STABILIZATION OF NETWORKED CONTROL SYSTEMS WITH RANDOM DELAYS

BY

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بسم الله الرحمن الرحيم

I lovingly dedicate this thesis to my parents, brothers & sister for all their love, support, encouragement & faith in me.

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THESIS ABSTRACT

Name: Mirza Hamedulla Baig.

Title: Stabilization of Networked Control Systems with Random Delays.

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In this thesis, a survey of the extensive research investigations performed on Networked systems subject to random delays is presented. The survey takes into consideration several technical views on the analysis and design procedures leading to stability results and outlines basic assumptions. In an NCS, the network can introduce unreliable and time-dependent levels of service because of delays, jitter, or losses. In turn, these network vagaries can jeopardize the stability, safety, and performance of the physical system. The primary objective of our research is to devise control algorithms to compensate for the vagaries of network service. Our strategies are targeted mainly at dealing with random networked delays. Results related to stability along with controller synthesis for networked systems will be provided. Some typical examples will also be provided to illustrate relevant issues.

ملخص الرسالة

الاسم الكامل: ميرزا حميد الله بيغ

عنوان الرسالة: استقرار نظم التحكم بالشبكات مع تأخير عشوائي

التخصص: هندسة النظم

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تقدم هذه الاطروحه دراسه بحثيه مستفيضه و متعمقه للنظم المستخدمه لشبكات الاتصالات فى عروه التغذيه العكسيه. وتركز الاطروحه أراء فنيه متنوعه لتحليل الاداء و تصميم المحاكمات مع ضمان الاتزان ومراعاه الفرضيات الاساسية. وحيث أن النظم المستخدمه لشبكات الاتصالات فى عروه التغذيه العكسيه تعانى بصفه رئيسيه من المشاكل المتعدده التى تعوق الاداء مثل التاخرالغير محدد للاشارات والتساقط العشوائى للرسائل والتقطيع الغير متزامن و التحديد الكمى لمتغيرات الحاله مما قد يؤدى الى عدم اتزانيه النظم . وتهدف هذه الاطروحه الى استنباط و تطوير اساليب لتصميم المحاكمات لمجابهة التأخر العشوائى للاشارات ثم تفرد لمحاكاه نظم عمليه و صناعية.

Chapter 1

INTRODUCTION

1.1 What is NCS?

Networked control systems are control systems comprised of the system to be controlled and of actuators, sensors, and controllers, the operation of which is coordinated via a shared communication network. These systems are typically spatially distributed, may operate in an asynchronous manner, but have their operation coordinated to achieve desired overall objectives.

Research on Networked control systems (NCS) has been the prime focus both in academia and in industrial applications for several decades. NCS has now developed into a multidisciplinary area. In this chapter, we provide an introduction to NCS and the different forms of NCS. The chapter begins with the history of NCS, different advantages of having such systems. As we proceed further, the chapter gives an insight to different challenges faced with building efficient, stable and secure NCS. We also discuss the different fields and research arenas, which are part of NCS and which work together to deal with different NCS issues. The following chapters provide a brief literature survey concerning each topic highlighting the recent trends in the evolution networked control systems.

For many years researchers have given us precise and optimum control strategies emerging from classical control theory, starting from open-loop control to sophisticated control strategies based on genetic algorithms. The advent of communication networks, however, introduced the concept of remotely controlling a system, which gave birth to networked control systems (NCS). The classical definition of NCS can be as follows: When a traditional feedback control system is closed via a communication channel, which may be shared with other nodes outside the control system, then the control system is called an NCS. An NCS can also be defined as a feedback control system wherein the control loops are closed through a real-time network. The defining feature of an NCS is that information (reference input, plant output, control input, etc.) is exchanged using a network among control system components (sensors, controllers, actuators, etc.), see Fig. 1.1.

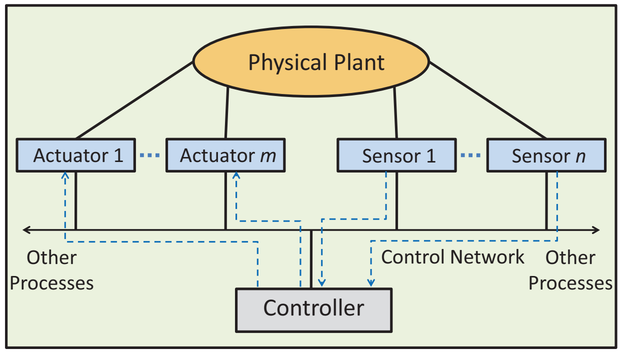

Figure 1.1: General NCS architecture

1.2 Advantages of NCS

For many years now, data networking technologies have been widely applied in industrial and military control applications. These applications include manufacturing plants, automobiles, and aircraft. Connecting the control system components in these applications, such as sensors, controllers, and actuators, via a network can effectively reduce the complexity of systems, with nominal economical investments. Furthermore, network controllers allow data to be shared efficiently. It is easy to fuse the global information to take intelligent decisions over a large physical space. They eliminate unnecessary wiring. It is easy to add more sensors, actuators and controllers with very little cost and without heavy structural changes to the whole system. Most importantly, they connect cyber space to physical space making task execution from a distance easily accessible (a form of tele-presence).

The use of a multipurpose shared network to connect spatially distributed elements results in flexible architectures and generally reduces installation and maintenance costs. Consequently, NCSs have been finding application in a broad range of areas such as mobile sensor networks [20], and automated highway systems and unmanned aerial vehicles [6], [22]. Due to other advantages, such as low cost of installation, ease of maintenance and great flexibility, networked control systems (NCSs) have been widely used in DC motor systems, dual-axis hydraulic positioning systems, and large scale transportation vehicles etc.

One of the biggest advantages of a system controlled over a network is scalability. As we talk about adding many sensors connected through the network at different locations, we can also have one or more actuators connected to one or more controllers through the network. For many years now, researchers have given us precise and optimum control strategies emerging from classical control theory, starting from PID control, optimal control, adaptive control, robust control, intelligent control and many other advanced forms of these control algorithms.

1.3 Limitations & drawbacks of NCS

Control and communications have traditionally been different areas with little overlap. Until the 1990s it was common to decouple the communication issues from consideration of state estimation or control problems. In particular, in the classic control and state estimation theory, the standard assumption is that all data transmission required by the algorithm can be performed with infinite

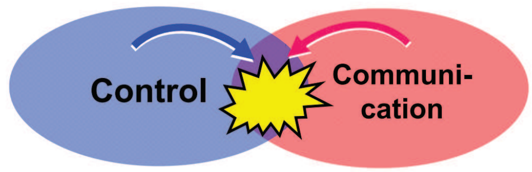

Figure 1.2: Networked Control: Control + Communication

precision in value. In such an approach, control and communication components are treated as totally independent. This considerably simplified the analysis and design of the overall system and mostly works well for engineering systems with large communication bandwidth.

NCSs lie at the intersection of control and communication theories. The classic control theory focuses on the study of interconnected dynamical systems linked through "ideal channels", whereas communication theory studies the transmission of information over "imperfect channels". A combination of these two frameworks is needed to model NCSs. We can broadly categorize NCS applications into two categories as (1) time-sensitive applications or time-critical control such as military, space and navigation operations; (2) time-insensitive or non-real-time control such as data storage, sensor data collection, e-mail, etc. However, network reliability is an important factor for both types of systems. After having an overview of different categories, components and applications of NCS, let us discuss the key issues that make NCSs distinct from other control systems from a controls perspective.

1.3.1 Limited Communication Bandwith

Any communication network can only carry a finite amount of information per unit of time. In many applications, this limitation poses significant constraints on the operation of NCSs. Examples of NCSs that are afflicted by severe communication limitations include unmanned air vehicles (UAVs), due to stealth requirements, power-starved vehicles such as planetary rovers, long-endurance energy-limited systems such as sensor networks, underwater vehicles, and large arrays of micro-actuators and sensors.

1.3.2 Network-induced Delay

To transmit a continuous-time signal over a network, the signal must be sampled, encoded in a digital format, transmitted over the network, and finally the data must be decoded at the receiver side. This process is significantly different from the usual periodic sampling in digital control. The overall delay between sampling and eventual decoding at the receiver can be highly variable because both the network access delays (i.e., the time it takes for a shared network to accept data) and the transmission delays (i.e., the time during which data are in transit inside the network) depend on highly variable network conditions such as congestion and channel quality. In some NCSs, the data transmitted are time stamped, which means that the receiver may have an estimate of the delays duration and take appropriate corrective action. A significant number of results have attempted to characterize a maximum upper bound on the sampling interval for which stability can be guaranteed. These results implicitly attempt to minimize the packet rate that is needed to stabilize a system through feedback.

1.3.3 Transmission losses (Packet Dropout)

A significant difference between NCSs and standard digital control is the possibility that data may be lost while in transit through the network. Typically, packet dropouts result from transmission errors in physical network links (which is far more common in wireless than in wired networks) or from buffer overflows due to congestion. Long transmission delays sometimes result in packet reordering, which essentially amounts to a packet dropout if the receiver discards "outdated" arrivals. Reliable transmission protocols, such as TCP, guarantee the eventual delivery of packets. However, these protocols are not appropriate for NCSs since the retransmission of old data is generally not very useful.

Normally feedback-controlled plants can tolerate a certain amount of data loss, but it is essential to determine whether the system is stable when only transmitting packets at a certain rate, and to compute the acceptable lower bounds on the packet transmission rates.

Having gone through a brief introduction to Networked Control Systems, we now move further and take a look at the key issue that the thesis deals with, i.e. stability of Networked Systems subject to random delays.

1.4 Stability of NCS with Random delays

Systems with time delay have attracted the interest of many researchers since the early 1900s. In the 1940s, some theorems were developed to check the stability of time delay systems in the frequency domain. The corresponding theorems in the time domain appeared in the 1950s and 1960s. In the last 20 years, the improvement in the computation tools gave an opportunity to develop new methods to check the stability of time delay systems.

The available tools to check the stability of time delay systems can be classified into two categories: delay-independent methods or delay-dependent methods. Delay-independent stability methods check whether the stability of a time delay system is preserved for a delay of any size or not. The methods in this category try to check if the magnitude of the delayed states does not affect the stability of the system, no matter what the value of that delay is. These methods are easier to derive, but they suffer some conservatism because: not all the systems have insignificant delayed states; in many cases the delay is fixed, and so applying these methods imposes unnecessary conditions and introduces additional complications; and lastly, delay-independent stability methods can be used only when the delay has a destabilizing effect. For these very reasons, many researchers have shifted their interests to the investigation of delay-dependent stability methods.

In contrast to delay-independent stability methods, delay-dependent stability methods require some information about the delay. This information serves one of the following two purposes:

- to check whether a given system, with some dynamics and delay information, is stable or not; or
- to check the maximum duration of delays in the presence of which a given a system, with some dynamics, can preserve its stability.

Generally, the second purpose is used to qualify any developed method. For implementation purposes, the conditions for time delay systems can only be sufficient. Different methods give different sets of conditions. In research, the commonly used delay types are:

1. Fixed Delay
   $\tau = \rho,\ \rho =$ constant.
2. Unknown Time-varying delay with an upper-bound
   $0 \leq \tau \leq \rho,\ \rho =$ constant.
3. Unknown time-varying delay with an upper-bound on its value and an upperbound on its rate of change
   $0 \leq \tau \leq \rho,\ \rho =$ constant,
   $\dot{\tau} \leq \mu,\ \mu =$ constant.
4. Delay that varies within some interval
   $h_1 \leq \tau \leq h_2,\ h_1,\ h_2 =$ constant.

5. Delay that varies within some interval with an upper-bound on its rate of change
   $h_1 \leq \tau \leq h_2$, $h_1,\ h_2 =$ constant,
   $\dot{\tau} \leq \mu$, $\mu =$ constant.

1.4.1 Stability using Lyapunov's Theorem

Based on Lyapunov's theorem, there are two main theorems to check the stability of time delay systems: the Lyapunov-Razumikhin theorem and the Lyapunov-Krasovskii theorem.

Lyapunov-Razumikhin Theorem

Because the evolution of the states in time delay systems depends on the current and previous states' values, their Lyapunov functions should become functionals (more details in Lyapunov-Krasovskii method discussed in the next section). The functional may complicate the formulation of the conditions and their analysis. To avoid such complications, Razumikhin developed a theorem which will construct Lyapunov functions but not as functionals. To apply the Razumikhin theorem, one should build a Lyapunov function $V(x(t))$. This $V(x(t))$ is equal to zero when $x(t) = 0$ and positive otherwise. The theorem does not require $\dot{V}$ to be less than zero always, but only when $V(x(t))$ becomes greater than or equal to a threshold $\bar{V}$. $\bar{V}$ is given by:

$$\bar{V} = \max_{\theta \in [-\tau, 0]} V(x(t+\theta))$$

Based on this condition, one can understand the theorem statement [78], which is:

Theorem 1.4.1 *Suppose f is a functional that takes time t and initial values x_t and gives a vector of n states $\dot{x}$, u, v and w are class $\mathcal{K}$ functions $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0$, v is strictly increasing. If there exists a continuously differentiable function $V : \mathcal{R}\ X\ \mathcal{R}^n \rightarrow \mathcal{R}$ such that:*

$$u(\|x\|) \leq V(t, x) \leq v(\|x\|) \tag{1.1}$$

and the time derivative of $V(x(t))$ along the solution $x(t)$ satisfies $\dot{V}(t, x) \leq -w(\|x\|)$ whenever $\bar{V} = V(t+\theta, x(t+\theta)) \leq V(t, x(t)), \theta \in [-\tau, 0]$; then the system is uniformly stable. If in addition $w(s) > 0$ for $s > 0$ and there exists a continuous non-decreasing function $p(s) > s$ for $s > 0$ such that $\dot{V}(t, x) \leq w(\|x\|)$ whenever $V(t+\theta, x(t+\theta)) \leq p(V(t, x(t)))$ for $\theta \in [-\tau, 0]$ then the system is uniformly asymptotically stable.

Here $\bar{V}$ serves as a measure for $V(x(t))$ in the interval from $t-\tau$ to t. If $V(x(t))$ is less than $\bar{V}$, $\dot{V}$ could be greater than zero. On the other hand, if $V(x(t))$ becomes greater than or equal to $\bar{V}$, then $\dot{V}$ must be less than zero, such that V will not grow beyond limits. In other words, according to the Razumikhin theorem, $\dot{V}$ need not be always less than zero, but the following conditions should be satisfied:

$$\dot{V} + a(V(x) - \bar{V}) \leq 0 \tag{1.2}$$

for $a > 0$. Therefore, there are three cases for the system to be stable:

1. $\dot{V} < 0$ and $V(x(t)) \geq V$. Here the states do not grow in magnitude;

2. $\dot{V} > 0$ but $V(x(t)) < V$. In this case, although $\dot{V}$ is positive (the values of the states increase), the Lyapunov function is limited by an upper bound; and

3. a case where both terms are negative.

The condition in (1.2) ensures uniform stability, i.e. the states may not reach the origin, but they are contained in some domain. To ensure the asymptotic stability, the condition should be:

$$\dot{V} + a(p(V(x(t))) - \bar{V}) < 0, \quad a > 0 \tag{1.3}$$

where $p(.)$ is a function with the property: $p(s) > s$.

This condition implies that when the system reaches some value which makes $p(V(x(t))) = \bar{V}$, then $\dot{V}$ should be negative and $V(x(t))$ will not reach $\bar{V}$. In the coming interval τ, $V(x)$ will never reach the old $\bar{V}(\bar{V}_{old})$. The maximum value

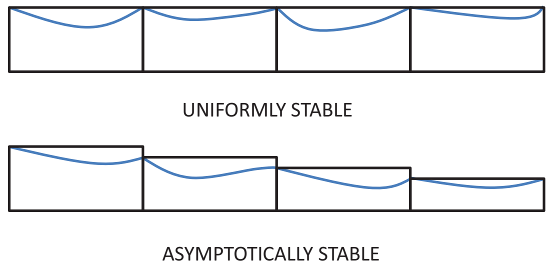

Figure 1.3: Lyapunov-Razumikhin Method

of V in this interval is the new $\bar{V}(\bar{V}_{new})$ which is less $\bar{V}_{old}$. With the passage of time, V keeps decreasing until the states reach the origin (see Figure 1.3).

Lyapunov-Krasovskii Theorem

While Razumikhin's theorem is based on constructing Lyapunov functions, the Lyapunov-Krasovskii theorem constructs functionals instead. Based on the Lyapunov theorem's concept, the function V is a measure of the system's internal energy. In time delay systems, the internal energy depends on the value of x_t, and it is reasonable to construct V which is a function of x_t (which is also a function). Because V is a function of another function, it becomes a functional. To ensure asymptotic stability, $\dot{V}$ should always be less than zero. The Lyapunov-Krasovskii theorem is discussed in more detail in the following section.

The remaining advantage of Razumikhin-based methods over Krasovskii is their relative simplicity, but Lyapunov-Krasovskii gives less conservative results. Before discussing the theorem, we have to define the following notations:

$$\begin{aligned} \phi &= x_t \\ \|\phi\|_c &= \max_{\theta \in [-\tau, 0]} x(t+\theta) \end{aligned} \tag{1.4}$$

The statement of the Lyapunov-Krasovskii theorem given in [78] is:

Theorem 1.4.2 *Suppose f is a functional that takes time t and initial values x_t and gives a vector of n states $\dot{x}$, u, v and w are class $\mathcal{K}$ functions $u(s)$ and $v(s)$ are positive for $s > 0$ and $u(0) = v(0) = 0$, v is strictly increasing. If there exists a continuously differentiable function V such that:*

$$u(\|\phi\|) \leq V(t, x_t) \leq v(\|\phi\|_c) \tag{1.5}$$

and the time derivative of V along the solution $x(t)$ satisfies $\dot{V}(t, x_t) \leq -w(\|\phi\|)$ for $\theta \in [-\tau, 0]$; then the system is uniformly stable. If in addition $w(s) > 0$ for $s > 0$ then the system is uniformly asymptotically stable.

It is clear that V is a functional and that $\dot{V}$ must always be negative.

As a conclusion of the section, this present thesis will use the Lyapunov-Krasovskii theorem to check the delay-dependent stability of uncertain continuous and discrete-time Networked systems. Since the stability of an NCS depends on the occurence of delays, the occurence of delays throughout this thesis is assumed to be governed by Bernoulli's Binomial distribution with varying probabilitis. The followin section presents a short summary on the Binomial distribution.

1.4.2 The Binomial Distribution

Many experiments in real life share the common element that their outcomes can be classified into one of two events, e.g. a coin can come up heads or tails; a child can be male or female; a person can die or not die; a person can be employed or unemployed. These outcomes are often labeled as "success" or "failure." Note that there is no connotation of "goodness" here - for example, in our context, when looking at a signal being transmitted, the statistician might label the signal as a "delayed" if the signal fails to reach on time and the signal as "non-delayed" if it reaches at the designated time. The usual notation is

p = probability of success, q = probability of failure = 1 - p.

Note that $p+q = 1$. In statistical terms, A Bernoulli trial is each repetition of an experiment involving only 2 outcomes. We are often interested in the result of independent, repeated bernoulli trials, i.e. the number of successes in repeated trials.

1. *independent* - the result of one trial does not affect the result of another trial.
2. *repeated* - conditions are the same for each trial, i.e. p and q remain

constant across trials. Hayes refers to this as a **stationary process**.

If p and q can change from trial to trial, the process is **nonstationary**. The term identically distributed is also often used.

Technically speanking, the Bernoulli distribution is a discrete data distribution that is used to describe a population of binary variable values. A simple Bernoulli random variable Y is described by the dichotomous relationship:

$$P(Y = 1) = p \tag{1.6}$$

$$P(Y = 0) = 1 - p \tag{1.7}$$

where $0 \leq p \leq 1$ This is denoted as:

$$Y = Ber(p) \tag{1.8}$$

The probability mass function f of the Bernoulli distribution is given by:

$$f(y; p) = \begin{cases} p & for\ y = 1 \\ 1 - p & for\ y = 0 \end{cases} \tag{1.9}$$

The mean and variance of the Bernoulli distribution are given by:

$$\mu = p \tag{1.10}$$

$$\sigma^2 = p(1 - p) \tag{1.11}$$

The Binomial distribution is a discrete data distribution that is used to model a population of counts for n different repetitions of a Bernoulli experiment. That is to say for:

$$X = (Y_1, Y_2, \ldots, Y_n) \tag{1.12}$$

With probabilities given by (1.6) and (1.7), then the probability of getting exactly x success in n trials is:

$$f(x; n, p) = (\frac{n}{x})p^x(1-p)^{n-x} \tag{1.13}$$

For $x = 0, 1, 2, \ldots n$ where

$$(\frac{n}{x}) = \frac{n!}{x!(n-x)!} \tag{1.14}$$

is the binomial coefficient. The mean and variance of the binomial distribution are given by:

$$\mu = p \tag{1.15}$$

$$\sigma^2 = p(1-p) \tag{1.16}$$

Having gone through a brief introduction on NCSs, let us take a quick look at the contributions made by this thesis in the following section.

1.5 Thesis Outline

The remainder of this thesis is divided into 6 chapters.

Chapter 2

The chapter is an overview of the recent results that deal with Networked Control Systems. It discusses briefly the various models that have been devised to deal with different network phenomena. The chapter is divided into sections, each section discussing a specific class of models based on the various network phenomena such as packet dropouts, transmission delays etc.

Chapter 3

In this chapter, we provide new results on NCS with nonstationary packet dropouts. We extend the work of [5] by developing an improved observer-based stabilizing control algorithm to estimate the states and control input through the construction of an augmented system where the original control input is regarded as a new state.

Chapter 4

In Chapter 4 we consider an NCS wherein nonstationary dropouts as well quantization losses are present in the communication network. The closed loop system is shown to be exponentially stable. The application of the proposed algorithm is demonstrated by means of suitable examples.

Chapter 5

This chapter extends the results obtained in the Chapter 2, considering an NCS with nonstationary packet dropouts and added nonlinearities. The closed-loop system is designed considering dynamic output feedback. Stability conditions are drawn using the Lyapunov Krasovskii functional and expressed in the form of LMIs.

Chapter 6

While all the results in the previous chapters were drawn considering a discrete time NCS, this chapter deals with continuous time networked systems with lossy communication networks. The closed loop is expressed as an augmented system and stability conditions are drawn with the help of Lyapunov theory.

Chapter 7

This chapter summarizes the main contributions of the thesis, provides very recent results in the area that the author became aware of by the time of completion of the thesis. Finally, suggestions for future work and developments are included in the last section of this chapter.

Notations: Capital letters denote matrices. Lower-case alphabet and Greek letters denote column vectors and scalars, respectively. $(.)^T$ and $(.)^H$ denote transpose and Hermitian transpose operations, respectively. I_n is the identity matrix of $n \times n$. 0_n is the zero matrix of dimension $n \times n$, $\mathrm{diag}[A]_1^N$ is a block diagonal with matrix with diagonal entries $A_i, i = 1, 2, \ldots, N$. In symmetric block matrices or long matrix expressions, we use * as an ellipsis for terms that are induced by symmetry, e.g.,

$$(*)\begin{bmatrix} (*)+R & S \\ (*) & Q \end{bmatrix} K \;=\; K^T \begin{bmatrix} R^T+R & S \\ S^T & Q \end{bmatrix} K$$

The l^{th} element of vector $u_i(k)$ is denoted as $u_i^{(l)}(k)$. In the discrete time domain, the time index is denoted by k, $k \in \mathbb{Z}$, $k \geq 0$. In a proof when the time index k is omitted for conciseness, $v(-\tau)$ denotes the vector $v(k-\tau)$. $\hat{u}$ denotes a sequence of predicted vectors of $u(j)$ starting from the current time step. $\breve{u}$ denotes a sequence of $u(-j)$ representing the historical data of u. $|Q|$ is the induced 1-norm of the matrix Q, which is defined as $|Q| = max\{\|Qv\|_2 : v \in \mathbb{R}^n, \|v\|_2 \leq 1\}$, $\|v\|_2$ is the L_2-norm of the vector v.

$\Re^n$ denote the n-dimensional space equipped with the norm $||.||$. We use W^t, W^{-1}, $\lambda_m(W)$ and $\lambda_M(W)$ to denote the transpose, the inverse, the minimum eigenvalue and the maximum eigenvalue of any square matrix W, respectively. We use $W < 0$ (≤ 0) to denote a symmetric negative definite (negative semidefinite) matrix W and I_j to denote the $n_j \times n_j$ identity matrix. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations.

Chapter 2

LITERATURE SURVEY

2.1 Introduction

Control systems with spatially distributed components have existed for several decades. Examples include control systems in chemical process plants, refineries, power plants, and airplanes. In the past, in such systems the components were connected via hardwired connections and the systems were designed to bring all the information from the sensors to a central location where the conditions were being monitored and decisions were made on how to control the system. The control policies then were implemented via the actuators, which could be valves, motors, etc. What is different today is that technology can put low-cost processing power at remote locations via microprocessors and that information can be transmitted reliably via shared digital networks or even wireless connections. These technology driven changes are fueled by the high costs of wiring and the difficulty in introducing additional components into the systems as the needs change.

The changes in the scope and implementation of control systems have caused two main changes in the emphasis in control system analysis and design. The first has to do with the explicit consideration of the interconnections; the network now must be considered explicitly as it affects significantly the dynamic behavior of the control system. The second change has to do with a renewed emphasis on distributed control systems. Because of these changes in control systems, several new concerns need to be addressed. Several areas such as communication protocols for scheduling and routing have become important in control when considering, for example, stability, performance, and reliability. Algorithms and software that are capable of dealing with hard and soft time constraints are very important in control implementation and design and so areas such as realtime systems from computer science are becoming increasingly important. There is also some reordering of priorities and importance of control concepts due to changes in importance to control applications. There had also been renewed emphasis on methodologies for increased autonomy that allows the system to run without feedback information for extended periods of time. At a more fundamental level, control theorists have been led to re-examine the open-(feedforward) versus closed-loop (feedback) control issues.

Technology advances, together with performance and cost considerations, are fueling the proliferation of networked control systems and, in turn, are raising fundamentally new questions in communications, information processing, and control dealing with the relationship between operations of the network and the quality of the overall systems operation. A wide range of research has recently been reported dealing with problems related to the distributed characteristics and the effect of the digital network in networked control systems.

The trend of modern industrial and commercial systems is to integrate computing, communication and control into different levels of factory operations and information processes. Rigorous research has been carried out in this domain to ensure better efficiency and stability of Networked Control Systems.

The literature review for this thesis also incorporates recent surveys carried out by the various authors in the following papers: [7], [8], [11], [12], [25], [23], [29], [28], [31], [27], [32], [38], [65], [67] and [72].

Feng-Li Lilian et al. in [11] discussed the impact of network architecture on control performance in a class of networked control systems (NCSs) and provided design considerations related to control quality of performance as well as network quality of service. The integrated network-control system changed the characteristics of time delays between application devices. This study first identified several key components of the time delay through an analysis of network protocols and control dynamics. The analysis of network and control parameters was used to determine an acceptable working range of sampling periods in an NCS.

Tipsuwan and Chow in [12] discussed in detail the effects of network delays on NCSs and surveyed a few networked control techniques to be used in an unstable NCS. They few assumptions these techniques used were that the Network communication was error-free, every frame or packet had a constant length and computational delay induced was constant and much lesser than the sampling period T. In 1988, Halevi and Ray proposed a fundamental technique named as the augmented deterministic discretetime model methodology to control a linear plant over a periodic delay network. The structure of the augmented discrete-time model is straight forward and easy. In addition, this methodology can be modified to support non-identical sampling periods of a sensor and a controller as mentioned by Liou and Ray in 1990.

In [38], the author presented a report to discuss the major contributions and the possible future challenges in the area of Networked Control Systems. He categorized activities in this field into control of networks, control over networks, and multi-agent systems. Control of networks is mainly concerned with providing a certain level of performance to a network data flow, while achieving efficient and fair utilization of network resources. Multi-agent systems deal with the study of how network architecture and interactions between network components influence global control goals.

In a more recent paper [67], Rachana and Chow discussed the different fields and research arenas in Networked control such as networking technology, network delay, network resource allocation, scheduling, network security in real-time NCSs, integration of components on a network, fault tolerance, etc. Greater emphasis has been laid on security in an NCS with a brief discussion on the development of efficient and scalable intrusion detection systems (IDS). Another key topic of discussion was the the integration of components of an NCS to achieve the global objectives.

The concept of networked control starting taking shape when Stilwell and Bishop [4], discussed a decentralized control strategy to control Platoons of underwater vehicles. They presented a control design methodology for regulating global functions of cooperating mobile systems. The application of relatively standard system-theoretic tools, such as decentralized control, led to a novel broadcast-only communication structure (single-source, unidirectional). More generally, methods presented there allowed the designer to determine what explicit communication strategies are sufficient for a stabilizing decentralized control to exist.

As the research on NCS progressed, researchers tried to focus on the more practical aspects of Networked control. Only then the various network based phenomena came into picture. The earliest phenomenon to be studied in depth was the problem of transmission losses or packet losses.

2.2 Models incorporating only Packet Losses

Pete and Raja [6] studied the effect of communication packet losses in the feedback loop of a control system. They particularly emphasized the vehicle control problems where information is communicated via a wireless local area network. They considered a simple packet-loss model for the communicated information and noted that the results for discrete-time linear systems with Markovian jumping parameters could be applied. The goal was to find a controller such that the closed loop is mean square stable for a given packet loss rate. LMI condition were developed for the existence of a stabilizing dynamic output feedback controller. In [22] they extended their work to develop an H_∞ optimal controller for discrete-time jump systems and derived sufficient conditions in terms of LMIs to satisfy the H_∞ norm requirements. The Markovian Jump Linear System (MLJS) model they used was of the form:

$$\begin{aligned} x(k+1) &= A_{\theta(k)}x(k) + B_{\theta(k)}u(k) \\ y(k) &= C_{\theta(k)}x(k) \\ x(0) = x_0, &\qquad \theta(0) = \theta_0 \end{aligned}$$

The $\theta(k)$ subscript denotes the time varying dependence of the state matrices via the network packet loss parameters. It is noted that the open loop system has two modes: $\theta = 0$ when the packet from sensor is dropped and $\theta = 1$

when the packet is received. However in their analysis, we note that the authors considered the effect of network (packet losses) only on the measurement channel and not on the actuation channel. This way the dropouts occured only while transmitting the plant output $y(k)$ to the controller.

