A MARKOV CHAIN STATE TRANSITION APPROACH TO ESTABLISHING CRITICAL PHASES FOR AUV RELIABILITY

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***Abstract*—** The deployment of complex autonomous underwater platforms for marine science comprises a series of sequential steps. Each step is critical to the success of the mission. In this paper we present a state transition approach, in the form of a Markov chain, which models the sequence of steps from pre-launch to operation to recovery. The aim is to identify the states and state transitions that present higher risk to the vehicle and hence to the mission, based on evidence and judgment. Developing a Markov chain consists of two separate tasks. The first defines the structure that encodes the sequence of events. The second task assigns probabilities to each possible transition. Our model comprises eleven discrete states, and includes distance-dependent underway survival statistics. The integration of the Markov model with underway survival statistics allows us to quantify the likelihood of success during each state and transition and consequently the likelihood of achieving the desired mission goals. To illustrate this generic process, the fault history of the Autosub3 autonomous underwater vehicle provides the information for different phases of operation. The method proposed here adds more detail to previous analyses; faults are discriminated according to the phase of the mission in which they took place.

***Index Terms*—Markov processes, Risk Analysis, Underwater vehicles.**

I. INTRODUCTION

NOWADAYS the execution of many marine science programmes involves the deployment and recovery of sophisticated mechatronics systems such as autonomous underwater vehicles (AUVs), remotely operated vehicles (ROVs), or undersea gliders. Common to all of these systems is that the deployment sequence from pre-launch checks to operation in the water to recovery consists of a series of sequential phases. With each phase there is an associated risk of loss. This risk is most obvious when the vehicle is subsurface executing its task, but is non-zero during all of the phases. For example, while ready on deck there is a possibility of loss due to an electrical fire. It is widely acknowledged that significant risks attend launch, and especially recovery. Indeed some insurers require the owner to co-insure during these particular phases [1]. Although each phase involves risk, there is also the ability to repair the system after a fault, failure or incident from all but two of the phases considered.

While the risks with the deployments of these sophisticated vehicles are becoming better understood and several studies have attempted to quantify the overall risks [2]-[7], until now no framework has been proposed to structure a risk analysis for underwater vehicles by the mission phases. We propose in this paper a Markov chain approach to model the risk in the different phases and to quantify risk for different scenarios. Markov chains were chosen because analytical results associated with the model facilitate analysis of the operating sequences before they are generated, indicating how the operation is likely to unfold. Furthermore, the Markov approach allows for the essential statistical dependence between phases. The Markov model was implemented in Matlab version 12 [8].

An early use of Markov chain theory in engineering was to estimate the reliability of hardware systems carrying out a simple task in a fixed environment [9]. The approach was extended to model multi-phased systems [10]. The term multi-phased system is used to describe a system that either changes structure, or whose failure characteristics change during its operation or mission. While the Markov chain presented in this paper models a structure (vehicle) that does not change, we argue that through the transitions between phases of the deployment process, the failure characteristics do alter. For example, the set of hazards, which determine risk, during launch and recovery phases are very different to the set of hazards when the vehicle is underway.

Markov chains have also been applied to the estimation of the reliability of software-based systems [11], where a model was used to generate statistical testing sequences that would ultimately help define the reliability of a software artefact. Recent research has focused on the integration of Markov chains with other mathematical approaches, such as Bayesian theory, Monte Carlo methods and event trees [12], [13]. The end result in all of these cases is a system that provides an insight into the specific problem that would be impossible to obtain by the use of Markov chains alone. Here we extend the formalism by embedding distance-related survival statistics into a Markov chain model applied to an underwater vehicle.

Throughout this paper the word ‘state’ is used to reference a state of the Markov model, and the word ‘phase’ is used to reference a phase of the deployment process, there is a one to one correspondence between a state and a phase.

The approach to devising the framework, and the questions that can be asked using the framework and Markov chain, are generic. They can be used with any item of apparatus. Here, the motivation was for the analysis of AUV mission risks, hence we draw extensively on data concerning the reliability of the Autosub3 AUV [7].

