

King Fahd University of Petroleum and Minerals

Aerospace Engineering Department

Astrodynamics

(AE 410)

Term Project

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1. PROJECT NATURE:

As mentioned in the abstract, predicting the birth of the moon crescent is important issue in Islamic Calendar since that is related to performing some Islamic religious occasions.

In This project I am going to find someway to help how to predict it the birth of the moon crescent without leaving any doubt by using some astronomical dynamic laws and knowledge that I got from this course as well as by the help of some other references related to the subject.

By the help of paper (Moon-Sun attitude sensor) made by Daniel Mortari and collecting the dates data of the last three years in Hijri and their corresponding dates in Gregorian to find the amount of the seen moon magnitude that could be noticed at the beginning of every month in Hijri calendar and then make the average value of this seen moon magnitude for two years and test it with the third year to accomplish a conclusion of how much the value of this seen moon magnitude to be our experimental standard value for later use of determining the moon crescent at the beginning of every month in Hijri calendar in future. Of course, the calculation is valid only for people living in our city (Dhahran) in Saudi Arabia.

Mat lab and Excel will be our tools in this paper. There will be three subroutine programs to calculate the location of the earth, moon and sun separately 24 hours, and the main program will use their locations that change with time to calculate the amount of seen moon magnitude for the first day of the thirty six months along the three years and then manipulating the data to get what is stated above.

2. BACKGROUND INFORMATION:

In this part we draw the simulation geometry that represents the earth moon sun motion and we define parameters that should be calculated and the equations used to calculate them.

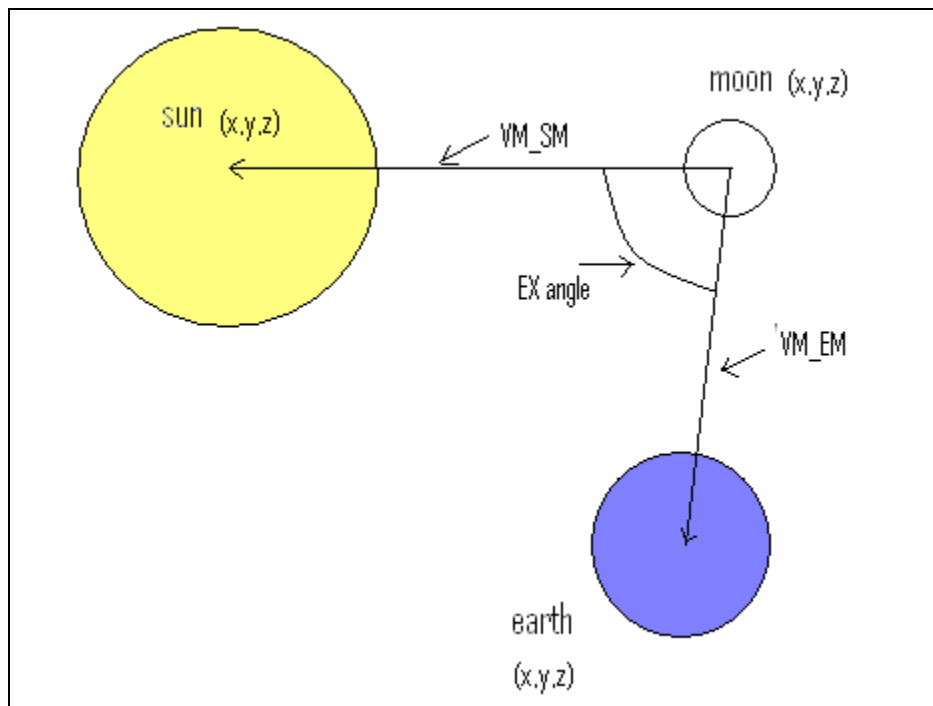


Figure 2.1 moon-sun-earth geometry

What we need in calculation is the EX (ξ) to find the phase (PHSE) that used to calculate the seen moon magnitude (M_{seen})

Where we find the vector joining the earth location to the moon location (VM_em) and the vector joining the sun location to the moon location (VM_sm) as shown in figure 2.1.

$$\rightarrow EX (\xi) = \text{COS}^{-1} (\text{VM_sm} \cdot \text{VM_em} / (|\text{VM_sm}| \times |\text{VM_em}|)) \dots\dots\dots(1)$$

Where:

EX (ξ) is the angle between the earth moon vector and the sun moon vector. It is changing according to the location of sun, moon and earth locations that change with time.

$$\rightarrow M_{\text{seen}} = 0.23 + 5 \log_{10} (|\text{VM_em}|) - 2.5 \log_{10} [\text{PX} (\xi)] \dots\dots\dots(2)$$

Where:

M_{seen} is the seen moon magnitude.

PX is the phase which is a function of the angle between the earth moon vector and the sun moon vector (EX).

The unit of (|VM_em|) should be in astronomical unit.

To find (PX) we need to interpolate among the historical data that we have as shown in table 3.1.

→ To find earth location:

We need to find the longitude and latitude of earth, which change with time as follows:

$$\text{Longitude} = \text{longitude_Dhahran} - \text{GST} \dots\dots\dots(3)$$

Where:

longitude_Dhahran is the longitude of our city which is equal to 50.1325 degrees.

And GST is the Greenwich Sidereal Time.

Where:

$$\text{GST} = \text{GST}_0 + \omega t \dots\dots\dots(4)$$

Where:

GST_0 is the angle (time) in degrees between Greenwich longitude (0 longitude) and the x-axis (vernal equinox) and it is calculated for 0 hours as:

$$\text{GST}_0 = 99.6910 + 36000.7689 * T + 0.0004 * T^2 \dots\dots\dots(5)$$

Where

$$T = \text{JD} / 36525 \dots\dots\dots(6)$$

And JD is the Julian day numbers and the unit of GST_0 is in degrees.

ω is the earth rotation in 24 hours and it is calculated to be (0.25 deg/min). and the time t is the time of the day in minutes and it is changing for 24 hours.

$$\text{Latitude} = \text{Latitude_Dhahran} \dots\dots\dots(7)$$

Where:

latitude_Dhahran is the latitude of our city which is equal to 26.3042 degrees.

$$X_E = R * \cos(\text{latitude}) * \cos(\text{longitude}) \dots \dots \dots (8)$$

$$Y_E = R * \cos(\text{latitude}) * \sin(\text{longitude}) \dots \dots \dots (9)$$

$$Z_E = R * \sin(\text{latitude}) \dots \dots \dots (10)$$

Where:

R is the mean earth radius which is equal to (6378000 meters).

And X_E , Y_E and Z_E are the earth position components.

➔ To find moon location:

By using the American Ephemeris that describes the moon mean motion with respect to earth as follows:

$$\theta_m = 270.434164 + 13.1763965268 * t - 8.5e-13 * t^2 + 3.9e-20 * t^3 \dots \dots \dots (11)$$

$$\omega_m = 334.329356 + 0.1114040803 * t - 7.739e-12 * t^2 - 2.6e-19 * t^3 \dots \dots \dots (12)$$

$$\Omega_m = 259.183275 - 0.0529539222 * t + 1.557e-12 * t^2 + 5e-20 * t^3 \dots \dots \dots (13)$$

$$i_m = 5.145396374 \dots \dots \dots (14)$$

Where all angles represent the moon orbit angles that are in degrees and time is the time difference in (days) between the time at which the computation is required and the reference time (Jan. 1, 1900, at 12:00; Julian date=2415010). So t = JD – 2415010.

The axes of the moon location can be found by first finding the orbital axes and then finding the transformation matrix that is used to find the inertial axes of the moon as follows:

$$X_{mo} = r * \cos(\theta_m) \dots \dots \dots (15)$$

$$Y_{mo} = r * \sin(\theta_m) \dots \dots \dots (16)$$

$$Z_{mo} = 0 \dots \dots \dots (17)$$

Where:

$$r = p / (1 + e * \cos(\theta_m)) \dots \dots \dots (18)$$

And:

$$p = a * (1 - e^2) \dots \dots \dots (19)$$

Where r is the distance between the earth and the moon in the earth-moon orbit and a is the semi major axis (circular orbital) of the earth-moon orbit. a is equal to (384400000 meters). And e represents the eccentricity which is equal to (0.054900489).

The transformation matrix calculated to be:

$$T = \begin{bmatrix} C(\omega_m)C(\Omega_m) - S(\omega_m)C(i_m)S(\Omega_m) & C(\omega_m)S(\Omega_m) + S(\omega_m)C(i_m)C(\Omega_m) & S(\omega_m)S(i_m) \\ -S(\omega_m)C(\Omega_m) - C(\omega_m)C(i_m)S(\Omega_m) & -S(\omega_m)S(\Omega_m) + C(\omega_m)C(i_m)C(\Omega_m) & C(\omega_m)S(i_m) \\ S(i_m)S(\Omega_m) & -S(i_m)C(\Omega_m) & C(i_m) \end{bmatrix} \dots \dots (20)$$

Where C and S represent cosine and sine the angles respectively.

$$M_I_axes = TT * \begin{bmatrix} X_{mo} \\ Y_{mo} \\ Z_{mo} \end{bmatrix} \dots\dots\dots(21)$$

Where:

M_I_axes are the inertial axes components of the moon and TT is the transpose of the transformation matrix.

➔ To find sun location:

The method is similar to what we did for the moon location. However the subroutine program that calculate the sun location was got from internet source.

All dates must be input in Julian day numbers. To change the date to Julian day we use the following method:

We enter the year (Y), the month (MO) and the day (D) to get the Julian day as follows:

$$A = (Y/100) \dots\dots\dots(22)$$

$$B = (A / 4) \dots\dots\dots(23)$$

$$C = (2 - A + B) \dots\dots\dots(24)$$

$$E = (365.25 * (Y + 4716)) \dots\dots\dots(25)$$

$$F = (30.6001 * (MO + 1)) \dots\dots\dots(26)$$

$$Jd = C + D + E + F - 1524.5 \dots\dots\dots(27)$$

3. CALCULATION ANALYSIS:

The experimental data for phase law is shown in table 3.1 as follows:

Table 3.1 experimental data for moon visual magnitude

EXF (deg)	PXEF
0	1
5	0.929
10	0.809
20	0.625
30	0.483
40	0.377
50	0.288
60	0.225
70	0.172
80	0.127
90	0.089
100	0.061
110	0.041
120	0.027
130	0.017
140	0.009
150	0.004
160	0.001

The seen moon magnitude for the above experimental data is shown in figure 3.1.

For the beginning of every month of years 1426, 1425 and 1424 in Hijri calendar and their corresponding dates in Gregorian calendar is shown in table 3.2, table 3.3 and table 3.4 as follows:

Table 3.2 (Year 1426 H)

Hijri (d-m-y)	Gregorian (d-m-y)
1 - 1 - 1426	10/2/2005
1 - 2 - 1426	11/3/2005
1 - 3 - 1426	10/4/2005
1 - 4 - 1426	9/5/2005
1 - 5 - 1426	8/6/2005
1 - 6 - 1426	7/7/2005
1 - 7 - 1426	6/8/2005
1 - 8 - 1426	4/9/2005
1 - 9 - 1426	4/10/2005
1 - 10 - 1426	3/11/2005
1 - 11 - 1426	3/12/2005
1 - 12 - 1426	1/1/2006

Table 3.3 (Year 1425 H)

Hijri (d-m-y)	Gregorian (d-m-y)
1 - 1 - 1425	21-2-2004
1 - 2 - 1425	22-3-2004
1 - 3 - 1425	20-4-2004
1 - 4 - 1425	20-5-2004
1 - 5 - 1425	19-6-2004
1 - 6 - 1425	18-7-2004
1 - 7 - 1425	17-8-2004
1 - 8 - 1425	15-9-2004
1 - 9 - 1425	15-10-2004
1 - 10 - 1425	14-11-2004
1 - 11 - 1425	13-12-2004
1 - 12 - 1425	12/1/2005

Table 3.4 (Year 1424 H)

Hijri (d-m-y)	Gregorian (d-m-y)
1 - 1 - 1424	4/3/2003
1 - 2 - 1424	3/4/2003
1 - 3 - 1424	2/5/2003
1 - 4 - 1424	1/6/2003
1 - 5 - 1424	1/7/2003
1 - 6 - 1424	30-7-2003
1 - 7 - 1424	28-8-2003
1 - 8 - 1424	27-9-2003
1 - 9 - 1424	26-10-2003
1 - 10 - 1424	25-11-2003
1 - 11 - 1424	24-12-2003
1 - 12 - 1424	23-1-2004

Now we start calculate M_{seen} for different months:

→ For year 1426 H:

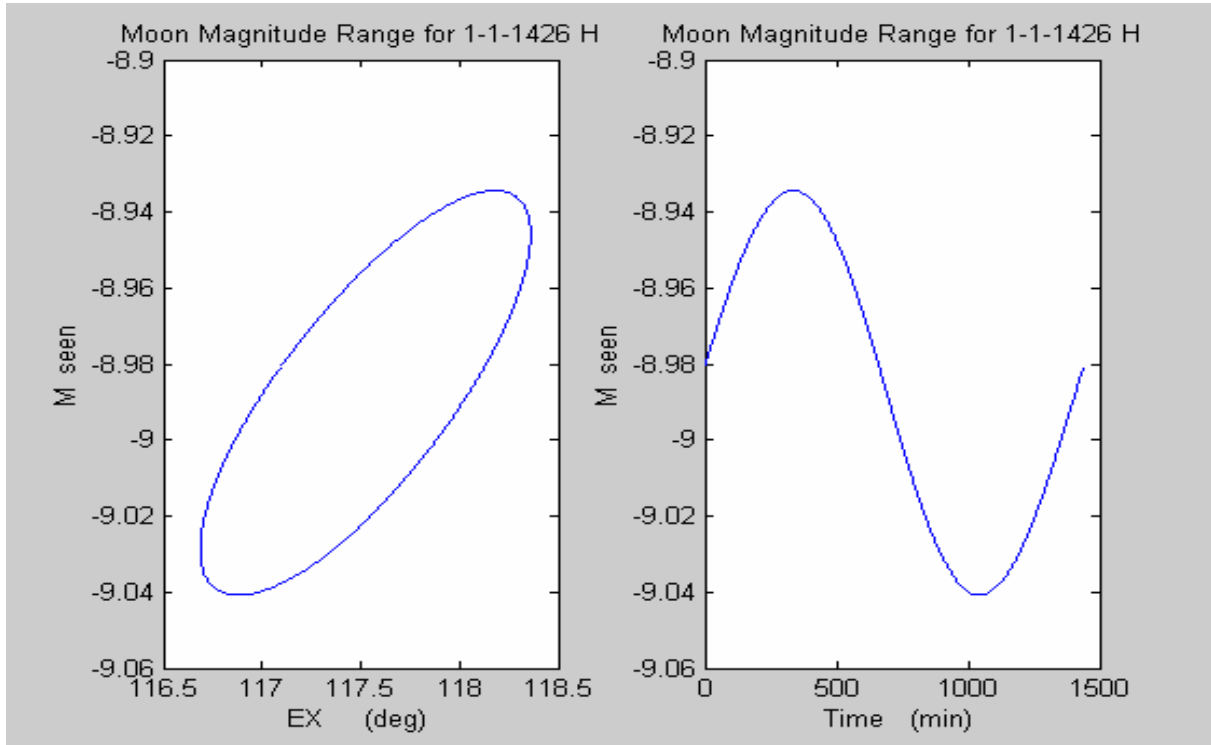


Figure 3.2 for 1-1-1426 H

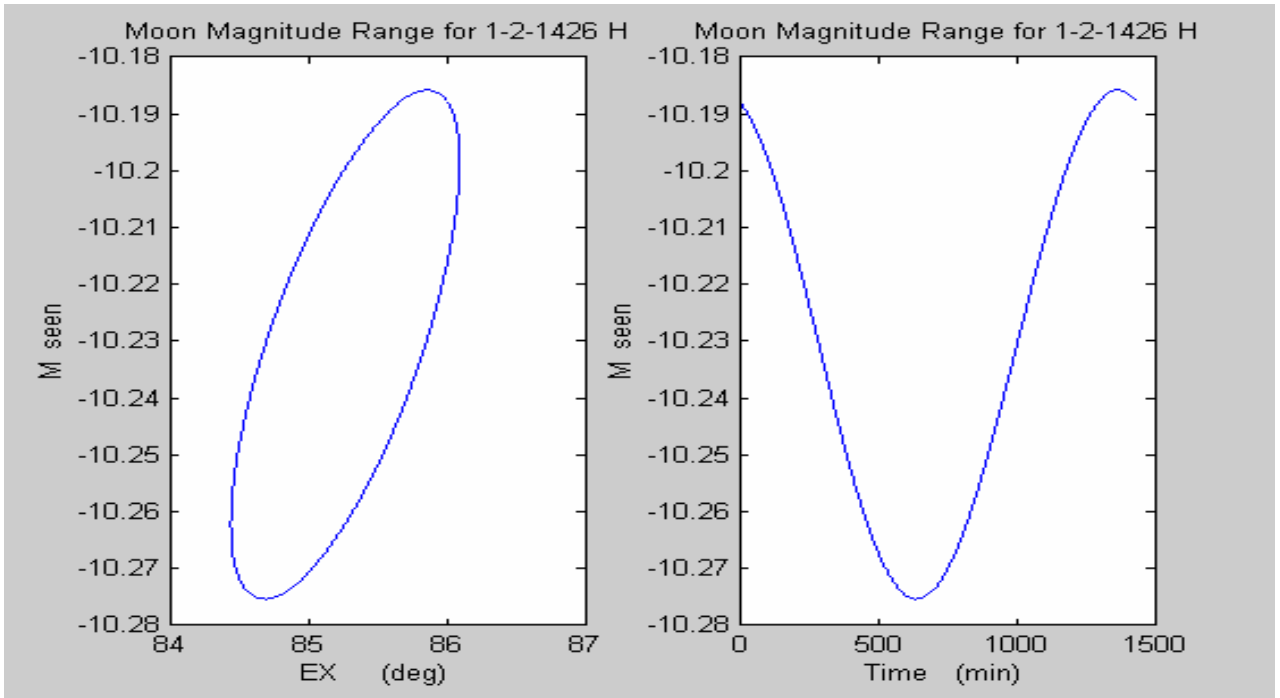


Figure 3.3 for 1-2-1426 H

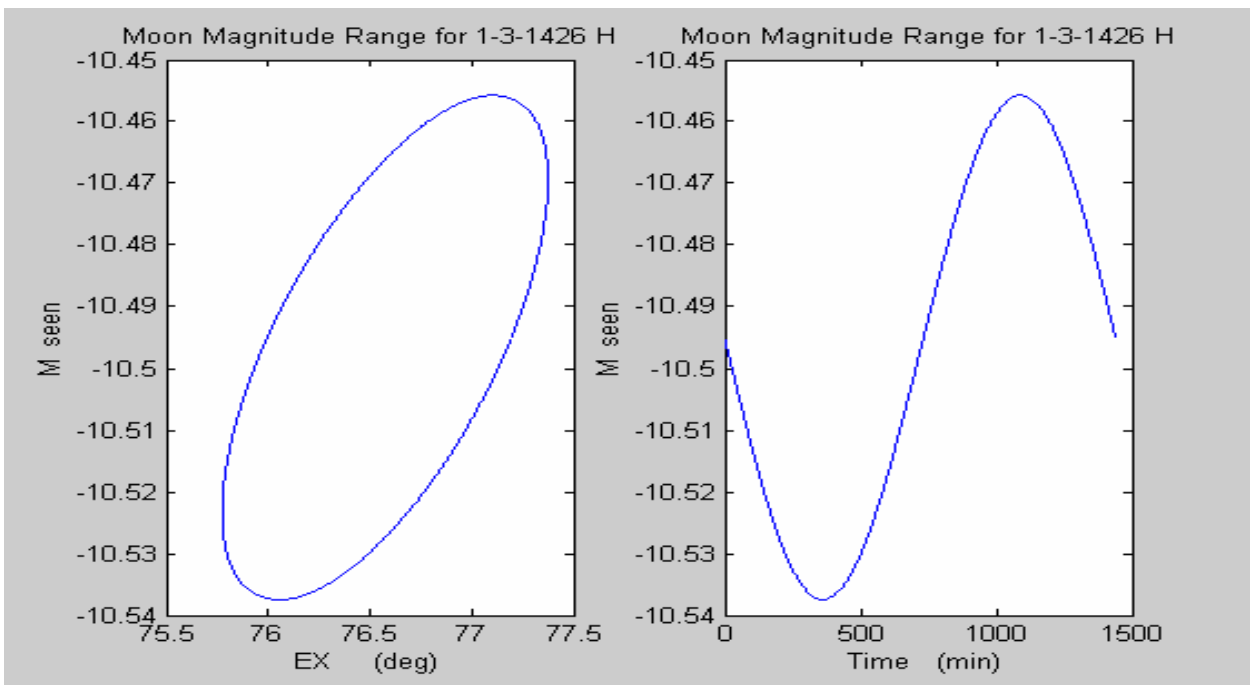


Figure 3.4 for 1-3-1426 H

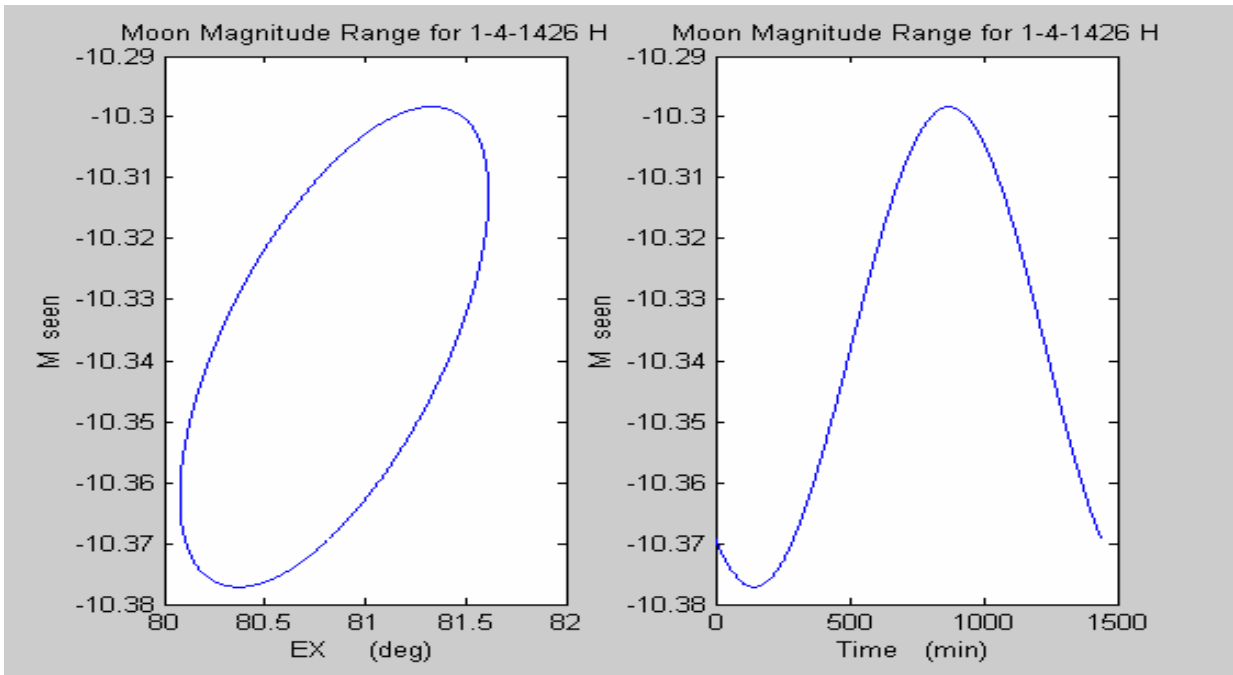


Figure 3.5 for 1-4-1426 H

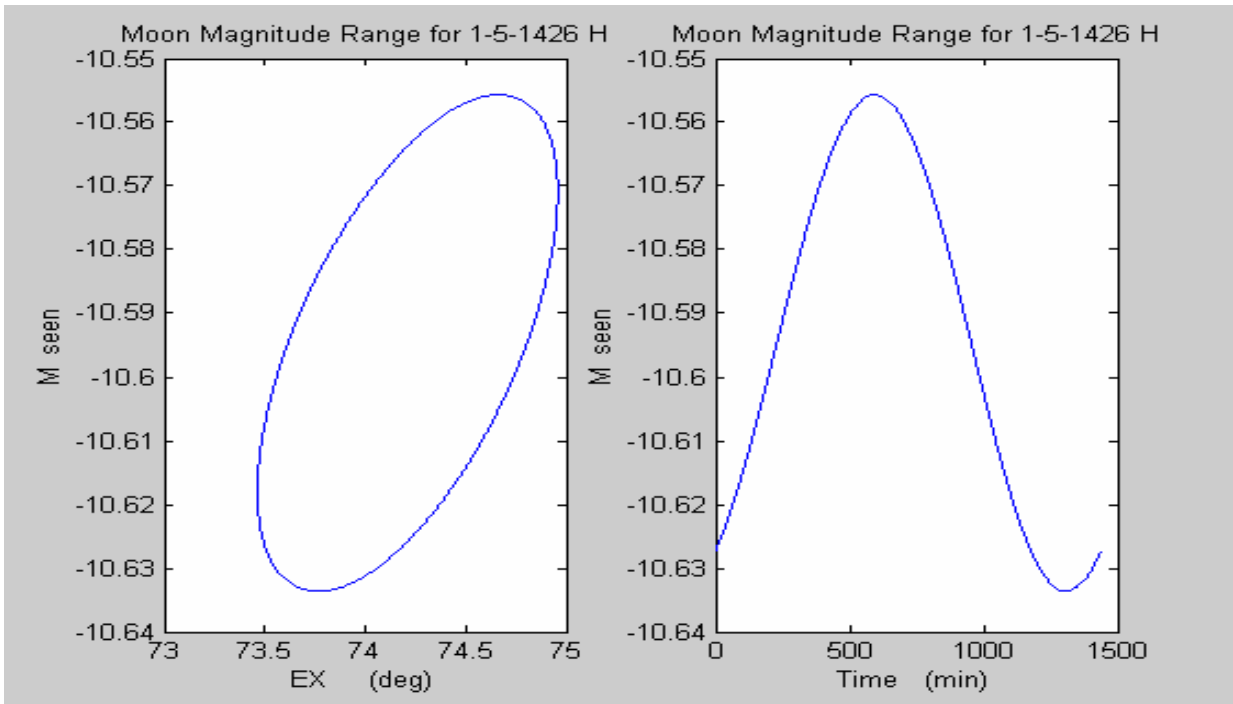


Figure 3.6 for 1-5-1426 H

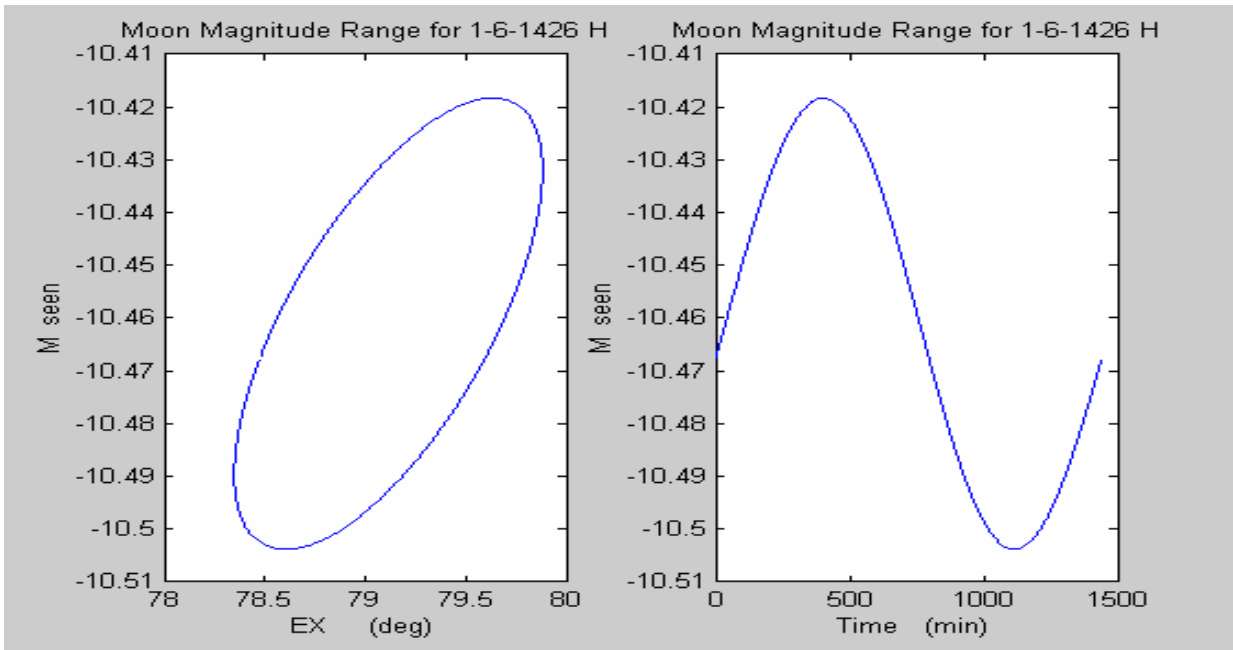


Figure 3.7 for 1-6-1426 H

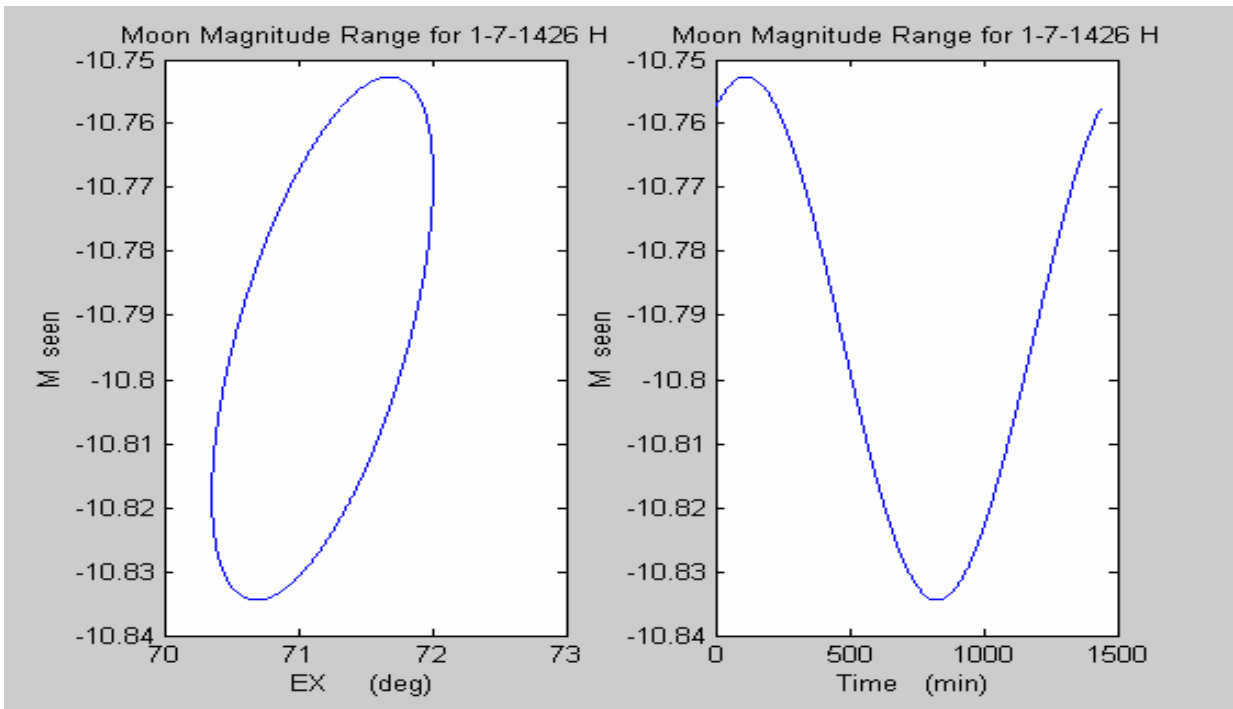


Figure 3.8 for 1-7-1426 H

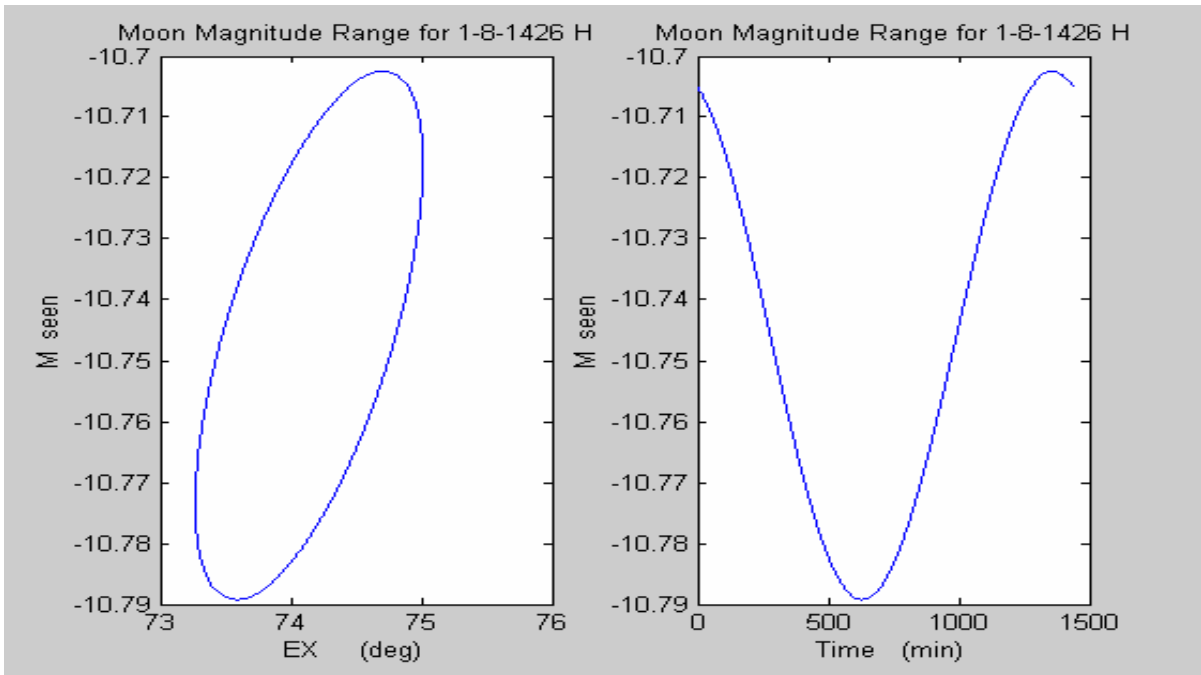


Figure 3.9 for 1-8-1426 H