In [10], $Walsh\ et\ al.$ introduced a novel control network protocol, try-once-discard (TOD), for multiple-input multiple-output (MIMO) networked control systems (NCSs), and provided, for the first time, an analytic proof of global exponential stability for both the new protocol and the more commonly used (statically scheduled) access methods. Their approach was to first design the controller using established techniques considering the network transparent, and then to analyze the effect of the network on closed-loop system performance. When implemented, the NCS would consist of multiple independent sensors and actuators competing for access to the network, with no universal clock available to synchronize their actions. Because the nodes act asynchronously, they allowed access to the network at anytime but they assumed each access occurs before a prescribed deadline, known as the maximum allowable transfer interval. Only one node may access the network at a time. This communication constraint imposed by the network was the main focus of the paper and a significant contribution to the research on networked control.

In [74] the authors considered the H_∞ filtering problem for a class of networked systems with packet losses. The networked filtering system is with packet losses is described as a discrete-time linear switched system. A sufficient condition fot the filtering error system to be exponentially stable is to ensure a prescribed

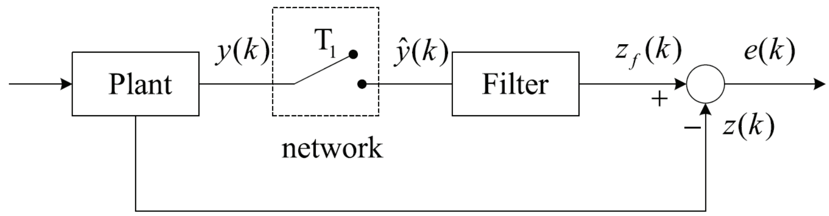

Figure 2.1: Network Filtering System, *Shu Yin et al. 2011*

H_∞ performance is derived by using piecewise continuous Lyapunov function approach and the average dwell time method. In this paper, the problem of H_∞ filtering for a class of networked systems with packet losses is investigated by using the deterministic system approach. By using the zero-input mechanism, i.e., the filter input holds at its last available value when a measurement is lost during the transmission. The block diagram of the model used is shown in Fig. 2.1

The plant used in [74] is described by:

$$\begin{aligned} x(k+1) &= A_p x(k) + B_p w(k) \\ y(k) &= C_p x(k) + D_p w(k) \\ z(k) &= H_p x(k) \end{aligned}$$

The filter input $\hat{y}(k)$ depends on the packet loss status in the network. In Fig. 2.1, the switch T_1 is used to represent the packet loss status. When T_1 is closed,

$\hat{y}(k) = y(k)$, and when T_1 is opened, $\hat{y}(k) = 0$. Therefore the packet loss dependent filter can be represented as follows:

$$\begin{aligned} x_f(k+1) &= A_{fi}x_f(k) + B_{fi}\hat{y}(k) \\ z_f(k) &= C_{fi}x_f(k) \\ i &\in M = \{0,\ 1\} \end{aligned}$$

When packet is lost, i.e. $i = 0$, the filter model is described by:

$$\begin{aligned} x_f(k+1) &= A_{f0}x_f(k) \\ z_f(k) &= C_{f0}x_f(k) \\ \hat{y}(k) &= 0 \end{aligned}$$

When packet is recieved, i.e. $i = 1$, the filter model is described by:

$$\begin{aligned} x_f(k+1) &= A_{f1}x_f(k) + B_{f1}\hat{y}(k) \\ z_f(k) &= C_{f1}x_f(k) \\ \hat{y}(k) &= y(k) \ = \ C_px(k) + D_pw(k) \end{aligned}$$

If we define the error signal as $e(k) = z(k) - z_f(k)$ and the augmented state as $\eta(k) = [x^T(k)x_f^T(k)\hat{y}^T(k-1)$, then the error system can be represented as the following discrete-time switched system:

$$S_{\sigma(k)} : \begin{cases} \eta(k+1) &= A_{\sigma(k)}\eta(k) + B_{\sigma(k)}w(k) \\ e(k) &= C_{\sigma(k)}\eta(k) \end{cases} \tag{2.1}$$

Where $\sigma(k) \in M$ represents the switching signal. Simultaneously researchers also started investigating another aspect of networks i.e. communication delays caused by the network.

Definition 2.2.1 *The closed loop system (2.1) is said to be exponentially stable if there exist constants $k > 0$ and $\lambda > 1$ such that the corresponding states satisfy $\|x(k)\| \leq k\lambda^{-(k-k_0)}\|x(k_0)\|,\ k \geq k_0$ where λ is called the decay rate.*

Definition 2.2.2 *Given a scalar $\gamma > 0$, the system (2.1) is said to be exponentially stable with an H_∞ performance level γ if it is exponentially stable and under zero initial condition, $\sum_{k=0}^{\infty} z^T(k)z(k) \leq \sum_{k=0}^{\infty} \gamma^2 w^T(k)w(k)$ for all nonzero $w(k) \in l_2[0, \infty)$.*

The authors then obtained the sufficient H_∞ conditions for the filtering error system from the following theorem.

Theorem 2.2.1 *Consider system (2.1), if there exist scalars $\gamma > 0$, $\mu > 1$, $\lambda > 1$, and $\varepsilon_1 > \varepsilon_0 > 1$, and matrices $P_i > 0$, $i \in M$, such that the following* *inequalities hold:*

$$1 - \varepsilon_i^{-} 2\mu \geq 0 \tag{2.2}$$

$$\begin{bmatrix} -P_i & 0 & P_i A_i & P_i B_i \\ \bullet & -I & C_i & 0 \\ \bullet & \bullet & -\varepsilon_i^{-2} P_i & 0 \\ \bullet & \bullet & \bullet & -\gamma^2 I \end{bmatrix} < 0 \tag{2.3}$$

$$P_i - \mu P_j \leq 0 \qquad i,\ j \in M \tag{2.4}$$

hold, and the average dwell time and the packet loss rate satisfy $\tau_a \geq \tau_a^* = \mathsf{ln}\mu/(2\mathsf{ln}\lambda)$ *and* $\alpha < \alpha^* = \mathsf{ln}(\lambda/\varepsilon_1)/\mathsf{ln}(\varepsilon_0/\varepsilon_1)$*, respectively. Then the system is exponentially stable with decay rate* λ^p *and ensures an* H_∞ *performance level* γ*, where* $\rho = 1 - \mathsf{ln}\mu/(2\tau_\mathsf{a}\mathsf{ln}\lambda)$*.*

A quantitative relation between the packet loss rate and the H_∞ filtering performance is then obtained, a mode-dependent full-order linear filter is designed by solving a convex optimization problem shown above.

2.3 Models incorporating only Network delays

In [13], the authors discussed the stability networked control systems with variable networked-induced delays. They introduced some novel concepts and designed feedback matrices and a switched strategy among them. An online algorithm was also presented by using gradient method for handling the random delays. They described the plant model as:

$$
\begin{aligned}
\dot{x}(t) &= Ax(t) + Bu(t),\ t \in [kh + \tau_k,\ (k+1)h + \tau_{k+1}) \\
y(t) &= Cx(t) \\
u(t^+) &= -K_i x(t - \tau_k),\ t \in \{kh + \tau_k,\ k = 0, 1, 2 \ldots\}
\end{aligned}
$$

Sampling the above system with period h and defining $z(kh) = [x^T(h),\ u^T((k-1)h)]^T$, yielded the following closed loop system:

$$
z((k+1)h) = \tilde{\Phi}(K_i)z(kh)\forall i = 1, 2, \ldots, p
$$

Jing et al. [17] described the stability problems of uncertain systems with arbitrarily varying and severe time-delays. By using unique LyapunovKrasovskii functionals, new stability conditions for a class of linear uncertain systems with a time-varying delays was obtained. Effectiveness of the proposed LyapunovKrasovskii functionals indicated that a proper distribution of the time delay in the LyapunovKrasovskii functionals is crucial to obtain less conservative criteria.

Srinivasgupta and Heinz [14] introduced an enhancement to the model predictive control (MPC) algorithm to address variable time delays that may occur in the control loop. These variable delays could arise from various sources such as measurement delays, human-in the-loop, and communication delays. The specific focus of their research was to investigate the effect of random communication delays on network-based process control systems.

Their experimental characterization of network communication delays revealed that they were mostly white, had a baseline minimum and approached wide-tailed distributions. They proposed the time-stamped model predictive control (TSMPC) algorithm, an extension to MPC that uses a communication delay model, along with time-stamping and buffering to improve reliability over networked-control systems. Experimental validation of this new algorithm resulted in improved performance over traditional MPC. Where time-stamping is not possible, accounting for the mean/median communication delay resulted in better performance, and this simplification was termed as the mean/median delay model predictive control (MDMPC).

In [16], the problem of robust H_∞ control for uncertain discrete systems with time-varying delays was considered. The system under consideration was subject to time-varying norm-bounded parameter uncertainties in both the state and measured output matrices. A full-order exponential stable dynamic output feedback controller which guarantees the exponential stability of the closed-loop system and reduces the effect of the disturbance input on the controlled output to a prescribed level for all admissible uncertainties was designed.

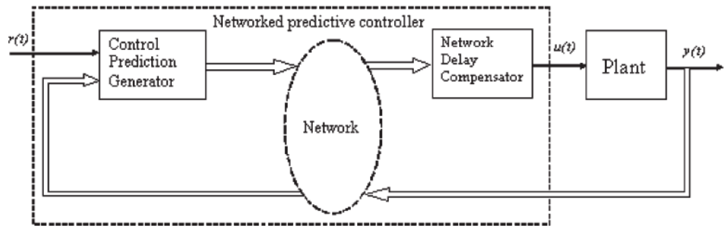

Figure 2.2: NPC Model, *Liu et al. 2007*

Nguang et al. [26] dealt with the problem of robust fault estimation for uncertain time-delay T-S fuzzy models and designed a delay-dependent fault estimator ensuring a prescribed H_∞ performance level for the fault estimation error, irrespective of the uncertainties and the time delays. Sufficient conditions for the existence of a robust fault estimator were developed in terms of linear matrix inequalities (LMIs). They also incorporated the characteristics of the Membership functions into the fault estimator design to reduce the conservativeness of neglecting those characteristics.

In [33], the design problem of an NCS with constant and random network delay in the forward and feedback channels, respectively, was considered. A novel networked predictive control (NPC) scheme was proposed to overcome the effects of network delay and data dropout. Stability criteria of closed-loop NPC systems were presented & necessary and sufficient conditions for the stability of closed-loop NCS with constant time delay were given. Furthermore, they showed that the closed-loop NPC system with bounded random network delay is stable if its corresponding switched system is stable. The block diagram of the proposed NPC model is shown in Fig. 2.2

The stabilisation problem for networked control systems with time-varying delays that may be smaller than one sampling period was studied in by Zhang et al. [34]. State feedback controllers were considered and the resulting closed-loop NCS is modelled as a discrete-time switched system. Criteria for exponential stability for the closed-loop NCS and design procedures for stabilising controllers were presented by using the average dwell time.

In the same year they extended the research to study time-varying delays that may be longer than one sampling period [39]. The timing diagram of the signals in the NCS is shown in Fig. 2.3 State feedback controllers were considered and the resulting closed-loop NCS is modelled as a discrete-time switched system. Conditions for exponential stability of the closed-loop NCS were however presented by using a different approach that combined the average dwell time and asynchronous dynamic system methods. In [45] they considered state feedback controllers, and modeled the closed-loop NCS as a switched delay system, which was then represented as an interconnected feedback system. A sufficient BIBO stability condition was derived for the closed-loop NCS by using the small gain theorem and the average dwell time technique. Similarly the problem of large delays was considered in [55], where the closed loop was modeled as a switched system and the exponential stability conditions were derived using the average dwell time method.

The research presented in [34] was extended by the authors in [69]. Though state feedback controllers were considered in this case also, the closed-loop NCS was described as a discrete-time linear uncertain system model, and the uncertain

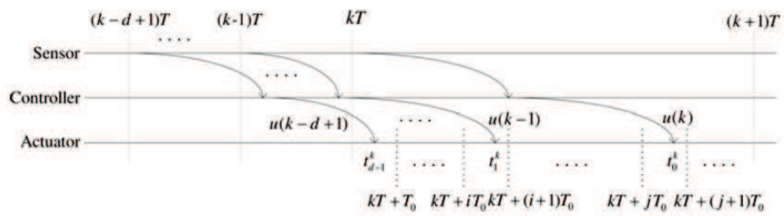

Figure 2.3: Timing diagram of signals in the NCS, *Zhang & Yu 2007*

part reflected the effect of the variation nature of the network-induced delays on the system dynamics. Then, the asymptotic stability condition for the obtained closed-loop NCS was derived to establish the quantitative relation between the stability of the closed-loop NCS and two delay parameters, namely, the allowable delay upper bound (ADB) and the allowable delay variation range (ADVR).

Xin-Gang Yu [35] considered the problem of robust H_∞ filtering for uncertain systems with time-varying distributed delays. The uncertainties under discussion are time varying but norm bounded. Based on the Lyapunov stability theory, sufficient condition for the existence of full order H_∞ filters was proposed by LMI approach such that the filtering error system is asymptotically stable and satisfies a prescribed attenuation level of noise.

Chen et al. [47] proposed a new approach for delay-dependent robust H_∞ stability analysis and control synthesis of uncertain systems with time-varying delay. The key features of the approach included the introduction of a new LyapunovKrasovskii functional, the construction of an augmented matrix with uncorrelated terms, and the employment of a tighter bounding technique. As a result, significant performance improvement was achieved in system analysis and synthesis without using either free weighting matrices or model transformation.

[49] was focussed on the design of robust sliding-mode control for time-delay systems with mismatched parametric uncertainties. Based on a delta-operator approach, a delay-dependent sufficient condition for the existence of linear sliding surfaces was given, and a reaching motion controller was also developed. The results obtained in this paper unified some previous related results of the continuous and discrete systems into the delta-operator systems framework. In [59] a similar problem of designing a sliding mode controller via static output feedback for a class of uncertain systems with mismatched uncertainty in the state matrix was considered. Firstly, they derived a new existence condition of linear sliding surface in terms of strict linear matrix inequalities and proposed an adaptive reaching control law such that the motion of the closed-loop system satisfies the reaching condition. They further considered the delay-type switching function, and a new robust stability condition was given in terms of LMIs for the reduced-order sliding mode dynamics.

In [76], the authors focussed on the stability analysis and robust design of uncertain discrete-time systems with time-varying delay. They developed both delay dependent and independent convex conditions to guarantee stability of the closed loop. Extensions to cope with decentralized control and output feedback control were also discussed. All the system matrices were assumed to be subject to polytopic disturbances and the proposed conditions employ parameter dependent Lyapunov-Krasovskii conditions.

2.4 Models incorporating multiple network phenomena

An iterative approach was proposed by Yu et al. in [18] to model networked control systems with arbitrary but finite data packet dropout as switched linear systems. Sufficient conditions were presented on the stability and stabilization of NCSs with packet dropout and network delays. The merit of the iterative approach is that the controllers can make full use of the previous information to stabilize NCSs when the current state measurements can not be transmitted by the network channel instantly.

In the same year they studied the problem of data packet dropout and transmission delays in NCSs in both continuous-time case and discrete-time case was studied in [19]. They modeled the NCSs with data packet dropout and delays as ordinary linear systems with input delays. For the continuous-time case, their technique was based on Lyapunov-Razumikhin function method. For the discrete-time case, they used the Lyapunov-Krasovskii based method. Attention was focused on the design of memoryless state feedback controllers that guaranteed stability of the closed-loop systems.

Yang et al. [24] designed a controller for networked systems with random communication delays. They categorized delays into two categories : i) from the controller to the plant, and ii) from the sensor to the controller, via a limited bandwidth communication channel. The random delays were modeled as a linear function of the stochastic variable satisfying Bernoulli random binary distribution. The observer-based controller was designed to exponentially stabilize the networked system in the sense of mean square, and also achieve the prescribed H_∞ disturbance attenuation level.

In [30], a constrained convex optimization problem was developed for the robust stabilization problem of a class of discrete-time networked control systems subject to non-linear perturbations under the effects of delays and data packet dropout. Such NCSs were modeled as discrete-time nonlinear systems with time-varying input delays. A sufficient condition was established in terms of a linear matrix inequalities which guaranteed stability of the NCS and at the same time maximized the non-linearity bound.

Yuan et al. [36] proposed a new method to model the networked control system with data dropout and transmission delays as an asynchronous dynamical system (ADS). Based on some assumptions, and by using Lyapunov stability theory, the sufficient conditions on the stability of such NCSs were presented in terms of LMIs. A similar approach based on the theory of asynchronous dynamic systems was applied for modelling an NCS in [37].

In [40], a new approach was proposed to study the modeling and control problems for the NCS with both network induced delay and packet-dropout. Different from the sampled data system approach presented earlier, a direct sampling scheme was applied to describe the closed-loop NCS as a time-delay switched linear system model by ignoring the inter-sample behavior of the NCS. Sampled data control approach was also used in [56] to deal with the stabilization problem of NCS with packet loss and bounded delays. A real-time networked control system was also constructed to test the stabilizing ability of the controller design in a real network environment.

Song et al. [46] considered a plant with network delays, dropouts and communication constraints wherein the plant has multiple sensor nodes and only one of them is allowed to communicate with the filter at each transmission instant, and the packet dropouts occur randomly. The filtering error system was modeled as a switched system with a stochastic parameter. Sufficient conditions were presented for the filtering error system to be mean-square exponentially stable and achieve a prescribed H_∞ performance. In [68] they proposed a new compensation scheme, upon which the filtering error system is modelled as a switched system by considering mode-dependent filters. Sufficient conditions were derived for the filtering error system to be exponentially stable and ensure a prescribed H_∞ disturbance attenuation level bound by using the average dwell-time method.

In [61] the authors addressed the problem of quantized feedback control for networked control systems. Considering the effects of the network such as delays, packet dropouts and signal quantization a sampled-data model of the closed loop feedback system based on the updating sequence of event-driven holder was formulated, from which a continuous system with two additive delay components in the state was developed. Subsequently by making use of a novel interval delay system approach, the stability analysis and control synthesis for NCSs

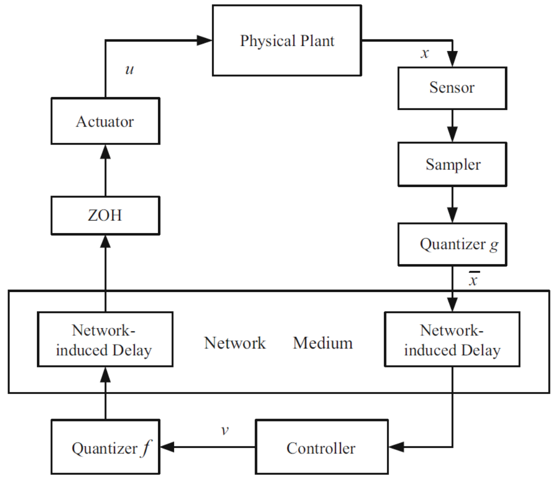

Figure 2.4: A typical networked control system with two quantizers, *Jianguo Dai 2010.*

with one/two static quantizers were solved accordingly. The block diagram of the networked system is shown in Fig. 2.4

Wu et al. [63] considered the stability of the discrete-time networked control systems with polytopic uncertainty, where a smart controller is updated with the buffered sensor information at stochastic intervals and the amount of the buffered data received by the controller under the buffer capacity constraint is also random. They established sufficient conditions to guarantee the exponential stability of generic switched NCSs and the exponential mean square stability of Markov-chain driven NCSs, respectively.

Considering MIMO NCSs where network is of limited access channels, a discrete-time switched delay model was formulated in [58] by constructing a novel piecewise Lyapunov-Krasovskii functional, a new stability criterion was developed in terms of linear matrix inequalities. On the basis of the obtained stability condition, a static output feedback controller was designed by applying an iterative algorithm.

H_∞ filter design for a class of networked control systems with multiple state delays via Takagi-Sugeno (T-S) fuzzy model was discussed in [73]. Packet losses induced by the limited bandwidth of communication networks, were also considered. The focus of this paper was on the analysis and design of a full-order H_∞ filter such that the filtering error dynamics is stochastically stable and a prescribed H_∞ attenuation level is guaranteed.

Schendel et al. [66] presented three discrete-time modelling approaches for networked control systems (NCSs) that incorporate time-varying sampling intervals, time-varying delays and dropouts. The focus of their work was on the extension of two existing techniques to describe dropouts, namely (i) dropouts modelled as prolongation of the delay and (ii) dropouts modelled as prolongation of the sampling interval, and the presentation of a new approach (iii) based on explicit dropout modelling using automata. Based on polytopic overapproximations of the resulting discrete-time NCS models, they provided LMI-based stability conditions for all three approaches.

NCSs under similar network conditions were studied in [64]. Communication constraints impose that, per transmission, only one sensor or actuator node can access the network and send its information. Which node is given access to the network at a transmission time was orchestrated by a so-called network protocol. The transmission intervals and transmission delays were described by a random process, having a continuous probability density function (PDF). By focussing on linear plants and controllers and periodic and quadratic protocols, the authors presented a modelling framework for NCSs based on stochastic discrete-time switched linear systems. Stability (in the mean-square) of these systems was analysed using convex overapproximations and a finite number of linear matrix inequalities. An extension to this paper was provided in [75]. The order in which nodes send their information is orchestrated by a network protocol, such as, the Round-Robin (RR) and the Try-Once-Discard (TOD) protocol. In this paper, they generalised the mentioned protocols to novel classes of so-called periodic and quadratic protocols. By focussing on linear plants and controllers, they presented a modelling framework for NCSs based on discrete-time switched linear uncertain systems. This framework allows the controller to be given in discrete time as well as in continuous time. To analyse stability of such systems for a range of possible transmission intervals and delays, with a possible nonzero lower bound, they proposed a new procedure to obtain a convex over-approximation in the form of a polytopic system with norm-bounded additive uncertainty.

Cloosterman et al. [70] presented a discrete-time model for networked control systems (NCSs) that incorporates all network phenomena: time-varying sampling intervals, packet dropouts and time-varying delays that may be both smaller and larger than the sampling interval. Based on this model, constructive

LMI conditions for controller synthesis were derived, such that stabilizing state-feedback controllers can be designed. Besides the proposed controller synthesis conditions a comparison was made between the use of parameter-dependent Lyapunov functions and Lyapunov-Krasovskii functions for stability analysis.

Luan et al. [5] designed an observer-based stabilizing controller for networked systems involving both random measurement and actuation delays. The developed control algorithm is suitable for networked systems with any type of delays. By the simultaneous presence of binary random delays and making full use of the delay information in the measurement model and controller design, new and less conservative stabilization conditions for networked control systems were derived. The criterion was formulated in the form of a nonconvex matrix inequality of which a feasible solution can be obtained by solving a minimization problem in terms of linear matrix inequalities. Below is a brief summary of their mathematical formulation. The LTI plant under consideration was assumed to be of the form:

$$\begin{aligned} x_p(k+1) &= Ax_p + Bu_p \\ y_p &= Cx_p \end{aligned} \tag{2.5}$$

where $x_p(k) \in \Re^n$ is the plants state vector and $u_p(k) \in \Re^m$ and $y_p(k) \in \Re^p$ are the plants control input and output vectors, respectively. The measurement subjected to random communication delay is given by

$$y_c(k) = (1 - \delta(k))y_p(k) + \delta(k)y_p(k - \tau_k^m) \tag{2.6}$$

where τ_k^m is the measurement delay, whose occurence is governed by the Bernoulli distribution, and $\delta(k)$ is Bernoulli distributed sequence with

$$\begin{aligned} Prob\{\delta(k) = 1\} &= E\{\delta(k)\} = \bar{\delta} \\ Prob\{\delta(k) = 0\} &= 1 - E\{\delta(k)\} = 1 - \bar{\delta} \end{aligned} \tag{2.7}$$

The following observer-based controller is designed when the full state vector is not available:

$$Observer \begin{cases} \hat{x}(k+1) = A\hat{x} + Bu_c(k) \\ \qquad\qquad +L(y_c(k) - \hat{y}_c(k)) \\ \hat{y}_c(k) = (1-\bar{\delta})C\hat{x}(k) + \bar{\delta}C\hat{x}(k - \tau_k^m) \end{cases} \tag{2.8}$$

$$Controller \begin{cases} u_c(k) = K\hat{x}(k) \\ u_p = (1-\alpha)u_c(k) + \alpha u_c(k - \tau_k^a) \end{cases} \tag{2.9}$$

where $\hat{x}(k) \in \Re^n$ is the estimate of the system (2.5), $\hat{y}_c(k) \in \Re^p$ is the observer output, and $L \in \Re^{n\times p}$ and $K \in \Re^{m\times n}$ are the observer gain and the controller gain, respectively. The stochastic variable α, mutually independent of δ, is also a Bernoulli distributed white sequence with

$$\begin{aligned} Prob\{\alpha(k) = 1\} &= E\{\alpha(k)\} = \bar{\alpha} \\ Prob\{\alpha(k) = 0\} &= 1 - E\{\alpha(k)\} = 1 - \bar{\alpha} \end{aligned} \tag{2.10}$$

where τ_k^a is the actuation delay. In this paper, assume that τ_k^a and τ_k^m are time varying and have the following bounded condition:

$$\begin{aligned} \underline{d}_m \leq \tau_k^m \leq \overline{d}_m \\ \underline{d}_a \leq \tau_k^a \leq \overline{d}_a \end{aligned} \tag{2.11}$$

The estimation error is defined by

$$e(k) = x_p(k) - \hat{x}(k) \tag{2.12}$$

Then it yields

$$
\begin{aligned}
x_p(k+1) \quad &= [A + (1-\bar{\alpha})BK]x_p(k) \\
&-(1-\bar{\alpha})BKe(k) \\
&+\bar{\alpha}BKx_p(k-\tau_k^a) - \bar{\alpha}BKe(k-\tau_k^a) \\
&-(\alpha-\bar{\alpha})BKx_p(k) + (\alpha-\bar{\alpha})BKe(k) \\
&+(\alpha-\bar{\alpha})BKx_p(k-\tau_k^a) \\
&-(\alpha-\bar{\alpha})BKe(k-\tau_k^a)
\end{aligned}
\tag{2.13}
$$

$$
\begin{aligned}
e(k+1) \quad &= [A - (1-\bar{\delta})LC]e(k) - \bar{\delta}LCe(k-\tau_k^m) \\
&+(\delta-\bar{\delta})LCx_p(k) \\
&-(\delta-\bar{\delta})LCx_p(k-\tau_k^m).
\end{aligned}
\tag{2.14}
$$

The aforementioned system (2.13) and (2.14) is equivalent to the following compact form:

$$
\begin{aligned}
\varepsilon(k+1) = \quad & (\bar{A}+\tilde{A})\varepsilon(k) + (\bar{B}+\tilde{B})\varepsilon(k-\tau_k^m) \\
& +(\bar{C}-\tilde{C})\varepsilon(k-\tau_k^a)
\end{aligned}
\tag{2.15}
$$

where

$$
\begin{aligned}
\varepsilon(k) &= [x_p^T(k) \quad e^T(k)]^T \\
\bar{A} &= \begin{bmatrix} A + (1-\bar{\alpha})BK & -(1-\bar{\alpha})BK \\ 0 & A - (1-\bar{\delta})LC \end{bmatrix} \\
\tilde{A} &= \begin{bmatrix} -(\alpha-\bar{\alpha})BK & (\alpha-\bar{\alpha})BK \\ (\delta-\bar{\delta})LC & 0 \end{bmatrix} \\
\bar{B} &= \begin{bmatrix} 0 & 0 \\ 0 & -\bar{\delta}LC \end{bmatrix} \begin{bmatrix} 0 & 0 \\ -(\delta-\bar{\delta})LC & 0 \end{bmatrix} \\
\bar{C} &= \begin{bmatrix} \bar{\alpha}BK & -\bar{\alpha}BK \\ 0 & 0 \end{bmatrix} \\
\tilde{C} &= \begin{bmatrix} (\alpha-\bar{\alpha})BK & -(\alpha-\bar{\alpha})BK \\ 0 & 0 \end{bmatrix}
\end{aligned}
$$

The aim of this paper is to design an observer based feedback stabilizing controller in the form of (2.8) and (2.9) such that the closed loop system is exponentially stable in the mean square. The work presented in the following chapter is an extension of the analysis carried out in [5] taking into consideration various addition factors that affect the practical working of Networked systems.

2.5 Conclusions

In this chapter, we have presented a survey of the main results pertaining to linear dynamical systems subject to saturation including actuator, output and state types . The survey has outlined basic assumptions and has taken into considerations several technical views on the analysis and design procedures leading to stability of networked systems under consideration. The key emphasis here was on NCSs subject to random delays. Previous results related stability of networked control systems have been provided. Some typical examples have been given to illustrate relevant issues.

Chapter 3

NETWORKED CONTROL SYSTEMS WITH NONSTATIONARY PACKET DROPOUTS

3.1 Introduction

The existence of time delays is commonly encountered in many dynamic systems, and time delay has been widely known to degrade the performance of the control systems [79]-[80]. There has been considerable research work appearing to address the control problem of networked systems in the presence of network delays. For example, Jiang et al. [41] proposed a methodology as an augmented deterministic discrete-time model to control a linear plant over a periodic delay network. Given that the network delays are time varying but bounded, the Lyapunov theory was employed in [10] to find the maximum delays that can be tolerated. However, in the aforementioned works, the network-induced delays have been commonly assumed to be deterministic, which is fairly unrealistic since delays resulting from network transmissions are typically time varying and random by nature.

Recently, researchers have started to model the random communication delays in various probabilistic ways and have tried to prove a version of stability such as the mean-square stability or the exponential mean-square stability. For example, in [14], the random communication delays have been considered as white in nature with known probability distributions. In [48], the time delay of NCSs was modeled as Markov chains such that the closed-loop systems are jump systems. In [23], the random delays were modeled as the Bernoulli binary distributed white sequence taking values of zero and one with certain probability. Among them, the binary random communication delay has received much research attention due to its practicality and simplicity in describing network-induced delays [2, 15, 5]. In [23], both the measurement and actuation delays are viewed as the Bernoulli distributed white sequence using a delay-free model with small random delays. An alternative model is developed in [44] using observer-based feedback control algorithm with time-varying delays occuring only in the channel from the sensor to the controller. This obviously does not accord with the practical situation in most NCSs, where another typical kind of network-induced delay often happens in the channel from the controller to the actuator. All the foregoing results are restricted to stationary dropouts which does not fully cover the common operational modes on networked systems.

In this chapter, we provide new results on NCS with nonstationary packet dropouts. We extend the work of [5] by developing an improved observer-based stabilizing control algorithm to estimate the states and control input through the construction of an augmented system where the original control input is regarded as a new state. Due to limited bandwidth communication channel, the simultaneous occurrence of measurement and actuation delays are considered using nonstationary random processes modeled by two mutually independent stochastic variables. Several properties of the developed approach are delineated. The observer-based controller is designed to exponentially stabilize the networked system and solved within the linear matrix inequality (LMI) framework. The theory is illustrated by simulation on a typical system.