The Markov chain representing AUV deployment sequence requires estimates for the transition probabilities between states. Griffiths and Trembranis [6] described an elicitation exercise for Autosub3, in which experts were asked to assign a probability of loss for the vehicle for each of 63 faults or incidents. It was possible to assign particular state transitions to many of the elicited probabilities. However, the judgments obtained in the elicitation exercise were not sufficient to populate all transition probabilities of the proposed Markov chain. For example, the study did not fully cover the pre-launch tests or incidents, neither did it consider whether ‘Loss’ was final, or whether salvage was possible and, if so, whether the salvaged vehicle could be returned to service or had to be scrapped. Therefore, where possible, we elicited judgments from two colleagues, senior engineers (Stephen McPhail and Peter Stevenson) with a vast amount of experience in Autosub3 development and deployment, for the transition probabilities which were not obtainable from the wider expert judgment study.

This paper is organised as follows. Section 2 gives a brief background on the Markov chain formalism; more detail concerning this formalism is given elsewhere in the literature [14]. Section 3 presents the Markov chain model that captures the sequence of activities undertaken during the deployment of an AUV together with details of the transition probabilities. In section 4 we present a test case illustrating applications of the approach and the nature of the analysis that can be supported by the Markov chain formalism. This includes estimating probability of loss at different phases and including answers to questions an operator might ask of the model. There is, for example, the obvious question, “Given that the vehicle passed its pre-launch check what is the likelihood of a successful open water mission of *x*km?”. There are also many questions related to particular phases, for instance, on the likely availability of the vehicle: “Prior to pre-launch tests, what is the likelihood of being able to have a mission start when needed?”. Or on the likelihood of a forced early recovery: “Prior to pre-launch tests, what is the likelihood of having to recover the vehicle immediately after launch?”. In section 5 we present our conclusions.

II. MARKOV CHAIN FORMALISM

In classical probability theory a set of possible outcomes E_1, E_2, … E_k. is given, with which there is associated a probability p_j, Pr (E_j)= p_j; the joint probability is defined by the multiplicative property Pr (E_1, E_2, … E_k)= $p_1 \cdot p_2 \cdot \ldots\ldots \cdot p_k$. The Markov chain theory introduces an assumption that simplifies this expression; it considers that the outcome of any trial depends on the outcome of the preceding trial[1] and only on it [14]. Thus, instead of associating a probability to an event, it associates a probability to a pair of events. For every pair of events (E_j, E_k) there is a corresponding transition probability pjk, where pjk is the probability of E_k occurring given that E_j occurred in the previous trial. According to Markov chain theory, in addition to p_{jk}, one must also define the probability of E_j occurring at the initial trial, aj0. Therefore for the initial trial the P{(E_j, E_k)}= $a_{j0} \cdot p_{jk}$. For the general case, considering a sequence of many transitions, given that event E_{j0} precedes E_{j1} which precedes E_{j2} and so on for the remaining events, the joint probability distribution is computed using the expression in 1:

$$P_r\left(E_{j0}, E_{j1}, ..., E_{jn}\right) = a_{j0} p_{j_0 j_1} p_{j_1 j_2} ... p_{j_{n-2} j_{n-1}} p_{j_{n-1} j_n} \quad 1.$$

The expression in 1 is a result of a sequential application of the condition probability assumption entailed by the Markov condition.

Generally speaking, a sequence of trials with possible outcomes E_1, E_2, …E_k is called a Markov chain if the probabilities of sample sequences are defined by 1. Quite often a state has more than one preceding state. When this is the case transition probabilities are arranged in a matrix, also denoted in the literature as a transition matrix or stochastic matrix [13]. The transition probability together with the initial state vector completely defines the Markov chain. The matrix in 2 is an example of a transition matrix for a Markov chain with k states.

$$P = \begin{Bmatrix} p_{00} & p_{01} & \cdots & p_{0k} \\ p_{10} & \cdots & \cdots & p_{1k} \\ \cdots & \cdots\cdot & \cdots & . \\ p_{k0} & p_{k1} & \cdots & p_{kk} \end{Bmatrix} \quad 2.$$

Considering the stochastic transition matrix in 2, p_{01} is the probability of moving from state 0 to state 1, p_{0i} is the probability of going from state 0 to state i, p_{ii} is the probability of the system not leaving state i in the next step. The stochastic matrix is useful to study the probability of a sequence of steps taking place before the sequence is generated, and is the basis of the analyses in section 4.

