Tuning of a Nonlinear Energy Sink using multi-stability

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Abstract. This work addresses the development of a nonlinear absorber based on the concept of a Nonlinear Energy Sink (NES) under different multi-stable configurations. The potential energy of the system can be written under a potential that possesses two or more stable equilibrium positions separated by saddle points. The study of the dynamics at different time scales leads to the computation of the Slow Invariant Manifold (SIM) where singular points indicate the possible occurrence of Strongly Modulated Responses (SMR). It is shown that, compared to a bi-stable NES, the threshold activation of a multistable NES can be significantly lowered with same dissipation ability.

Introduction

Since the seminal papers by Vakakis *et al* [1, 2], targeted energy transfer, or energy pumping, has become a subject of growing interest. Despite highly efficient energy dissipation, the main drawback is that the higher the frequency of the primary linear system to control, the higher the amplitude for activation of non linear passive dissipation. Recent theoretical and numerical works by Manevitch *et al* [3], Romeo *et al* [4] and experimental work by Mattei *et al* [5] showed that a bi-stable NES (B-NES) provides improved robustness in frequency and amplitude range over existing NESs by lowering the activation threshold. We show that, by a proper shaping of the system underlying Hamiltonian, the threshold activation of a tri-stable NES can be significantly lowered with same dissipation ability.

Description of the model

The system under consideration is composed by a linear structure of unit mass and stiffness, damping λ_1 whose displacement is $x(t)$ under sinusoidal forcing with amplitude F and angular frequency Ω. This system is coupled to a strongly nonlinear oscillator of mass ϵ, damping λ_N and linear stiffness κ_N whose displacement is $y(t)$. The nonlinear stiffness is described by a polynomial function $F_N(y(t))$ whose degree and coefficients depend on the problem under consideration. The barycentric coordinates $v(t) = x(t) + \epsilon y(t)$ and $w(t) = x(t) - y(t)$ are solutions of:

$$\ddot{v}(t) + \epsilon\lambda_1 \frac{\dot{v}(t) + \epsilon\dot{w}(t)}{1+\epsilon} + \frac{v(t) + \epsilon w(t)}{1+\epsilon} = \epsilon F \sin(\Omega t), \quad (1)$$

$$\ddot{w}(t) + \epsilon\lambda_1 \frac{\dot{v}(t) + \epsilon\dot{w}(t)}{1+\epsilon} + \frac{v(t) + \epsilon w(t)}{1+\epsilon} + \lambda_N(1+\epsilon)\dot{w}(t) + \kappa_N(1+\epsilon)\mathcal{F}\left(-w(t)\right) = 0. \quad (2)$$

Results

We consider the problem close the linear resonance: $\Omega = 1 + \epsilon\sigma$, where σ is a detuning parameter. The motion of the system is analysed by using the complexification-averaging method of Manevitch under 1:1 resonance. The complex variables of Manevitch are introduced under polar form $\Phi_1(t)\exp(\imath\Omega t) = \dot{v}(t) + \imath\Omega v(t)$ and $\Phi_2(t)\exp(\imath\Omega t) = \dot{w}(t) + \imath\Omega w(t)$. To save place, in the rest of the text, the time dependence of Φ_1, Φ_2, v and w is cancelled. These representations are introduced into Eq. (1) and Eq. (2) and an averaging over the fast frequency is done. For ϵ small, a multiple-scale expansion with fast time $t_0 = \epsilon$ and slow time $t_1 = \epsilon t$, $d/dt = \partial/\partial t_0 + \epsilon\partial/\partial t_1$ and $\Phi_i = \phi_i + \epsilon\psi_i$, $i = 1, 2$ gives to the leading order ϵ^0:

$$\frac{\partial\phi_1}{\partial t_0} = 0, \frac{\partial\phi_2}{\partial t_0} + \frac{\imath}{2}\left(\phi_2 - \phi_1\right) + \frac{\lambda_N}{2}\phi_2 - \frac{\imath\kappa_N}{2}\phi_2\mathcal{F}\left(\phi_2\right) = 0, \quad (3)$$

where $\mathcal{F}\left(\phi_2\right)$ is a nonlinear function that depends on the non-linearity of the NES. The first relation of system (3) gives $\phi_1 = \phi_1(t_1, t_2, \cdots)$. Introducing this result in the second relation of system (3) shows that the fixed points of this equation depends only on slow time t_1 and are given by

$$\frac{\imath}{2}\left(\phi_2 - \phi_1\right) + \frac{\lambda_N}{2}\phi_2 - \frac{\imath\kappa_N}{2}\phi_2\mathcal{F}\left(\phi_2\right) = 0 \quad (4)$$

ϕ_1 and ϕ_2 are written under polar form $\phi_i = A_i\exp(\imath\theta_i)$. Now to solve Eq. (4), the non-linearity shall be defined. In a previous study [5], the bi-stable NES (B-NES) was described using a Helmholtz-Duffing nonlinear stiffness given by $F_N(x) = x + 3/2x^2 + 1/2x^3$. One can show that $\mathcal{F}(\phi_2) = \exp(\imath\theta_i)(1 + 3/8|A_2|^2)$. Then Eq. (4) becomes $\imath A_1\exp(\imath\theta_1) = (\lambda_N A_2 - \imath(1 - \kappa_N(1 + 3/8|A_2|^2)A_2))\exp(\imath\theta_2)$.
By conjugation and side by side multiplication, one obtains the Slow Invariant Manifold (SIM) on the system. Let us denote $Z_1 = A_1^2$ and $Z_2 = A_2^2$, the SIM is given by $Z_1 = \lambda_N^2 Z_2 + (1 - \kappa_N(1 + 3/8Z_2))^2 Z_2$.