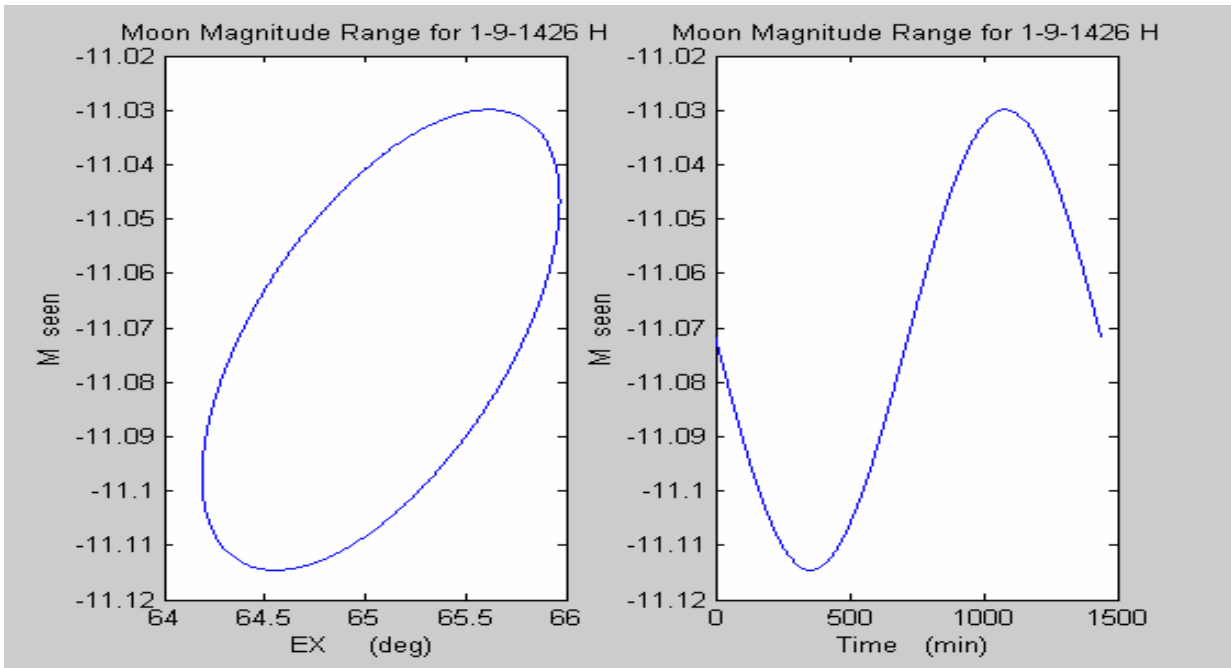


Figure 3.10 for 1-9-1426 H

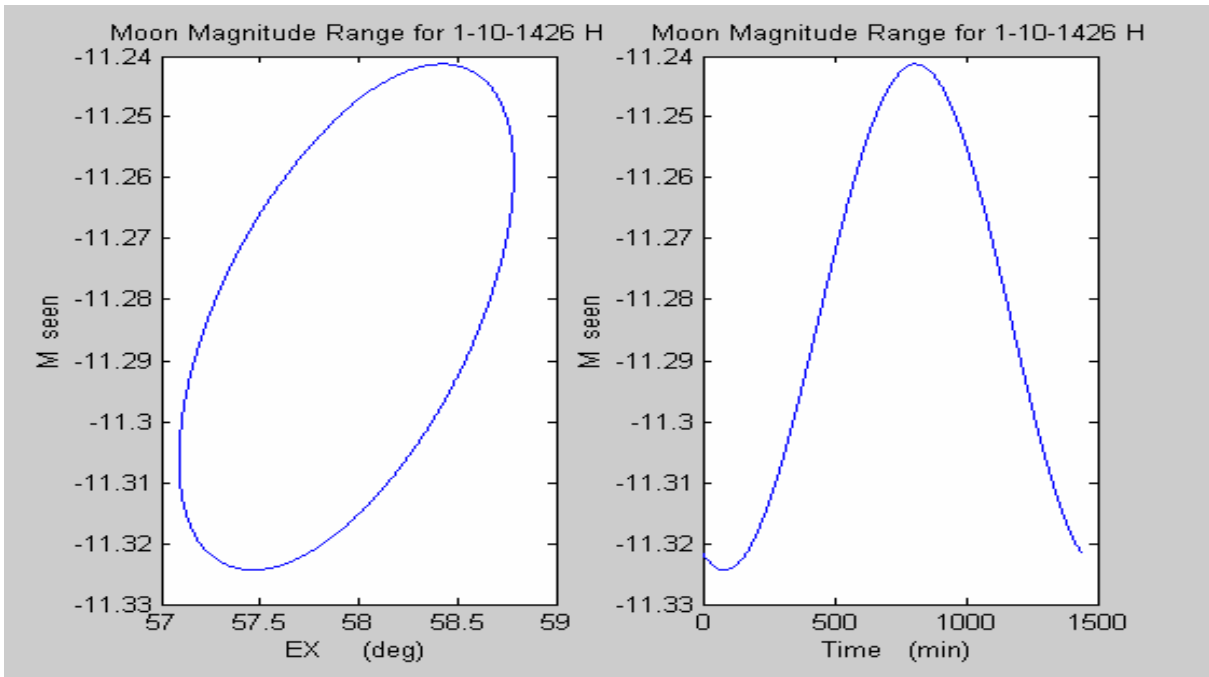


Figure 3.11 for 1-10-1426 H

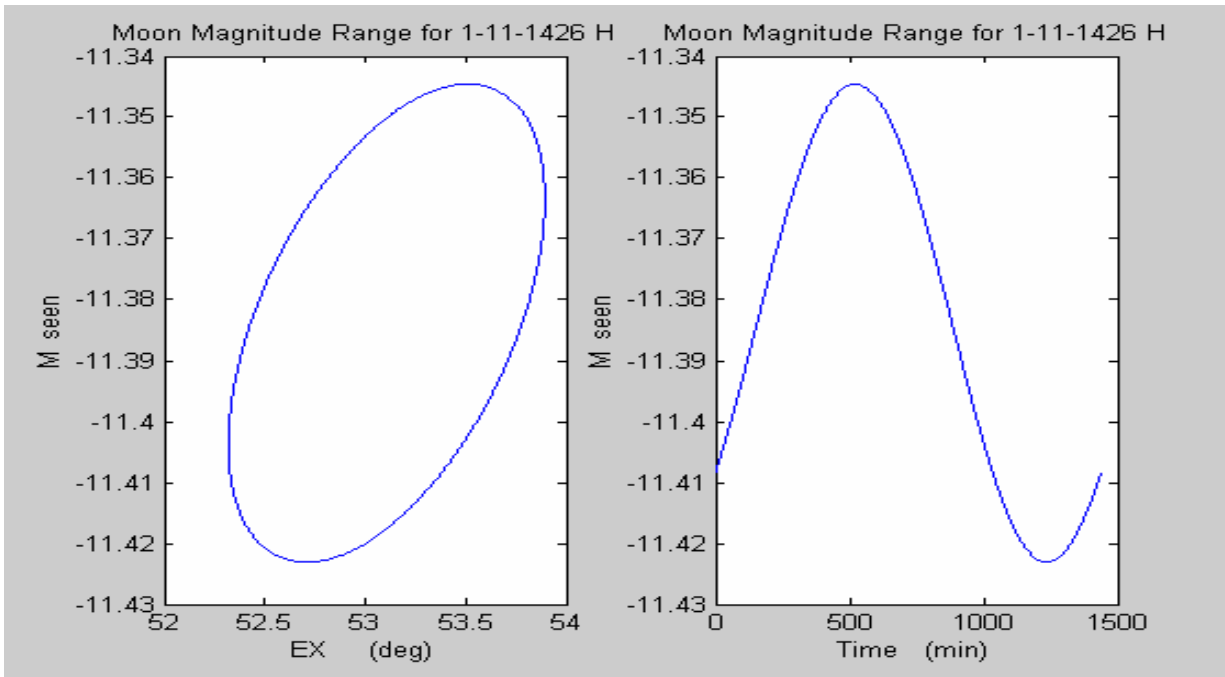


Figure 3.12 for 1-11-1426 H

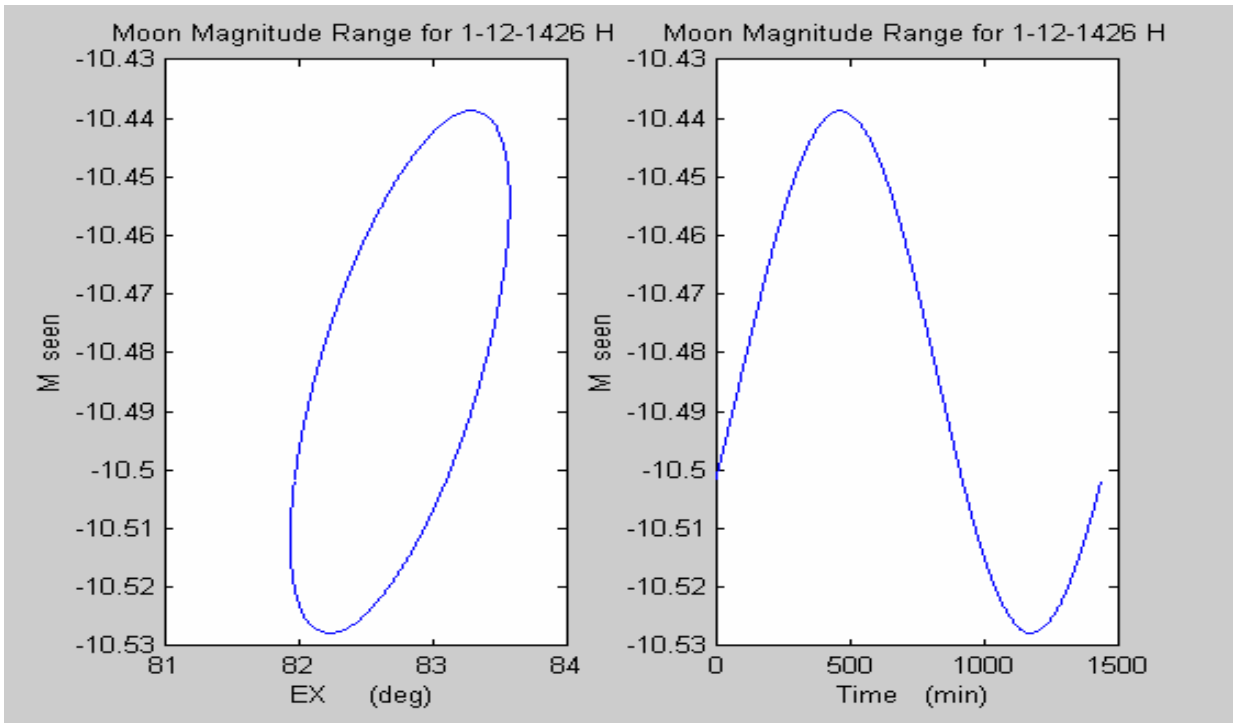


Figure 3.13 for 1-12-1426 H

As shown in above figures from figure 3.2 to figure 3.12 the seen moon magnitude is shown for every month for 24 hours of year 1246 H; this value indicates that the moon could be seen within this range. They give the minimum and the maximum value of M_{seen} . The data is tabulated in table 3.5.

