\ldots+p_{i, c}^{K_{i}-1-m}\right\}
\end{aligned}
$$

Again using (C.2),

$$
\begin{aligned}
S U M 2 & =\left(\frac{1-p_{i, c}}{1-p_{i, c}^{K}}\right)\left(\frac{2^{m_{i}} W-1}{2}\right) p_{i, c}^{m_{i}} \frac{p_{i, c}^{K-m}-1}{p_{i, c}-1} \\
& =\left(\frac{1}{1-p_{i, c}^{K}}\right)\left(\frac{2^{m_{i}} W-1}{2}\right)\left(p_{i, c}^{m}-p_{i, c}^{K}\right)
\end{aligned}
$$

Substituting SUM1 and SUM2 in (C.1) gives the overall average size of the contention window,

$$
\begin{array}{r}
W_{i, a v g}=\frac{1}{1-p_{i, c}^{K_{i}}} \times\left(\frac{W_{i}\left(1-p_{i, c}\right)\left(1-\left(2 p_{i, c}\right)^{m_{i}}\right)}{2\left(1-2 p_{i, c}\right)}-\frac{1-p_{i, c}^{m_{i}}}{2}\right. \\
\left.+\frac{\left(2^{m_{i}} W_{i}-1\right)\left(p_{i, c}^{m_{i}}-p_{i, c}^{K_{i}}\right)}{2}\right)
\end{array}
$$

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