3.2 Problem Formulation

Consider the NCS with random communication delays, where the sensor is clock driven and the controller and the actuator are event driven. The discrete-time linear time-invariant plant model is as follows:

$$x_p(k+1) = Ax_p + Bu_p, \quad y_p = Cx_p \tag{3.1}$$

where $x_p(k) \in \mathfrak{R}^n$ is the plants state vector and $u_p(k) \in \mathfrak{R}^m$ and $y_p(k) \in \mathfrak{R}^p$ are the plants control input and output vectors, respectively. A, B, and C are known as real matrices with appropriate dimensions. We assume for a more general case that the measurement with a randomly varying communication delay is described by

$$y_c(k) = \begin{cases} y_p(k - \tau_k^m), \ \delta(k) = 1 \\ y_p(k), \ \delta(k) = 0 \end{cases} \tag{3.2}$$

where τ_k^m stands for *measurement delay*, the occurrence of which satisfies the Bernoulli distribution, and $\delta(k)$ is Bernoulli distributed white sequence. It order to capture the current practice of computer communication management that experiences different time-dependent operational modes, we let

$$Prob\{\delta(k) = 1\} = p_k$$

Table 3.1: Pattern of p_k

p_k	q_1	q_2	$\cdots$	q_{n-1}	q_n
$Prob(p_k = q)$	r_1	r_2	$\cdots$	r_{n-1}	r_n

where p_k assumes discrete values, see Table 3.1. Two particular classes can be considered:

Class 1: p_k has the probability mass function where $q_r - q_{r-1} =$ constant for $r = 2, ..., n$. This covers a wide range of cases including

1. If there is no information about the likelihood of different values, we use the uniform discrete distribution, $r_i = 1/n,\ i = 1, 2, ..., n,$ see Fig. 3.1.

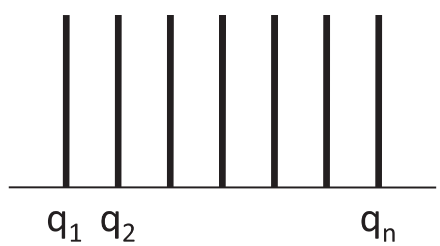

Figure 3.1: Uniform discrete distribution

2. If it is suspected that p_k follows a symmetric triangle distribution, we use the following function: i) For n even, $r_i = a + jd,\ j = 0, 1, ..., n/2$ and $r_i = a + (n-j)d,\ j = 0, 1, ..., n/2+1, n/2+2, ..., n,$ where $na + dn(n-1)/4 = 1$, ii) For n odd, $r_i = a + jd,\ j = 0, 1, ..., (n-1)/2$ and $r_i = a + (n-j)d,\ j = 0, 1, ..., (n+1)/2, (n+2)/2, ..., n,$ where $na + dn(n-1)^2/4 = 1$

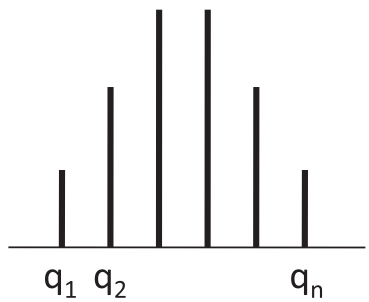

Figure 3.2: Symmetric triangle distribution: n even

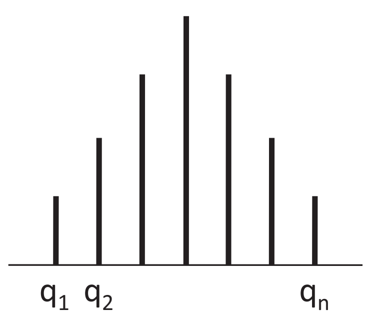

Figure 3.3: Symmetric triangle distribution: n odd

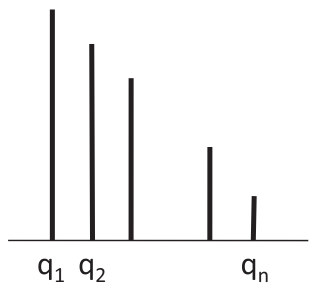

Figure 3.4: Decreasing linear function distribution

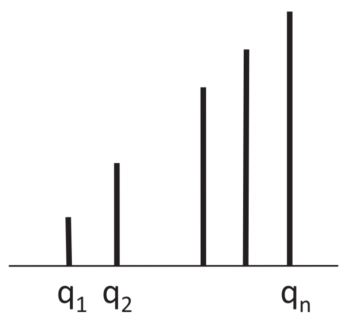

Figure 3.5: Increasing linear function distribution

3. If it is suspected that r_i is a decreasing linear function, we use $r_i = a - jd,\ j = 0, 1, ..., n$ where $na - dn(n-1)/2 = 1$

4. If it is suspected that r_i is a increasing linear function, we use $r_i = a - (n-j)d,\ j = 0, 1, ..., n$ where $na - dn(n-1)/2 = 1$

Class 2: $p_k = X/n,\ n > 0$ and $0 \leq X \leq n$ is a random variable that follows the Bionomial distribution $\mathbf{B}(q, n),\ q > 0$, that is

$$\begin{aligned} Prob(p_k = (ax+b)/n) &= \begin{pmatrix} n \\ x \end{pmatrix} q^x (1-q)^{n-x}, \quad b > 0, \\ x &= 0, 1, 2, ..., n, \quad an + b < n \end{aligned}$$

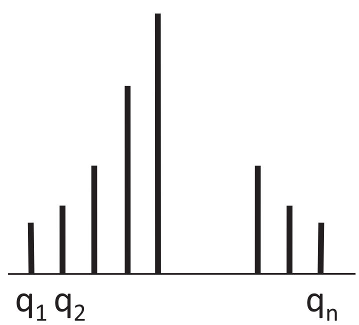

Figure 3.6: Bionomial distribution

Remark 3.2.1 *It is significant to note that the case $Prob\{\delta(k) = 1\} = \bar{\delta}$, where $\bar{\delta}$ is a constant value, is widely used in majority of results on NCS. In this chapter, we focus on nonstationary dropouts.*

When the full state information is not available and the time delay occurs on the actuation side, it is desirable to design the following observer-based controller [5]:

Table 3.2: Pattern of s_k

s_k	s_1	s_2	$\cdots$	s_{n-1}	s_n
$Prob(s_k = t)$	t_1	t_2	$\cdots$	t_{n-1}	t_n

$$
\begin{aligned}
&Observer: \\
&\hat{x}(k+1) = A\hat{x} + Bu_c(k) + L(y_c(k) - \hat{y}_c(k)) \\
&\hat{y}_c(k) = \begin{cases} C\hat{x}(k), \ \delta(k) = 0 \\ C\hat{x}(k - \tau_k^m), \ \delta(k) = 1 \end{cases}
\end{aligned} \tag{3.3}
$$

$$
\begin{aligned}
&Controller: \\
&u_c(k) = K\hat{x}(k) \\
&u_p = \begin{cases} u_c(k), \ \alpha(k) = 0 \\ u_c(k - \tau_k^a), \ \alpha(k) = 1 \end{cases}
\end{aligned} \tag{3.4}
$$

where $\hat{x}(k) \in \Re^n$ is the estimate of the system (3.1), $\hat{y}_c(k) \in \Re^p$ is the observer output, and $L \in \Re^{n\times p}$ and $K \in \Re^{m\times n}$ are the observer and controller gains, respectively, and τ_k^a is the *actuation delay*. The stochastic variable $\alpha(k)$, mutually independent of δ, is also a Bernoulli distributed white sequence with

$$Prob\{\alpha(k) = 1\} = s_k$$

where s_k assumes discrete values. By similarity, a particular class is that s_k has some probability mass function as in Table 3.2, where $s_r - s_{r-1} =$ constant for $r = 2, ..., n$.

With reference to the implementation of the algorithm that we propose, the application can be summed up in the following way:

1. We assume that the measurement delay and actuation delay occur in the measurement and actuation channel respectively.
2. Therefore the plant output $y_p(k)$ and generated control input $u_c(k)$ are delayed by indefinite but bounded time periods τ_k^m and τ_k^a respectively.
3. The receiving buffer at the end of the communication channel is designed to incorporate the most recent values of plant output (coming from the measurement channel) and the control input (coming from the actuation channel). Assuming two delayed packets arriving at the receiving buffer at the same time, the older packet is discarded, and the relatively new packet used to carry out the control operation. These discarded packets are what we term as *nonstationary dropouts*.

In this chapter, we assume that τ_k^a and τ_k^m are time-varying and have the following bounded condition:

$$\tau_m^- \leq \tau_k^m \leq \tau_m^+, \;\; \tau_a^- \leq \tau_k^a \leq \tau_a^+ \tag{3.5}$$

Define the estimation error by $e(k) = x_p(k) - \hat{x}(k)$. Then, it yields

$$
\begin{aligned}
x_p(k+1) &= \begin{cases} Ax_p(k) + BKx_p(k-\tau_k^\alpha) \\ -BKe(k-\tau_k^\alpha), \qquad \alpha(k) = 1, \\ (A+BK)x_p(k) - BKe(k), \\ \qquad\qquad\qquad\qquad \alpha(k) = 0, \end{cases} &(3.6)\\
e(k+1) &= x_p(k+1) - \hat{x}(k+1) \\
&= \begin{cases} Ae(k) - LCe(k-\tau_k^m), \quad \delta(k) = 1, \\ (A-LC)e(k), \qquad\qquad \delta(k) = 0 \end{cases} &(3.7)
\end{aligned}
$$

In terms of $\xi(k) = [x_p^T(k) \quad e^T(k)]^T,$ system (3.6) and (3.7) can be cast into the form:

$$
\xi(k+1) = \mathsf{A}_j\xi(k) + \mathsf{B}_j\xi(k-\tau_k^m) + \mathsf{C}_j\xi(k-\tau_k^a) \qquad (3.8)
$$

where $\{\mathsf{A}_j,\ \mathsf{B}_j,\ \mathsf{C}_j,\ j = 1, ..., 4\}$ and j is an index identifying one of the following pairs $\{(\delta(k) = 1,\ \alpha(k) = 1),\ (\delta(k) = 1,\ \alpha(k) = 0),\ (\delta(k) = 0,\ \alpha(k) =$

$0), \ (\delta(k)=0, \ \alpha(k)=1)\}$:

$$
\begin{aligned}
\mathsf{A}_1 &= \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \ \mathsf{A}_2 = \begin{bmatrix} A+BK & -BK \\ 0 & A \end{bmatrix}, \\
\mathsf{A}_3 &= \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \ \mathsf{A}_4 = \begin{bmatrix} A & 0 \\ 0 & A-LC \end{bmatrix}, \\
\mathsf{B}_1 &= \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}, \ \mathsf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathsf{B}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathsf{B}_4 = \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}, \\
\mathsf{C}_1 &= \begin{bmatrix} 0 & 0 \\ 0 & -LC \end{bmatrix}, \ \mathsf{C}_2 = \begin{bmatrix} 0 & 0 \\ 0 & -LC \end{bmatrix}, \\
\mathsf{C}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \ \mathsf{C}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (3.9)
\end{aligned}
$$

Remark 3.2.2 *It is remarked for simulation processing that we can express (3.6)-(3.7) in the form*

$$
\begin{aligned}
& x_p(k+1) = s_k[Ax_p(k) + BKx_p(k-\tau_k^\alpha) \\
& -BKe(k-\tau_k^\alpha)] \\
& +(1-s_k)[(A+BK)x_p(k) - BKe(k)] \qquad (3.10) \\
& e(k+1) = p_k[Ae(k) - LCe(k-\tau_k^m)] \\
& +(1-p_k)[(A-LC)e(k)] \qquad (3.11)
\end{aligned}
$$

where the values of the random variables $p_k,\ s_k$ are generated in the manner discussed earlier.

Remark 3.2.3 *It is important to note from (3.9) that*

$$\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j = \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, j = 1, .., 4 \tag{3.12}$$

The interpretation of this result is that $\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j$ represents the fundamental matrix of the delayed system (3.8), which must be independent of the mode of operation. This will help in simplifying the control design algorithm.

Our aim here is to design an observer based feedback stabilizing controller in the form of (3.3) and (3.4) such that the closed loop system (3.8) is exponentially stable in the mean square. Our approach is based on the concepts of switched time-delay systems [81]. For simplicity in exposition, we introduce

$$\begin{aligned}
\sigma_1(k) &= Prob\{\delta(k) = 1,\ \alpha(k) = 1\},\ \ \hat{\sigma}_1 = \mathbb{E}[\sigma_1] \\
\sigma_2(k) &= Prob\{\delta(k) = 1,\ \alpha(k) = 0\},\ \ \hat{\sigma}_2 = \mathbb{E}[\sigma_2] \\
\sigma_3(k) &= Prob\{\delta(k) = 0, \alpha(k) = 0\},\ \ \hat{\sigma}_3 = \mathbb{E}[\sigma_3] \\
\sigma_4(k) &= Prob\{\delta(k) = 0,\ \alpha(k) = 1\},\ \ \hat{\sigma}_4 = \mathbb{E}[\sigma_4]
\end{aligned} \tag{3.13}$$

where $\mathbb{E}[\sigma_i]$ is the expected value of $\sigma_i,\ i = 1, .., 4$. Since we assume that $\delta(k)$ and $\alpha(k)$ are independent random variables, then it follows from (3.13) that

$$\begin{aligned}
&\hat{\sigma}_1 = \mathbb{E}[p_k]\mathbb{E}[s_k], \quad \hat{\sigma}_2 = \mathbb{E}[p_k]\mathbb{E}[1 - s_k] \\
&\hat{\sigma}_3 = \mathbb{E}[1 - p_k]\mathbb{E}[1 - s_k], \quad \hat{\sigma}_4 = \mathbb{E}[1 - p_k]\mathbb{E}[s_k]
\end{aligned} \tag{3.14}$$

3.3 Main Results

In this section, we will thoroughly investigate the stability analysis and controller synthesis problems for the closed-loop system (3.8). First, let us deal with the stability analysis problem and derive a sufficient condition under which the closed-loop system (3.8) with the given controller (3.3) and (3.4) is exponentially stable in the mean square. Extending on [82], the following Lyapunov function candidate is constructed to establish the main theorem:

$$\begin{aligned}
V(\xi(k)) &= \sum_{i=1}^{5} V_i(\xi(k)) \qquad (3.15)\\
V_1(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \xi^T(k) P \xi(k),\ P > 0 \\
V_2(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i),\ Q_j = Q_j^T > 0
\end{aligned}$$

$$\begin{aligned}
V_3(\xi(k)) &= \sum_{j=1}^{4}\sigma_j \sum_{i=k-\tau_k^a}^{k-1} \xi^T(i)Q_j\xi(i) \\
V_4(\xi(k)) &= \sum_{j=1}^{4}\sigma_j \sum_{\ell=-\tau_m^+ +2}^{-\tau_m^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i)Q_j\xi(i) \\
V_5(\xi(k)) &= \sum_{j=1}^{4}\sigma_j \sum_{\ell=-\tau_a^+ +2}^{-\tau_a^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i)Q_j\xi(i) \qquad (3.16)
\end{aligned}$$

It is not difficult to show that there exist real scalars $\mu > 0$ and $\upsilon > 0$ such that

$$\mu\|\xi\|^2 \leq V(\xi(k)) \leq \upsilon\|\xi(k)\|^2 \qquad (3.17)$$

Remark 3.3.1 *By carefully considering Remark 3.2.3 in view of model (3.8), it is justified to select matrix P to be the same for all operational modes, hence independent of j, while keeping matrix Q_j dependent on mode j.*

We now present the analysis result for system (3.8) to be exponentially stable.

Theorem 3.1: *Let the controller and observer gain matrices K and L be given. The closed-loop system (3.8) is exponentially stable if there exist matrices $0 < P,\ 0 < Q_j^T = Q_j,\ j = 1, .., 4$ and matrices $R_i,\ S_i,$ and $M_i,\ i = 1, 2,$ such that* *the following matrix inequality holds*

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & \Lambda_{2j} \\ \bullet & \Lambda_{3j} \end{bmatrix} < 0 \tag{3.18}$$

$$\Lambda_{1j} = \begin{bmatrix} \Psi_j + \Phi_{j1} & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - \hat{\sigma}_j Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - \hat{\sigma}_j Q_j \end{bmatrix}$$

$$\Lambda_{2j} = \begin{bmatrix} -R_1 + M_1^T - \Phi_{j2} & -R_2 + M_2^T - \Phi_{j3} \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix}$$

$$\Lambda_{3j} = \begin{bmatrix} -M_1 - M_1^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & -M_2 - M_2^T + \Phi_{j6} \end{bmatrix} \tag{3.19}$$

where

$$\begin{aligned}
\Psi_j &= -P + \hat{\sigma}_j(\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)Q_j + R_1 + R_1^T + R_2 + R_2^T \\
\Phi_{j1} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) \\
\Phi_{j2} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P \mathsf{B}_j \\
\Phi_{j3} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P \mathsf{C}_j, \quad \Phi_{j5} = \mathsf{B}_j^T P \mathsf{C}_j \\
\Phi_{j4} &= \mathsf{B}_j^T \hat{\sigma}_j P \mathsf{B}_j, \qquad \Phi_{j6} = \mathsf{C}_j^T \hat{\sigma}_j P \mathsf{C}_j
\end{aligned}$$

Proof: Defining $y(k) = x(k+1) - x(k)$, one has

$$\xi(k - \tau_k^m) = \xi(k) - \sum_{i=k-\tau_k^m}^{k-1} y(i) \tag{3.20}$$

$$\xi(k - \tau_k^a) = \xi(k) - \sum_{i=k-\tau_k^a}^{k-1} y(i) \tag{3.21}$$

Then the system (3.8) can be transformed into

$$\xi(k+1) = (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)\xi(k) - \mathsf{B}_j\lambda(k) - \mathsf{C}_j\gamma(k) \tag{3.22}$$

where

$$\lambda(k) = \sum_{i=k-\tau_k^m}^{k-1} y(i), \quad \gamma(k) = \sum_{i=k-\tau_k^a}^{k-1} y(i).$$

Evaluating the difference of $V_1(\xi(k))$ along the solution of system (3.22), we have

$$\begin{aligned} &\mathbb{E}[\Delta V_1(\xi(k))] = \mathbb{E}[V_1(\xi(k+1))] - V_1(\xi(k)) \\ &= \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)[\Phi_{j1} - P]\xi(k) - 2\xi^T(k)\Phi_{j2}\lambda(k) \\ &-2\xi^T(k)\Phi_{j3}\gamma(k) + \lambda^T(k)\Phi_{j4}\lambda(k) + 2\lambda^T(k)\Phi_{j5}\gamma(k) + \gamma^T(k)\Phi_{j6}\gamma(k)\Big] \end{aligned} \tag{3.23}$$

A straightforward computation gives

$$
\begin{aligned}
&\mathbb{E}[\Delta V_2(\xi(k)] = \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\sum_{i=k+1-\tau_{k+1}^m}^{k} \xi^T(i) Q_j \xi(i) - \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
&= \xi^T(k) Q \xi(k) - \xi(k-\tau_k^m) Q_j \xi(k-\tau_k^m) \\
&+ \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i) Q_j \xi(i) - \sum_{i=k+1-\tau_k^m}^{k-1} \xi(i) Q_j \xi(i) \Bigg]
\end{aligned}
\tag{3.24}
$$

In view of

$$
\begin{aligned}
& \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
= \quad & \sum_{i=k+1-\tau_{k+1}^m}^{k-\tau_k^m} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
\leq \quad & \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i)
\end{aligned}
\tag{3.25}
$$

We readily obtain

$$
\begin{aligned}
\mathbb{E}[\Delta V_2(\xi(k))] \quad & \leq \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\xi^T(k) Q_j \xi(k) - \xi^T(k-\tau_k^m) Q_j \xi(k-\tau_k^m) \\
& + \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i) \Bigg]
\end{aligned}
\tag{3.26}
$$

Following parallel procedure, we get

$$\mathbb{E}[\Delta V_3(\xi(k))] \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k) Q_j \xi(k)$$

$$-\xi^T(k-\tau_k^a) Q_j \xi(k-\tau_k^a) + \sum_{i=k+1-\tau_a^+}^{k-\tau_a^-} \xi^T(i) Q_j \xi(i) \Big] \tag{3.27}$$

Finally

$$\mathbb{E}[\Delta V_4(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\sum_{\ell=-\tau^+{}_m+2}^{-\tau^-{}_m+1} [\xi^T(k) Q_j \xi(k)$$

$$-\xi^T(k+\ell-1) Q_j \xi(k+\ell-1)] \Big]$$

$$= \sum_{j=1}^{4} \hat{\sigma}_j \Big[(\tau^+{}_m - \tau^-{}_m) \xi^T(k) Q_j \xi(k) - \sum_{i=k+1-\tau^+{}_m}^{k-\tau^-{}_m} \xi^T(i) Q_j \xi(i) \Big] \tag{3.28}$$

$$\mathbb{E}[\Delta V_5(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[(\tau^+{}_a - \tau^-{}_a) \xi^T(k) Q_j \xi(k)$$

$$- \sum_{i=k+1-\tau^+{}_a}^{k-\tau^-{}_a} \xi^T(i) Q_j \xi(i) \Big] \tag{3.29}$$

It follows from (3.20) and (3.21) that:

$$\xi(k) - \xi(k-\tau_k^m) - \lambda(k) = 0 \tag{3.30}$$

$$\xi(k) - \xi(k-\tau_k^a) - \gamma(k) = 0 \tag{3.31}$$

Therefore, for any appropriately dimensioned matrices R_i, S_i and M_i, $i = 1, 2$, we have the following equations:

$$\begin{aligned}
&2[\xi^T(k)R_1 + \xi^T(k-\tau_k^m)S_1 + \lambda^T(k)M_1] \\
&\times[\xi(k) - \xi(k-\tau_k^m) - \tau(k)] = 0 \qquad (3.32) \\
&2[\xi^T(k)R_2 + \xi^T(k-\tau_k^a)S_2 + \gamma^T(k)M_2] \\
&\times[\xi(k) - \xi(k-\tau_k^a) - \gamma(k)] = 0 \qquad (3.33)
\end{aligned}$$

On combining (3.23)–(3.33), we reach

$$\begin{aligned}
&\mathbb{E}[\Delta V(\xi(k))] \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)\Psi_j\xi(k) \\
&+\xi^T(k)(-2R_1 + 2S_1^T)\xi(k-\tau_k^m) \\
&+\xi^T(k)(-2R_2 + 2S_2^T)\xi(k-\tau_k^a) \\
&+\xi^T(k)(-2R_1 + 2M_1^T - 2\Phi_{j2})\lambda(k) \\
&+\xi^T(k)(-2R_2 + 2M_2^T - 2\Phi_{j3})\gamma(k) \\
&+\xi^T(k-\tau_k^m)(-S_1 - S_1^T - \hat{\sigma}_j Q_j)\xi(k-\tau_k^m) \\
&+\xi^T(k-\tau_k^m)(-2S_1 - 2M_1^T)\lambda(k) \\
&+\xi^T(k-\tau_k^a)(-S_2 - S_2^T - \hat{\sigma}_j Q_j)\xi(k-\tau_k^a) \\
&+\xi^T(k-\tau_k^a)(-2S_2 - 2M_2^T)\gamma(k) \\
&+\lambda^T(k)(-M_1 - M_1^T + \Phi_{j4})\lambda(k) \\
&+\gamma^T(k)(-M_2 - M_2^T + \Phi_{j5})\gamma(k) \\
&+\lambda^T(k)\Phi_{j6}\gamma(k)\Big] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k)\widetilde{\Lambda}_j\zeta(k)\Big] \qquad (3.34)
\end{aligned}$$

where

$$\zeta(k) = \begin{bmatrix} \zeta_1^T & \zeta_2^T \end{bmatrix}^T, \ \zeta_2 = \begin{bmatrix} \lambda^T(k) & \gamma^T(k) \end{bmatrix}^T$$
$$\zeta_1 = \begin{bmatrix} \xi^T(k) & \xi^T(k-\tau_k^m) & \xi^T(k-\tau_k^a) \end{bmatrix}^T \tag{3.35}$$

and $\widetilde{\Lambda}_j$ corresponds to Λ_j in (3.19) by Schur complements. If $\Lambda_j < 0, \ j = 1,..,4$ holds, then

$$\begin{aligned} &\mathbb{E}[V(\xi(k+1)) - V(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k)\widetilde{\Lambda}_j\zeta(k)\Big] \\ &\leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[-\widetilde{\Lambda}_{min}(\widetilde{\Lambda}_j)\zeta^T(k)\zeta(k)\Big] \\ &< -\sum_{j=1}^{4} \hat{\sigma}_j \Big[\beta_j\zeta^T(k)\zeta(k)\Big] \end{aligned} \tag{3.36}$$

where

$$0 < \beta_j < min\big[\lambda_{min}(\Lambda_j), max\{\lambda_{max}(P), \ \lambda_{max}(Q_j)\}\big]$$

Inequality (3.36) implies that $\mathbb{E}[V(\xi(k+1)) - V(\xi(k))] < -\phi V(\xi(k)), \ 0 < \phi < 1.$ In the manner of [23], we get

$$||\xi(k)||^2 \leq \frac{\upsilon}{\kappa}||\xi(0)||^2(1-\phi)^k + \frac{\lambda}{\mu\phi}$$

Therefore, it can be verified that the closed-loop system (3.8) is exponentially stable. This completes the proof.

A solution to the problem of the observer-based stabilizing controller design is provided by the following theorem:

Theorem 3.2: *Let the delay bounds* $\tau_m^+,\ \tau_m^-,\ \tau_a^+,\ \tau_a^-$ *be given. Evaluate the quantities* $\hat{\sigma}_j,\ j = 1,..,4$. *Then the closed-loop system (3.8) is exponentially stable if there exist matrices* $0 < X,\ Y_1,\ Y_2,\ 0 < \Xi_j,\ j = 1,..,4$ *and matrices* $\Pi_i,\ \Upsilon_i$ *and* $\Gamma_i,\ i = 1,2,$ *such that the following matrix inequality holds for*

$j = 1, .., 4$:

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j \\ \bullet & \Lambda_{3j} & 0 \\ \bullet & \bullet & -\hat{\sigma}_j \widehat{X} \end{bmatrix} < 0 \tag{3.37}$$

$$\widehat{X} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \tag{3.38}$$

$$\widehat{\Psi}_j = -X + \hat{\sigma}_j(\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)\Xi_j + \Pi_1 + \Pi_1^T + \Pi_2 + \Pi_2^T$$

$$\widehat{\Lambda}_{1j} = \begin{bmatrix} \widehat{\Psi}_j & -\Pi_1 + \Upsilon_1^T & -\Pi_2 + \Upsilon_2^T \\ \bullet & -\Upsilon_1 - \Upsilon_1^T - \hat{\sigma}_j \Xi_j & 0 \\ \bullet & \bullet & -\Upsilon_2 - \Upsilon_2^T - \hat{\sigma}_j \Xi_j \end{bmatrix}$$

$$\widehat{\Lambda}_{2j} = \begin{bmatrix} -\Pi_1 + \Gamma_1^T & -\Pi_2 + \Gamma_2^T \\ -\Upsilon_1 - \Gamma_1^T & 0 \\ 0 & -\Upsilon_2 - \Gamma_2^T \end{bmatrix}, \ \widehat{\Lambda}_{3j} = \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & -\Gamma_2 - \Gamma_2^T \end{bmatrix}$$

$$\widehat{\Omega}_j = \begin{bmatrix} \widehat{\Omega}_{1j} & 0 & 0 & -\widehat{\Omega}_{4j} & -\widehat{\Omega}_{5j} \end{bmatrix}$$

$$\widehat{\Omega}_{1j} = \begin{bmatrix} XA^T + Y_1^T B^T & 0 \\ XA^T & XA^T - Y_2^T \end{bmatrix}, \ \forall\, j$$

$$\widehat{\Omega}_{4j} = \begin{bmatrix} Y_1^T B^T & 0 \\ 0 & 0 \end{bmatrix}, \quad j = 1, 4, \quad \widehat{\Omega}_{5j} = \begin{bmatrix} 0 & 0 \\ 0 & -Y_2^T \end{bmatrix}, \quad j = 1, 2$$

$$\widehat{\Omega}_{4j} = 0, \ j = 2, 3, \quad \widehat{\Omega}_{5j} = 0, \ j = 3, 4 \tag{3.39}$$

where the gain matrices are given by

$$K = Y_1 X^{-1}, \;\; L = Y_2 X^{-1} C^{\dagger}$$

Proof: Define

$$\Omega_j = \begin{bmatrix} (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) & 0 & -\mathsf{B}_j & -\mathsf{C}_j \end{bmatrix}^T$$

then matrix inequality (3.18) can be expressed as

$$\Lambda_j = \widetilde{\Lambda} + \Omega_j P \Omega_j^T < 0 \tag{3.40}$$

$$\begin{aligned}
\widetilde{\Lambda}_j &= \begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} \\ \bullet & \widetilde{\Lambda}_{3j} \end{bmatrix} \; < \; 0 \\
\widetilde{\Lambda}_{1j} &= \begin{bmatrix} \Psi_j & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - Q_j \end{bmatrix} \\
\widetilde{\Lambda}_{2j} &= \begin{bmatrix} -R_1 + M_1^T & -R_2 + M_2^T \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix} \\
\widetilde{\Lambda}_{3j} &= \begin{bmatrix} -M_1 - M_1^T & 0 \\ \bullet & -M_2 - M_2^T \end{bmatrix}
\end{aligned} \tag{3.41}$$

Setting $\widehat{X} = P^{-1}$, invoking Schur complements, we write matrix Λ_j in (3.40) equivalently as

$$\begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} & \Omega_j \\ \bullet & \widetilde{\Lambda}_{3j} & 0 \\ \bullet & \bullet & -\widehat{X} \end{bmatrix} < 0 \qquad (3.42)$$

Applying the congruence transformation

$$T_j = diag[\widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ I]$$

to matrix inequality in (3.42) and manipulating using (3.38) and

$$\begin{aligned} \Xi_j &= \widehat{X} Q_j \widehat{X},\ \Pi_j = \widehat{X} R_j \widehat{X},\ \Upsilon_j = \widehat{X} S_j \widehat{X}, \\ \Gamma_j &= \widehat{X} M_j \widehat{X} \end{aligned}$$

we readily obtain matrix inequality (3.37) subject (3.39).