Table 3.5 M_{seen} for year 1426 H

Hijri (d-m-y)	$M_{\text{seen_mean}}$	$M_{\text{seen_max}}$	$M_{\text{seen_min}}$
1 - 1 - 1426	-8.9874	-9.0408	-8.9343
1 - 2 - 1426	-10.2303	-10.2753	-10.1858
1 - 3 - 1426	-10.4964	-10.5375	-10.4558
1 - 4 - 1426	-10.3375	-10.3772	-10.2983
1 - 5 - 1426	-10.5944	-10.6336	-10.5557
1 - 6 - 1426	-10.4608	-10.5038	-10.4183
1 - 7 - 1426	-10.7933	-10.8345	-10.7526
1 - 8 - 1426	-10.7455	-10.7892	-10.7024
1 - 9 - 1426	-11.0719	-11.1146	-11.0299
1 - 10 - 1426	-11.2825	-11.3244	-11.2413
1 - 11 - 1426	-11.3835	-11.4231	-11.3446
1 - 12 - 1426	-10.4829	-10.5278	-10.4387
Average	-10.5722	-10.61515	-10.52980833

→ For year 1425 H:

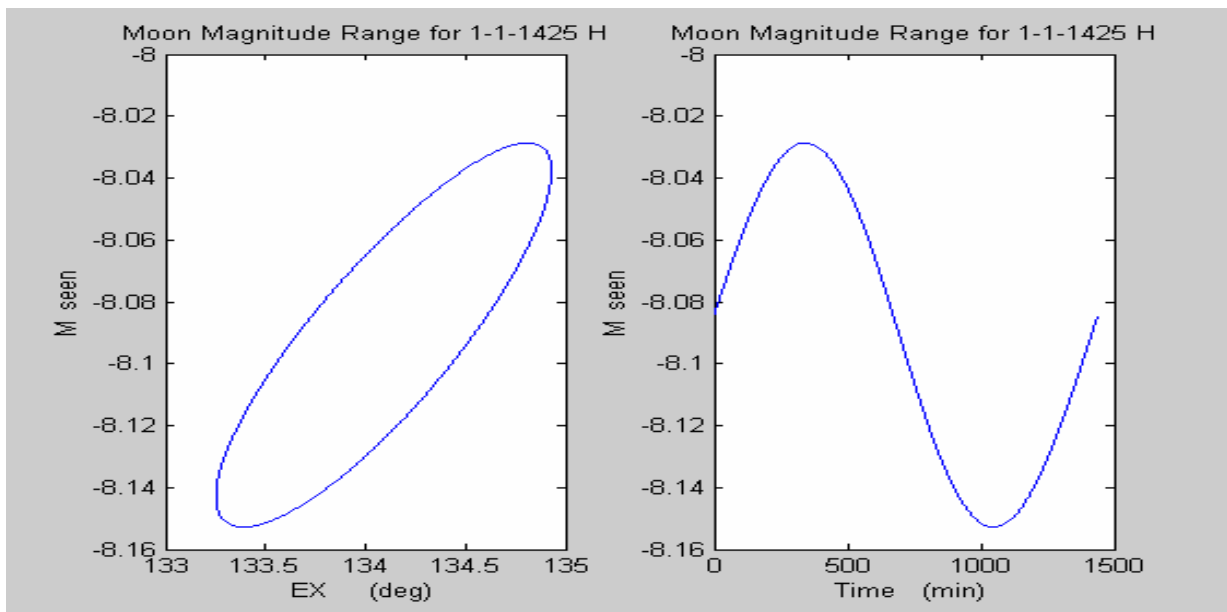


Figure 3.14 for 1-1-1425 H

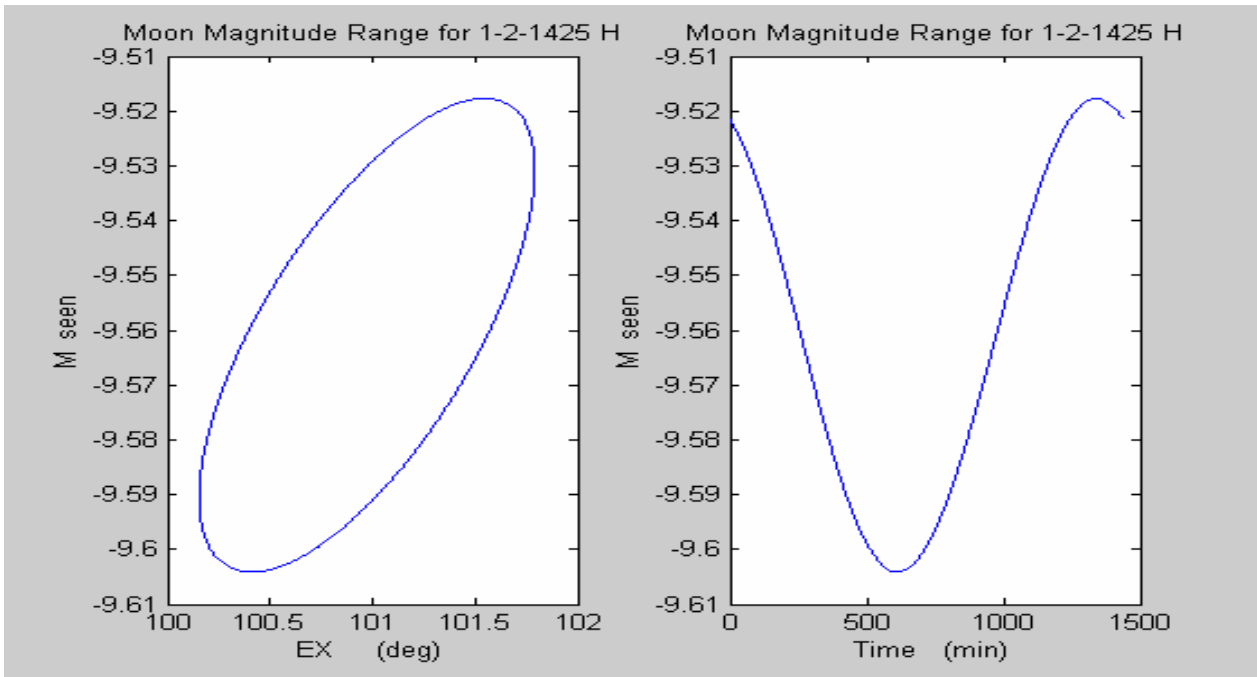


Figure 3.15 for 1-2-1425 H

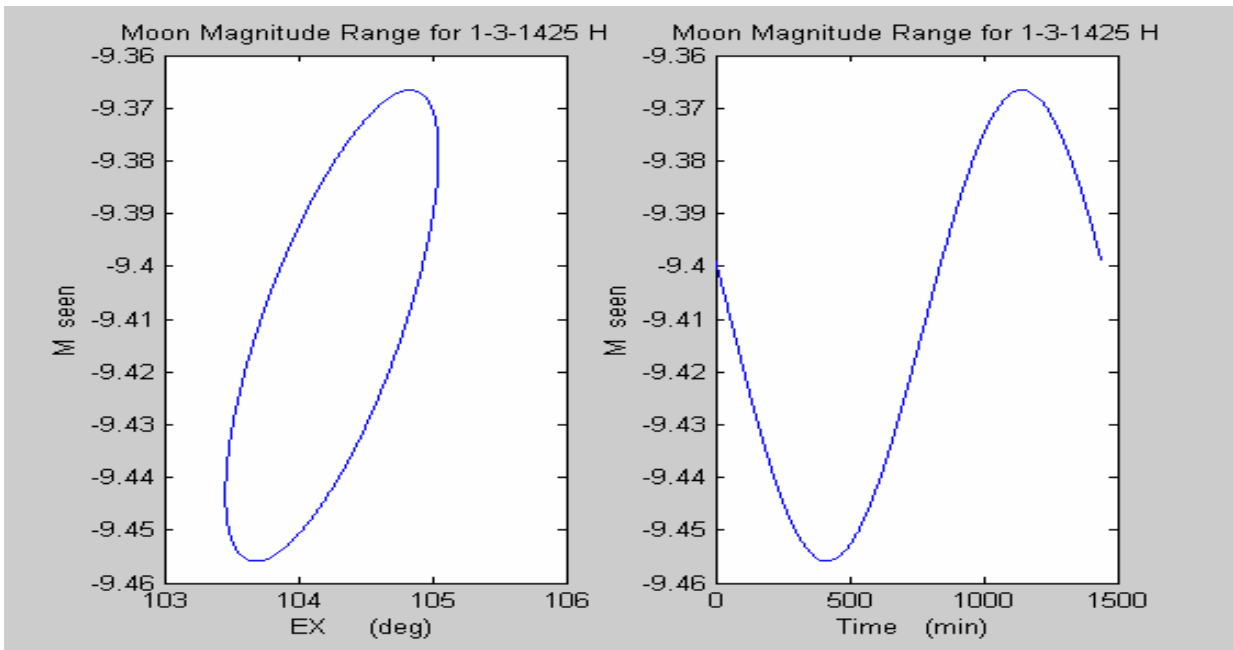


Figure 3.16 for 1-3-1425 H

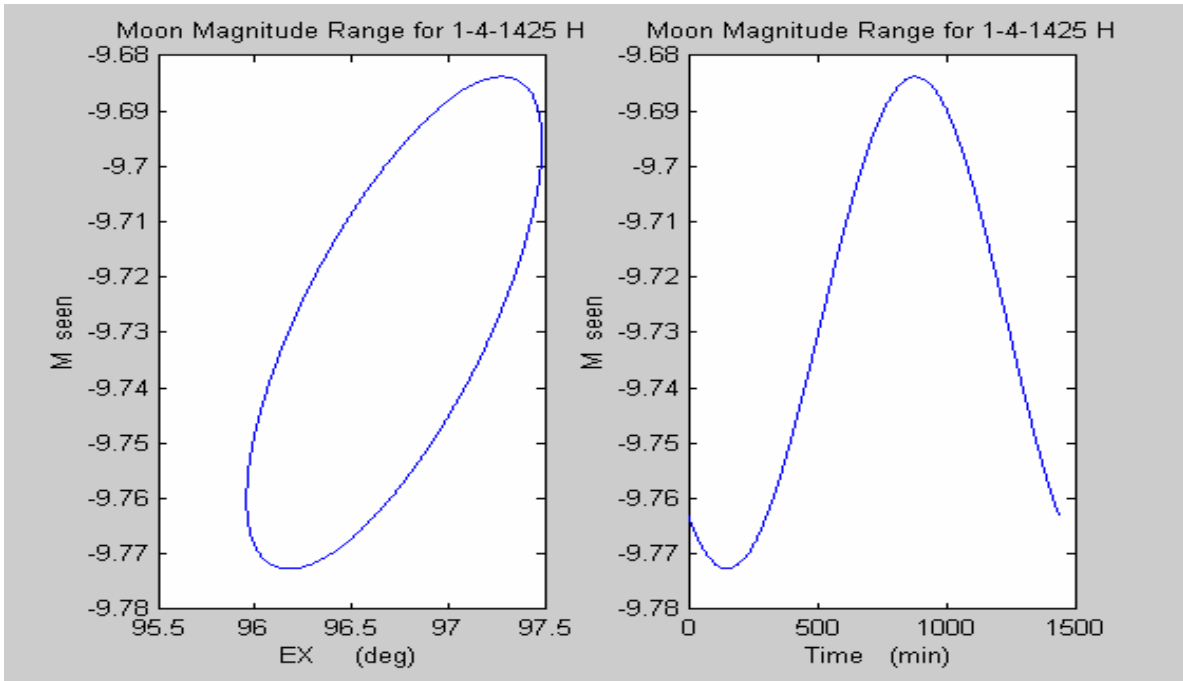