As shown by Iurasov in is thesis [6], the B-NES most efficient energy dissipation is observed for an inter well chaotic strongly modulated response under 1:1 resonance. This occurs for an excitation level sufficient to cross the unstable saddle point limiting the wells. To lower this limit, one aims at lowering the amplitude of the unstable saddle point. To do this, a new unstable saddle point is imposed to the potential function (see Fig. 1). This leads to a tri-stable NES (T-NES) which nonlinear stiffness is given by $F_N(x) = x + 9/2x^2 + 13/2x^3 + 15/4x^4 + 3/4x^5$. Let us denote by Y_1 and Y_2 the square of the amplitude of the fixed points of the system with a T-NES. The SIM

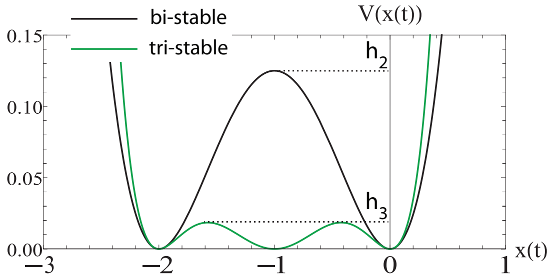

Figure 1: Potential functions for the bi-stable ($h_2 = 1/8$) and tri-stable ($h_3 = 1/54$) NES.

is given by $Y_1 = \lambda_N^2 Y_2 + \left(1 - \kappa_N \left(1 + 39/8Y_2 + 15/16\,(Y_2)^2\right)\right)^2 Y_2$.

The two SIMs are strictly monotones for $\kappa_n \geq 1$. For $\kappa_n < 1$ and $\lambda_N < (1-\kappa_N)/\sqrt{3}$ for the B-NES and $\lambda_N \lessapprox (1-\kappa_N)/\sqrt{3}$ for the T-NES both SIMs are composed of three branches separated by critical points. These points are given, for the B-NES by $Z_2^{1,2} = 8/(9\kappa_N)(2(1-\kappa_N) \mp \sqrt{(1-\kappa_N)^2 - 3\lambda_N^2})$ and by $Y_2^1 = -39/25 + \sqrt{(400 + 4163\kappa_N)/\kappa_N}/(5\sqrt{3}) + \mathcal{O}(\lambda_N^2)$ and $Y_2^2 = -15/5 + \sqrt{(80 + 427\kappa_N)/\kappa_N}/(5\sqrt{3}) + \mathcal{O}(\lambda_N^2)$ for the T-NES. The stability of the various branches is estimated by a linear perturbation analysis. The results are presented in Fig. 2. Both topologies of SIM allow SMR and therefore efficient dissipation. The ratio of the first critical point on each SIM is $Z_2^1/Y_2^1 \approx 13$, indicating that the lower threshold of the pumping is reduced by a factor $\sqrt{Z_2^1/Y_2^1} \approx 3.6$.

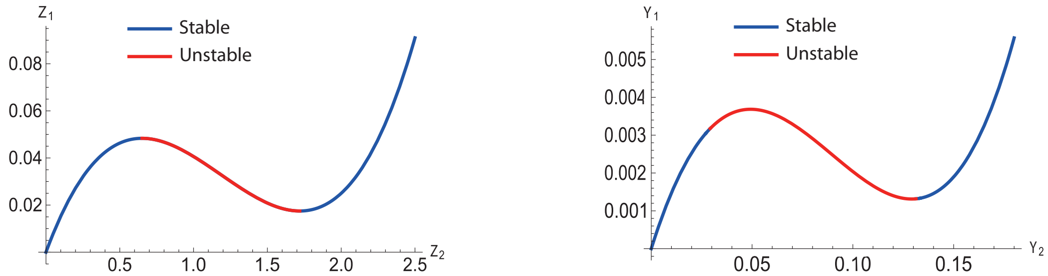

Figure 2: SIM the B-NES (left) and T-NES (right). $\kappa_N = 0.6$ and $\lambda_N = 0.1$

Conclusions

Using analytical approximations, it has been shown that, by a proper shaping of the wells of the potential of the underlying Hamiltonian of the system, the topology of the SIM of a T-NES is translated to a significantly lower level that that of a B-NES. Then, compared to a B-NES, the threshold activation of a T-NES can be significantly lowered with same dissipation ability.

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