Remark 3.3.2 *The selection of $\widehat{X}$ as given by (3.38) has the advantage of converting the solution of bilinear matrix inequalities to that of seeking the feasibility of linear matrix inequalities and hence avoiding iterative procedures. It should be noted that the LMI (3.37) dependens of the average dropout patterns identified by (3.14), which is quite useful in illustrating different operating conditions of the communications network.*

Remark 3.3.3 *It is remarked that the implementation of* **Theorem 3.2** *is online in nature as it requires cally random generators to pick-up numbers corresponding to the scalars* $\hat{\sigma}_1,\ \ldots,\ \hat{\sigma}_4$ *and to evaluate the probabilities in model (3.10) and (3.11) to compute the state and error trajectories. This represents a salient feature not shared by other methods for networked control design under unreliable communication links.*

3.4 Numerical Simulation

In this section, we aim to demonstrate the effectiveness and applicability of the developed control design method and provide the simulation results on three representative examples

3.4.1 Uninterruptible power system

We study the networked control problem for the uninterruptible power system (UPS). Our objective here is to control the pulsewidth-modulated inverter, such that the output ac voltage is kept at the desired setting and undistorted [83]. We consider an UPS with 1 kVA, the discrete-time model (3.1) of which can be obtained with a sampling time of 10 ms at a half-load operating point as follows

:

$$A = \begin{bmatrix} 0.9226 & -0.633 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$C = \begin{bmatrix} 23.737 & 20.287 & 0 \end{bmatrix}$$

In what follows, we apply the new algorithm with nonstationary dropouts of **Theorem 3.2** to obtain the controller and observer gain matrices as:

$$K = [-0.0809 \;\; 0.0190 \;\; -0.0001], \quad ||K|| = 0.0831$$
$$L = [-0.0035 \;\; -0.0023 \;\; -0.0002]^T, \quad ||L|| = 0.0042$$

Had we considered stationary dropouts and applied the algorithm of [5] by taking the occurrence probabilities of the random measurement delay and the actuation delay to be $\bar{\delta} = \bar{\alpha} = 0.1$, we would obtained the controller and observer gain matrices as:

$$K = [-0.2178 \;\; 0.1729 \;\; 0.0269], \quad ||K|| = 0.2794,$$
$$L = [0.0117 \;\; 0.0299 \;\; 0.0230]^T, \quad ||L|| = 0.0395$$

For the purpose of simulation, we assume that the measurement delay and the actuation delay vary as $\tau_k^m \in [0.5 \longrightarrow 5]$ and $\tau_k^a \in [1 \longrightarrow 10]$, respectively. With the help of the 'variable fraction delay' block in Matlab Simulink software (Matlab 7.0) to handle discrete time-varying delays τ_k^m and τ_k^a and under the initial conditions $x_{0_p} = [1 \quad 0 \quad 0]^T$ and $\hat{x}_0 = [0 \quad 0 \quad 0]^T$, the simulation results of the state responses are given in Fig. 3.7, where the dotted lines denote the state responses using the control algorithm proposed in [5]. The Bernoulli sequences $\alpha(k)$ and $\delta(k)$ are depicted in Fig. 3.8.

It can be observed from Fig. 3.7 that, with the new control algorithm that we developed, not only does the dynamical behavior of the NCS take less time to converge to zero but also the system oscillation is smaller. In other words, compared with the control algorithm presented in [5], the new developed control algorithm has the advantages of faster response, smaller overshoot, and higher control precision. The feedback gain required for regulation is also much smaller. In addition, the simulations were carried out on the above system considering nonstationary packet dropouts where the probability distributions of p_k and s_k follows a symmetric triangle distribution with $n = 100$, the delay sequences $\alpha(k)$ and $\delta(k)$ for nonstationary packet dropouts are similar to that of Fig. 3.10 and the response of system states is shown in Fig. 3.9.

3.4.2 Autonomous underwater vehicle

A dynamic model of autonomous underwater vehicle (AUV) was described in [84] where the control objective is to ensure that the motion of the AUV is stable

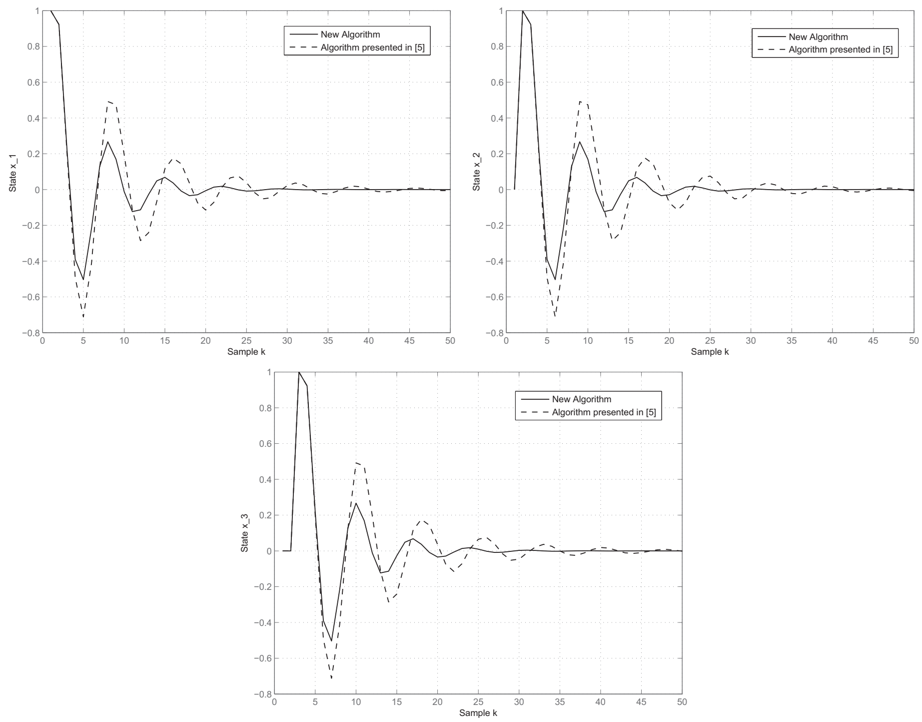

Figure 3.7: State trajectories for stationary dropouts (UPS)

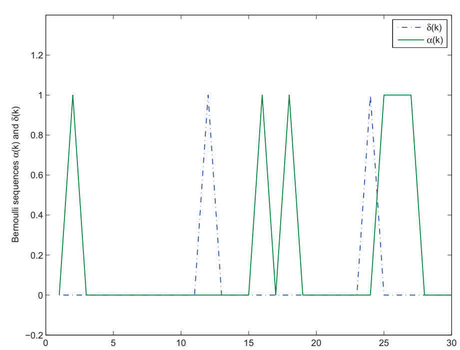

Figure 3.8: Bernoulli sequences $\alpha(k)$ and $\delta(k)$

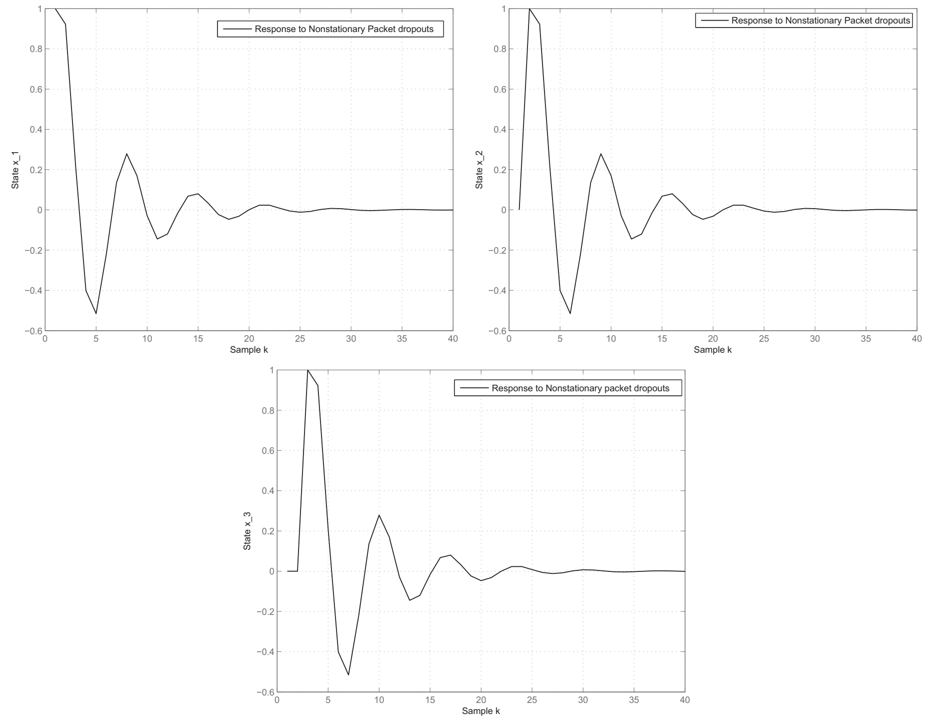

Figure 3.9: State trajectories for nonstationary dropouts (UPS)

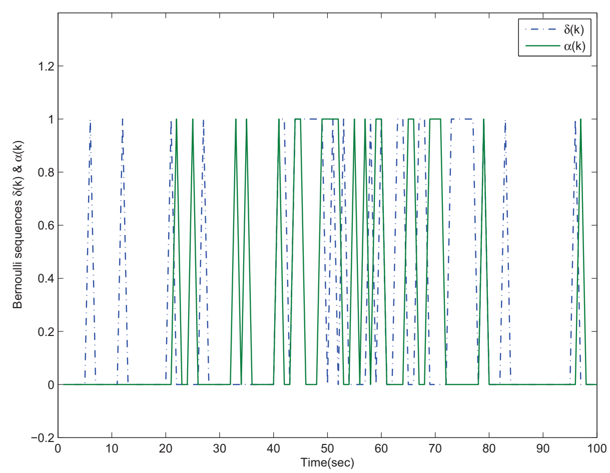

Figure 3.10: Delay sequences $\alpha(k)$ and $\delta(k)$ with nonstationary probabilities

at a prescribed setting. The discrete-time model is given by follows :

$$
\begin{aligned}
A &= \begin{bmatrix} -0.14 & -0.69 & 0 \\ -0.19 & -0.048 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0.056 \\ -0.23 \\ 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

Following parallel lines, we apply the new algorithm with nonstationary dropouts of **Theorem 3.2** to obtain the controller and observer gain matrices as:

$$
\begin{aligned}
K &= [-0.0809 \quad 0.0190 \quad -0.0001], \quad ||K|| = 0.0098 \\
L &= [-0.0035 \quad -0.0023 \quad -0.0002]^T, \quad ||L|| = 0.2126
\end{aligned}
$$

Where for the case of stationary dropouts, we applied the algorithm of [5] by taking the occurrence probabilities of the random measurement delay and the actuation delay to be $\bar{\delta} = \bar{\alpha} = 0.5$, we then obtained the controller and observer gain matrices as:

$$
\begin{aligned}
K &= [-0.4548 \quad 0.0036 \quad 0.3584], \quad ||K|| = 0.5791 \\
L &= [-0.2479 \quad -0.3181 \quad -0.0147]^T, \quad ||L|| = 0.4035
\end{aligned}
$$

For the purpose of simulation, we assume that the measurement delay and the actuation delay vary as $\tau_k^m \in [1 \longrightarrow 5]$ and $\tau_k^a \in [3 \longrightarrow 9]$, respectively. Likewise, we employ the 'variable fraction delay' block in Matlab Simulink software

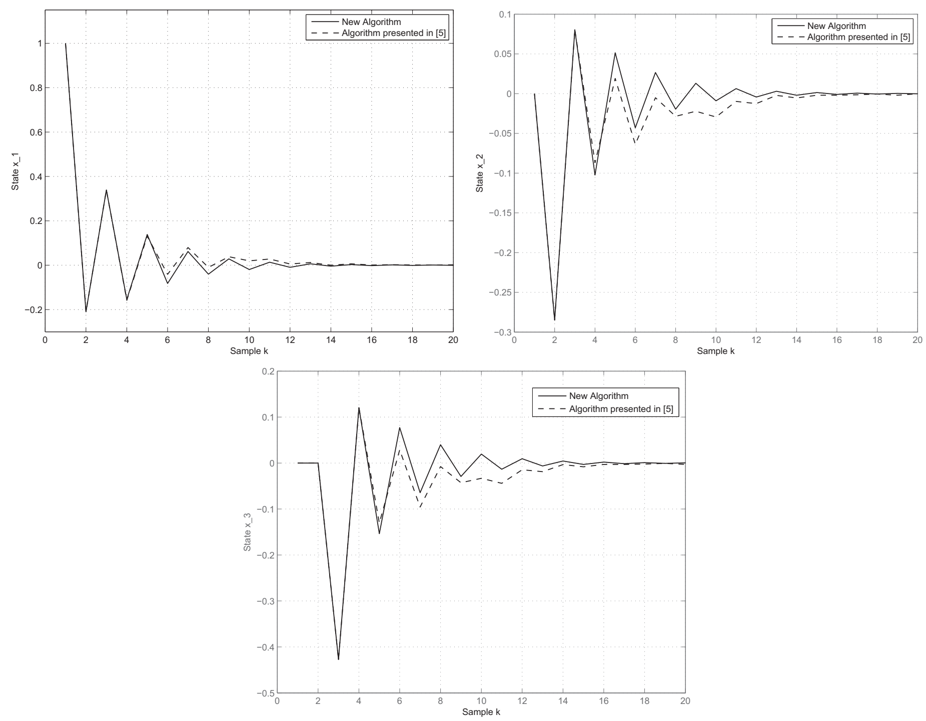

Figure 3.11: State trajectories for stationary dropouts (AUV)

(Matlab 7.0) to handle discrete time-varying delays τ_k^m and τ_k^a and under the initial conditions $x_{0_p} = [1 \quad 0 \quad 0]^T$ and $\hat{x}_0 = [0 \quad 0 \quad 0]^T$. The simulation results of the state responses are given in Fig. 3.11, where the dotted lines denote the state responses using the control algorithm reported in [5].

It can be observed from Fig. 3.11 by comparing the developed control algorithm with the control algorithm presented in [5] that our control method has the advantages of faster response, smaller overshoot, and higher control precision. The feedback gain required for regulation is also much smaller.

The response of systems states with nonstationary dropouts is shown in Fig.

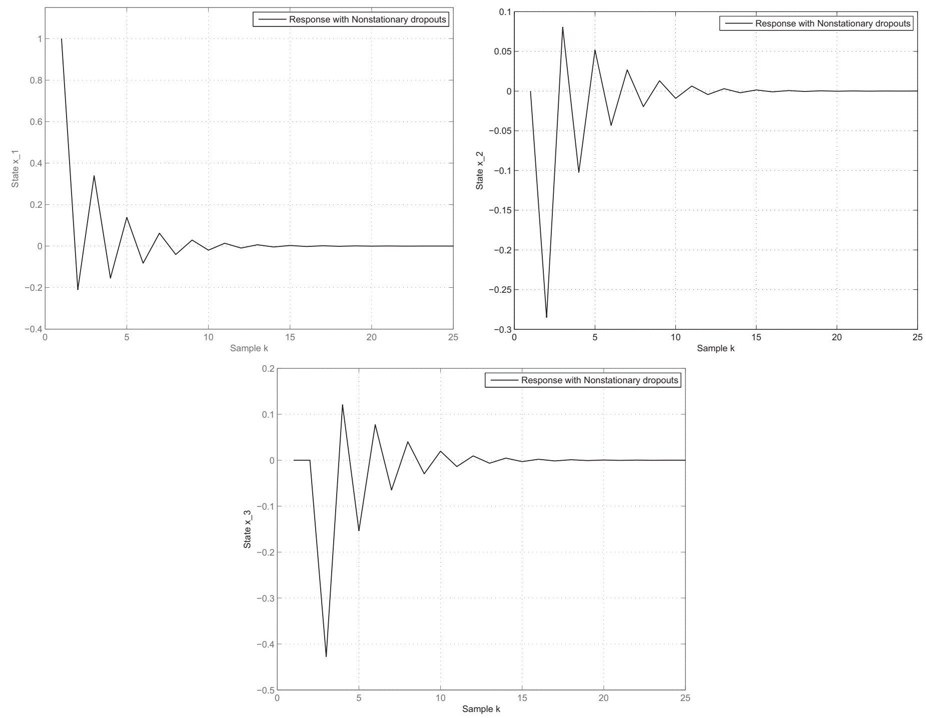

Figure 3.12: State trajectories for nonstationary dropouts (AUV)

3.12.

3.5 Conclusion

The stability analysis and controller synthesis problems are thoroughly investigated for NCSs with time-varying delays and subject to nonstationary packet dropouts. Attention is focused on the design of the new observer-based controller such that the resulting closed-loop system is exponentially stable in the mean-square sense. The effectiveness of the proposed results has been shown through several numerical examples. One of our future research topics would be the study of NCSs with both network-induced random delays and presence of quantization, while we also intend to study the behaviour of the NCS when it is subjected to bounded nonlinearities and disturbances.

Chapter 4

NETWORKED CONTROL SYSTEMS WITH QUANTIZATION AND PACKET DROPOUTS

4.1 Introduction

It becomes increasingly apparent that networked control systems (NCSs), where sensors, controllers and plants are connected over a communication network, provide appealing advantages in several applications covering a broad range of areas. Considerable attention has been devoted to the stability and control of NCSs; see for example, Jiang, Mao, and Shi (2010), Seiler and Sengupta (2005), Walsh, Ye, and Bushnell (2002), Yang, Xia, and Shi (2011), Yin, Yu, and Zhang (2010) and Zhao, Liu, and Rees (2009) and the references therein. Nevertheless, it is worth mentioning that the insertion of communication networks in control loops leads to some inevitable phenomena including random delay, packet dropout, quantization errors and so on, which may result in system performance deterioration and have been primarily highlighted in the literature.

Time delays commonly exist in practical NCSs (Liu & Yang, 2011; Shi, Mahmoud, Nguang, & Ismail, 2006; Wang, Ho, Liu, & Liu, 2009; Wu, Su, Shi, & Qiu, 2011), which are of discrete nature.

Quantization always exists in computer-based control systems and quantization errors have adverse effects on the NCSs performance. In early 1990s, quantized state feedback was employed to stabilize an unstable linear system by Delchamps (1990). Since then, there is a new trend of research on the quantization effect on NCSs where a quantizer is regarded as an information coder. Consequently, it is necessary to conduct an analysis on the quantizers and understand how much effect the quantization makes on the overall systems.

On the other hand, due to the limited transmission capacity of the network, one of the challenging issues that has inevitably emerged is data loss (Liu, 2010; Shen, Wang, & Hung, 2010; Zhao et al., 2009). Recently, there have been three main methods to deal with control input data loss for real-time NCSs, that is to use zero control input, keep the previous one, or use the predictive control sequence (Liu, 2010). In NCSs, it is assumed that the measurement signals are quantized before being communicated.

Recently, researchers have started to model the random communication delays in various probabilistic ways and have tried to prove a version of stability such as the mean-square stability or the exponential mean-square stability. For example, in [14], the random communication delays have been considered as white in nature with known probability distributions. In [48], the time delay of NCSs was modeled as Markov chains such that the closed-loop systems are jump systems. In [23], the random delays were modeled as the Bernoulli binary distributed white sequence taking values of zero and one with certain probability. Among them, the binary random communication delay has received much research attention due to its practicality and simplicity in describing network-induced delays [2, 15, 5]. In [23], both the measurement and actuation delays are viewed as the Bernoulli distributed white sequence using a delay-free model with small random delays. An alternative model is developed in [44] using observer-based feedback control algorithm with time-varying delays occurring only in the channel from the sensor to the controller. This obviously does not accord with the practical situation in most NCSs, where another typical kind of network-induced delay often happens in the channel from the controller to the actuator. All the foregoing results are restricted to stationary dropouts which does not fully cover the common operational modes on networked systems.

In this paper we assume that the output measurements from the plant undergo logarithmic quantization before reaching the controller. An observer is designed to estimate the states of the plant from these quantized measurements. We are also considering random delays occuring in the measurement and actuation channel simultaneously. The control system is designed to render the closed-loop system with all the vagaries stable in the mean square sense. The Lyapunov Krasovkii functionals are deployed and the stability conditions are expressed in the form of LMIs. A practical example has also been considered for simulation to illustrate the effectiveness of the developed strategies.

4.2 Problem Formulation

Consider the NCS with random communication delays, where the sensor is clock driven and the controller and the actuator are event driven. The discrete-time linear time-invariant plant model is as follows:

$$x_p(k+1) = Ax_p + Bu_p, \quad y_p = Cx_p \tag{4.1}$$

where $x_p(k) \in \Re^n$ is the plants state vector and $u_p(k) \in \Re^m$ and $y_p(k) \in \Re^p$ are the plants control input and output vectors, respectively. A, B, and C are known as real matrices with appropriate dimensions. With reference to Fig. 4.1, the measured output $y_p(k)$ is transmitted through a logarithmic quantizer that

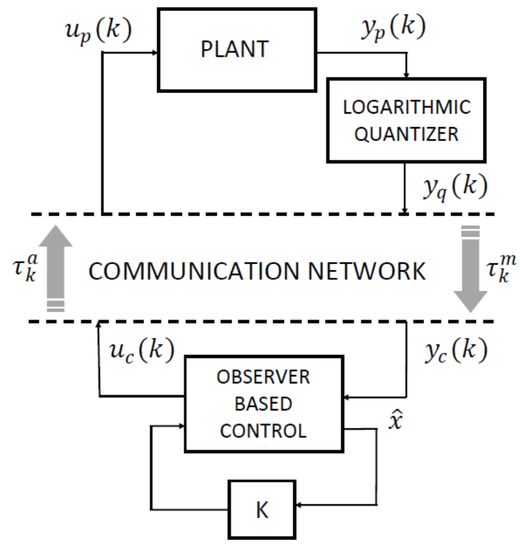

Figure 4.1: System Block Diagram

yields $y_q(k)$. Let the set of quantized levels be described as

$$\begin{aligned} \mathcal{V} &= \{\pm v_j,\ v_j = \varrho^j v_0,\ j = 0, \pm 1,\ \pm 2, ...\} \cup \{0\}, \\ & 0 < \varrho < 1,\ v_0 > 0 \end{aligned}$$

where the parameter ϱ is called the quantization density, and the logarithmic quantizer $\mathbf{q}(.)$ is defined by

$$\mathbf{q}(\nu) = \begin{cases} v_j, & if\ v_{mj} < \nu \leq v_{Mj}, \\ 0, & if\ \nu = 0, \\ -\mathbf{q}(-\nu), & if\ \nu < 0 \end{cases} \tag{4.2}$$

where $\omega = (1 - \varrho)/(1 + \varrho)$, $v_{mj} = v_j/(1 + \omega)$, $v_{Mj} = v_j/(1 - \omega)$. Note that the quantizing effects can be transformed for a given quantization density ϱ into sector bounded format as

$$\mathbf{q}(\nu) \; - \; \nu \; = \; \mathbf{\Delta}\nu, \;\; ||\mathbf{\Delta}|| \; \leq \; \omega \tag{4.3}$$

Based on the quantized signals, the controller will be designed such that the desired dynamic performance of system (4.1) is achieved while the data packet dropout arises. Toward our goal, we assume for a more general case that the measurement with a randomly varying communication delay is described by

$$y_c(k) = \begin{cases} y_q(k), \;\; \delta(k) = 0 \\ y_q(k - \tau_k^m), \;\; \delta(k) = 1 \end{cases}$$

which in view of $y_q(k) = \mathbf{q}(y_p(k)) = (1 + \mathbf{\Delta})y_p(k)$, it becomes

$$y_c(k) = \begin{cases} (1 + \mathbf{\Delta})y_p(k), \;\; \delta(k) = 0 \\ (1 + \mathbf{\Delta})y_p(k - \tau_k^m), \;\; \delta(k) = 1 \end{cases} \tag{4.4}$$

where τ_k^m stands for *measurement delay*, the occurrence of which satisfies the Bernoulli distribution, and $\delta(k)$ is Bernoulli distributed white sequence. We let

$$Prob\{\delta(k) = 1\} = p_k$$

where p_k assumes discrete values. Two particular classes can be considered:

Class 1: p_k has the probability mass function where $q_r - q_{r-1} =$ constant for $r = 2, ..., n$. This covers a wide range of cases including uniform discrete distribution, symmetric triangle distribution, decreasing linear function or increasing linear function.

Class 2: $p_k = X/n,\ n > 0$ and $0 \leq X \leq n$ is a random variable that follows the Bionomial distribution $\mathbf{B}(q, n),\ q > 0$.

Remark 4.2.1 *It is significant to note that the case* $Prob\{\delta(k) = 1\} = \bar{\delta}$, *where* $\bar{\delta}$ *is a constant value, is widely used in majority of results on NCS. But here we focus on nonstationary dropouts.*

Taking into consideration the time delay that occurs on the actuation side, we proceed to design the following observer-based controller:

$$\begin{aligned}
&Observer: \\
&\hat{x}(k+1) = A\hat{x} + Bu_c(k) + L(y_c(k) - \hat{y}_c(k)) \\
&\hat{y}_c(k) = \begin{cases} C\hat{x}(k), & \delta(k) = 0 \\ C\hat{x}(k - \tau_k^m), & \delta(k) = 1 \end{cases}
\end{aligned} \tag{4.5}$$

$$\begin{aligned}
&Controller: \\
&u_c(k) = K\hat{x}(k) \\
&u_p = \begin{cases} u_c(k), & \alpha(k) = 0 \\ u_c(k - \tau_k^a), & \alpha(k) = 1 \end{cases}
\end{aligned} \tag{4.6}$$

where $\hat{x}(k) \in \Re^n$ is the estimate of the system (4.1), $\hat{y}_c(k) \in \Re^p$ is the observer output, and $L \in \Re^{n\times p}$ and $K \in \Re^{m\times n}$ are the observer and controller gains, respectively, and τ_k^a is the *actuation delay.* The stochastic variable $\alpha(k)$, mutually independent of δ, is also a Bernoulli distributed white sequence with

$$Prob\{\alpha(k) = 1\} = s_k$$

where s_k assumes discrete values. By similarity, a particular class is that s_k has some probability mass function as in Table II, where $s_r - s_{r-1} =$ constant for $r = 2, ..., n$. It is assumed that τ_k^a and τ_k^m are time-varying and have the following bounded condition:

$$\tau_m^- \le \tau_k^m \le \tau_m^+, \ \ \tau_a^- \le \tau_k^a \le \tau_a^+ \tag{4.7}$$

Define the estimation error by $e(k) = x_p(k) - \hat{x}(k)$. Then, it yields

$$x_p(k+1) \;=\; \begin{cases} (A+BK)x_p(k) & \\ -BKe(k), & \alpha(k)=0, \\ Ax_p(k) + BKx_p(k-\tau_k^\alpha) & \\ -BKe(k-\tau_k^\alpha), & \alpha(k)=1, \end{cases} \tag{4.8}$$

$$\begin{aligned} e(k+1) &= x_p(k+1) - \hat{x}(k+1) \\ &= \begin{cases} (A-LC)e(k) & \\ -LC\mathbf{\Delta}x_p(k), \ \delta(k)=0, & \\ Ae(k) - LCe(k-\tau_k^m) & \\ -LC\mathbf{\Delta}x_p(k-\tau_k^m), \ \delta(k)=1, & \end{cases} \end{aligned} \tag{4.9}$$

In terms of $\xi(k) = [x_p^T(k) \quad e^T(k)]^T$, system (4.8) and (4.9) can be cast into the form:

$$\xi(k+1) = \mathsf{A}_j\xi(k) + \mathsf{B}_j\xi(k-\tau_k^m) + \mathsf{C}_j\xi(k-\tau_k^a) \tag{4.10}$$

where $\{\mathsf{A}_j,\ \mathsf{B}_j,\ \mathsf{C}_j,\ j = 1, ..., 4\}$ and j is an index identifying one of the following pairs $\{(\delta(k) = 1,\ \alpha(k) = 1),\ (\delta(k) = 1,\ \alpha(k) = 0),\ (\delta(k) = 0,\ \alpha(k) = 0),\ (\delta(k) = 0,\ \alpha(k) = 1)\}$:

$$\begin{aligned}
\mathsf{A}_1 &= \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix},\ \mathsf{A}_2 = \begin{bmatrix} A+BK & -BK \\ 0 & A \end{bmatrix}, \\
\mathsf{A}_3 &= \begin{bmatrix} A+BK & -BK \\ -LC\mathbf{\Delta} & A-LC \end{bmatrix}, \\
\mathsf{A}_4 &= \begin{bmatrix} A & 0 \\ -LC\mathbf{\Delta} & A-LC \end{bmatrix}, \\
\mathsf{B}_1 &= \begin{bmatrix} 0 & 0 \\ -LC\mathbf{\Delta} & -LC \end{bmatrix},\ \mathsf{B}_2 = \begin{bmatrix} 0 & 0 \\ -LC\mathbf{\Delta} & -LC \end{bmatrix}, \\
\mathsf{B}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \mathsf{B}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathsf{C}_1 &= \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix},\ \mathsf{C}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathsf{C}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \mathsf{C}_4 = \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}
\end{aligned} \tag{4.11}$$

Remark 4.2.2 *It is remarked for simulation processing that we can express (4.8)-(4.9) in the form*

$$\begin{aligned} x_p(k+1) &= s_k[Ax_p(k) + BKx_p(k-\tau_k^\alpha) \\ &- BKe(k-\tau_k^\alpha)] \\ &+ (1-s_k)[(A+BK)x_p(k) - BKe(k)] \qquad (4.12) \\ e(k+1) &= p_k[Ae(k) - LCe(k-\tau_k^m) \\ &- LC\mathbf{\Delta}x_p(k-\tau_k^m)] \\ &+ (1-p_k)[(A-LC)e(k) - LC\mathbf{\Delta}x_p(k)] \qquad (4.13) \end{aligned}$$

where the values of the random variables $p_k,\ s_k$ are generated in the manna r discussed earlier.

Remark 4.2.3 *It is important to note from (4.11) that*

$$\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j = \begin{bmatrix} A+BK & -BK \\ -LC\mathbf{\Delta} & A-LC \end{bmatrix}, j = 1, .., 4 \qquad (4.14)$$

The interpretation of this result is that $\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j$ represents the fundamental matrix of the delayed system (4.10), which must be independent of the mode of operation. This will help in simplifying the control design algorithm.