Figure 3.17 for 1-4-1425 H

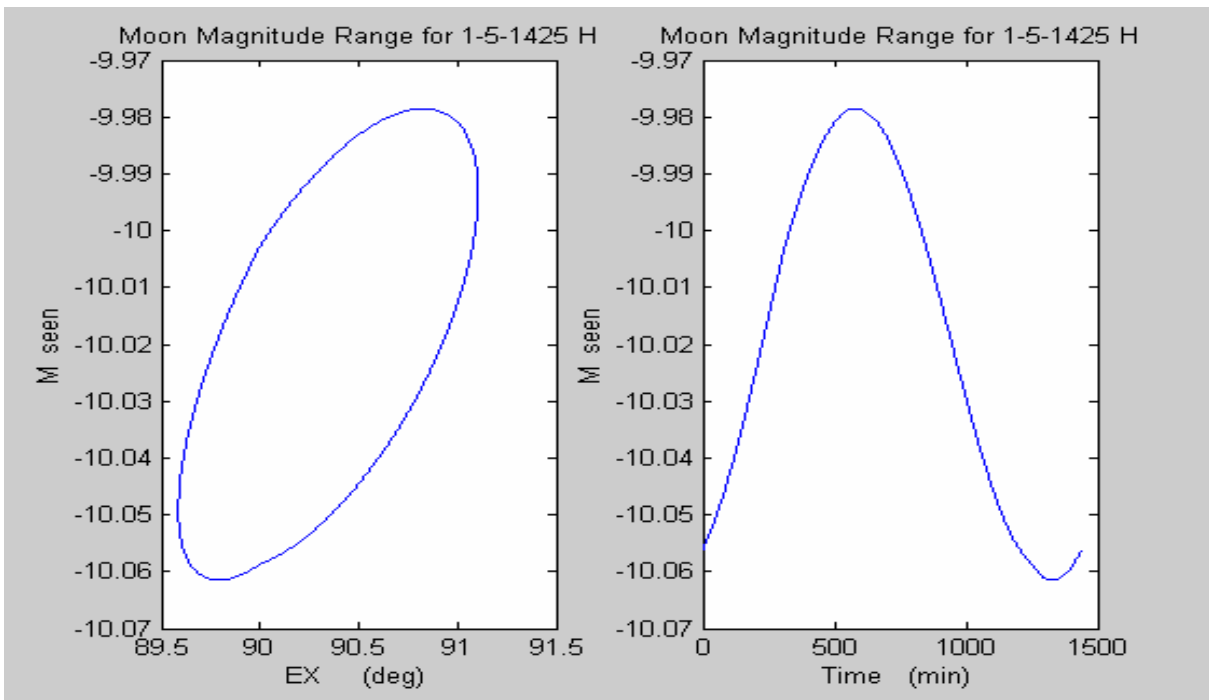


Figure 3.18 for 1-5-1425 H

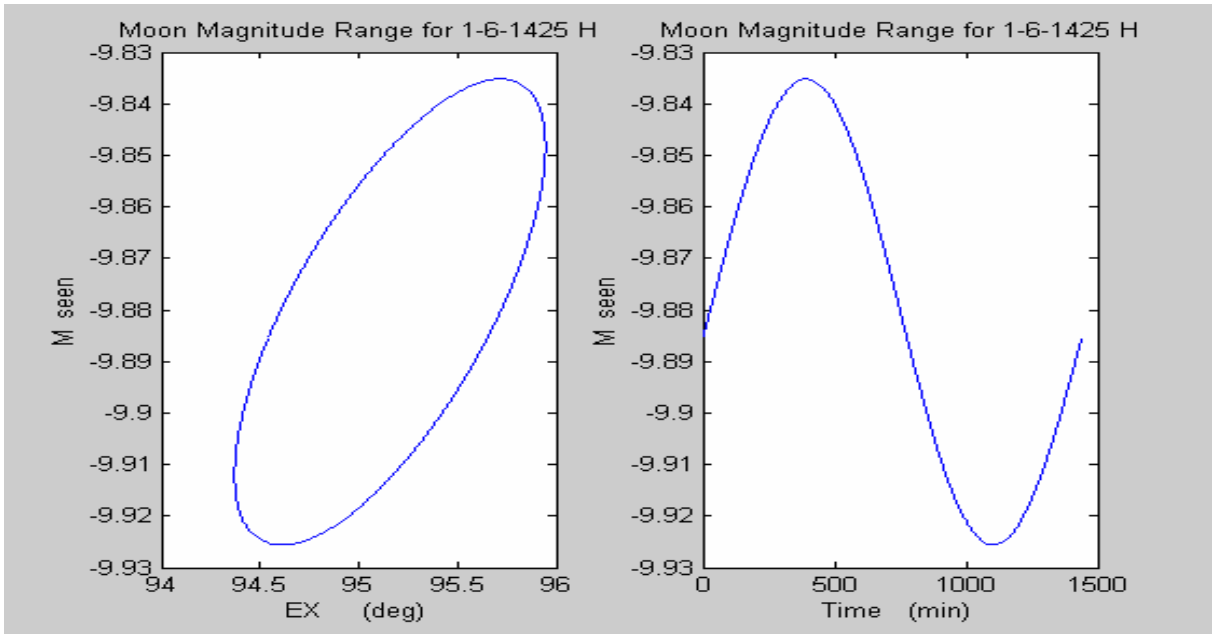


Figure 3.19 for 1-6-1425 H

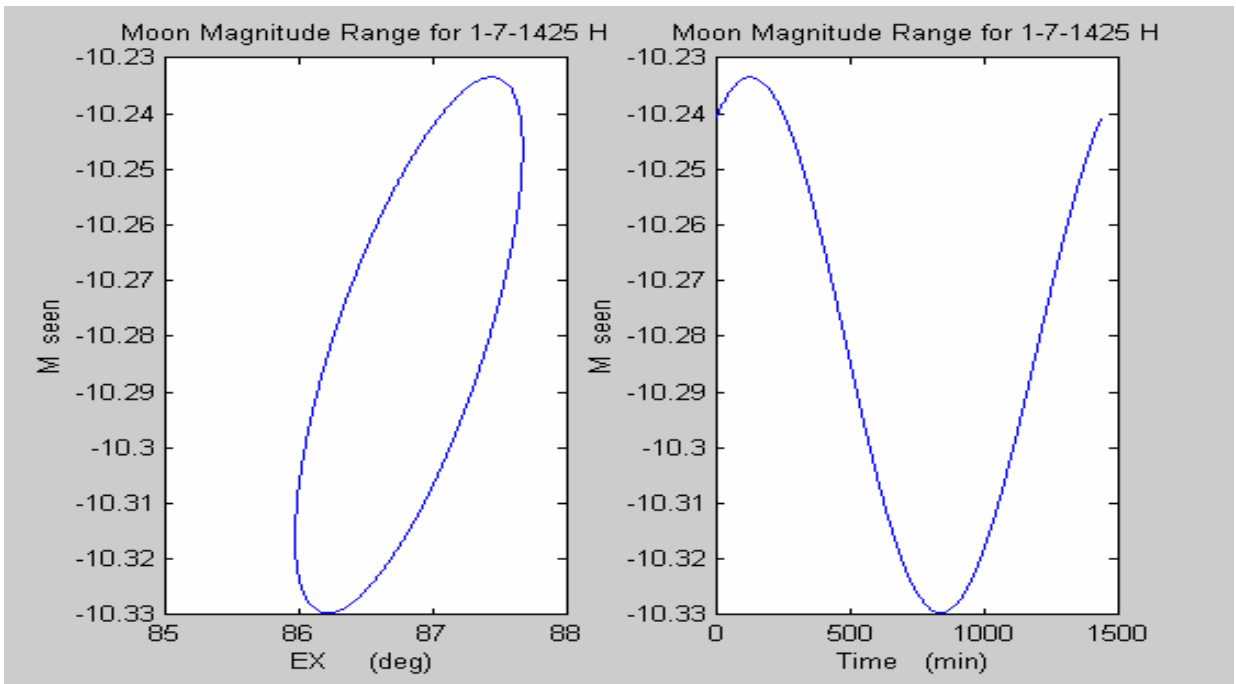


Figure 3.20 for 1-7-1425 H

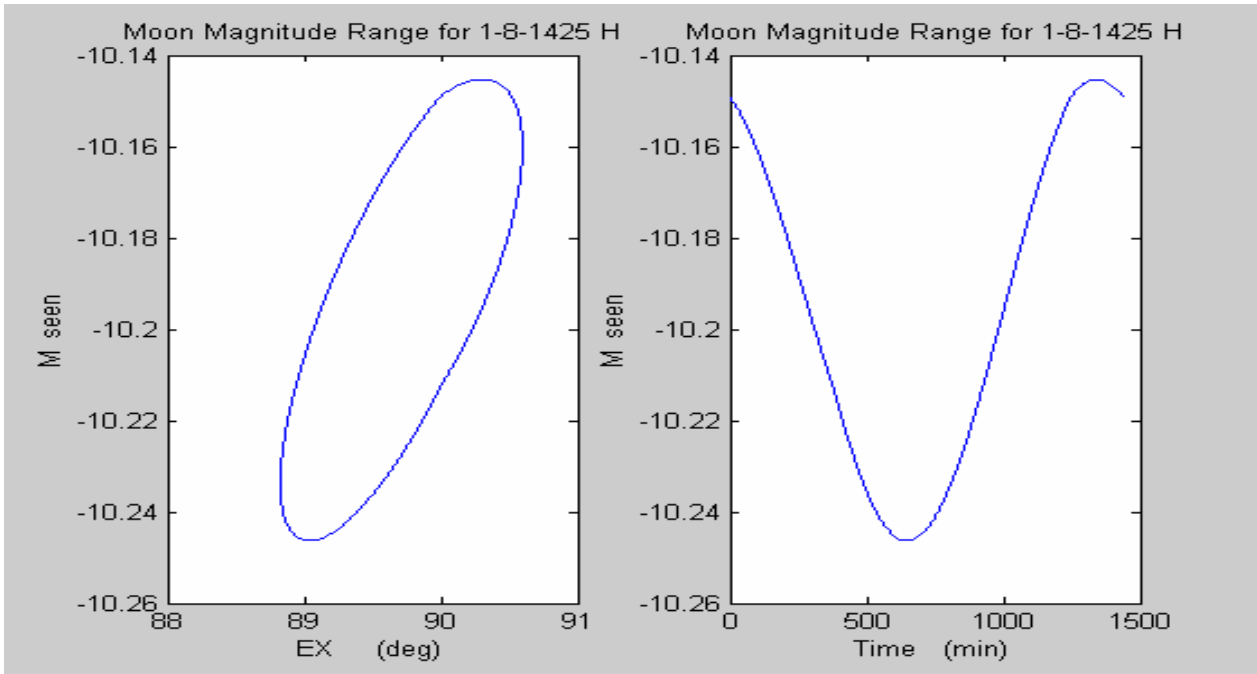


Figure 3.21 for 1-8-1425 H

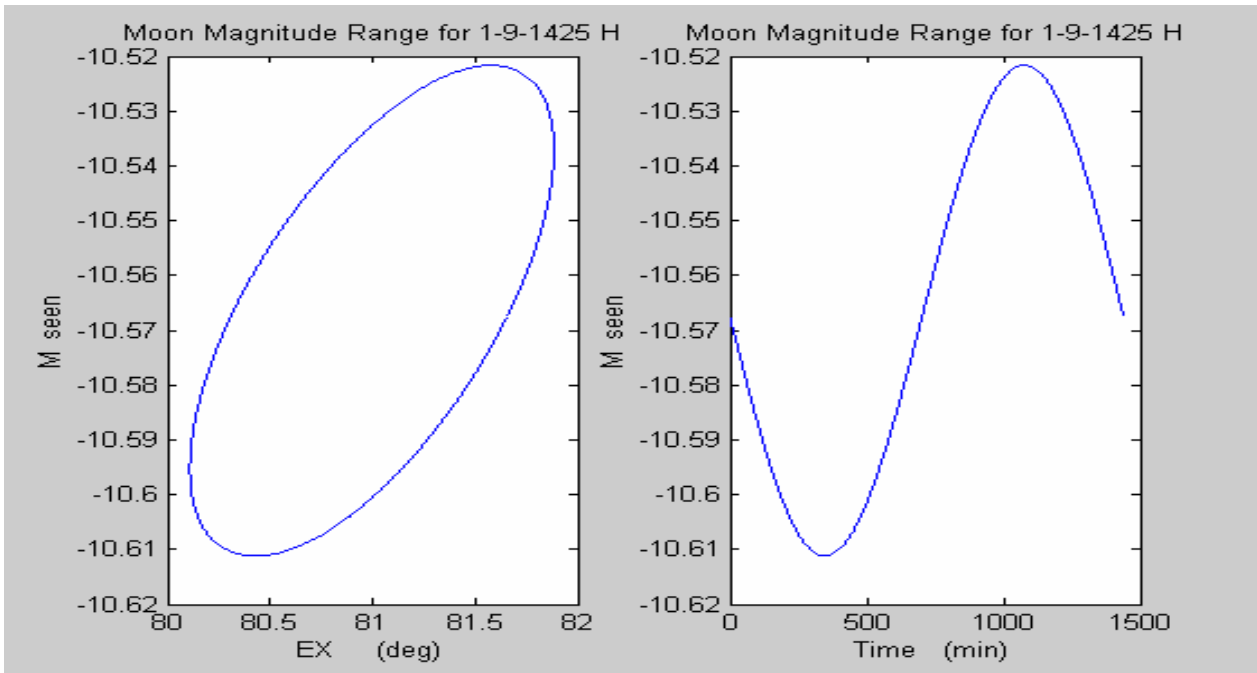


Figure 3.22 for 1-9-1425 H

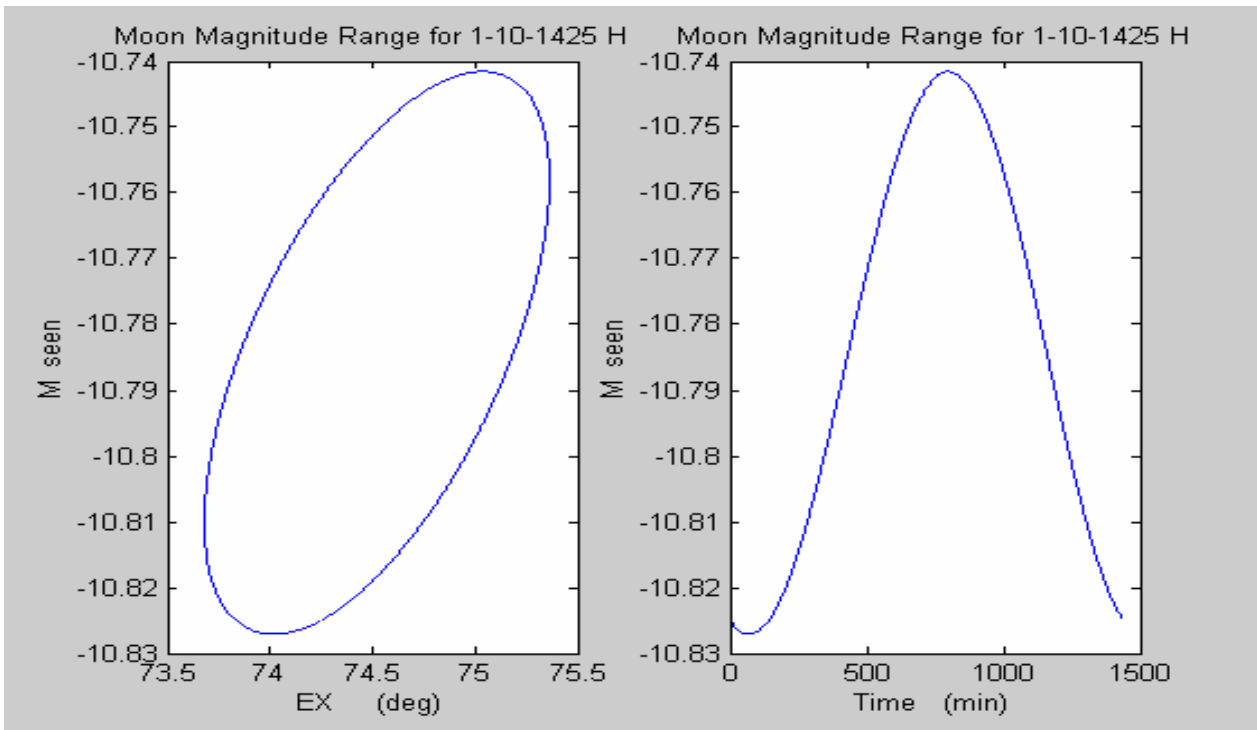


Figure 3.23 for 1-10-1425 H

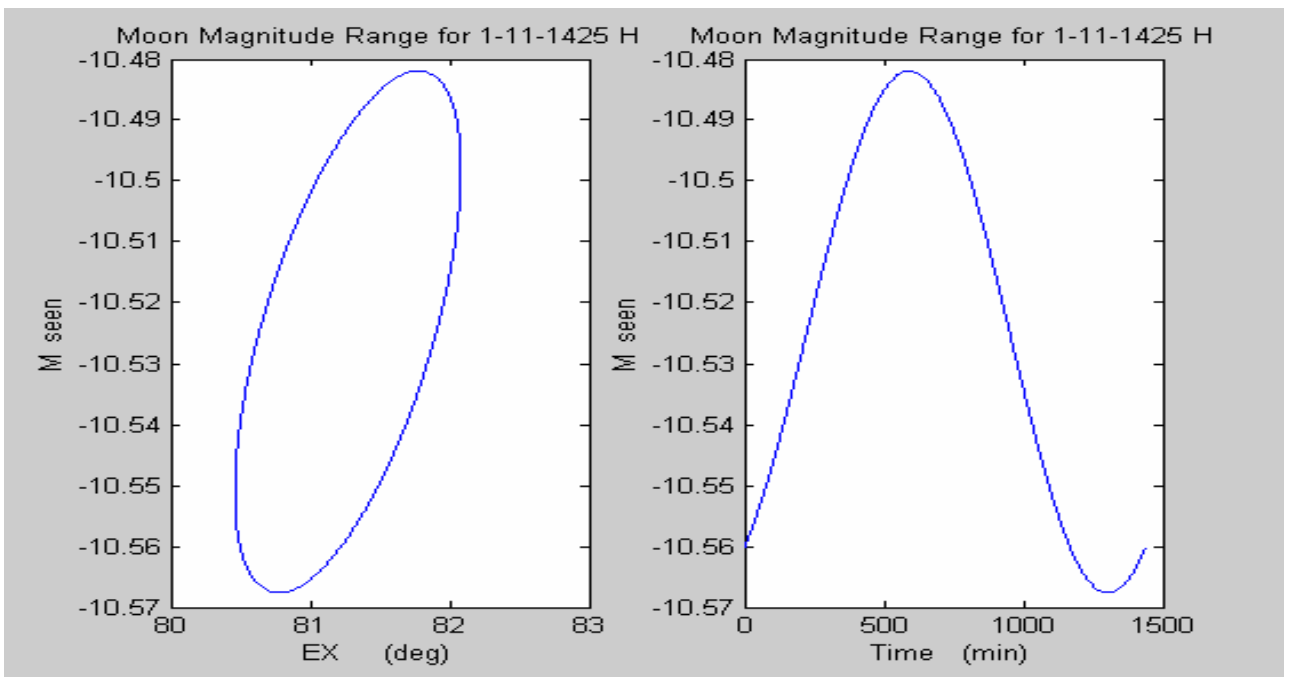


Figure 3.24 for 1-11-1425 H

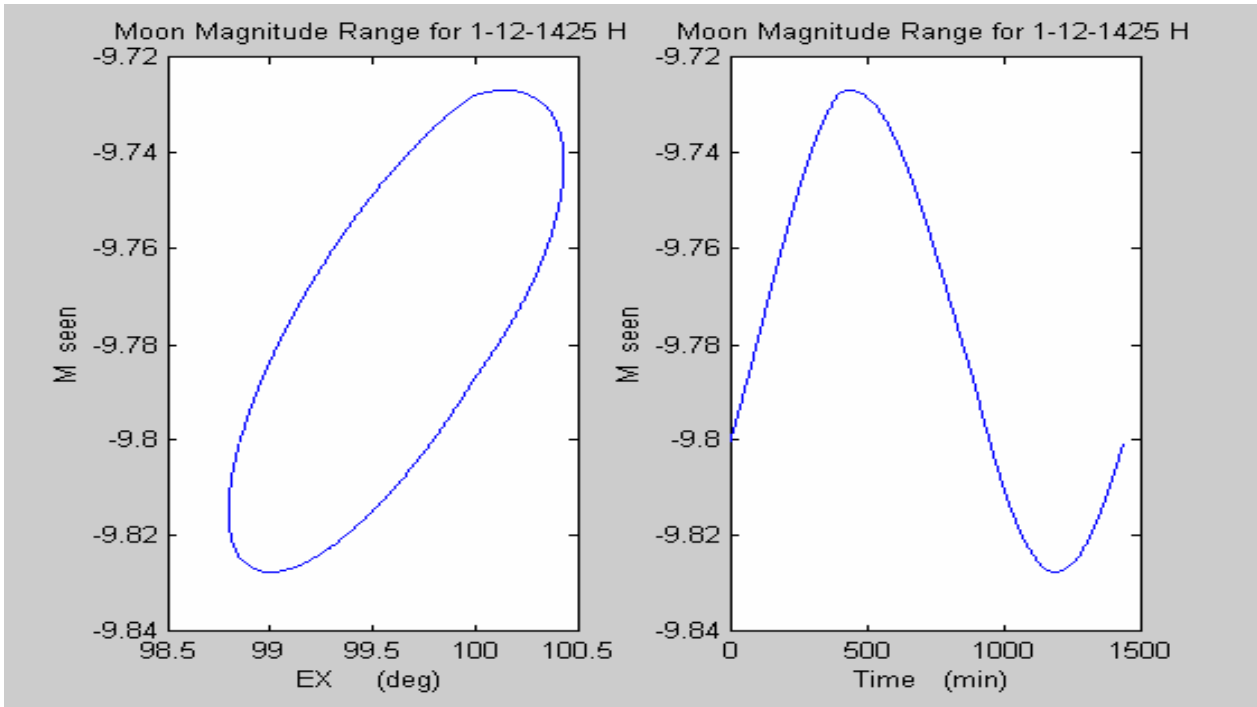


Figure 3.25 for 1-12-1425 H

As shown in above figures from figure 3.13 to figure 3.25 the seen moon magnitude is shown for every month for 24 hours of year 1245 H; this value indicates that the moon could be seen within this range. They give the minimum and the maximum value of M_{seen} . The data is tabulated in table 3.6.