Our aim here is to design an observer based feedback stabilizing controller in the form of (4.5) and (4.6) such that the closed loop system (4.10) is exponentially stable in the mean square. Our approach is based on the concepts of switched time-delay systems [81]. For simplicity in exposition, we introduce

$$\begin{aligned}
\sigma_1(k) &= Prob\{\delta(k) = 1,\ \alpha(k) = 1\},\ \ \hat{\sigma}_1 = \mathbb{E}[\sigma_1] \\
\sigma_2(k) &= Prob\{\delta(k) = 1,\ \alpha(k) = 0\},\ \ \hat{\sigma}_2 = \mathbb{E}[\sigma_2] \\
\sigma_3(k) &= Prob\{\delta(k) = 0, \alpha(k) = 0\},\ \ \hat{\sigma}_3 = \mathbb{E}[\sigma_3] \\
\sigma_4(k) &= Prob\{\delta(k) = 0,\ \alpha(k) = 1\},\ \ \hat{\sigma}_4 = \mathbb{E}[\sigma_4]
\end{aligned} \tag{4.15}$$

where $\mathbb{E}[\sigma_i]$ is the expected value of $\sigma_i,\ i = 1, .., 4$. Since we assume that $\delta(k)$ and $\alpha(k)$ are independent random variables, then it follows from (4.15) that

$$\begin{aligned}
\hat{\sigma}_1 &= \mathbb{E}[p_k]\mathbb{E}[s_k],\ \ \hat{\sigma}_2 = \mathbb{E}[p_k]\mathbb{E}[1 - s_k] \\
\hat{\sigma}_3 &= \mathbb{E}[1 - p_k]\mathbb{E}[1 - s_k],\ \ \hat{\sigma}_4 = \mathbb{E}[1 - p_k]\mathbb{E}[s_k]
\end{aligned} \tag{4.16}$$

4.3 Main Results

In this section, we will thoroughly investigate the stability and controller synthesis problems for the closed-loop system (4.10). First, let us deal with the stability analysis problem and derive a sufficient condition under which the closed-loop system (4.10) with the given controller (4.5) and (4.6) is exponentially stable in the mean square. Extending on [82], the following Lyapunov function candidate is constructed to establish the main theorem:

$$
\begin{aligned}
V(\xi(k)) &= \sum_{i=1}^{5} V_i(\xi(k)) && (4.17)\\
V_1(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \xi^T(k) P \xi(k), \; P > 0 \\
V_2(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i), \; Q_j = Q_j^T > 0 \\
V_3(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{i=k-\tau_k^a}^{k-1} \xi^T(i) Q_j \xi(i) \\
V_4(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{\ell=-\tau_m^+ +2}^{-\tau_m^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i) Q_j \xi(i) \\
V_5(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{\ell=-\tau_a^+ +2}^{-\tau_a^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i) Q_j \xi(i) && (4.18)
\end{aligned}
$$

It is not difficult to show that there exist real scalars $\mu > 0$ and $\upsilon > 0$ such that

$$\mu \|\xi\|^2 \leq V(\xi(k)) \leq \upsilon \|\xi(k)\|^2 \tag{4.19}$$

Remark 4.3.1 *By carefully considering Remark 4.2.3 in view of model (4.10), it is justified to select matrix P to be the same for all operational modes, hence independent of j, while keeping matrix Q_j dependent on mode j.*

We now present the analysis result for system (4.10) to be exponentially stable.

Theorem 3.1: *Let the controller and observer gain matrices K and L be given. The closed-loop system (4.10) is exponentially stable if there exist matrices* $0 <$

$P,\ 0 < Q_j^T = Q_j,\ j = 1,..,4$ and matrices $R_i,\ S_i,$ and $M_i,\ i = 1,2,$ such that the following matrix inequality holds

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & \Lambda_{2j} \\ \bullet & \Lambda_{3j} \end{bmatrix} \ < \ 0 \tag{4.20}$$

$$\Lambda_{1j} =$$

$$\begin{bmatrix} \Psi_j + \Phi_{j1} & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - \hat{\sigma}_j Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - \hat{\sigma}_j Q_j \end{bmatrix}$$

$$\Lambda_{2j} = \begin{bmatrix} -R_1 + M_1^T - \Phi_{j2} & -R_2 + M_2^T - \Phi_{j3} \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix}$$

$$\Lambda_{3j} = \begin{bmatrix} -M_1 - M_1^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & -M_2 - M_2^T + \Phi_{j6} \end{bmatrix} \tag{4.21}$$

where

$$\begin{aligned} \Psi_j &= -P + \hat{\sigma}_j(\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)Q_j \\ &+ R_1 + R_1^T + R_2 + R_2^T \\ \Phi_{j1} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) \\ \Phi_{j2} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P \mathsf{B}_j \\ \Phi_{j3} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T \hat{\sigma}_j P \mathsf{C}_j,\ \Phi_{j5} = \mathsf{B}_j^T P \mathsf{C}_j \\ \Phi_{j4} &= \mathsf{B}_j^T \hat{\sigma}_j P \mathsf{B}_j,\ \Phi_{j6} = \mathsf{C}_j^T \hat{\sigma}_j P \mathsf{C}_j \end{aligned}$$

Proof: Defining $y(k) = x(k+1) - x(k)$, one has

$$\xi(k - \tau_k^m) = \xi(k) - \sum_{i=k-\tau_k^m}^{k-1} y(i) \tag{4.22}$$

$$\xi(k - \tau_k^a) = \xi(k) - \sum_{i=k-\tau_k^a}^{k-1} y(i) \tag{4.23}$$

Then the system (4.10) can be transformed into

$$\xi(k+1) = (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)\xi(k) - \mathsf{B}_j\lambda(k) - \mathsf{C}_j\gamma(k) \tag{4.24}$$

where

$$\lambda(k) = \sum_{i=k-\tau_k^m}^{k-1} y(i), \quad \gamma(k) = \sum_{i=k-\tau_k^a}^{k-1} y(i).$$

Evaluating the difference of $V_1(\xi(k))$ along the solution of system (4.24), we have

$$\begin{aligned}
&\mathbb{E}[\Delta V_1(\xi(k))] = \mathbb{E}[V_1(\xi(k+1))] - V_1(\xi(k)) \\
&= \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)[\Phi_{j1} - P]\xi(k) - 2\xi^T(k)\Phi_{j2}\lambda(k) \\
&-2\xi^T(k)\Phi_{j3}\gamma(k) + \lambda^T(k)\Phi_{j4}\lambda(k) \\
&+2\lambda^T(k)\Phi_{j5}\gamma(k) + \gamma^T(k)\Phi_{j6}\gamma(k) \Big]
\end{aligned} \tag{4.25}$$

A straightforward computation gives

$$
\begin{aligned}
&\mathbb{E}[\Delta V_2(\xi(k)] = \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\sum_{i=k+1-\tau_{k+1}^m}^{k} \xi^T(i) Q_j \xi(i) \\
&- \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
&= \xi^T(k) Q \xi(k) - \xi(k-\tau_k^m) Q_j \xi(k-\tau_k^m) \\
&+ \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i) Q_j \xi(i) - \sum_{i=k+1-\tau_k^m}^{k-1} \xi(i) Q_j \xi(i) \Bigg]
\end{aligned} \tag{4.26}
$$

In view of

$$
\begin{aligned}
& \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
= & \sum_{i=k+1-\tau_{k+1}^m}^{k-\tau_k^m} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
\leq & \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i)
\end{aligned} \tag{4.27}
$$

We readily obtain

$$
\begin{aligned}
&\mathbb{E}[\Delta V_2(\xi(k))] \leq \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\xi^T(k) Q_j \xi(k) \\
&-\xi^T(k-\tau_k^m) Q_j \xi(k-\tau_k^m) \\
&+ \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i) \Bigg]
\end{aligned} \tag{4.28}
$$

Following parallel procedure, we get

$$\mathbb{E}[\Delta V_3(\xi(k))] \leq \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\xi^T(k) Q_j \xi(k)$$

$$-\xi^T(k-\tau_k^a) Q_j \xi(k-\tau_k^a) + \sum_{i=k+1-\tau_a^+}^{k-\tau_a^-} \xi^T(i) Q_j \xi(i)\Bigg] \tag{4.29}$$

Finally

$$\mathbb{E}[\Delta V_4(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[\sum_{\ell=-\tau^+{}_m+2}^{-\tau^-{}_m+1} [\xi^T(k) Q_j \xi(k)$$

$$-\xi^T(k+\ell-1) Q_j \xi(k+\ell-1)]\Bigg]$$

$$= \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[(\tau^+{}_m - \tau^-{}_m) \xi^T(k) Q_j \xi(k)$$

$$- \sum_{i=k+1-\tau^+{}_m}^{k-\tau^-{}_m} \xi^T(i) Q_j \xi(i)\Bigg] \tag{4.30}$$

$$\mathbb{E}[\Delta V_5(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Bigg[(\tau^+{}_a - \tau^-{}_a) \xi^T(k) Q_j \xi(k)$$

$$- \sum_{i=k+1-\tau^+{}_a}^{k-\tau^-{}_a} \xi^T(i) Q_j \xi(i)\Bigg] \tag{4.31}$$

It follows from (4.22) and (4.23) that:

$$\xi(k) - \xi(k-\tau_k^m) - \lambda(k) = 0 \tag{4.32}$$

$$\xi(k) - \xi(k-\tau_k^a) - \gamma(k) = 0 \tag{4.33}$$

Therefore, for any appropriately dimensioned matrices R_i, S_i and M_i, $i = 1, 2$, we have the following equations:

$$
\begin{aligned}
&2[\xi^T(k)R_1 + \xi^T(k-\tau_k^m)S_1 + \lambda^T(k)M_1] \\
&\times[\xi(k) - \xi(k-\tau_k^m) - \tau(k)] = 0 \qquad (4.34) \\
&2[\xi^T(k)R_2 + \xi^T(k-\tau_k^a)S_2 + \gamma^T(k)M_2] \\
&\times[\xi(k) - \xi(k-\tau_k^a) - \gamma(k)] = 0 \qquad (4.35)
\end{aligned}
$$

On combining (4.25)–(4.35), we reach

$$
\begin{aligned}
&\mathbb{E}[\Delta V(\xi(k))] \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)\Psi_j\xi(k) \\
&+\xi^T(k)(-2R_1 + 2S_1^T)\xi(k-\tau_k^m) \\
&+\xi^T(k)(-2R_2 + 2S_2^T)\xi(k-\tau_k^a) \\
&+\xi^T(k)(-2R_1 + 2M_1^T - 2\Phi_{j2})\lambda(k) \\
&+\xi^T(k)(-2R_2 + 2M_2^T - 2\Phi_{j3})\gamma(k) \\
&+\xi^T(k-\tau_k^m)(-S_1 - S_1^T - \hat{\sigma}_jQ_j)\xi(k-\tau_k^m) \\
&+\xi^T(k-\tau_k^m)(-2S_1 - 2M_1^T)\lambda(k) \\
&+\xi^T(k-\tau_k^a)(-S_2 - S_2^T - \hat{\sigma}_jQ_j)\xi(k-\tau_k^a) \\
&+\xi^T(k-\tau_k^a)(-2S_2 - 2M_2^T)\gamma(k) \\
&+\lambda^T(k)(-M_1 - M_1^T + \Phi_{j4})\lambda(k) \\
&+\gamma^T(k)(-M_2 - M_2^T + \Phi_{j5})\gamma(k) \\
&+\lambda^T(k)\Phi_{j6}\gamma(k) \Big] = \sum_{j=1}^{4} \hat{\sigma}_j \left[\zeta^T(k)\widetilde{\Lambda}_j\zeta(k) \right] \qquad (4.36)
\end{aligned}
$$

where

$$\zeta(k) = \begin{bmatrix} \zeta_1^T & \zeta_2^T \end{bmatrix}^T, \ \zeta_2 = \begin{bmatrix} \lambda^T(k) & \gamma^T(k) \end{bmatrix}^T$$
$$\zeta_1 = \begin{bmatrix} \xi^T(k) & \xi^T(k-\tau_k^m) & \xi^T(k-\tau_k^a) \end{bmatrix}^T \tag{4.37}$$

and $\widetilde{\Lambda}_j$ corresponds to Λ_j in (4.21) by Schur complements. If $\Lambda_j < 0, \ j = 1,..,4$ holds, then

$$\begin{aligned} &\mathbb{E}[V(\xi(k+1)) - V(\xi(k))] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k)\widetilde{\Lambda}_j\zeta(k) \Big] \\ &\leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[-\widetilde{\Lambda}_{min}(\widetilde{\Lambda}_j)\zeta^T(k)\zeta(k) \Big] \\ &< -\sum_{j=1}^{4} \hat{\sigma}_j \Big[\beta_j\zeta^T(k)\zeta(k) \Big] \end{aligned} \tag{4.38}$$

where

$$0 < \beta_j < min\big[\lambda_{min}(\Lambda_j), max\{\lambda_{max}(P), \ \lambda_{max}(Q_j)\}\big]$$

Inequality (4.38) implies that $\mathbb{E}[V(\xi(k+1)) - V(\xi(k))] < -\phi V(\xi(k)), \ 0 < \phi < 1$. In the manner of [23], we get

$$||\xi(k)||^2 \leq \frac{\upsilon}{\kappa}||\xi(0)||^2(1-\phi)^k + \frac{\lambda}{\mu\phi}$$

Therefore, it can be verified that the closed-loop system (4.10) is exponentially stable. This completes the proof.

A solution to the problem of the observer-based networked feedback stabilizing controller design is provided by the following theorem:

Theorem 3.2: *Let the delay bounds* $\tau_m^+,\ \tau_m^-,\ \tau_a^+,\ \tau_a^-$ *be given. Evaluate the quantities* $\hat{\sigma}_j,\ j = 1,..,4$. *Then the closed-loop system (4.10) is exponentially stable if there exist matrices* $0 < X,\ Y_1,\ Y_2,\ 0 < \Xi_j,\ j = 1,..,4$ *and matrices* $\Pi_i,\ \Upsilon_i$ *and* $\Gamma_i,\ i = 1,2,$ *such that the following matrix inequality holds for* $j = 1,..,4$*:*

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j \\ \bullet & \Lambda_{3j} & 0 \\ \bullet & \bullet & -\hat{\sigma}_j \widehat{X} \end{bmatrix} < 0 \tag{4.39}$$

$$\widehat{X} = \begin{bmatrix} X & 0 \\ X & X \end{bmatrix} \tag{4.40}$$

$$\widehat{\Psi}_j = -X + \hat{\sigma}_j(\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)\Xi_j +$$
$$\Pi_1 + \Pi_1^T + \Pi_2 + \Pi_2^T$$

$$\widehat{\Lambda}_{1j} = \begin{bmatrix} \widehat{\Psi}_j & -\Pi_1 + \Upsilon_1^T & -\Pi_2 + \Upsilon_2^T \\ \bullet & -\Upsilon_1 - \Upsilon_1^T - \hat{\sigma}_j \Xi_j & 0 \\ \bullet & \bullet & -\Upsilon_2 - \Upsilon_2^T - \hat{\sigma}_j \Xi_j \end{bmatrix}$$

$$\widehat{\Lambda}_{2j} = \begin{bmatrix} -\Pi_1 + \Gamma_1^T & -\Pi_2 + \Gamma_2^T \\ -\Upsilon_1 - \Gamma_1^T & 0 \\ 0 & -\Upsilon_2 - \Gamma_2^T \end{bmatrix}$$

$$\widehat{\Lambda}_{3j} = \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & -\Gamma_2 - \Gamma_2^T \end{bmatrix}$$

$$\widehat{\Omega}_j = \begin{bmatrix} \widehat{\Omega}_{1j} & 0 & 0 & -\widehat{\Omega}_{4j} & -\widehat{\Omega}_{5j} \end{bmatrix}^T$$

$$\widehat{\Omega}_{1j} = \begin{bmatrix} XA^T + Y_1^T B^T & -\boldsymbol{\Delta} Y_2^T \\ XA^T & XA^T - (1 + \boldsymbol{\Delta}) Y_2^T \end{bmatrix}, \ \forall\, j$$

$$\widehat{\Omega}_{4j} = \begin{bmatrix} Y_1^T B^T & 0 \\ 0 & 0 \end{bmatrix}, \quad j = 1, 4$$

$$\widehat{\Omega}_{5j} = \begin{bmatrix} 0 & 0 \\ 0 & -Y_2^T \end{bmatrix}, \quad j = 1, 2$$

$$\widehat{\Omega}_{4j} = 0,\ j = 2, 3, \quad \widehat{\Omega}_{5j} = 0, \quad j = 3, 4 \tag{4.41}$$

where the gain matrices are given by

$$K = Y_1 X^{-1}, \quad L = Y_2 X^{-1} C^{\dagger}$$

Proof: Define

$$\Omega_j = \left[\begin{array}{cccc} (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) & 0 & -\mathsf{B}_j & -\mathsf{C}_j \end{array} \right]^T$$

then matrix inequality (4.20) can be expressed as

$$\Lambda_j = \widetilde{\Lambda} + \Omega_j P \Omega_j^T < 0 \tag{4.42}$$

$$\widetilde{\Lambda}_j = \begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} \\ \bullet & \widetilde{\Lambda}_{3j} \end{bmatrix} < 0$$

$$\widetilde{\Lambda}_{1j} = \begin{bmatrix} \Psi_j & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - Q_j \end{bmatrix}$$

$$\widetilde{\Lambda}_{2j} = \begin{bmatrix} -R_1 + M_1^T & -R_2 + M_2^T \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix}$$

$$\widetilde{\Lambda}_{3j} = \begin{bmatrix} -M_1 - M_1^T & 0 \\ \bullet & -M_2 - M_2^T \end{bmatrix} \tag{4.43}$$

Setting $\widehat{X} = P^{-1}$, invoking Schur complements, we write matrix Λ_j in (4.42) equivalently as

$$\begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} & \Omega_j \\ \bullet & \widetilde{\Lambda}_{3j} & 0 \\ \bullet & \bullet & -\widehat{X} \end{bmatrix} < 0 \tag{4.44}$$

Applying the congruence transformation

$$T_j = diag[\widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ I]$$

to matrix inequality in (4.44) and manipulating using (4.40) and

$$\begin{aligned} \Xi_j &= \widehat{X} Q_j \widehat{X},\ \Pi_j = \widehat{X} R_j \widehat{X},\ \Upsilon_j = \widehat{X} S_j \widehat{X}, \\ \Gamma_j &= \widehat{X} M_j \widehat{X} \end{aligned}$$

we readily obtain matrix inequality (4.39) subject (4.41).

Remark 4.3.2 *The selection of $\widehat{X}$ as given by (4.40) has the advantage of converting the solution of bilinear matrix inequalities to that of seeking the feasibility of linear matrix inequalities and hence avoiding iterative procedures. It should be noted that the LMI (4.39) depend ens of the average dropout patterns identified by (4.16), which is quite useful in illustrating different operating conditions of the communications network.*

Remark 4.3.3 *It is remarked that the implementation of* **Theorem 3.2** *is on-line in nature as it requires calling random generators to pick-up numbers corresponding to the scalars $\hat{\sigma}_1,\ ...,\ \hat{\sigma}_4$ and to evaluate the probabilities in model (4.12) and (4.13) to compute the state and error trajectories. This represents a salient feature not shared by other methods for networked control design under unreliable communication links.*

4.4 Numerical Simulation

A quadruple-tank process consisting of four interconnected water tanks and two pumps is considered for simulation. Its manipulated variables are voltages to the pumps and the controlled variables are the water levels in the two lower tanks. The quadruple-tank process is being built by considering the concept of two double-tank processes. The quadruple tank system presents a multi-input-multi-output (MIMO) system. A schematic description of the four tank system can be visualized by Figure 4.2. The system has two control inputs (pump throughputs) which can be manipulated to control the water level in the tanks. The two pumps are used to transfer water from a sump into four overhead tanks. In [85], an appropriate model is presented with the control objective being to regulate the level in the four-tanks at a desired setting and undistorted. The system matrices are given by:

$$
\begin{aligned}
A &= \begin{bmatrix} -0.0278 & 0 & 0.0206 & 0 \\ 0 & -0.0233 & 0 & 0.0141 \\ 0 & 0 & -0.0206 & 0 \\ 0 & 0 & 0 & -0.0141 \end{bmatrix}, \\
B &= \begin{bmatrix} 5 & 0 \\ 0 & 6.667 \\ 0 & 10 \\ 11.667 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

The mass functions of random variables p_k and q_k are selected to follow sym-

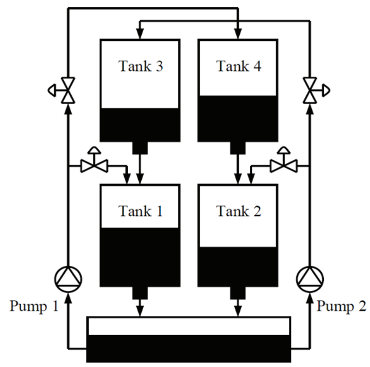

Figure 4.2: Schematic diagram of quadruple tank system

metric triangle distribution using 300 sample values, it is found that

$$\begin{aligned} \hat{\sigma}_1 &= 0.0100, \quad \hat{\sigma}_2 = 0.0964 \\ \hat{\sigma}_3 &= 0.8100, \quad \hat{\sigma}_4 = 0.0837 \end{aligned}$$

Next, setting $\varrho = 0.333$ and applying the new Algorithm, we obtain the controller and observer gain matrices as follows:

$$\begin{aligned} K &= \begin{bmatrix} -0.1204 & -0.1107 & 0.0897 & 0.3033 \\ -0.0018 & 0.0315 & 0.1174 & -0.0090 \end{bmatrix} \\ L^t &= \begin{bmatrix} -0.0127 & 0.1024 & -0.0297 & -0.3498 \\ 0.0156 & 0.0116 & 0.0245 & -0.0876 \end{bmatrix} \end{aligned}$$

$$||K|| = 0.3654, \quad ||L|| = 0.4093$$

Similarly, with $\varrho = 0.667$ and applying the new Algorithm, we obtain the controller and observer gain matrices as follows:

$$
\begin{aligned}
K &= \begin{bmatrix} -0.0002 & -0.0067 & -0.1262 & 0.1099 \\ 0.0002 & 0.0049 & 0.0851 & -0.0105 \end{bmatrix} \\
L^t &= \begin{bmatrix} -0.0300 & -0.0356 & -0.0047 & 0.0108 \\ -0.0879 & -0.1043 & -0.0083 & -0.0560 \end{bmatrix}
\end{aligned}
$$

$$||K|| = 0.1813, \quad ||L|| = 0.1535$$

On the other hand, with $\bar{\delta}$ $\bar{\alpha}$ following the same distribution and the measurement and actuation delays varying as $\tau_k^m \in [1 \longrightarrow 5]$ and $\tau_k^a \in [3 \longrightarrow 9]$, the algorithm presented in the previous chapter was implemented and controller and observer gain matrices obtained are as follows:

$$
\begin{aligned}
K &= \begin{bmatrix} -0.0440 & 0.0037 & -0.2525 & 0.1129 \\ -0.0596 & 0.0549 & 0.1034 & -1.0081 \end{bmatrix} \\
L &= \begin{bmatrix} -0.3836 & -0.0889 & 0.0635 & 0.1217 \\ 0.0424 & 0.0189 & -0.0379 & -0.0375 \end{bmatrix}
\end{aligned}
$$

$$||K|| = 1.0261, \quad ||L|| = 0.4213$$

Invoking the 'variable fraction delay' block in Matlab Simulink software (Matlab 7.0) to handle discrete time-varying delays τ_k^m and τ_k^a and under the initial con-

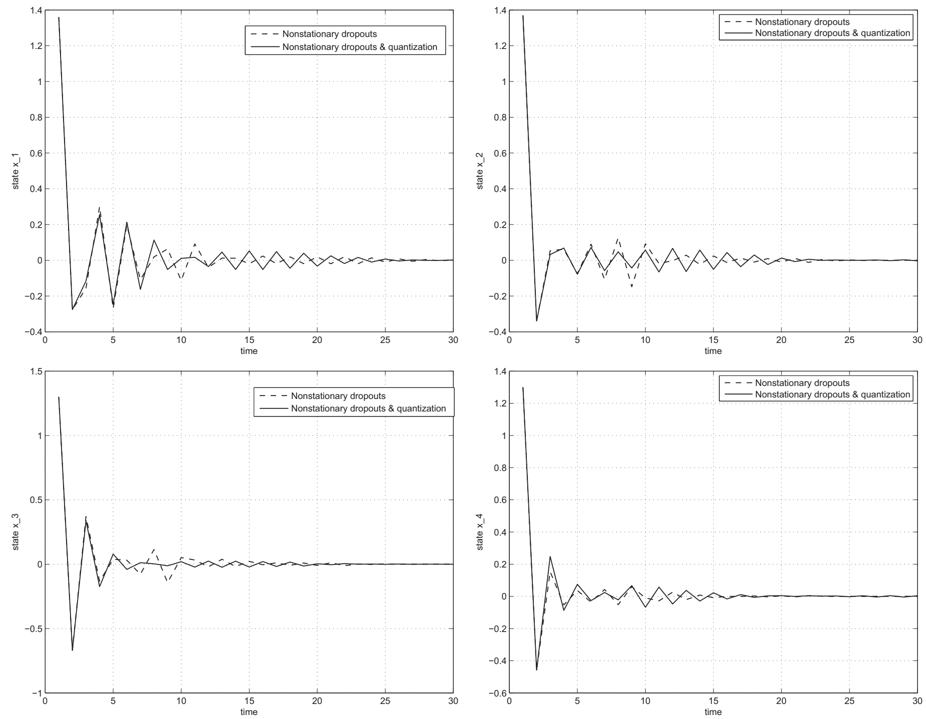

Figure 4.3: State trajectories for systems with and without quantization

ditions $x_{0_p} = [1.36 \quad 1.37 \quad 1.3 \quad 1.3]^T$ and $\hat{x}_0 = [0 \quad 0 \quad 0 \quad 0]^T$, the simulation results of the state responses are given in Fig. 4.3, where the dotted lines denote the state responses of the system without quantization. It is quite visible that the system response with quantization takes more time to settle and shows more oscillations.

On the other hand, the response of quantized systems states with two distinct values of ϱ, 0.333 and 0.667 is shown in Fig. 4.4.

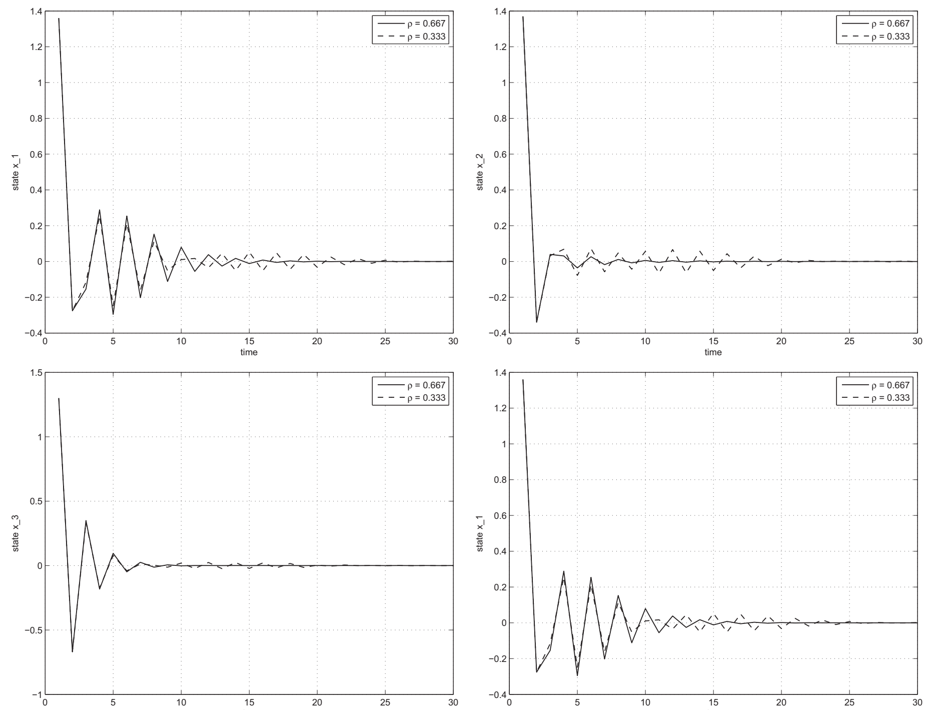

Figure 4.4: Quantized tate trajectories for nonstationary dropouts

4.5 Conclusion

The stability analysis and controller synthesis problems are thoroughly investigated for NCSs with time-varying delays and and subject to quantization and nonstationary packet dropout. Attention has been focused on the design of the new observer-based controller such that the resulting closed-loop system is exponentially stable in the mean-square sense. The effectiveness of the proposed results has been shown through a typical numerical example. The effect of quantization on a system response has also been demonstrated.

Chapter 5

NETWORKED FEEDBACK CONTROL FOR NONLINEAR SYSTEMS WITH NONSTATIONARY PACKET DROPOUTS

5.1 Introduction

Every advancement in technology has it's pros and cons. Similarly, despite the huge advantage and widespread use of networked communication in control systems, the introduction of the networks in the control loops makes the analysis of networked control systems very complicated and adds a great deal of uncertainity to the system behavior. The justification to the above statement is the fact that the network itself is a dynamic system subject to various shortcomings, such as data dropout, limited bandwidth, time delay, and quantization.

Nonlinear time-delay systems have widely been used to describe propagation and transport phenomena in engineering and practical applications as mentioned in [94]. The investigation into the adverse effects caused by the delays on the performance of any control system has drawn considerable interest since the presence of delays may induce complexity and uncertainity, especially in nonlinear systems as shown in [93], [95], [96], [97]. In this context, the stability conditions for linear time-delay systems are broadly classified into two main categories: delay-independent, which are not influenced by the arbitrary size of the delays [98] and delay-dependent, which include information on the size and occurence of the delay, see [95]–[99]. In [96] the authors initially reported results on deriving delay-dependent stability and stabilization criteria using Leibniz–Newton formula. Some improved mehods as in [100, 101] were obtained recently to deal with the problem of robust control design of uncertain time-delay systems.

In recent years, there has been considerable development in the control theory for nonlinear time-delay systems and a large number of methods generalizing some aspects of the so-called differential geometric approach have been developed [102], including backstepping, adaptive control, observer-based control and also by using state-predictors for controlling nonlinear time-delay systems and can be found in [15]–[103] . Relevant results were also reported in [104], [105], [106], [107].

With this chapter we take a step further in the development of feedback stabilization methods for nonlinear discrete-time NCSs with random packet dropouts and delays. We develop an improved observer-based stabilizing control algorithm through the construction of an augmented system where the original control input is regarded as a new state. Due to limited bandwidth communication channel, the simultaneous occurrence of measurement and actuation delays are considered using nonstationary random processes modeled by two mutually independent stochastic variables. The developed stability conditions are represented in the form of a convex optimization problem and the results are tested by simulation on a real-time example.