Table 3.6 M_{seen} for year 1425 H

Hijri (d-m-y)	M_{seen_mean}	M_{seen_max}	M_{seen_min}
1 - 1 - 1425	-8.0908	-8.1528	-8.0287
1 - 2 - 1425	-9.5607	-9.6041	-9.5177
1 - 3 - 1425	-9.411	-9.4559	-9.3666
1 - 4 - 1425	-9.728	-9.7727	-9.6838
1 - 5 - 1425	-10.0198	-10.0615	-9.9785
1 - 6 - 1425	-9.8802	-9.9256	-9.8352
1 - 7 - 1425	-10.2813	-10.3296	-10.2334
1 - 8 - 1425	-10.1953	-10.2463	-10.1453
1 - 9 - 1425	-10.5661	-10.6112	-10.5217
1 - 10 - 1425	-10.784	-10.8271	-10.7416
1 - 11 - 1425	-10.5245	-10.5676	-10.4819
1 - 12 - 1425	-9.7769	-9.8275	-9.7268
Average	-9.90155	-9.948491667	-9.8551

→ For year 1424 H:

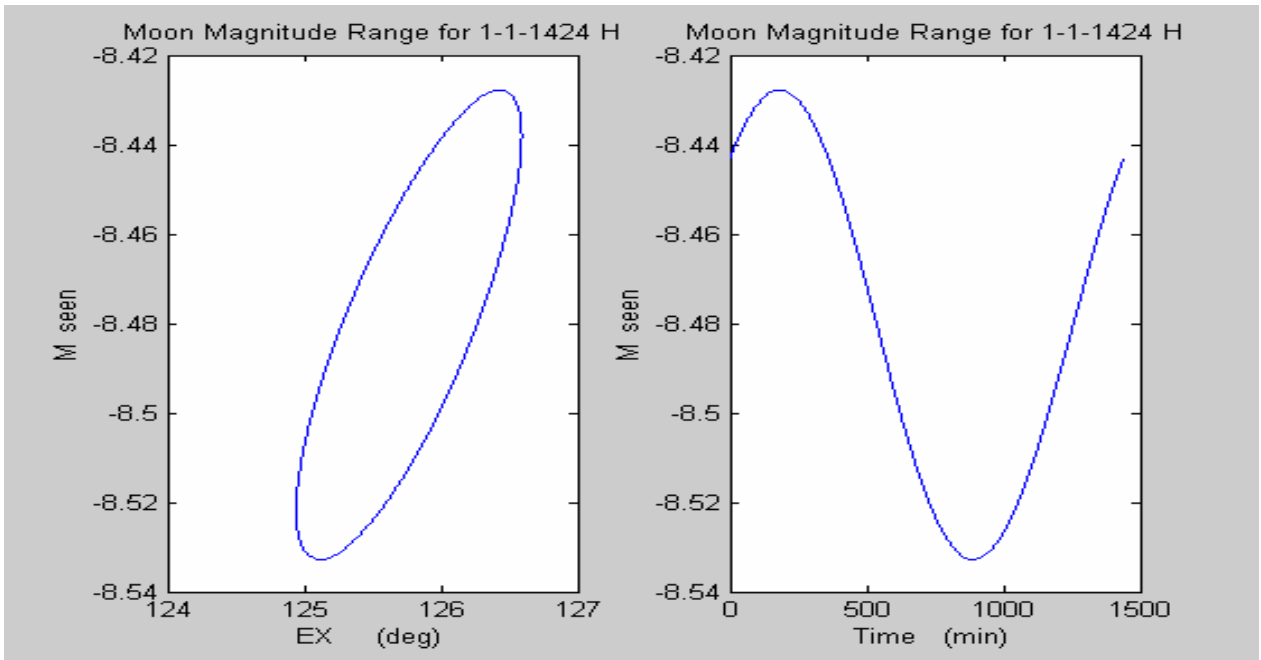


Figure 3.26 for 1-1-1424 H

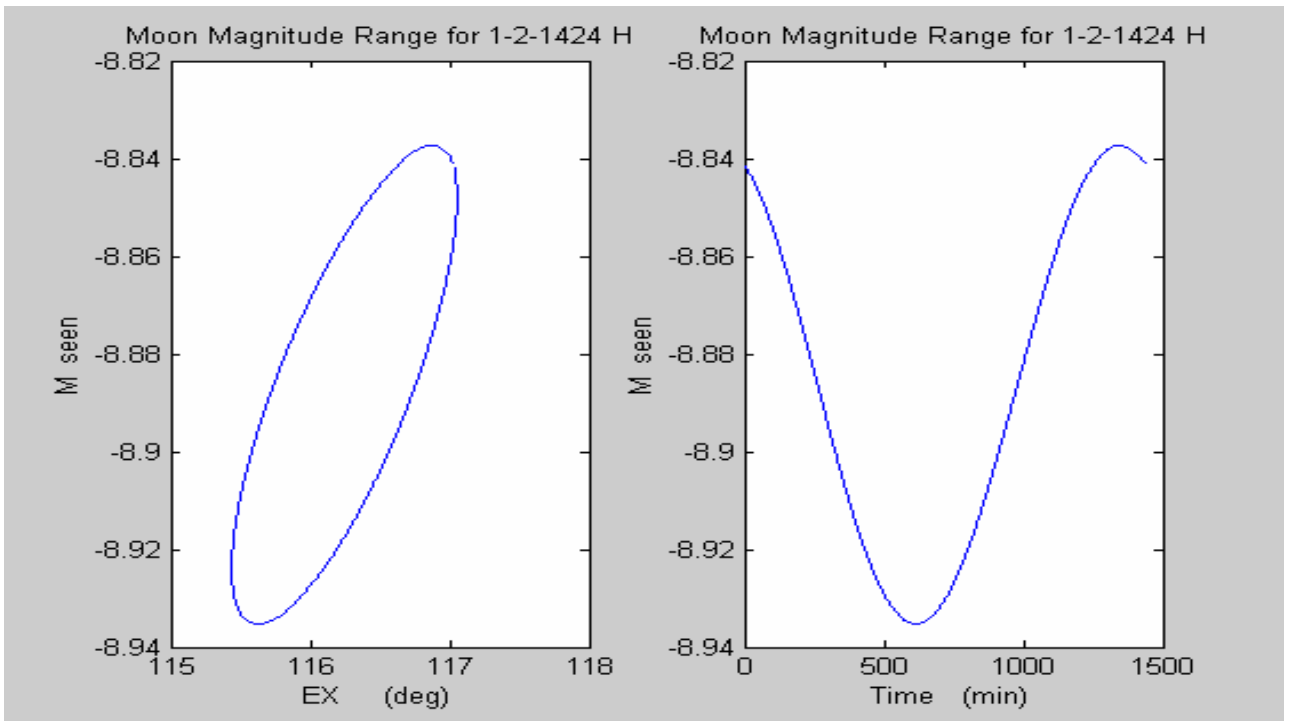


Figure 3.27 for 1-2-1424 H

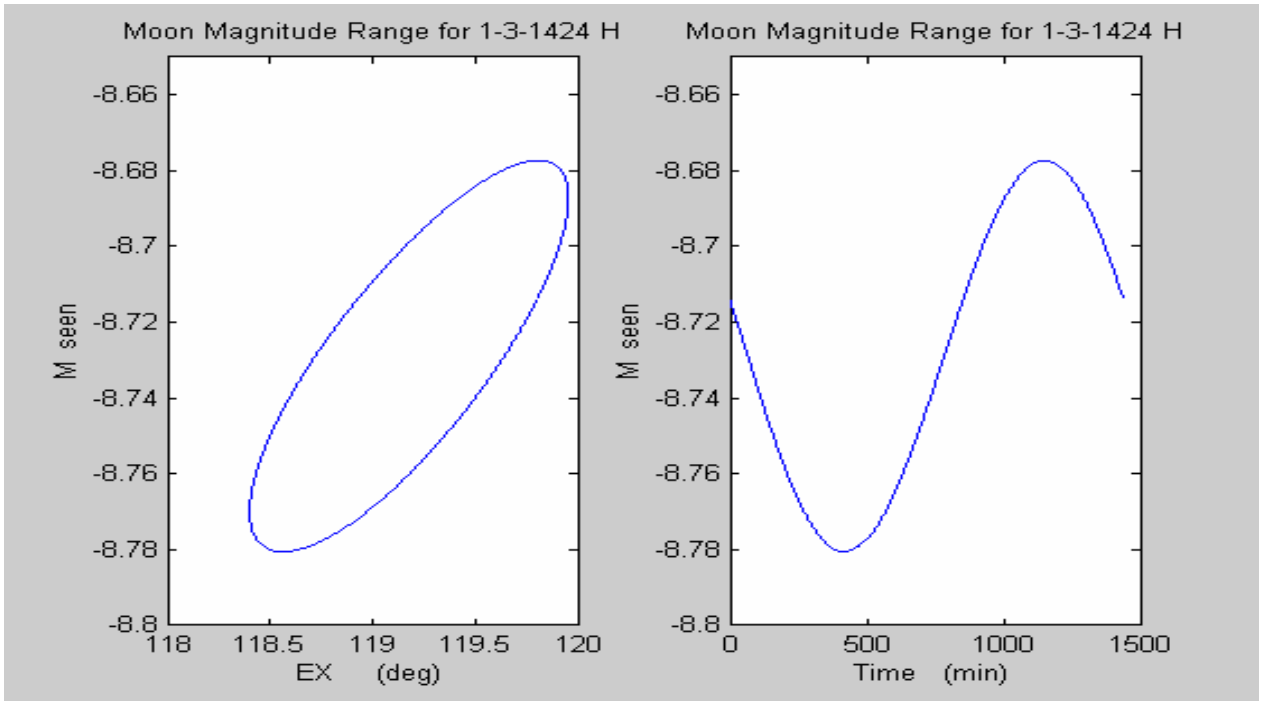


Figure 3.28 for 1-3-1424 H

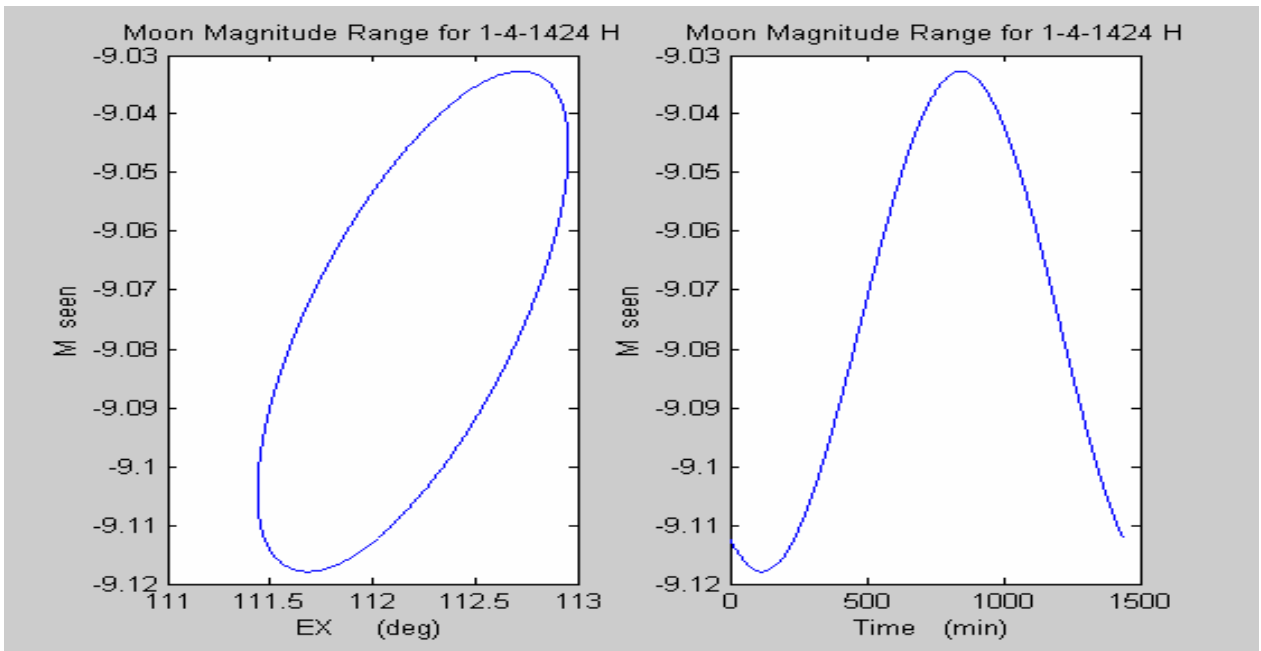


Figure 3.29 for 1-4-1424 H

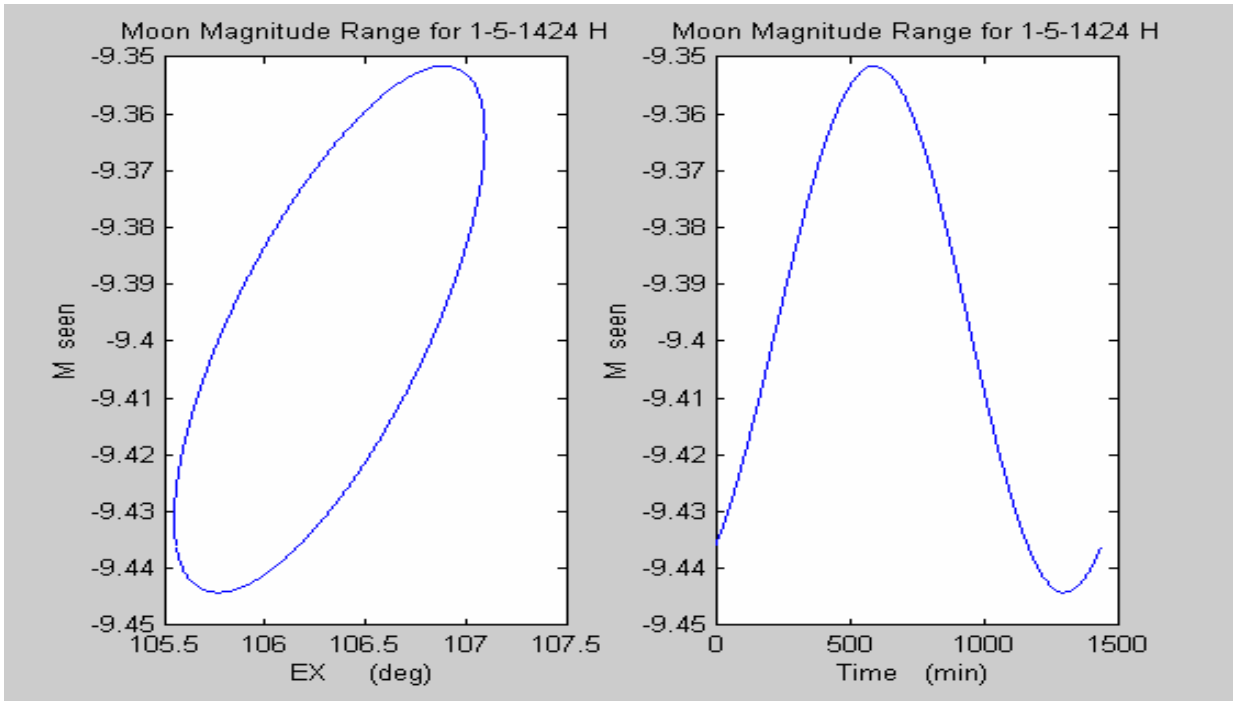


Figure 3.30 for 1-5-1424 H

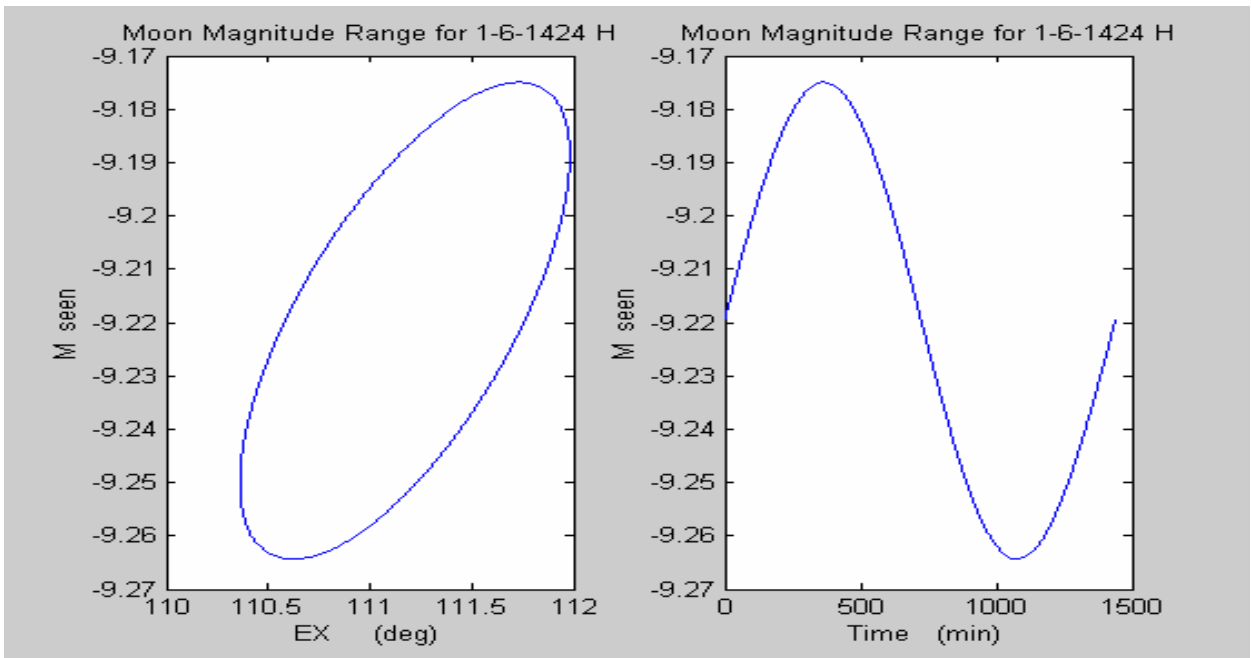


Figure 3.31 for 1-6-1424 H

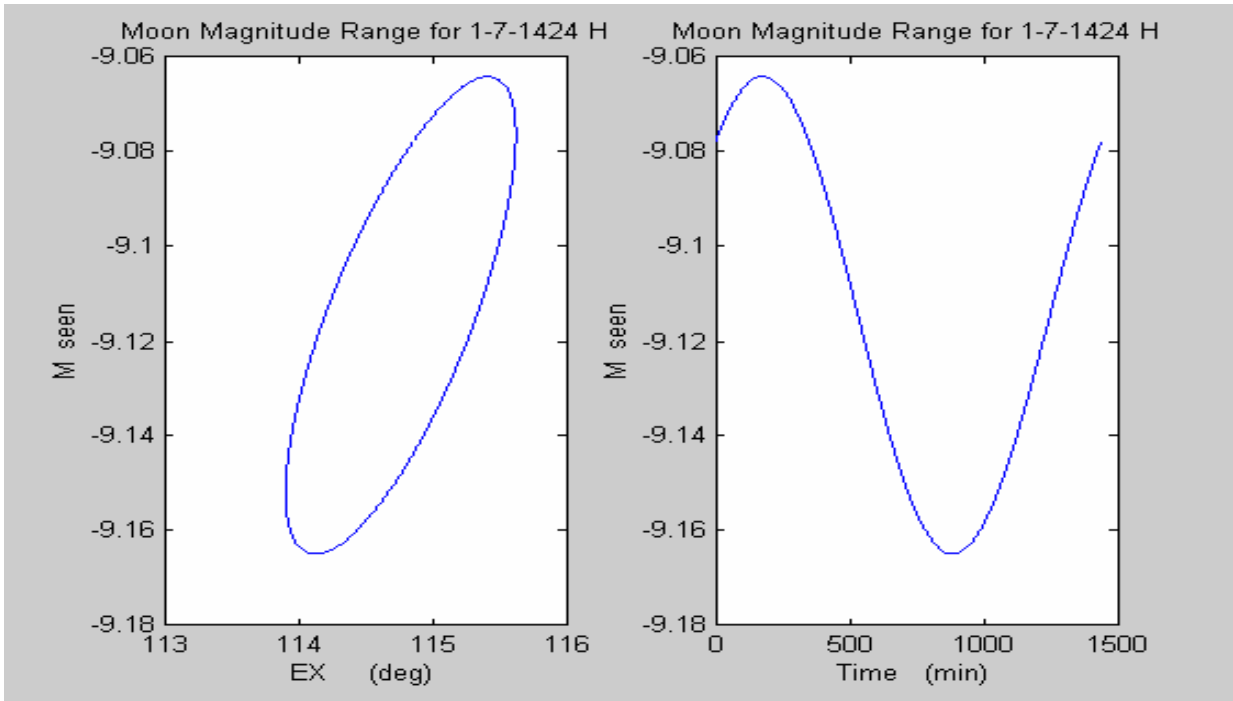


Figure 3.32 for 1-7-1424 H

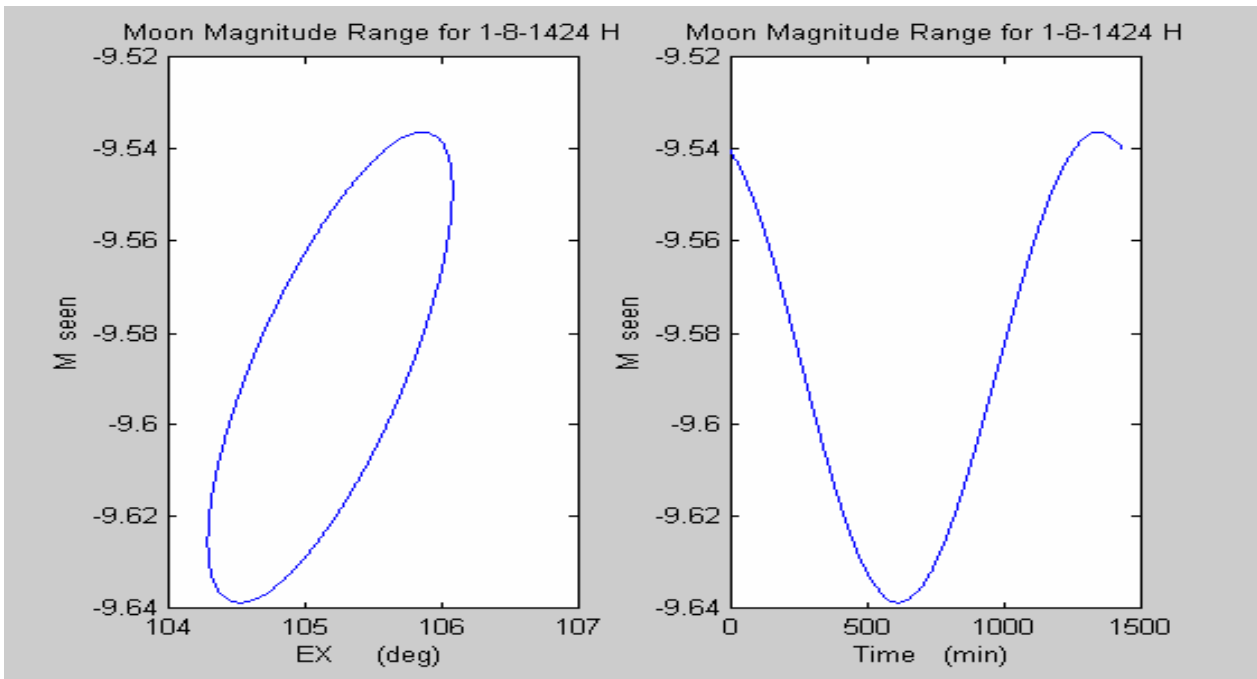


Figure 3.33 for 1-8-1424 H

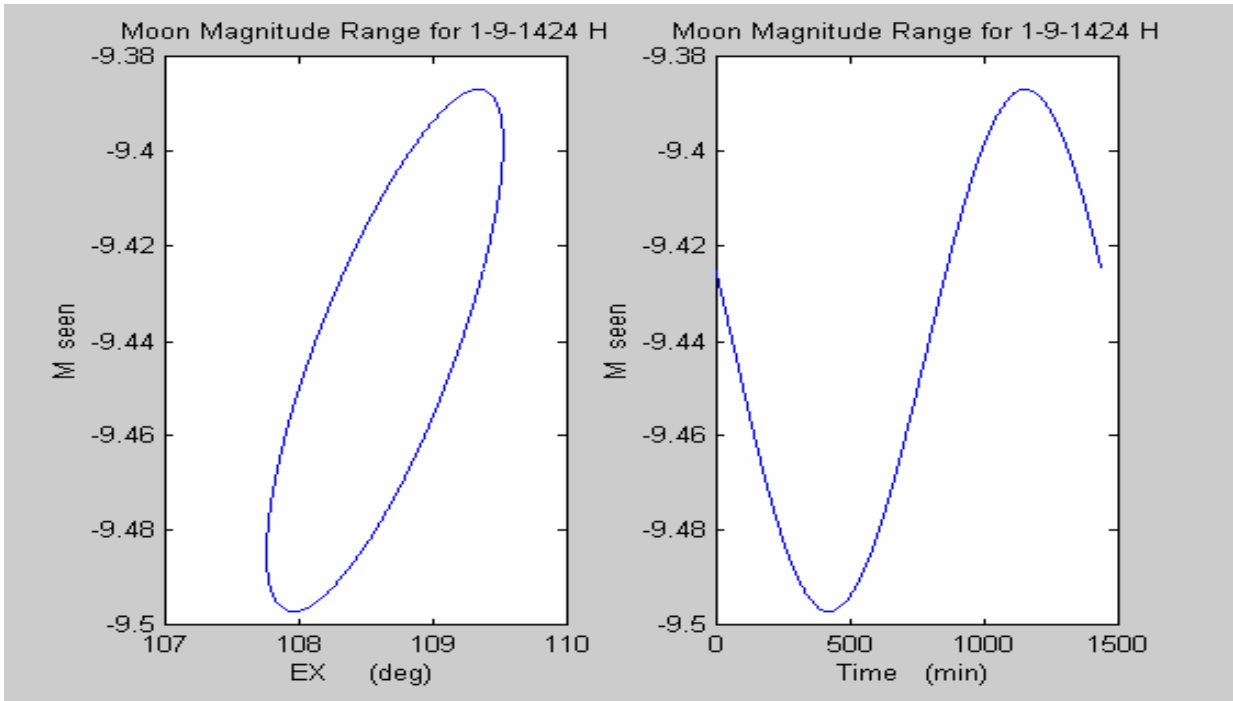


Figure 3.34 for 1-9-1424 H

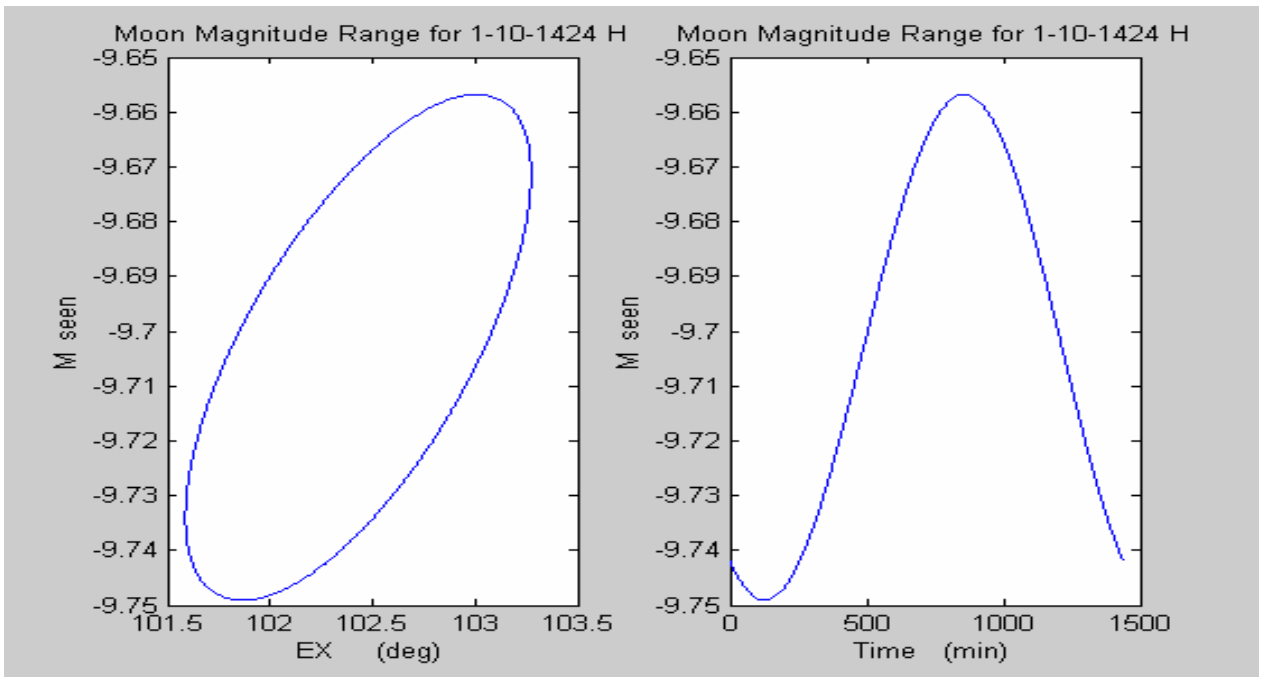


Figure 3.35 for 1-10-1424 H

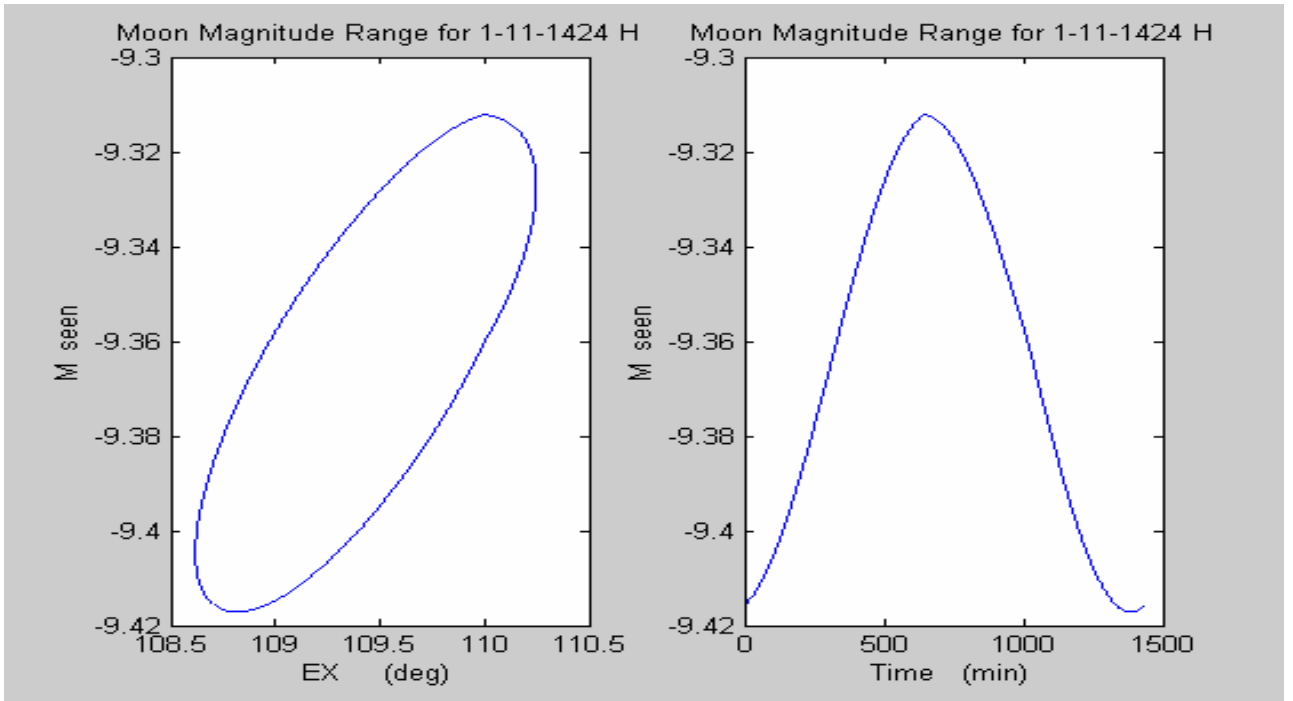


Figure 3.36 for 1-11-1424 H

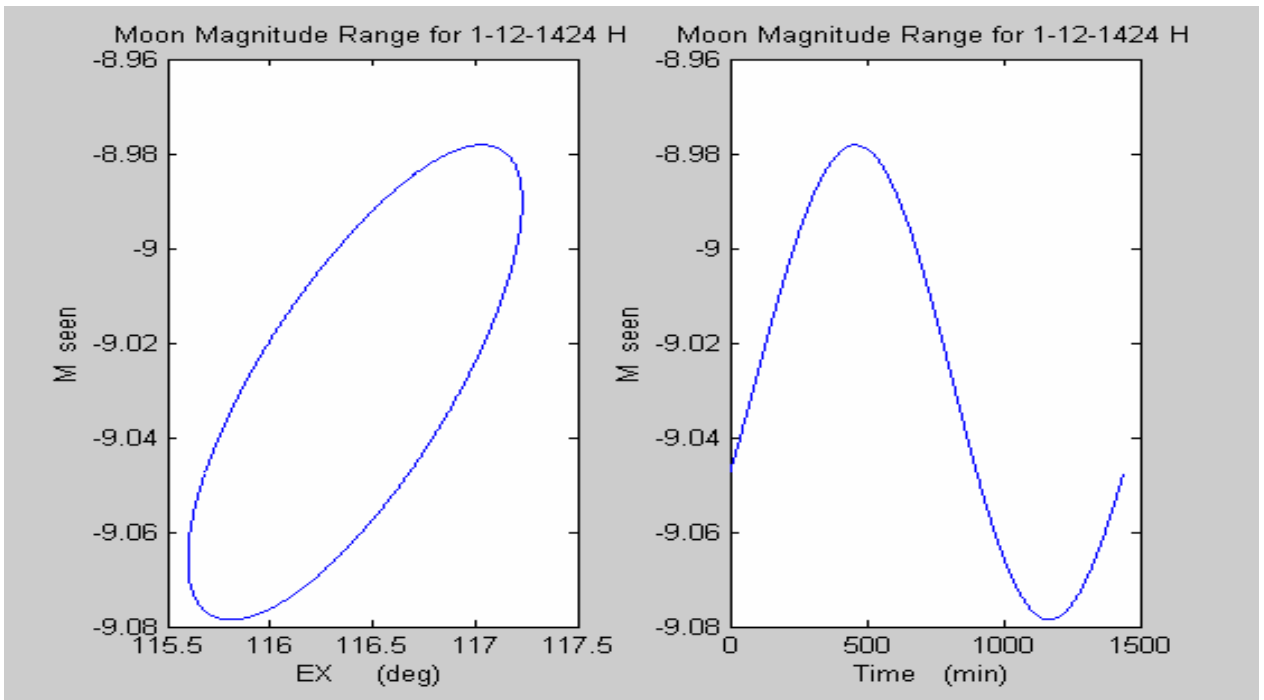


Figure 3.37 for 1-12-1424 H

As shown in above figures from figure 3.26 to figure 3.37 the seen moon magnitude is shown for every month for 24 hours of year 1244 H; this value indicates that the moon could be seen within this range. They give the minimum and the maximum value of M_{seen} . The data is tabulated in table 3.7.

Table 3.7 M_{seen} for year 1424 H

Hijri (d-m-y)	$M_{\text{seen_mean}}$	$M_{\text{seen_max}}$	$M_{\text{seen_min}}$
1 - 1 - 1424	-8.4802	-8.5328	-8.4278
1 - 2 - 1424	-8.8861	-8.9352	-8.8373
1 - 3 - 1424	-8.7288	-8.7805	-8.6773
1 - 4 - 1424	-9.0751	-9.1179	-9.0327
1 - 5 - 1424	-9.3977	-9.4442	-9.3516
1 - 6 - 1424	-9.2194	-9.2644	-9.175
1 - 7 - 1424	-9.1143	-9.1649	-9.0641
1 - 8 - 1424	-9.5872	-9.6386	-9.5362
1 - 9 - 1424	-9.4418	-9.497	-9.3868
1 - 10 - 1424	-9.7027	-9.7491	-9.6568
1 - 11 - 1424	-9.3649	-9.4171	-9.3122
1 - 12 - 1424	-9.0282	-9.0786	-8.9781
Average	-9.168866667	-9.218358333	-9.119658333

→ We calculate the average values of both years 1424 H and 1425 H and use the new value and we compare it to what we get in year 1426 H and calculate the percentage error as shown in table 3.8.

Table 3.8 average value of years 1424 and 1425 H

Year (H)	M_seen_mean	M_seen_max	M_seen_min
1424	-9.168866667	-9.218358333	-9.119658333
1425	-10.19083229	-10.11551979	-9.8551
Average	-9.679849479	-9.666939063	-9.487379167

While the values of year 1426 H are shown in table 3.9 as follows:

Table 3.9 average value of year 1426 H

Year 1426 H	M_seen_mean	M_seen_max	M_seen_min
Average	-10.82077778	-10.86290556	-10.77925648

4. DISCUSSION:

By looking to tables 3.8 and 3.9 we note that the values of moon seen magnitude M_{seen} for the three years are very close to each other and it is about 10 magnitude which indicates actually to same result in experimental data done by Mr. Daniel Mortari (ref.1).so if we pick the point -10 for example and we look at the corresponding angle value to it in figure 3.1, we find it to be approximately 95 deg which actually matches the above figures for the different three years.

Also by looking to the mean value of M_{seen} for the different years, we notice that it lays between the mean minimum and mean maximum value of M_{seen} for the three years.