5.2 Problem Formulation

Consider the NCS with random communication delays, where the sensor is clock driven and the controller and the actuator are event driven. The plant model under consideration is given by:

$$x_p(k+1) = Ax_p(k) + Bu_p(k) + f_0(x_k, k) + \Gamma_0 w(k),$$
$$y_p(k) = Cx_p(k)$$
$$z_p(k) = Gx_p(k) + \Phi_0 w(k) \tag{5.1}$$

where $x_p(k) \in \Re^n$ is the plants state vector, $u_p(k) \in \Re^m$ and $y_p(k) \in \Re^p$ are the plants control input and output vectors, respectively, $w(k) \in \Re^q$ is the disturbance input which belongs to $\ell_2[0, \infty)$ and $z_p(k) \in \Re^q$ is the controlled output. A, B, C, G, Γ_0 and Φ_0 are known as real matrices with appropriate dimensions.

The unknown function $f_0(x_p, k) \in \Re^n$ is a vector-valued time-varying nonlinear perturbation with $f_0(0, k) = 0$ and satisfies the following Lipschitz condition for all $(x_p, k), (\hat{x}_p, k)$:

$$\|f_0(x_p, k) - f_0(\hat{x}_p, k)\| \leq \alpha \|F(x_p - \hat{x}_p)\| \tag{5.2}$$

for some constant $\alpha > 0$ and $F \in \Re^n \times \Re^n$ is a constant matrix. Note that as a consequence of (5.2) we have

$$\|f_0(x_p, k)\| \leq \alpha \|F\ x_p\| \tag{5.3}$$

Equivalently stated, condition (5.3) implies that

$$f_0^t(x_p, k) f_0(x_p, k) - \alpha^2 x_p^t F^t F x_p \leq 0 \tag{5.4}$$

Towards our goal, we assume for a more general case that the measurement with a randomly varying communication delay is described by

$$y_c(k) = \begin{cases} y_p(k), & \delta(k) = 0 \\ y_p(k - \tau_k^m), & \delta(k) = 1 \end{cases} \tag{5.5}$$

where τ_k^m stands for *measurement delay*, the occurrence of which satisfies the Bernoulli distribution, and $\delta(k)$ is Bernoulli distributed white sequence. It order to capture the current practice of computer communication management that experiences different time-dependent operational modes, we let

$$Prob\{\delta(k) = 1\} = p_k$$

where p_k assumes discrete values. Two particular classes can be considered:

Class 1: p_k has the probability mass function where $q_r - q_{r-1} =$ constant for $r = 2, ..., n$. This covers a wide range of cases including uniform discrete distribution, symmetric triangle distribution, decreasing linear function or increasing linear function.

Class 2: $p_k = X/n,\ n > 0$ and $0 \leq X \leq n$ is a random variable that follows the Bionomial distribution $\mathbf{B}(q, n),\ q > 0$.

Remark 5.2.1 *It is significant to note that the case* $Prob\{\delta(k) = 1\} = \bar{\bar{\delta}}$*, where* $\bar{\bar{\delta}}$ *is a constant value, is widely used in majority of results on NCS. Here we focus on nonstationary dropouts.*

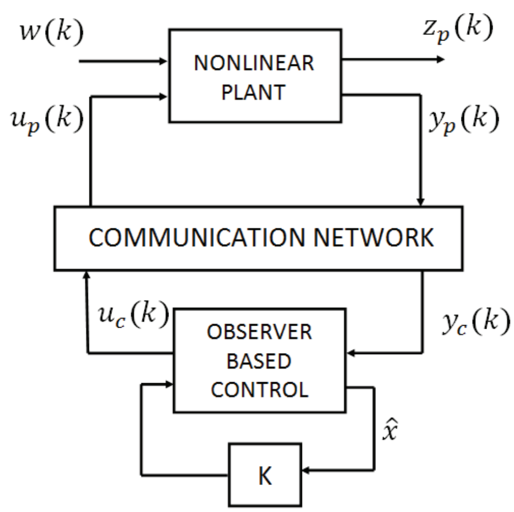

Figure 5.1: Block diagram of Nonlinear NCS

Taking into consideration the time delay that occurs on the actuation side, we proceed to design the following observer-based controller:

$$
\begin{aligned}
&Observer:\\
&\hat{x}(k+1) = A\hat{x}(k) + Bu_c(k) + L(y_c(k) - \hat{y}_c(k))\\
&\hat{y}_c(k) = \begin{cases} C\hat{x}(k), & \delta(k) = 0 \\ C\hat{x}(k-\tau_k^m), & \delta(k) = 1 \end{cases}
\end{aligned} \tag{5.6}
$$

$$
\begin{aligned}
&Controller:\\
&u_c(k) = K\hat{x}(k)\\
&u_p = \begin{cases} u_c(k), & \alpha(k) = 0 \\ u_c(k-\tau_k^a), & \alpha(k) = 1 \end{cases}
\end{aligned} \tag{5.7}
$$

where $\hat{x}(k) \in \mathfrak{R}^n$ is the estimate of the system (5.1), $\hat{y}_c(k) \in \mathfrak{R}^p$ is the observer output, and $L \in \mathfrak{R}^{n\times p}$ and $K \in \mathfrak{R}^{m\times n}$ are the observer and controller gains, respectively, and τ_k^a is the *actuation delay*. The stochastic variable $\alpha(k)$, mutually independent of δ, is also a Bernoulli distributed white sequence with

$$Prob\{\alpha(k) = 1\} = s_k$$

where s_k assumes discrete values. By similarity, a particular class is that s_k has some probability mass function as in Table II, where $s_r - s_{r-1} =$ constant for $r = 2, ..., n$. It is assumed that τ_k^a and τ_k^m are time-varying and have the following bounded condition:

$$\tau_m^- \leq \tau_k^m \leq \tau_m^+, \;\; \tau_a^- \leq \tau_k^a \leq \tau_a^+ \tag{5.8}$$

Define the estimation error by $e(k) = x_p(k) - \hat{x}(k)$. Then, it yields

$$x_p(k+1) = \begin{cases} (A+BK)x_p(k) \\ -BKe(k) + f_0(x_p, k) \\ +\Gamma_0 w(k), \qquad \alpha(k) = 0, \\ \\ Ax_p(k) + BKx_p(k - \tau_k^{\alpha}) \\ -BKe(k - \tau_k^{\alpha}) + f_0(x_p, k) \\ +\Gamma_0 w(k), \qquad \alpha(k) = 1, \end{cases} \tag{5.9}$$

$$\begin{aligned} e(k+1) &= x_p(k+1) - \hat{x}(k+1) \\ &= \begin{cases} (A-LC)e(k) \\ +f_0(x_p, k) + \Gamma_0 w(k), \qquad \delta(k) = 0, \\ \\ Ae(k) - LCe(k - \tau_k^m) \\ +f_0(x_p, k) + \Gamma_0 w(k), \qquad \delta(k) = 1, \end{cases} \end{aligned} \tag{5.10}$$

In terms of $\xi(k) = [x_p^T(k) \quad e^T(k)]^T$, system (5.9) and (5.10) can be cast into the form:

$$\begin{aligned} \xi(k+1) = \mathsf{A}_j \xi(k) + \mathsf{B}_j \xi(k - \tau_k^m) + \mathsf{C}_j \xi(k - \tau_k^a) \\ + \hat{f}(x_p, k) + \widehat{\Gamma} w(k) \end{aligned} \tag{5.11}$$

where $\{\mathsf{A}_j,\ \mathsf{B}_j,\ \mathsf{C}_j,\ j = 1, ..., 4\}$ and j is an index identifying one of the following pairs $\{(\delta(k) = 1,\ \alpha(k) = 1),\ (\delta(k) = 1,\ \alpha(k) = 0),\ (\delta(k) = 0,\ \alpha(k) =$

$0),\ (\delta(k)=0,\ \alpha(k)=1)\}$:

$$
\begin{aligned}
\mathsf{A}_1 &= \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix},\ \mathsf{A}_2 = \begin{bmatrix} A+BK & -BK \\ 0 & A \end{bmatrix}, \\
\mathsf{A}_3 &= \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, \\
\mathsf{A}_4 &= \begin{bmatrix} A & 0 \\ 0 & A-LC \end{bmatrix}, \\
\mathsf{B}_1 &= \begin{bmatrix} 0 & 0 \\ 0 & -LC \end{bmatrix},\ \mathsf{B}_2 = \begin{bmatrix} 0 & 0 \\ 0 & -LC \end{bmatrix}, \\
\mathsf{B}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \mathsf{B}_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathsf{C}_1 &= \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix},\ \mathsf{C}_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \\
\mathsf{C}_3 &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \mathsf{C}_4 = \begin{bmatrix} BK & -BK \\ 0 & 0 \end{bmatrix}, \\
\widehat{\Gamma} &= \begin{bmatrix} \Gamma_0 & 0 \\ 0 & \Gamma_0 \end{bmatrix},\ \widehat{G}^T = \begin{bmatrix} G \\ 0 \end{bmatrix},\ \widehat{\Phi}^T = \begin{bmatrix} \Phi_0 \\ 0 \end{bmatrix}
\end{aligned}
\tag{5.12}
$$

Remark 5.2.2 *It is remarked for simulation processing that we can express*

(5.9)-(5.10) in the form

$$\begin{aligned} x_p(k+1) &= s_k[Ax_p(k) + BKx_p(k-\tau_k^\alpha) \\ &+ BKe(k-\tau_k^\alpha) + f_0(x_p,k) + \Gamma_0 w(k)] \\ &+ (1-s_k)[(A+BK)x_p(k) - BKe(k) \\ &+ f_0(x_p,k) + \Gamma_0 w(k)] \qquad (5.13) \\ e(k+1) &= p_k[Ae(k) - LCe(k-\tau_k^m) + f_0(x_p,k) \\ &+ \Gamma_0 w(k)] \\ &+ (1-p_k)[(A-LC)e(k) + f_0(x_p,k) \\ &+ \Gamma_0 w(k)] \qquad (5.14) \end{aligned}$$

where the values of the random variables p_k, s_k are generated in the manner discussed earlier.

Remark 5.2.3 *It is important to note from (5.13) that*

$$\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j = \begin{bmatrix} A+BK & -BK \\ 0 & A-LC \end{bmatrix}, j = 1, .., 4 \qquad (5.15)$$

The interpretation of this result is that $\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j$ represents the fundamental matrix of the delayed system (5.11), which must be independent of the mode of operation. This will help in simplifying the control design algorithm.

Our aim here is to design a feedback stabilizing controller in the form of equations (5.6) and (5.7) such that the closed-loop system (5.11) is exponentially stable using the Lyapunov theory. Our approach is based on the concepts of switched time-delay systems [81].

5.3 Main Results

This section investigates the stability analysis and controller synthesis problems for the closed-loop system (5.11). At first, the sufficient conditions are obtained under which the closed-loop system (5.11) with the given controller (5.6) and (5.7) is exponentially stable in the mean square. Extending on [82], the following Lyapunov function candidate is constructed to establish the main theorem:

$$V(\xi(k)) = \sum_{i=1}^{5} V_i(\xi(k)) \tag{5.16}$$

$$\begin{aligned}
V_1(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \xi^T(k) P \xi(k),\ P > 0 \\
V_2(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i),\ Q_j = Q_j^T > 0 \\
V_3(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{i=k-\tau_k^a}^{k-1} \xi^T(i) Q_j \xi(i) \\
V_4(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{\ell=-\tau_m^+ +2}^{-\tau_m^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i) Q_j \xi(i) \\
V_5(\xi(k)) &= \sum_{j=1}^{4} \sigma_j \sum_{\ell=-\tau_a^+ +2}^{-\tau_a^- +1} \sum_{i=k+\ell-1}^{k-1} \xi^T(i) Q_j \xi(i) \qquad (5.17)
\end{aligned}$$

It is not difficult to show that there exist real scalars $\mu > 0$ and $\upsilon > 0$ such that

$$\mu \|\xi\|^2 \leq V(\xi(k)) \leq \upsilon \|\xi(k)\|^2 \qquad (5.18)$$

Remark 5.3.1 *By carefully considering Remark 5.2.3 in view of model (5.11), it is justified to select matrix P to be the same for all operational modes, hence independent of j, while keeping matrix Q_j dependent on mode j.*

We now present the analysis result for system (5.11) to be exponentially stable.

Theorem 3.1: *Let the controller and observer gain matrices K and L be given. The closed-loop system (5.11) is exponentially stable if there exist matrices $0 < P,\ 0 < Q_j^T = Q_j,\ j = 1, .., 4$ and matrices $R_i,\ S_i,$ and $M_i,\ i = 1, 2,$ such that* *the following matrix inequality holds*

$$\Lambda_j = \begin{bmatrix} \Lambda_{1j} & \Lambda_{2j} \\ \bullet & \Lambda_{3j} \end{bmatrix} < 0 \tag{5.19}$$

$$\Lambda_{1j} = \begin{bmatrix} \Psi_j + \Phi_{j1} & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - Q_j \end{bmatrix}$$

$$\Lambda_{2j} = \begin{bmatrix} -R_1 + M_1^T - \Phi_{j2} & -R_2 + M_2^T - \Phi_{j3} \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix}$$

$$\Lambda_{3j} = \begin{bmatrix} -M_1 - M_1^T + \Phi_{j4} & \Phi_{j5} \\ \bullet & -M_2 - M_2^T + \Phi_{j6} \end{bmatrix} \tag{5.20}$$

where

$$\begin{aligned} \Psi_j &= -P + (\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)Q_j \\ &+ R_1 + R_1^T + R_2 + R_2^T + \rho\alpha^2 \widehat{F}^T \widehat{F} \\ \Phi_{j1} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) \\ \Phi_{j2} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P \mathsf{B}_j \\ \Phi_{j3} &= (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P \mathsf{C}_j, \ \Phi_{j5} = \mathsf{B}_j^T P \mathsf{C}_j \\ \Phi_{j4} &= \mathsf{B}_j^T P \mathsf{B}_j, \ \Phi_{j6} = \mathsf{C}_j^T P \mathsf{C}_j \end{aligned}$$

Proof: Defining $y(k) = x(k+1) - x(k)$, one has

$$\xi(k - \tau_k^m) = \xi(k) - \sum_{i=k-\tau_k^m}^{k-1} y(i) \tag{5.21}$$

$$\xi(k - \tau_k^a) = \xi(k) - \sum_{i=k-\tau_k^a}^{k-1} y(i) \tag{5.22}$$

Then the system (5.11) can be transformed into

$$\begin{aligned} \xi(k+1) = (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)\xi(k) + \hat{f}(x_p, k) + \widehat{\Gamma} w(k) \\ -\mathsf{B}_j \lambda(k) - \mathsf{C}_j \gamma(k) \end{aligned} \tag{5.23}$$

where

$$\lambda(k) = \sum_{i=k-\tau_k^m}^{k-1} y(i), \quad \gamma(k) = \sum_{i=k-\tau_k^a}^{k-1} y(i).$$

Evaluating the difference of $V_1(\xi(k))$ along the solution of system (5.23), we have

$$\begin{aligned}
&\Delta V_1(\xi(k)) = V_1(\xi(k+1)) - V_1(\xi(k)) \\
&= \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)[\Phi_{j1} - P]\xi(k) - 2\xi^T(k)\Phi_{j2}\lambda(k) \\
&- 2\xi^T(k)\Phi_{j3}\gamma(k) + \lambda^T(k)\Phi_{j4}\lambda(k) \\
&+ 2\lambda^T(k)\Phi_{j5}\gamma(k) + \gamma^T(k)\Phi_{j6}\gamma(k) \\
&+ 2\xi^T(k)(\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P\hat{f} \\
&+ 2\xi^T(k)(\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P\widehat{\Gamma}w(k) \\
&- 2\lambda^T(k)\mathsf{B}_j^T P\hat{f} - 2\lambda^T(k)\mathsf{B}_j^T P\widehat{\Gamma}w(k) \\
&- 2\gamma^T(k)\mathsf{C}_j^T P\hat{f} - 2\gamma^T(k)\mathsf{C}_j^T P\widehat{\Gamma}w(k) \\
&+ \hat{f}^T P\hat{f} + 2\hat{f}^T P\widehat{\Gamma}w(k) \\
&+ w^T(k)\widehat{\Gamma}^T P\widehat{\Gamma}w(k)\Big]
\end{aligned} \tag{5.24}$$

A straightforward computation gives

$$\begin{aligned}
&\Delta V_2(\xi(k) = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\sum_{i=k+1-\tau_{k+1}^m}^{k} \xi^T(i)Q_j\xi(i) \\
&- \sum_{i=k-\tau_k^m}^{k-1} \xi^T(i)Q_j\xi(i) \\
&= \xi^T(k)Q\xi(k) - \xi(k-\tau_k^m)Q_j\xi(k-\tau_k^m) \\
&+ \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i)Q_j\xi(i) - \sum_{i=k+1-\tau_k^m}^{k-1} \xi(i)Q_j\xi(i)\Big]
\end{aligned} \tag{5.25}$$

In view of

$$\begin{aligned}
& \sum_{i=k+1-\tau_{k+1}^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
= \quad & \sum_{i=k+1-\tau_{k+1}^m}^{k-\tau_k^m} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) \\
\leq \quad & \sum_{i=k+1-\tau_k^m}^{k-1} \xi^T(i) Q_j \xi(i) + \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i) \qquad (5.26)
\end{aligned}$$

We readily obtain

$$\begin{aligned}
& \Delta V_2(\xi(k)) \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k) Q_j \xi(k) \\
& -\xi^T(k-\tau_k^m) Q_j \xi(k-\tau_k^m) \\
& + \sum_{i=k+1-\tau_m^+}^{k-\tau_m^-} \xi^T(i) Q_j \xi(i) \Big] \qquad (5.27)
\end{aligned}$$

Following parallel procedure, we get

$$\begin{aligned}
& \Delta V_3(\xi(k)) \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k) Q_j \xi(k) \\
& -\xi^T(k-\tau_k^a) Q_j \xi(k-\tau_k^a) + \sum_{i=k+1-\tau_a^+}^{k-\tau_a^-} \xi^T(i) Q_j \xi(i) \Big] \qquad (5.28)
\end{aligned}$$

Finally

$$\Delta V_4(\xi(k)) = \sum_{j=1}^{4}\hat{\sigma}_j\Bigg[\sum_{\ell=-\tau^+{}_m+2}^{-\tau^-{}_m+1}[\xi^T(k)Q_j\xi(k)$$

$$-\xi^T(k+\ell-1)Q_j\xi(k+\ell-1)]\Bigg]$$

$$= \sum_{j=1}^{4}\hat{\sigma}_j\Bigg[(\tau^+{}_m - \tau^-{}_m)\xi^T(k)Q_j\xi(k) - \sum_{i=k+1-\tau^+{}_m}^{k-\tau^-{}_m}\xi^T(i)Q_j\xi(i)\Bigg] \quad (5.29)$$

$$\Delta V_5(\xi(k)) = \sum_{j=1}^{4}\hat{\sigma}_j\Bigg[(\tau^+{}_a - \tau^-{}_a)\xi^T(k)Q_j\xi(k)$$

$$-\sum_{i=k+1-\tau^+{}_a}^{k-\tau^-{}_a}\xi^T(i)Q_j\xi(i)\Bigg] \quad (5.30)$$

It follows from (5.21) and (5.22) that:

$$\xi(k) - \xi(k-\tau_k^m) - \lambda(k) = 0 \quad (5.31)$$

$$\xi(k) - \xi(k-\tau_k^a) - \gamma(k) = 0 \quad (5.32)$$

Therefore, for any appropriately dimensioned matrices R_i, S_i and M_i, $i = 1, 2$, we have the following equations:

$$2[\xi^T(k)R_1 + \xi^T(k-\tau_k^m)S_1 + \lambda^T(k)M_1]$$

$$\times[\xi(k) - \xi(k-\tau_k^m) - \tau(k)] = 0 \quad (5.33)$$

$$2[\xi^T(k)R_2 + \xi^T(k-\tau_k^a)S_2 + \gamma^T(k)M_2]$$

$$\times[\xi(k) - \xi(k-\tau_k^a) - \gamma(k)] = 0 \quad (5.34)$$

Taking (5.4) into consideration and forming the Lipschitz condition for the above system we get

$$-\rho\hat{f}^t\hat{f} + \rho\alpha^2\xi^t\widehat{F}^t\widehat{F}\xi \geq 0 \tag{5.35}$$

On combining (5.24)–(5.35), we reach

$$
\begin{aligned}
&\Delta V(\xi(k)) \leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[\xi^T(k)\Psi_j\xi(k) \\
&+ \sum_{j=1}^{4} \xi^T(k)(-2R_1 + 2S_1^T)\xi(k-\tau_k^m) \\
&+\xi^T(k)(-2R_2 + 2S_2^T)\xi(k-\tau_k^a) \\
&+\xi^T(k)(-2R_1 + 2M_1^T - 2\Phi_{j2})\lambda(k) \\
&+\xi^T(k)(-2R_2 + 2M_2^T - 2\Phi_{j3})\gamma(k) \\
&+\xi^T(k-\tau_k^m)(-S_1 - S_1^T - Q_j)\xi(k-\tau_k^m) \\
&+\xi^T(k-\tau_k^m)(-2S_1 - 2M_1^T)\lambda(k) \\
&+\xi^T(k-\tau_k^a)(-S_2 - S_2^T - Q_j)\xi(k-\tau_k^a) \\
&+\xi^T(k-\tau_k^a)(-2S_2 - 2M_2^T)\gamma(k) \\
&+\lambda^T(k)(-M_1 - M_1^T + \Phi_{j4})\lambda(k) \\
&+\gamma^T(k)(-M_2 - M_2^T + \Phi_{j5})\gamma(k) \\
&+\lambda^T(k)\Phi_{j6}\gamma(k) \\
&+ 2\xi^T(k)(\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P\hat{f} \\
&+ 2\xi^T(k)(\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j)^T P\widehat{\Gamma}w(k) \\
&- 2\lambda^T(k)\mathsf{B}_j^T P\hat{f} - 2\lambda^T(k)\mathsf{B}_j^T P\widehat{\Gamma}w(k) \\
&- 2\gamma^T(k)\mathsf{C}_j^T P\hat{f} - 2\gamma^T(k)\mathsf{C}_j^T P\widehat{\Gamma}w(k) \\
&+ \hat{f}^T P\hat{f} + 2\hat{f}^T P\widehat{\Gamma}w(k) - \rho\hat{f}^t\hat{f} + \rho\alpha^2\xi^t\widehat{F}^t\widehat{F}\xi \\
&+ w^T(k)\widehat{\Gamma}^T P\widehat{\Gamma}w(k)\Big] = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k)\widetilde{\Lambda}_j\zeta(k)\Big]
\end{aligned} \tag{5.36}
$$

where

$$\zeta(k) = \left[\begin{array}{ccc} \zeta_1^T & \zeta_2^T & \zeta_3^T \end{array} \right]^T, \ \zeta_2 = \left[\begin{array}{cc} \lambda^T(k) & \gamma^T(k) \end{array} \right]^T$$
$$\zeta_1 = \left[\begin{array}{ccc} \xi^T(k) & \xi^T(k-\tau_k^m) & \xi^T(k-\tau_k^a) \end{array} \right]^T$$
$$\zeta_3 = \left[\begin{array}{cc} \hat{f}^T(x_p, k) & w^T(k) \end{array} \right]^T \tag{5.37}$$

and $\widetilde{\Lambda}_j$ corresponds to Λ_j in (5.20) by Schur complements. If $\Lambda_j < 0, \ j = 1, .., 4$ holds, then

$$V(\xi(k+1)) - V(\xi(k)) = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k) \widetilde{\Lambda}_j \zeta(k) \Big]$$
$$\leq \sum_{j=1}^{4} \hat{\sigma}_j \Big[-\widetilde{\Lambda}_{min}(\widetilde{\Lambda}_j) \zeta^T(k) \zeta(k) \Big]$$
$$< -\sum_{j=1}^{4} \hat{\sigma}_j \Big[\beta_j \zeta^T(k) \zeta(k) \Big] \tag{5.38}$$

where

$$0 < \beta_j < min\big[\lambda_{min}(\Lambda_j), max\{ \lambda_{max}(P), \ \lambda_{max}(Q_j) \} \big]$$

Inequality (5.38) implies that $V(\xi(k+1)) - V(\xi(k)) < -\phi V(\xi(k)), \ 0 < \phi < 1$. In the manner of [23], we get

$$||\xi(k)||^2 \leq \frac{\upsilon}{\kappa} ||\xi(0)||^2 (1-\phi)^k + \frac{\lambda}{\mu\phi}$$

Consider the performance measure

$$J_K = \sum_{k=0}^{K} \Big(z_p^T(k) z_p(k) - \gamma^2 w^T(k) w(k) \Big) \tag{5.39}$$

For any $w(k) \in \ell[0, \infty) \neq 0$ and zero initial condition, we have

$$\begin{aligned} J_K &= \sum_{k=0}^{K} \Big(z_p^T(k) z_p(k) - \gamma^2 w^T(k) w(k) \\ &\qquad + \Delta V(x)|_{(1)} - \Delta V(\xi(k))|_{(1)} \Big) \\ &\leq \sum_{k=0}^{K} \Big(z_p^T(k) z_p(k) - \gamma^2 w^T(k) w(k) + \Delta V(x)|_{(1)} \Big) \end{aligned}$$

where $\Delta V(x)|_{(1)}$ defines the Lyapunov difference along the solutions of system (5.1). Proceeding as before we get

$$\begin{aligned} & z_p^T(k) z_p(k) - \gamma^2 w^T(k) w(k) + \Delta V(\xi(k))|_{(1)} \\ &\qquad = \sum_{j=1}^{4} \hat{\sigma}_j \Big[\zeta^T(k) \bar{\Lambda}_j \zeta(k) \Big] \end{aligned} \tag{5.40}$$

where $\bar{\Lambda}_j$ corresponds to the $\widetilde{\Lambda}_j$ in (5.20) by Schur complements. It is readily seen that

$$z_p^T(k) z_p(k) - \gamma^2 w^T(k) w(k) + \Delta V(\xi(k))|_{(1)} < 0$$

for arbitrary $k \in [0, K)$, which implies for any $w(k) \in \ell_2[0, \infty) \neq 0$ that $J < 0$ leading to $\|z_p(k)\|_2 < \gamma |w(k)\|_2$ and the proof is completed.

A solution to the problem of the observer-based networked feedback stabilizing controller design is provided by the following theorem:

Theorem 3.2: *Let the delay bounds* $\tau_m^+,\ \tau_m^-,\ \tau_a^+,\ \tau_a^-$ *be given. Then the closed-loop system (5.11) is exponentially stable if there exist matrices* $0 < X,\ Y_1,\ Y_2,\ 0 < \Xi_j,\ j = 1, .., 4$ *and matrices* $\Pi_i,\ \Upsilon_i$ *and* $\Gamma_i,\ i = 1, 2,$ *such that the following matrix inequality holds for* $j = 1, .., 4$*:*

$$\begin{bmatrix} \widehat{\Lambda}_{1j} & \widehat{\Lambda}_{2j} & \widehat{\Omega}_j & \widehat{\Omega}_j & 0 & \widehat{X}\widehat{G}^T & \alpha\widehat{X}\widehat{F}^T \\ \bullet & \widehat{\Lambda}_{3j} & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \widehat{X} - \rho I & \widehat{X}\widehat{\Gamma}^T & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -\widehat{X} & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\mu I \end{bmatrix} < 0 \tag{5.41}$$

$$\widehat{X} = \begin{bmatrix} X & X \\ X & X \end{bmatrix} \tag{5.42}$$

$$\begin{aligned}
\widehat{\Psi}_j &= -\widehat{X} + (\tau_m^+ - \tau_m^- + \tau_a^+ - \tau_a^- + 2)\Xi_j \\
&\quad +\Pi_1 + \Pi_1^T + \Pi_2 + \Pi_2^T \\
\widehat{\Lambda}_{1j} &= \begin{bmatrix} \widehat{\Psi}_j & -\Pi_1 + \Upsilon_1^T & -\Pi_2 + \Upsilon_2^T \\ \bullet & -\Upsilon_1 - \Upsilon_1^T - \Xi_j & 0 \\ \bullet & \bullet & -\Upsilon_2 - \Upsilon_2^T - \Xi_j \end{bmatrix} \\
\widehat{\Lambda}_{2j} &= \begin{bmatrix} -\Pi_1 + \Gamma_1^T & -\Pi_2 + \Gamma_2^T \\ -\Upsilon_1 - \Gamma_1^T & 0 \\ 0 & -\Upsilon_2 - \Gamma_2^T \end{bmatrix} \\
\widehat{\Lambda}_{3j} &= \begin{bmatrix} -\Gamma_1 - \Gamma_1^T & 0 \\ \bullet & -\Gamma_2 - \Gamma_2^T \end{bmatrix} \\
\widehat{\Omega}_j &= \begin{bmatrix} \widehat{\Omega}_{1j} & 0 & 0 & -\widehat{\Omega}_{4j} & 0 \end{bmatrix}^T \\
\widehat{\Omega}_{1j} &= \begin{bmatrix} XA^T & XA^T - Y_2^T \\ XA^T & XA^T - Y_2^T \end{bmatrix} \ \forall\, j \\
\widehat{\Omega}_{4j} &= \begin{bmatrix} 0 & -Y_2^T \\ 0 & -Y_2^T \end{bmatrix} \ \forall\, j = 1, 2 \\
\widehat{\Omega}_{4j} &= 0 \quad \forall\, j = 3, 4,
\end{aligned} \tag{5.43}$$

where the gain matrices are given by

$$K = Y_1 X^{-1}, \quad L = Y_2 X^{-1} C^{\dagger}$$

Proof: Define

$$\Omega_j = \begin{bmatrix} (\mathsf{A}_j + \mathsf{B}_j + \mathsf{C}_j) & 0 & -\mathsf{B}_j & -\mathsf{C}_j \end{bmatrix}^T$$

then matrix inequality (5.19) can be expressed as

$$\begin{aligned}
\Lambda_j &= \widetilde{\Lambda}_j + \Omega_j P \Omega_j^T < 0 \qquad (5.44)\\
\widetilde{\Lambda}_j &= \begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} \\ \bullet & \widetilde{\Lambda}_{3j} \end{bmatrix} < 0 \\
\widetilde{\Lambda}_{1j} &= \begin{bmatrix} \Psi_j & -R_1 + S_1^T & -R_2 + S_2^T \\ \bullet & -S_1 - S_1^T - Q_j & 0 \\ \bullet & \bullet & -S_2 - S_2^T - Q_j \end{bmatrix} \\
\widetilde{\Lambda}_{2j} &= \begin{bmatrix} -R_1 + M_1^T & -R_2 + M_2^T \\ -S_1 - M_1^T & 0 \\ 0 & -S_2 - M_2^T \end{bmatrix} \\
\widetilde{\Lambda}_{3j} &= \begin{bmatrix} -M_1 - M_1^T & 0 \\ \bullet & -M_2 - M_2^T \end{bmatrix} \qquad (5.45)
\end{aligned}$$

Setting $\widehat{X} = P^{-1}$, invoking Schur complements, we write matrix Λ_j in (5.44) equivalently as

$$\begin{bmatrix} \widetilde{\Lambda}_{1j} & \widetilde{\Lambda}_{2j} & \Omega_j & \Omega_j P & 0 & \widehat{G}^T & \alpha\widehat{F}^T \\ \bullet & \widetilde{\Lambda}_{3j} & 0 & 0 & 0 & 0 & 0 \\ \bullet & \bullet & \widehat{X} - \rho I & \widehat{\Gamma}^T P & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & -P & 0 & 0 & 0 \\ \bullet & \bullet & \bullet & \bullet & -\gamma^2 I & \widehat{\Phi}^T & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & -I & 0 \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & -\mu I \end{bmatrix} < 0 \tag{5.46}$$

Applying the congruence transformation

$$T_j = diag[\widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ \widehat{X},\ I,\ \widehat{X},\ I,\ I,\ I]$$

to matrix inequality in (5.46) and manipulating using (5.42) and

$$\begin{aligned} \Xi_j &= \widehat{X} Q_j \widehat{X},\ \Pi_j = \widehat{X} R_j \widehat{X},\ \Upsilon_j = \widehat{X} S_j \widehat{X}, \\ \Gamma_j &= \widehat{X} M_j \widehat{X} \end{aligned}$$

we readily obtain matrix inequality (5.41) subject (5.43).

Remark 5.3.2 *The selection of $\widehat{X}$ as given by (5.42) has the advantage of converting the solution of bilinear matrix inequalities to that of seeking the feasibility* *of linear matrix inequalities and hence avoiding iterative procedures.*

5.4 Numerical Simulation

The system under consideration for simulation is a quadruple-tank process consisting of four water tanks that are interconnected and the flow through the pipes is controlled by two pumps as shown in Fig. 4.2 in the previous chapter. By controlling the voltages to the pumps we aim at regulating the water levels in the two lower tanks. The four tank system is being built by considering the concept of two double-tank processes. The quadruple tank system presents a multi-input-multi-output (MIMO) system. A schematic description of the four tank system can be visualized by the Figure shown. The system has two control inputs (pump throughputs) which can be manipulated to control the water level in the tanks. The two pumps are used to transfer water from a sump into four overhead tanks. In [85], an appropriate model is presented with the control objective being to regulate the level in the four-tanks at a desired setting and undistorted. The system matrices are given by:

$$
\begin{aligned}
A &= \begin{bmatrix} -0.0278 & 0 & 0.0206 & 0 \\ 0 & -0.0233 & 0 & 0.0141 \\ 0 & 0 & -0.0206 & 0 \\ 0 & 0 & 0 & -0.0141 \end{bmatrix}, \\
B &= \begin{bmatrix} 5 & 0 \\ 0 & 6.667 \\ 0 & 10 \\ 11.667 & 0 \end{bmatrix}, \\
C &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.
\end{aligned}
$$

Nonlinearities were added to the system by the following matrices:

$$
\begin{aligned}
F &= \begin{bmatrix} 2.9875 & 2.1215 & 4.7685 & 3.4080 \\ 1.6765 & 2.1470 & 2.2895 & 2.3165 \\ 1.4960 & 0.6245 & 1.2025 & 1.0610 \\ 2.2630 & 0.1220 & 3.8195 & 0.4925 \end{bmatrix}, \\
\Gamma_0 &= \begin{bmatrix} 0.0987 & 0.0863 & 0.1091 & 0.0999 \\ 0.1210 & 0.1000 & 0.1256 & 0.1100 \\ 0.0898 & 0.0918 & 0.1000 & 0.0888 \\ 0.1090 & 0.1220 & 0.1235 & 0.1156 \end{bmatrix}, \\
G &= \begin{bmatrix} 0.0905 & 0.0703 & 0.1064 & 0.1064 \\ 0.1006 & 0.0108 & 0.1072 & 0.0539 \\ 0.0141 & 0.0309 & 0.0175 & 0.0889 \\ 0.1015 & 0.0608 & 0.1078 & 0.0158 \end{bmatrix}, \\
\Phi_0 &= \begin{bmatrix} 0.0987 & 0.0863 & 0.1091 & 0.0999 \\ 0.1210 & 0.1000 & 0.1256 & 0.1100 \\ 0.0898 & 0.0918 & 0.1000 & 0.0888 \\ 0.1090 & 0.1220 & 0.1235 & 0.1156 \end{bmatrix}.
\end{aligned}
$$

In the above system the control signal is transmitted through a network, and due to the limited bandwidth of the network, it gives rise to probabilistic signal delays. The mass functions of the random variables p_k and s_k are selected to follow symmetric triangle distribution using 300 sample values. The measurement and actuation delays were bounded as follows: $0.5 < \tau_k^m < 5$ and $1 < \tau_k^a < 10$. The 'variable fraction delay' block in Matlab Simulink software (Matlab 7.0) was used to handle the discrete time-varying delays.

The variable μ was chosen to be less than 0.1 in each of the cases illustrated below. The value of α was varied and the system behavior was studied in each case. Setting $\alpha = 0.5$ and applying the new algorithm, we obtain the controller and observer gain matrices as follows:

$$
\begin{aligned}
K &= \begin{bmatrix} -0.1327 & -0.0349 & -0.0268 & -0.0452 \\ 0.0146 & 0.0333 & 0.0023 & -0.2489 \end{bmatrix} \\
L^T &= \begin{bmatrix} -0.2156 & -0.0415 & 0.0188 & 0.0207 \\ 0.0062 & -0.0159 & -0.0019 & 0.0327 \end{bmatrix}
\end{aligned}
$$
$$\|K\| = 0.2546, \quad \|L\| = 0.2213$$

Similarly with $\alpha = 1$ and applying the new algorithm, we obtain the controller and observer gain matrices as follows:

$$
\begin{aligned}
K &= \begin{bmatrix} -0.1199 & -0.0308 & -0.0125 & -0.0646 \\ 0.0087 & 0.0405 & 0.0151 & -0.2502 \end{bmatrix} \\
L^T &= \begin{bmatrix} -0.2065 & -0.0399 & 0.0200 & 0.0226 \\ 0.0020 & -0.0271 & -0.0001 & 0.0345 \end{bmatrix}
\end{aligned}
$$

$$\|K\| = 0.2615, \quad \|L\| = 0.2126$$

With $\alpha = 1.5$ and applying the new algorithm, we obtain the controller and observer gain matrices as follows:

$$
\begin{aligned}
K &= \begin{bmatrix} -0.1275 & -0.0441 & -0.0237 & -0.0403 \\ 0.0117 & 0.0389 & 0.0132 & -0.2712 \end{bmatrix} \\
L^T &= \begin{bmatrix} -0.2024 & -0.0416 & 0.0201 & 0.0188 \\ 0.0025 & -0.0233 & -0.0029 & 0.0326 \end{bmatrix}
\end{aligned}
$$

$$\|K\| = 0.2763, \quad \|L\| = 0.2085$$

Further it was noted that as we increase the value of α to 2, the networked control system was rendered unstable. The comparative state responses of the system at different values of α are shown in Fig. 5.2. As we vary the value of α there was not much variation in the norms of the controller and observer gain matrices, but the difference in the state trajectories was notable. As the value of α is increased, the system takes slightly longer to settle to steady state and most importantly the steady state error in the response increases. We then compare the response of the nonlinear system for a given value of α, say 0.5, with the linear system, subject to similar delays and nonstationary packet dropouts. For

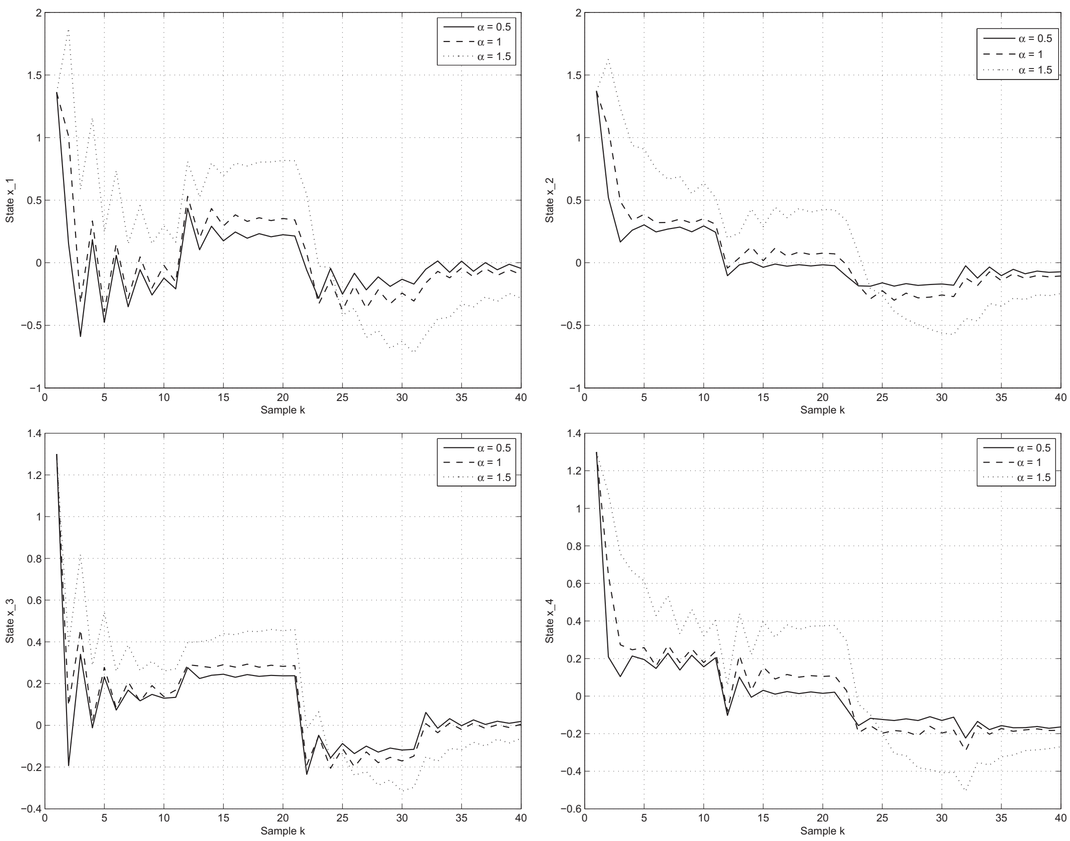

Figure 5.2: State trajectories for different values of α

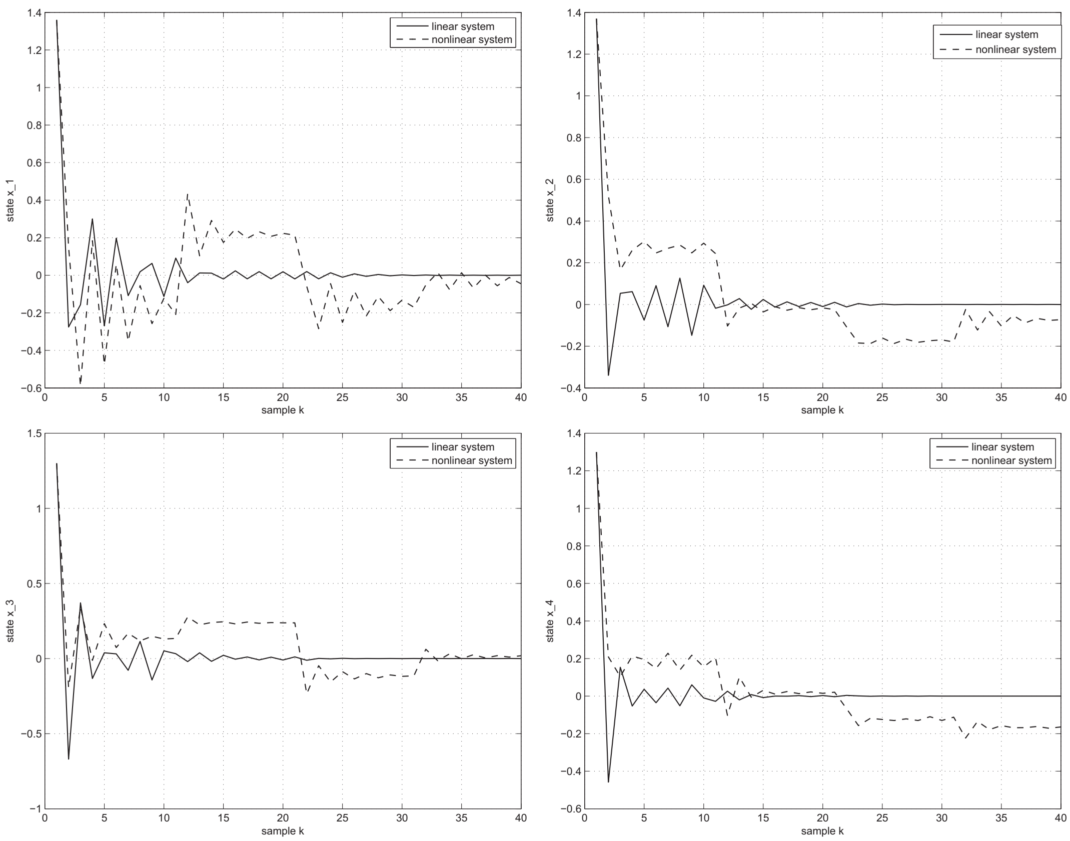

Figure 5.3: Comparative response of nonlinear and linear system

the linear case, we obtain the controller and observer gain matrices as follows:

$$K = \begin{bmatrix} -0.0440 & 0.0037 & -0.2525 & 0.1129 \\ -0.0596 & 0.0549 & 0.1034 & -1.0081 \end{bmatrix}$$

$$L^T = \begin{bmatrix} -0.3836 & -0.0889 & 0.0635 & 0.1217 \\ 0.0424 & 0.0189 & -0.0379 & -0.0375 \end{bmatrix}$$

$$\|K\| = 1.0261 \quad \|L\| = 0.4213$$

The comparative plots are shown in Fig. 5.3. We see that the linear system settles to steady state much quicker than the nonlinear system and the steady state error is zero in case of the linear system.

5.5 Conclusion

The stability analysis and controller synthesis problems are thoroughly investigated for nonlinear NCSs with time-varying delays and subject to nonstationary packet dropouts. The main focus of the study was design of the new observer-based controller such that the resulting closed-loop system is exponentially stable in the mean-square sense. The proposed results were validated by simulation on a realtime example and comparison has been drawn with a linear system subject to similar network phenomena.

Chapter 6

OBSERVER BASED CONTROL FOR NETWORK SYSTEMS OVER LOSSY COMMUNICATION CHANNEL

6.1 Introduction

Many complicated control systems, such as process plants, surveillance aircrafts, and space shuttles, wireless communication networks are employed to exchange information and control signals between spatially distributed system components, like supervisory computers, controllers, and intelligent inputoutput (I/O) devices (e.g., smart sensors and actuators). Each of the system components connected via a serial communication channel is labelled as a networked control system (NCS).

Data is exchanged between networked nodes in the form of discrete units called packets. Therefore any continuous-time signal is sampled before transmission over the network. Data packets may be lost at any point between the source and the destination and are termed as dropouts. Packet dropout may occur along the network as a result of various uncertainties and noise in the channels. It may also occur at the destination when out of order delivery takes place. In reliable transmission protocols that guarantee the eventual delivery of packets, data is resent repeatedly to ensure the delivery of each packet. However NCSs should operate with non-reliable transport protocols since transmission of old data is retudant and meaningless in process control plants. Networked control is being used for many real-time applications for a number of reasons [108], which include their low cost of operation owing to reduced hardware, lesser power requirements, easier installation and maintenance, and greater flexibility and reliability.

In [10] Walsh *et al.* introduced a novel control network protocol, try-once-discard (TOD), for multiple-input multiple-output NCSs, and provided, for the first time, an analytic proof of global exponential stability for both the new protocol and the more commonly used (statically scheduled) access methods.

An approach was initially proposed by Yu *et al.* in [18] to model networked control systems with arbitrary but finite data packet dropout as switched linear systems. In the same year they studied the problem of data packet dropout and transmission delays in NCSs in both continuous-time case and discrete-time case in [19]. They modeled the NCSs with data packet dropout and delays as ordinary linear systems with input delays. The stabilisation problem for networked control systems with time-varying delays smaller than one sampling period was studied in by Zhang *et al.* [34]. Later they extended the research to study delays longer than one sampling period [39]. In [45] they considered state feedback controllers, and modeled the closed-loop NCS as a switched delay system.

In the recent past Luan *et al.* [5] developed an observer-based controller for networked systems involving two major types of delays namely: random measurement and actuation delays. The occurence of delays was governed by a Binary Bernoulli distribution. An observer-based controller was also designed by [111] for a system incorporating packet dropouts besides communication delays. A similar work was also presented in [112] where the authors considered the $\mathcal{H}_\infty$ control problem for interconnected continuous-time delay systems and an added network phenomenon of quantization.

We intend to design an observer-based output feedback controller that remotely stabilizes a linear plant subject to network phenomena, i.e., delays, sampling, and packet dropouts in the (sensor) measurement and actuation channels. The chapter is organized as follows: In section II we introduce the plant structure and the intended controller framework. We design the closed loop model considering dynamic output feedback. In section III we find a sufficient condition for asymptotic stability of system in the form of LMIs. In the last section we illustrate the use of our method by means of a real-time example. The following are the few key assumptions that we make while carrying out the analysis:

A1 : The sensor is time-driven, has a receiving buffer which contains the most recently received data packet from the sensor and the sampling period is h^m.

A2 : The actuator is time-driven and has a receiving buffer which contains the most recently received data packet from the controller. The actuator reads the buffer periodically at a sampling rate of h^a.

A3 : The time-varying measurement delay at time step $k \in \mathbb{N}$ is denoted by τ_k^m, and is bounded by $\varphi_1 < \tau_k^m < \varrho_1$

A4 : The time-varying actuation delay at time step $l \in \mathbb{N}$ is denoted by τ_l^a, and is bounded by $\varphi_2 < \tau_l^a < \varrho_2$

A5 : Every sampled data is time stamped, so the controller and actuator always use the most recent data packet.

$\mathsf{A6}$: It is assumed that n^m and n^a dropouts can occur in the measurement and actuation channel respectively.

Lemma 6.1.1 *(The* **S** *Procedure) [110] : Denote the set* $\mathsf{Z} = \{z\}$ *and let* $\mathcal{F}(z),\ \mathcal{Y}_1(z),\ \mathcal{Y}_2(z),\ \ldots, \mathcal{Y}_k(z)$ *be some functionals or functions. Define domain* D *as*

$$\mathsf{D} = \{z \in \mathsf{Z} : \mathcal{Y}_1(z) \geq 0,\ \mathcal{Y}_2(z) \geq 0, ..., \mathcal{Y}_k(z) \geq 0\}$$

and the two following conditions:

(I) $\mathcal{F}(z) > 0,\ \forall\ z \in \mathsf{D}$,

(II) $\exists\ \varepsilon_1 \geq 0,\ \varepsilon_2 \geq 0, ..., \varepsilon_k \geq 0$ *such that* $\mathcal{S}(\varepsilon, z) = \mathcal{F}(z) - \sum_{j=1}^{k}\ \varepsilon_j\ \mathcal{Y}_j(z)\ > 0\ \forall\ z\ \in\ \mathsf{Z}$

Then (II) *implies* (I).

Lemma 6.1.2 *(The Integral Inequality) [109] : For any constant matrix* $0 < \Sigma \in \Re^{n\times n}$*, scalar* $\tau_* < \tau(t) < \tau^+$ *and vector function* $\dot{x} : [t - \tau^+, t - \tau_*] \to \Re^n$ *such that the following integration is well-defined, then it holds that*

$$\begin{aligned} &-(\tau^+ - \tau_*) \int_{t-\tau^+}^{t-\tau_*}\ \dot{x}^T(s)\ \Sigma\ \dot{x}(s)\ ds \\ &\leq\ -\left[x(t-\tau_*)\ -\ x(t-\tau^+)\right]^T \Sigma \left[x(t-\tau_*)\ -\ x(t-\tau^+)\right] \end{aligned}$$

Lemma 6.1.2 is frequently called the "integral inequality" and it is derived from Jensen's inequality [109]. Sometimes, the arguments of a function will be omitted when no confusion can arise.

6.2 Problem Statement

We consider a linear state-space model of a system given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{6.1}$$

$$y(t) = Cx(t) \tag{6.2}$$

where $x(t) \in \Re^n$ is the system state vector, $u(t) \in \Re^m$ is the control input to the system and the $y(t) \in \Re^p$ is the output of the system. $A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$ and $C \in \Re^{p\times m}$ are real and constant matrices of appropriate dimensions.

The measurements of the plant output $y(t)$ are sampled at a time interval h^m so they are available at the other end of the network at time instants kh^m where $k \in \mathbb{N}$. However in our case we assume that the network is subject to delays as well. The variable measurement delay encountered by each sample is assumed to be τ_k^m. Therefore the measurements subject to delays arrive at the controller at time instants $kh^m + \tau_k^m$. The time delay is also subject to upper and lower bounds $\varphi_1 < \tau_k^m < \varrho_1$. A block diagram of the network model under study can be seen in Fig. 6.1.

The estimate of the plant's state is generated by the observer as follows:

$$\begin{aligned}\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L(y(kh^m) - C\hat{x}(kh^m))\\ \forall\ \ t \in [kh^m + \tau_k^m,\ \ (k+1)h^m + \tau_{k+1}^m)\end{aligned} \tag{6.3}$$

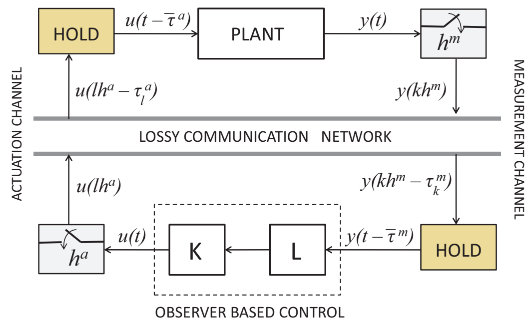

Figure 6.1: Feedback NCS with Observer-based control

Where $u(t)$ is the last successfully receieved measurement. The controller sends updates to the actuation unit at time intervals of h^a (sampling time). But due to the delays present in the network, the updates are received by the actuation unit at time instants $lh^a + \tau_l^a$ where $l \in \mathbb{N}$ and τ_l^a is the bounded actuation delay such that $\varphi_2 < \tau_l^a < \varrho_2$.

6.3 Output-Feedback Design

We now develop the observer based controller for the above mentioned Networked Control System. The control input to the linear plant is defined as shown below:

$$u(t) = K\hat{x}(lh^a) \tag{6.4}$$

where the gain matrix K will be selected to guarantee that the closed-loop system is stable.

Keeping the delays and nature of the system in mind, we formulate the delay differential equations as follows:

$$\begin{aligned} \bar{\tau}^m &= t - kh^m, \ \ \forall\, t \in [kh^m + \tau_k^m,\ (k+1)h^m + \tau_{k+1}^m) \\ \bar{\tau}^a &= t - lh^a, \ \ \forall\, t \in [lh^a + \tau_l^a,\ (l+1)h^a + \tau_{l+1}^a) \end{aligned} \tag{6.5}$$

The range of $\bar{\tau}^m$ and $\bar{\tau}^a$ is defined by

$$\begin{aligned} \bar{\tau}^m &\in [\mathbf{min}\{\tau_k^m\},\ \ h^m + \mathbf{max}\{\tau_{k+1}^m\}),\ \ \dot{\bar{\tau}}^m = 1 \\ \bar{\tau}^a &\in [\mathbf{min}\{\tau_l^a\},\ \ h^a + \mathbf{max}\{\tau_{l+1}^a\}),\ \ \dot{\bar{\tau}}^a = 1 \end{aligned} \tag{6.6}$$

We can now reformulate the equation of the observer in (6.3) as follows:

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L(y(t - \bar{\tau}^m) - C\hat{x}(t - \bar{\tau}^m)) \\ u(t) &= K\hat{x}(t - \bar{\tau}^a) \end{aligned} \tag{6.7}$$

Fig. 6.2.a shows the variation of $\bar{\tau}^m$ with time. Packet dropouts are treated as delays which grow beyond bounds. If there are n^m dropouts in the measurement

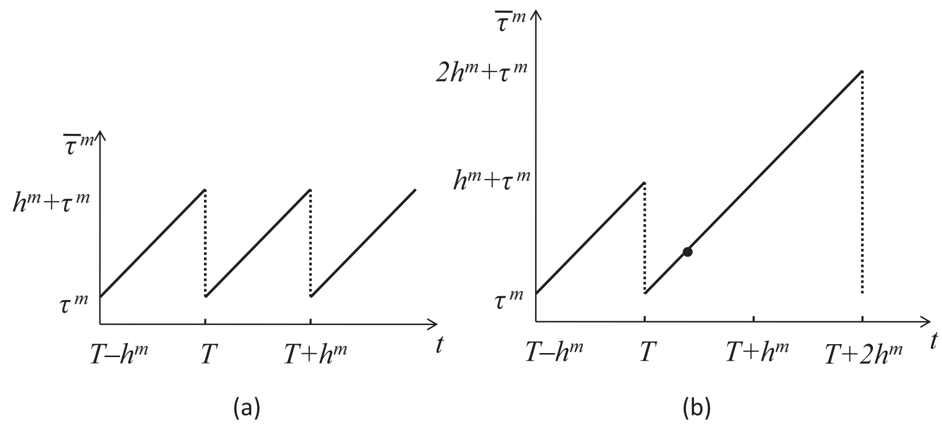

Figure 6.2: Evolution of $\bar{\tau}^s$ with respect to time (a) No packet dropout occurs, (b) Packet sent at kh^m is dropped.

channel and n^a dropouts in the actuation channel, then

$$\bar{\tau}^m \in [\mathbf{min}\{\tau_k^m\}, \ \ (n^m+1)h^m + \mathbf{max}\{\tau_{k+1}^m\})$$
$$\bar{\tau}^a \in [\mathbf{min}\{\tau_l^a\}, \ \ (n^a+1)h^a + \mathbf{max}\{\tau_{l+1}^a\})$$

Therefore it should be noted that any dropouts in the channel will be reflected in the system in the form of delays which are multiples of the sampling time period. Fig. 6.2.b shows the variation in $\bar{\tau}^m$ if the packet at a given instant kh^m is dropped. Defining the estimation error in the system as $e(t) = x(t) - \hat{x}(t)$, assuming the augmented state vector $\xi(t) = [\hat{x}^T(t) \ \ e^T(t)]^T$ using eq. (6.1), eq.

(6.2) and eq. (6.7), the closed loop is defined by

$$\dot{\xi}(t) = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix} \xi(t) + \begin{bmatrix} 0 & LC \\ 0 & -LC \end{bmatrix} \xi(t - \bar{\tau}^m) + \begin{bmatrix} -BK & 0 \\ 0 & 0 \end{bmatrix} \xi(t - \bar{\tau}^a) \tag{6.8}$$

where

$$\mathcal{A} = \begin{bmatrix} A & 0 \\ 0 & A \end{bmatrix}, \ \mathcal{B}_1 = \begin{bmatrix} 0 & LC \\ 0 & -LC \end{bmatrix}, \ \mathcal{B}_2 = \begin{bmatrix} -BK & 0 \\ 0 & 0 \end{bmatrix}$$

To facilitate further development, we consider the case where the output matrix C is assumed to be of full row rank and $C^{\dagger}$ represents the right-inverse. Introducing the Lyapunov-Krasovskii functional for the given system:

$$\begin{aligned}
V(t) &= \sum_{i=1}^{5} V_i(t) \\
V_1(t) &= \xi_j^T(t)\mathcal{P}_j\xi(t), \\
V_2(t) &= \sum_{j=1}^{2} \int_{t-\varphi_j}^{t} \xi^T(s)\mathcal{Q}_j\xi(s)\, ds, \\
V_3(t) &= \sum_{j=1}^{2} \varphi_j \int_{-\varphi_j}^{0} \int_{t+s}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_j\dot{\xi}(\alpha) d\alpha\, ds, \\
V_4(t) &= \sum_{j=1}^{2} (\varrho_j - \varphi_j) \int_{-\varrho_j}^{-\varphi_j} \int_{t+s}^{t} \dot{\xi}^T(\alpha)\mathcal{S}_j\dot{\xi}_j(\alpha) d\alpha\, ds, \\
V_5(t) &= \sum_{j=1}^{2} \int_{t-\varrho_j}^{t} \xi^T(s)\mathcal{R}_j\xi(s)\, ds
\end{aligned} \tag{6.9}$$

where $j = 1$ applies to measurement delays and $j = 2$ applies to actuation delays.

$0 < \mathcal{P}_j = \mathcal{P}_j^T,\ 0 < \mathcal{W}_j = \mathcal{W}_j^T,\ 0 < \mathcal{Q}_j = \mathcal{Q}_j^T,\ 0 < \mathcal{R}_j = \mathcal{R}_j^T,\ 0 < \mathcal{S}_j = \mathcal{S}_j^T$ are weighting matrices of appropriate dimensions. The theorem below establishes the main control design.