Also we test the result we got from the year 1426 H (table 3.9) and the average result we got from years 1425 H and 1424 H (table 3.8) to calculate the percentage error to see weather the difference is in the acceptable range or there is a big difference in the value of moon seen magnitude (M_{seen}).

$$\text{Error \%} = ((-10.82077778 - (-9.679849479)) / -10.82077778) \times 100$$

$$= 10.5 \%$$

We note that the error is very small and neglected and we can say that it is acceptable. Actually that indicates that for the last three years people were almost correct in predicting the beginning of months. The small error may be due to mistakes in predicting the crescent correctly in some months and also due to the approximations in experimental data since the relation that calculates M_{seen} (eq.2) depends on long series to improve its accuracy not only on the given terms, and also we could say that a part of the error came from the computer calculation like rounding and so on. But as a final word, I say we got a great result by using this method and we have now a reference that we can make sure that the moon crescent could be seen only in the approximated range we got here (-10.86290556 , -9.487379167); and we can determine whether the moon crescent could be seen or not in specific day and specific time if we get some data from before that includes the time history for certain months as a next step in this project in soon future.

5. CONCLUSION:

As a conclusion, I say that the purpose of the project was achieved with interesting results by using this method to predict the birth of the moon crescent. And we can get benefit from it in the future in different aspects like serving our religion and Islamic nation in a great manner. However for me I thank Allah for all the success in doing this paper and the success in applying the principles and knowledge of this course and contribute them to related practical beneficial application in our daily life. It was very powerful tool in learning and it helped me a lot in understanding the course contents. Finally, I thank the person who was behind this work and who spent lots of his valuable time and effort for giving me all support and great help **Dr. Ayman Kassem**.

APPENDIX

→Main program:

```
%Enter the year
Y=2004
%Enter the month
MO=1
%Enter the day
D=23

                                % calculating the Julian date:
A=floor(Y/100);
B=floor(A/4);
C=floor(2-A+B);
E=floor(365.25*(Y+4716));
F=floor(30.6001*(MO+1));
jd=C+D+E+F-1524.5

for k=1:1440 % CALCULATING THE LOCATION OF EARTH, MOON AND SUN
FOR 24 HOURS FOR EVERY MINIUTE

    RE(k,:)=earth(jd,k);

    RM(k,:)=moon(jd,k/1440);

    RS(k,:)=sun(jd,k/1440);

    DR_ME=RE(k,:)-RM(k,:);
```

```

VM_ME(k)=norm(DR_ME) ;

DR_MS=RS(k,:)-RM(k,:);

VM_MS(k)=norm(DR_MS);

EX(k)=(acos((dot(DR_ME,DR_MS))/(VM_MS(k)*VM_ME(k))))*180/pi ; %in deg

PEXF=[1 0.929 0.809 0.625 0.483 0.377 0.288 0.225 0.172 0.127 0.089 0.061
0.041 0.027 0.017 0.009 0.004 0.001];

EXF=[0 5 10 20 30 40 50 60 70 80 90 100 110 120 130 140 150 160 ]; %in
deg

PHSE(k)= interp1 (EXF,PEXF,EX(k));

% M_seen is the seen moon magnitude

M_seen(k)= 0.23+5*log10(abs(VM_ME(k)/1.4959965e11))-2.5*log10(PHSE(k));
% we divided VM_ME by the astronomical unit to mach the experimental data

end

subplot(1,2,1),plot(EX,M_seen);title('Moon Magnitude Range for 1-12-1424
H');xlabel('EX (deg)');ylabel('M _ seen')
subplot(1,2,2),plot(M_seen);title('Moon Magnitude Range for 1-12-1424
H');xlabel('Time (min)');ylabel('M _ seen')

```

```