Theorem 6.3.1 *Given the bounds $\varphi_1,\ \ \varphi_2 > 0$ and $\varrho_1,\ \ \varrho_2 > 0$. System (6.1)-(6.2) is delay-dependent asymptotically stable if there exist weighting matrices $0 < \mathcal{X},\ \mathcal{Y}_1,\ \mathcal{Y}_2,\ \ 0 < \Lambda_{11},\ \Lambda_{12},\ \Lambda_{21},\ \Lambda_{22},\ \Lambda_{31},\ \Lambda_{32},\ \Lambda_{41},\ \Lambda_{42}$ satisfying the following LMI*

$$\widetilde{\Pi} = \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 \\ \bullet & \Pi_4 & \Pi_5 \\ \bullet & \bullet & \Pi_6 \end{bmatrix} < 0 \tag{6.10}$$

$$\Pi_1 = \begin{bmatrix} \Pi_{jo} & 0 & \mathcal{B}_1\widehat{\mathcal{X}} & \Lambda_{21} \\ \bullet & -\Pi_{c1} & \Lambda_{31} & 0 \\ \bullet & \bullet & -\Pi_{m1} & \Lambda_{31} \\ \bullet & \bullet & \bullet & -\Pi_{n1} \end{bmatrix},$$

$$\Pi_2 = \begin{bmatrix} \varphi_1\widehat{\mathcal{X}}\mathcal{A}^T & (\varrho_1-\varphi_1)\widehat{\mathcal{X}}\mathcal{A}^T & 0 & \mathcal{B}_2\widehat{\mathcal{X}} \\ 0 & 0 & 0 & 0 \\ \varphi_1\widehat{\mathcal{X}}\mathcal{B}_1^T & (\varrho_1-\varphi_1)\widehat{\mathcal{X}}\mathcal{B}_1^T & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Pi_3 = \begin{bmatrix} \Lambda_{22} & \varphi_2\widehat{\mathcal{X}}\mathcal{A}^T & (\varrho_2-\varphi_2)\widehat{\mathcal{X}}\mathcal{A}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Pi_4 = \begin{bmatrix} -2\widehat{\mathcal{X}} + \Lambda_{21} & 0 & 0 & 0 \\ \bullet & -2\widehat{\mathcal{X}} + \Lambda_{31} & 0 & 0 \\ \bullet & \bullet & -\Pi_{c2} & \Lambda_{32} \\ \bullet & \bullet & \bullet & -\Pi_{m2} \end{bmatrix},$$

$$\Pi_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Lambda_{32} & \varphi_2 \widehat{\mathcal{X}} \mathcal{B}_2^T & (\varrho_2 - \varphi_2)\widehat{\mathcal{X}} \mathcal{B}_2^T \end{bmatrix}$$

$$\Pi_6 = \begin{bmatrix} -\Pi_{n2} & 0 & 0 \\ \bullet & -2\widehat{\mathcal{X}} + \Lambda_{22} & 0 \\ \bullet & \bullet & -2\widehat{\mathcal{X}} + \Lambda_{32} \end{bmatrix},$$

$$\widehat{\mathcal{X}} = \begin{bmatrix} \mathcal{X} & \mathcal{X} \\ \mathcal{X} & \mathcal{X} \end{bmatrix}$$

$$\Pi_{jo} = \mathcal{A}\widehat{\mathcal{X}} + \widehat{\mathcal{X}}\mathcal{A}^T + \Lambda_{11} + \Lambda_{12} + \Lambda_{41} + \Lambda_{42} - \Lambda_{21} - \Lambda_{22},$$

$$\Pi_{c1} = \Lambda_{11} + \Lambda_{31}, \qquad \Pi_{c2} = \Lambda_{12} + \Lambda_{32},$$

$$\Pi_{n1} = \Lambda_{21} + \Lambda_{31} + \Lambda_{41}, \quad \Pi_{n2} = \Lambda_{22} + \Lambda_{32} + \Lambda_{42},$$

$$\Pi_{m1} = 2\Lambda_{31}, \qquad \Pi_{m2} = 2\Lambda_{32}, \tag{6.11}$$

Here we should note that the gain matrices are given by $K = \mathcal{Y}_1\mathcal{X}^{-1}$ *and* $L = \mathcal{Y}_2\mathcal{X}^{-1}C^{\dagger}$.

Proof : Computing the time-derivative of the Lyapunov function $V(t)$ along the solutions of (6.1) gives us:

$$\begin{aligned}
&\dot{V}(t) = 2\xi^T(t)\mathcal{P}\dot{\xi}(t) + \xi^T(t)[\mathcal{Q}_1 + \mathcal{R}_1 + \mathcal{Q}_2 + \mathcal{R}_2]\xi(t) \\
&-\xi^T(t-\varphi_1)\mathcal{Q}_1\xi(t-\varphi_1) - \xi^T(t-\varphi_2)\mathcal{Q}_2\xi(t-\varphi_2) - \xi^T(t-\varrho_1)\mathcal{R}_1\xi(t-\varrho_1) \\
&-\xi^T(t-\varrho_2)\mathcal{R}_2\xi(t-\varrho_2) + \dot{\xi}^T(t)[\varphi_1^2\mathcal{W}_1 + (\varrho_1-\varphi_1)^2\mathcal{S}_1]\dot{\xi}(t) \\
&+\dot{\xi}^T(t)[\varphi_2^2\mathcal{W}_2 + (\varrho_2-\varphi_2)^2\mathcal{S}_2]\dot{\xi}(t) - \int_{t-\varphi_1}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_1\dot{\xi}(\alpha)d\alpha \\
&-\int_{t-\varphi_2}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_2\dot{\xi}(\alpha)d\alpha - \int_{t-\varrho_1}^{t-\varphi_1} \dot{\xi}^T(\alpha)\mathcal{S}_1\dot{\xi}(\alpha)d\alpha - \int_{t-\varrho_2}^{t-\varphi_2} \dot{\xi}^T(\alpha)\mathcal{S}_2\dot{\xi}(\alpha)d\alpha \\
&\leq 2\xi^T(t)\mathcal{P}\dot{\xi}(t) + \xi^T(t)[\mathcal{Q}_1 + \mathcal{R}_1 + \mathcal{Q}_2 + \mathcal{R}_2]\xi(t) - \xi^T(t-\varphi_1)\mathcal{Q}_1\xi(t-\varphi_1) \\
&-\xi^T(t-\varphi_2)\mathcal{Q}_2\xi(t-\varphi_2) - \xi^T(t-\varrho_1)\mathcal{R}_1\xi(t-\varrho_1) - \xi^T(t-\varrho_2)\mathcal{R}_2\xi(t-\varrho_2) \\
&+\dot{\xi}^T(t)[\varphi_1\mathcal{W}_1 + (\varrho_1-\varphi_1)\mathcal{S}_1]\dot{\xi}(t) + \dot{\xi}^T(t)[\varphi_2\mathcal{W}_2 + (\varrho_2-\varphi_2)\mathcal{S}_2]\dot{\xi}(t) \\
&-\varphi_1\int_{t-\varrho_1}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_1\dot{\xi}(\alpha)d\alpha - \varphi_2\int_{t-\varrho_2}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_2\dot{\xi}(\alpha)d\alpha \\
&-(\varrho_1-\varphi_1)\int_{t-\varrho_1}^{t-\varphi_1} \dot{\xi}^T(\alpha)\mathcal{S}_1\dot{\xi}(\alpha)d\alpha - (\varrho_2-\varphi_2)\int_{t-\varrho_2}^{t-\varphi_2} \dot{\xi}^T(\alpha)\mathcal{S}_2\dot{\xi}(\alpha)d\alpha
\end{aligned} \tag{6.12}$$

Applying **Lemma** 6.1.1, we get

$$\begin{aligned}
&-\varphi_j\int_{t-\varphi_j}^{t} \dot{\xi}^T(\alpha)\mathcal{W}_j\dot{\xi}(\alpha)d\alpha \\
&\leq \begin{bmatrix} \xi(t) \\ \xi(t-\varphi_j) \end{bmatrix}^T \begin{bmatrix} -\mathcal{W}_j & \mathcal{W}_j \\ \bullet & -\mathcal{W}_j \end{bmatrix} \begin{bmatrix} \xi(t) \\ \xi(t-\varphi_j) \end{bmatrix}
\end{aligned} \tag{6.13}$$

Similarly,

$$
\begin{aligned}
& -(\varrho_j - \varphi_j) \int_{t-\varrho_j}^{t-\varphi_j} \dot{\xi}^T(\alpha) \mathcal{S}_j \dot{\xi}(\alpha) d\alpha \\
= \;& -(\varrho_j - \varphi_j) \Big[\int_{t-\tau}^{t-\varphi} \dot{\xi}^T(\alpha) \mathcal{S} \dot{\xi}(\alpha) d\alpha + \int_{t-\varrho_j}^{t-\tau_j} \dot{\xi}^T(\alpha) \mathcal{S}_j \dot{\xi}(\alpha) d\alpha \Big] \\
\leq \;& -(\tau_j - \varphi_j) \Big[\int_{t-\tau_j}^{t-\varphi_j} \dot{\xi}^T(\alpha) \mathcal{S}_j \dot{\xi}(\alpha) d\alpha \Big] - (\varrho_j - \tau_j) \Big[\int_{t-\varrho_j}^{t-\tau_j} \dot{\xi}^T(\alpha) \mathcal{S}_j \dot{\xi}(\alpha) d\alpha \Big] \\
\leq \;& -\Big(\int_{t-\tau_j}^{t-\varphi_j} \dot{\xi}^T(\alpha) d\alpha \Big) \mathcal{S}_j \Big(\int_{t-\tau_j}^{t-\varphi_j} \dot{\xi}(\alpha) d\alpha \Big) \\
& -\Big(\int_{t-\varrho_j}^{t-\tau_j} \dot{\xi}^T(\alpha) d\alpha \Big) \mathcal{S}_j \Big(\int_{t-\varrho_j}^{t-\tau_j} \dot{x}_j(\alpha) d\alpha \Big) \\
= \;& -[\xi(t-\varphi_j) - \xi(t-\tau_j)]^T \mathcal{S}_j [\xi(t-\varphi_j) - \xi(t-\tau_j)] \\
& \quad -[\xi(t-\tau_j) - \xi(t-\varrho_j)]^T \mathcal{S}_j [\xi(t-\tau_j) - \xi(t-\varrho_j)] \qquad (6.14)
\end{aligned}
$$

where $j = 1$ applies to measurement delay and $j = 2$ applies to actuation delay. From equations (6.9)-(6.14) by applying Schur complements and using **Lemma 6.1.1**, we get

$$
\begin{aligned}
\dot{V}(t) &\leq \zeta^T(t)\, \Xi\, \zeta(t), \\
\zeta(t) &= \begin{bmatrix} \zeta_1^T(t) & \zeta_2^T(t) \end{bmatrix}^T, \\
\zeta_1(t) &= \begin{bmatrix} \xi^T(t) & \xi^T(t-\varphi_1) & \xi^T(t-\varrho_1) \end{bmatrix}^T, \\
\zeta_2(t) &= \begin{bmatrix} \xi^T(t-\varphi_2) & \xi^T(t-\varrho_2) \end{bmatrix}^T \qquad (6.15)
\end{aligned}
$$

where Ξ corresponds to $\widetilde{\Pi}$ in (6.10) by means of schur complement operations. If $\widetilde{\Pi} < 0$ so is $\Xi < 0$, leading to $\dot{V}(t) \leq -\omega\ ||\zeta||^2$. This establishes the internal asymptotic stability.

To compute that the feedback gains, we apply Schur complements and rewrite Ξ as

$$\widehat{\Xi} = \begin{bmatrix} \Xi_1 & \Xi_2 & \Xi_3 \\ \bullet & \Xi_4 & \Xi_5 \\ \bullet & \bullet & \Xi_6 \end{bmatrix} < 0 \tag{6.16}$$

$$\Xi_1 = \begin{bmatrix} \Xi_{os} & 0 & \mathcal{P}\mathcal{B}_1 & \mathcal{W}_1 \\ \bullet & -\Xi_{c1} & \mathcal{S}_1 & 0 \\ \bullet & \bullet & -\Xi_{m1} & \mathcal{S}_1 \\ \bullet & \bullet & \bullet & -\Xi_{n1} \end{bmatrix},$$

$$\Xi_2 = \begin{bmatrix} \varphi_1\mathcal{A}^T & (\varrho_1-\varphi_1)\mathcal{A}^T & 0 & \mathcal{P}\mathcal{B}_2 \\ 0 & 0 & 0 & 0 \\ \varphi_1\mathcal{B}_1^T & (\varrho_1-\varphi_1)\mathcal{B}_1^T & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Xi_3 = \begin{bmatrix} \mathcal{W}_2 & \varphi_2\mathcal{A}^T & (\varrho_2-\varphi_2)\mathcal{A}^T \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$
\Xi_4 = \begin{bmatrix} -\mathcal{W}_1^{-1} & 0 & 0 & 0 \\ \bullet & -\mathcal{S}_1^{-1} & 0 & 0 \\ \bullet & \bullet & -\Xi_{c2} & -\mathcal{S}_2 \\ \bullet & \bullet & \bullet & -\Xi_{m2} \end{bmatrix},
$$

$$
\Xi_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathcal{S}_2 & \varphi_2 \mathcal{B}_2^T & (\varrho_2 - \varphi_2)\mathcal{B}_2^T \end{bmatrix}
$$

$$
\Xi_6 = \begin{bmatrix} -\Xi_{n2} & 0 & 0 \\ \bullet & -\mathcal{W}_2^{-1} & 0 \\ \bullet & \bullet & \mathcal{S}_2^{-1} \end{bmatrix},
$$

$$
\begin{aligned}
&\Xi_{os} = \mathcal{P}\mathcal{A} + \mathcal{A}^T\mathcal{P} + \mathcal{Q}_1 + \mathcal{Q}_2 + \mathcal{R}_1 + \mathcal{R}_2 - \mathcal{W}_1 - \mathcal{W}_2, \\
&\Xi_{c1} = \mathcal{Q}_1 + \mathcal{S}_1, \qquad \Xi_{c2} = \mathcal{Q}_2 + \mathcal{S}_2, \\
&\Xi_{n1} = \mathcal{W}_1 + \mathcal{S}_1 + \mathcal{R}_1, \;\; \Xi_{n2} = \mathcal{W}_2 + \mathcal{S}_2 + \mathcal{R}_2, \\
&\Xi_{m1} = 2\mathcal{S}_1, \qquad \Xi_{m2} = 2\mathcal{S}_2.
\end{aligned} \tag{6.17}
$$

Then we define $\widehat{\mathcal{X}} = \mathcal{P}^{-1}$ and apply the congruent transformation

$$
T = diag[\; \widehat{\mathcal{X}} \;\; \widehat{\mathcal{X}} \;\; \widehat{\mathcal{X}} \;\; \widehat{\mathcal{X}} \;\; I \;\; I \;\; \widehat{\mathcal{X}} \;\; \widehat{\mathcal{X}} \;\; \widehat{\mathcal{X}} \;\; I \;\; I \;]
$$

Using the algebraic matrix inequalities $-\mathcal{W}_j^{-1} \leq -2\mathcal{X}_j + \Lambda_{2j}$, $-\mathcal{S}_j^{-1} \leq -2\mathcal{X}_j + \Lambda_{3j}$ in addition to the matrix definitions (6.11), we obtain LMI (6.10) by Schur complements. This concludes the proof.

6.4 Example

Consider an LTI plant model quadruple-tank process consisting of four water tanks [85]. These are interconnected and the flow through the pipes is regulated by using two pumps. Fig. 4.2 shows a schematic diagram of a typical quadruple tank process. The aim of the control strategy is to regulate the level of water in the two lower tanks by controlling the electrical power to the pumps. The quadruple tank system presents a multiple-input-multiple-output (MIMO) system. The system has two control inputs which can be manipulated to control the water level in the tanks. The two pumps are used to transfer water from a sump into four overhead tanks. The model used for simulation was a continuous version of the discrete-time system shown below:

$$A = \begin{bmatrix} -0.0278 & 0 & 0.0206 & 0 \\ 0 & -0.0233 & 0 & 0.0141 \\ 0 & 0 & -0.0206 & 0 \\ 0 & 0 & 0 & -0.0141 \end{bmatrix},$$

$$B = \begin{bmatrix} 5 & 0 \\ 0 & 6.667 \\ 0 & 10 \\ 11.667 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

The simulation was carried out on Matlab R2008b, wherein, for each given case the LMIs in *Theorem 6.3.1* were solved to obtain the gain matrices, following which the data from the workspace was imported into a Simulink Model and the plant was run to obtain the results. For the convenience of simulation, it was assumed that the sampling rate in the measurement as well as the actuation channel is equal, i.e. $h^m = h^a = 0.1$sec. For the purpose of illustration, 4 different cases have been shown below:

Case I

$\varrho_1 = 0.3$, $\varrho_2 = 3.8$, $\varphi_1 = 0.4$, $\varphi_2 = 4.0$, $n^m = 0$, $n^a = 0$

$$K = \begin{bmatrix} 0.5783 & -0.0658 & -0.5752 & 0.4191 \\ -0.1633 & 0.2681 & 0.6544 & -0.1536 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.1861 & -0.5918 & -0.0235 & -0.1784 \\ -0.2267 & 0.3555 & 0.0208 & 0.2400 \end{bmatrix}$$

$\|K\| = 1.1269$, $\|L\| = 0.7975$

Case II

$\varrho_1 = 0.5$, $\varrho_2 = 3.5$, $\varphi_1 = 0.4$, $\varphi_2 = 5.0$, $n^m = 1$, $n^a = 1$

$$K = \begin{bmatrix} 0.5122 & -0.1001 & -0.4978 & 0.4990 \\ -0.1194 & 0.3504 & 0.5649 & -0.2188 \end{bmatrix}$$

$$L = \begin{bmatrix} 0.1851 & -0.4786 & 0.0093 & -0.1133 \\ -0.2136 & 0.0904 & 0.0168 & 0.1585 \end{bmatrix}$$

$\|K\| = 1.0681,\ \|L\| = 0.5664$

Case III

$\varrho_1 = 0.5,\ \varrho_2 = 4,\ \varphi_1 = 0.6,\ \varphi_2 = 5.5,\ n^m = 3,\ n^a = 3$

$$K = \begin{bmatrix} 0.4599 & -0.1501 & -0.4368 & 0.5793 \\ -0.1289 & 0.4367 & 0.5835 & -0.2802 \end{bmatrix}$$

$$L = \begin{bmatrix} -0.0201 & -0.4142 & 0.0391 & -0.0863 \\ -0.2350 & -0.0453 & -0.0301 & 0.1346 \end{bmatrix}$$

$\|K\| = 1.1133,\ \|L\| = 0.4267$

Case IV

$\varrho_1 = 0.3,\ \varrho_2 = 3.8,\ \varphi_1 = 0.4,\ \varphi_2 = 4.0,\ n^m = 5,\ n^a = 5$

$$K = \begin{bmatrix} 0.5150 & -0.1212 & -0.4952 & 0.5306 \\ -0.1420 & 0.3792 & 0.6365 & -0.2310 \end{bmatrix}$$

$$L = \begin{bmatrix} -0.0484 & -0.2057 & 0.3326 & -0.1342 \\ -0.1525 & -0.0533 & -0.0981 & 0.1097 \end{bmatrix}$$

$\|K\| = 1.1309$, $\|L\| = 0.4239$

Case V

$\varrho_1 > 15$ and/or $\varrho_2 > 12$.

After carrying out simulations for various values of delay bounds and packet dropouts, it was concluded that the system is driven unstable at values of $\varrho_1 > 15$ and $\varrho_2 > 12$. The response of the system in **Case V** has been shown in Fig. 6.3.

6.5 Conclusions

This chapter provides a new output feedback stabilization technique for networked systems subject to network phenomena such as sampling, delays and

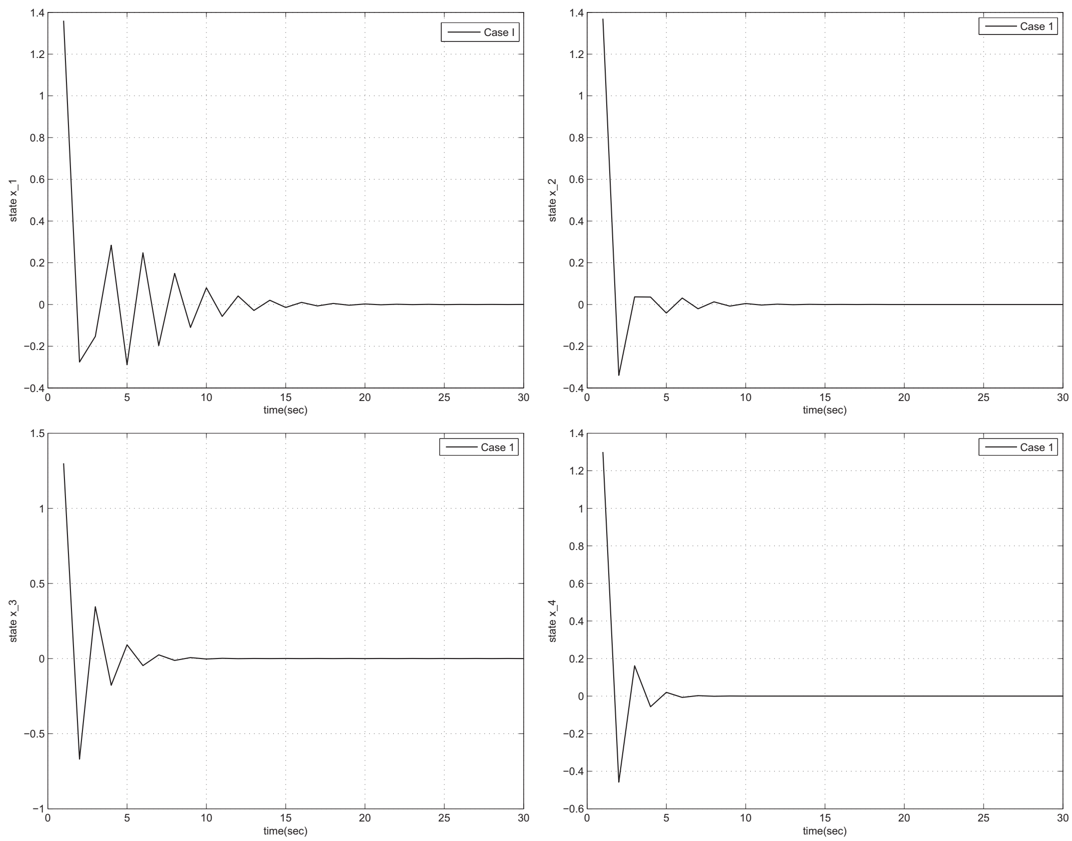

Figure 6.3: Response of system to conditions given in *Case I*

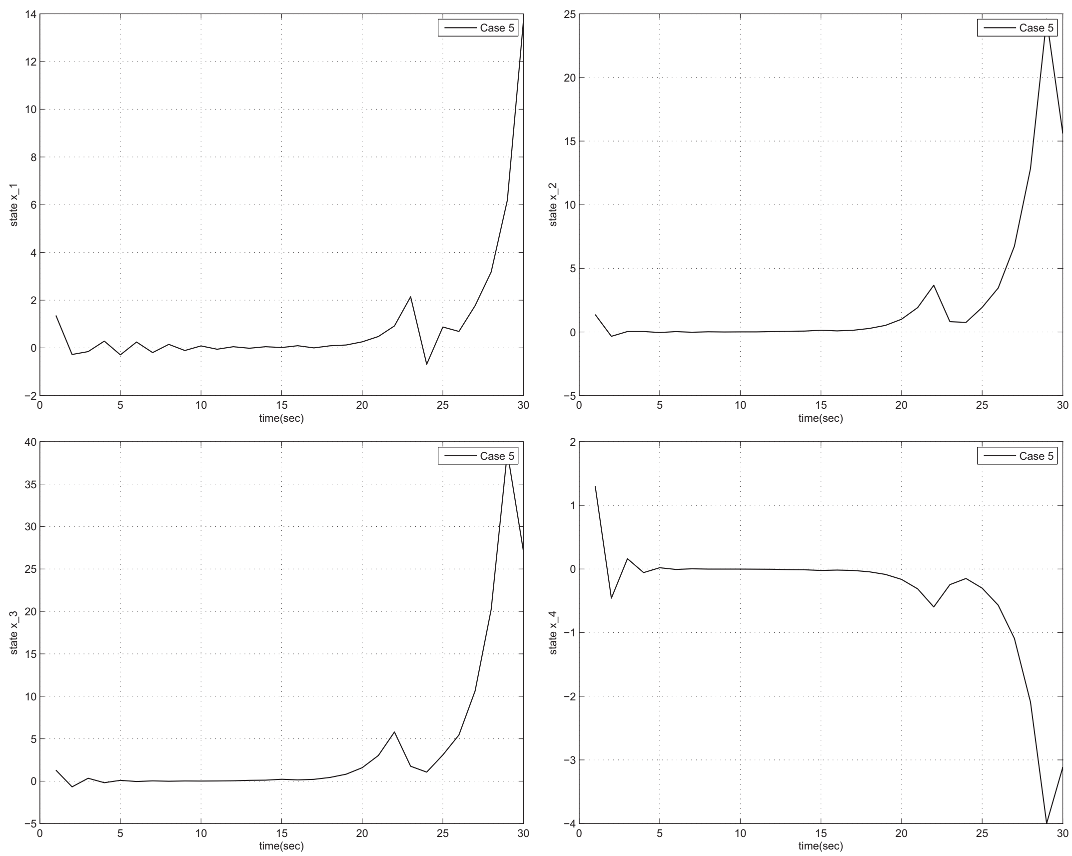

Figure 6.4: Response of system to conditions given in *Case V*

packet dropouts based on the Lyapunov-Krasovskii theory. Numerical simulations have also been used to illustrate the developed techniques. We look forward to extending our results to robust control problem by incorporating noise and designing $\mathcal{H}_\infty$ or $\mathcal{H}_2$ controllers. Taking into consideration the tremendous amount of research that has gone into networked controller, the problem of controller saturation can also incorporated in such NCS.

Chapter 7

CONCLUSIONS AND FUTURE WORK

To sum up the work presented in the thesis, we have provided new control design strategies for networked control systems with the perspective of practical applications, incorporating greater amount of randomness in network behavior and paying significant attention to statistical analysis of networked systems. Another notable fact here is that the design strategies throughout were based on the assumption that all the states of the system are not available for measurement, which generalizes our approach in the control of networked systems.

The most recent developments in networked control were concerned with probabilistic approach to occurence of random delays, however to the best of the authors knowledge, the probabilities were generally assumed to be constant which is slightly an optimistic assumption. Various factors could lead to non-uniform behaviour of the network ranging from ambient temperature to network traffic and aging of communication equipment. We have addressed this fact by considering nonstationary probabilities.

We extended the work of Luan et al. [5] by developing an improved observer-based stabilizing control algorithm to estimate the states and control input through the construction of an augmented system where the original control input was regarded as a new state. The occurrence of measurement and actuation delays were considered using nonstationary random processes modeled by two mutually independent stochastic variables. The observer-based controller was designed to exponentially stabilize the networked system and the developed stability conditions were represented in the form of a convex optimization problem and the results wre tested by simulation on a real-time examples. The simulations revealed that the newly developed strategies provided faster speed of response with lesser control inputs compared to the conventional techniques and were more effective in stabilizing NCSs.

In the next part of the thesis we considered the quantization problem. Quantization as we know generally exists in computer-based control systems and quantization errors have adverse effects on the performance of NCSs. It can delineate a system and result in chaotic behaviour. We assume that the output measurements from the plant are subjected to logarithmic quantization before they reach the controller. An observer-based controller is designed to provide the necessary control and render the closed-loop system stable in the presence of quantization as well as random delays in both the measurement and actuation channels of the NCS.

With the next chapter we took a step further in the development of feedback stabilization methods for nonlinear discrete-time NCSs with random packet dropouts and delays. The presence of disturbance inputs was also acknowledged. The linearities were assumed to be bounded however and the system behaviour was studied with varied amounts of nonlinearities and disturbances entering the system. The behaviour of the nonlinear system was compared with the linear system with similar delays occuring in their networks.

In the last part we took into consideration the analysis of continuous time NCSs with Lossy networks. The Lyapunov Krasovskii functionals were deployed to obtain the stability conditions, expressed in the form of LMIs. Simulations were carried out under different conditions in several cases to study the variation in system behaviour.

The area of networked control is still a raw arena for research advances, and constitutes a new paradigm for control engineers to explore. Hence our work in this thesis has the great potential to be further expanded in several directions. Suggestions for future work in brief would be

- With reference to Chapter 5, where we considered bounded nonlinearities, controller design for the non-linear networked systems with unbounded nonlinearities can be further investigated.

- In Chapter 4 of the thesis we have considered quantization only in the measurement channel, however, stability analysis and control design for NCSs subject to quantization in both measurement and actuation channel, with nonstationary dropouts is still left to be explored.

- A lot of study still needs to be done into statistical models to see how they can be applied to occurence of random delays in the networked systems.

Finally we would like to reiterate that the analysis presented in this thesis is general in the sense that no specific underlying network type, structure or operating protocol has been taken into consideration. Hence the results can be applied to wired or wireless type of networks, with the knowledge that several specific models will have to be devised for the latter.

Nomenclature

Discrete Case

$x_p(k)$ - plant state vector ($\in \Re^n$)

$u_p(k)$ - plant input vector ($\in \Re^m$)

$y_p(k)$ - plant state vector ($\in \Re^p$)

τ_k^m - bounded measurement delay ($\tau_m^- \le \tau_k^m \le \tau_m^+$)

τ_k^a - bounded actuation delay ($\tau_a^- \le \tau_k^a \le \tau_a^+$)

$\delta(k)$ - bernoulli sequence governing the occurence of measurement delay

$\alpha(k)$ - bernoulli sequence governing the occurence of actuation delay

$y_c(k)$ - plant output reaching the controller

$u_c(k)$ - control signal generated by controller

p_k - probability that measurement delay occurs (not constant)

s_k - probability that actuation delay occurs (not constant)

A - plant state matrix ($\in \Re^{n \times n}$)

B - plant input matrix ($\in \Re^{n \times m}$)

C - plant output matrix ($\in \Re^{m \times p}$)

K - state-feedback gain matrix ($\in \Re^{m\times n}$)

L - observer gain matrix ($\in \Re^{m\times p}$)

$w(k)$ - ℓ_2 disturbance input ($\in \Re^{q}$)

$z(k)$ - controlled plant input ($\in \Re^{q}$)

$f_0(x, k)$ - time-varying nonlinear perturbation ($\in \Re^{n}$)

Γ_0 - disturbance matrix ($\in \Re^{q\times n}$)

G - matrix that realtes the states to the controlled outputs ($\in \Re^{n\times q}$)

Φ_0 - matrix that relates the disturbances to the controlled outputs ($\in \Re^{q\times q}$)

Continuous Case

$x(t)$ - plant state vector ($\in \Re^{n}$)

$u(t)$ - plant input vector ($\in \Re^{m}$)

$y(t)$ - plant state vector ($\in \Re^{p}$)

h^m - sensor sampling period

h^m - actuator sampling period

τ_k^m - bounded measurement delay ($\psi_1 \leq \tau_k^m \leq \rho_1$)

τ_k^a - bounded actuation delay ($\psi_2 \leq \tau_k^a \leq \rho_2$)

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