MEAN = mean(M_seen)
```

```
MIN = max(M_seen)% we take care about the negative sine of (M_seen), it is just a notation.
```

```
MAX = min(M_seen)
```

→Earth Subroutine program:

```
function rearth=earth(jd,t)
```

```
EARTH_RADIUS=6378e3 ; % in meters
```

```
T=jd/36525;
```

```
EARTH_ROT=0.25 ; %in deg/min
```

```
long_Dhahran_deg = 50.1325 ; %in deg;
```

```
lat_Dhahran_deg = 26.3042 ; %in deg;
```

```
long_Dhahran = 50.1325*pi/180 ;%in rad;
```

```
lat_Dhahran = 26.3042*pi/180 ; %in rad;
```

```
GST_0=99.6910+36000.7689*T+0.0004*T^2; %in deg
```

```
GST = GST_0 + EARTH_ROT * t; %in deg
```

```

long_e = long_Dhahran - GST*pi/180 ; % in rad;
lat_e = lat_Dhahran ; %in rad; it is fixed (no change in latitude with time)

rearth(1)=EARTH_RADIUS*cos(lat_e)*cos(long_e);
rearth(2)=EARTH_RADIUS*cos(lat_e)*sin(long_e);
rearth(3)=EARTH_RADIUS*sin(lat_e);

```

→Moon Subroutine program:

```

% t : time difference in (days) between the time at which the computation
% is required and the the reference time (Jan. 1, 1900, at 12:00; Julian date=2415010)
% theta : the rotation angle
%
% i : inclination;

function rmoon = moon ( jd,k);

    twopi    =  2.0*pi;
    deg2rad  =  pi/180.0;

    t=jd-2415010;

```

```

theta=270.434164+13.1763965268*t-8.5e-13*t^2+3.9e-20*t^3 ; %deg;

s_omega=334.329356+0.1114040803*t-7.739e-12*t^2-2.6e-19*t^3 ; %deg;

c_omega=259.183275-0.0529539222*t+1.557e-12*t^2+5e-20*t^3 ; %deg

i=5.145396374 ; %deg

theta_m = deg2rad *theta; %rad

s_omega_m = deg2rad * s_omega ; %rad

c_omega_m = deg2rad * c_omega ; %rad

i_m = deg2rad * i ; %rad

aa=384400e3 ; %(average distance between the moon and earth in meters)
ee=0.054900489 ; %(eccentricity)
pp=aa.*(1-ee.^2);
rr=pp./(1+ee.*cos(theta_m));

xx=rr.*cos(theta_m);
yy=rr.*sin(theta_m);
zz=[0];

```



```

T=[cos(s_omega_m).*cos(c_omega_m)-
sin(s_omega_m).*cos(i_m).*sin(c_omega_m),cos(s_omega_m).*sin(c_omega_m)+sin(s_
omega_m).*cos(i_m).*cos(c_omega_m),sin(s_omega_m).*sin(i_m);

-sin(s_omega_m).*cos(c_omega_m)-cos(s_omega_m).*cos(i_m).*sin(c_omega_m),-
sin(s_omega_m).*sin(c_omega_m)+cos(s_omega_m).*cos(i_m).*cos(c_omega_m),cos(s
_omega_m).*sin(i_m);

sin(i_m).*sin(c_omega_m), -sin(i_m).*cos(c_omega_m),  cos(i_m)];

TT=T';

rmoon(1)=TT(1,:)*[xx;yy;zz];
rmoon(2)=TT(2,:)*[xx;yy;zz];
rmoon(3)=TT(3,:)*[xx;yy;zz];

```

→ Sun Subroutine program:

```

% sun axes
% t : time.
% M : meananomaly
% i : inclination;
function rsun = sun ( jd,k );

    twopi    =  2.0*pi;
    deg2rad  =  pi/180.0;

    t= ( jd - 2451545.0 )/ 36525.0;

    meanlong= 280.4606184 + 36000.77005361 *t;
    meanlong= meanlong-360*fix(meanlong/360.0); %deg

    M= 357.5277233 + 35999.05034 *t;
    M= M*deg2rad-twopi*fix(M*deg2rad/twopi); %rad

    if ( M < 0.0 )
        M= twopi + M;
    end

    long= meanlong + 1.914666471 *sin(M) ...
        + 0.019994643 *sin(2.0 *M); %deg

    i= 23.439291 - 0.0130042 *t; %deg

    meanlong = meanlong*deg2rad;
    if ( meanlong < 0.0 )

```

```

    meanlong= twopi + meanlong;
end
long = long *deg2rad;
i= i *deg2rad;

% ----- find magnitude of sun vector components -----
magr= 1.000140612 - 0.016708617 *cos( M ) ...
      - 0.000139589 *cos( 2.0 *M ); % in au's
magr=magr*1.4959965*10^11; % in meters

rsun(1)= magr*cos( long );
rsun(2)= magr*cos(i)*sin(long);
rsun(3)= magr*sin(i)*sin(long);

```

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1. Moon –sun attitude sensor, by Daniele Mortari, journal spacecraft and rockets 1997 0022-4650 vol.34 no.3 (360-364).
2. Spaceflight dynamics, by William E. Wiesel.
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