III. MARKOV CHAIN STATE MODEL FOR AUV OPERATIONS

The deployment process is represented as a sequence of discrete states. The time spent in each state is not critical to the analysis, except when underway, which we deal with in a different way. When underway, time is proportional to distance, and distance is an input to a model of survival statistics. With respect to other states, it is assumed that the process will take as much time as necessary in order

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[1] Trial is here defined as an experiment where the final outcome depends only on chance.

to decide whether or not the process transits to the next phase. However, when underway the transition to recovery or loss depends on the mission distance, which is generally directly related to time [10]. Other applications of Markov models have used the concept of sojourn time to model the time spent in each state [15]. At this stage we do not have sufficient information to model the time take for an AUV deployment. It is assumed that the operators take as much time as required in each state.

A. The Markov Chain Model

The Markov chain model of an AUV deployment process comprises eleven states (X1 to X11), corresponding to eleven possible phases of the deployment. The proposed Markov chain is depicted in Figure 1; the name of each node is abbreviated to facilitate the model description and subsequent analysis. Where faults or incidents can lead to loss we provide real examples, most of which have not been documented in the peer reviewed literature.

The deployment process starts at the initial pre-test state at a deployment location (Dp). This is the vehicle embarked onto a ship, or delivered to a launch site. The vehicle is ready to be tested and the power is on. A test sequence is carried out, which comprises a set of visual and systems checks. If the tests are unsuccessful, the transition is a loop back to this state, implying fault identification and rectification before another test is carried out in an attempt to transition to the next state. While some may argue that there are prior states that should be considered, for example preparation at base, or selection of the operations team, we have assumed that the effects of these factors can be, and have been, subsumed into the statistics of the phases listed. This is a simplification, and in part, a necessary simplification to enable use of the Markov approach. In reality, selection of the operations team can have an impact during all subsequent phases, which cannot be modelled using the Markov approach alone. In this case a Bayesian mathematical formulation could be used for updating the transition probabilities in light of new observations concerning the experience of the deployment team.

The next state is the post-test state, ready to launch (Dr). The successful act of taking the vehicle from its carrier frame and into the water, be it over the side of a ship or lowered from a quay, for example, takes the vehicle to the Overboard state (O). This may be by purpose-built or general-purpose gantry or by hand for small vehicles. Problems can occur in this state necessitating recovery or they can lead to damage, for example, due to a fire or explosion in the energy source; although this should be a rare occurrence. The Markov chain model recognizes this possibility with the transition from state Dr to a state where the vehicle would have to be salvaged (S). From Overboard (O) the vehicle is ready for pre-dive checks. The vehicle is now floating in the water, as it is common for AUVs to be positively buoyant. If the AUV passes all pre-dive checks on the surface, a command is sent to the AUV to start diving (Dv). If these checks are not passed, the decision may be made to recover the vehicle (R). Operations on the surface next to the deployment platform carry significant risk [1]. There is the possibility that the AUV will be lost, for example, if the AUV is caught by the vessel's propeller, as has happened to a Norwegian AUV. During the Diving (Dv) phase some mechanical and communication tests may be carried out. New risks emerge, such as from failure of a component that has not been called upon until this phase, for example a stern plane actuator whose failure can force the AUV to perform an uncontrolled dive, a situation that has lead to temporary loss (L), as happened with Autosub2 AUV in the Celtic Sea, off the UK coast. The Markov chain model captures this phenomenon with the link Dv -> L. During the dive (Dv) to the holding/test pattern phase (Sh) the AUV is usually within telemetry range, giving the engineers the opportunity, if necessary, to send the 'abort to surface' signal.

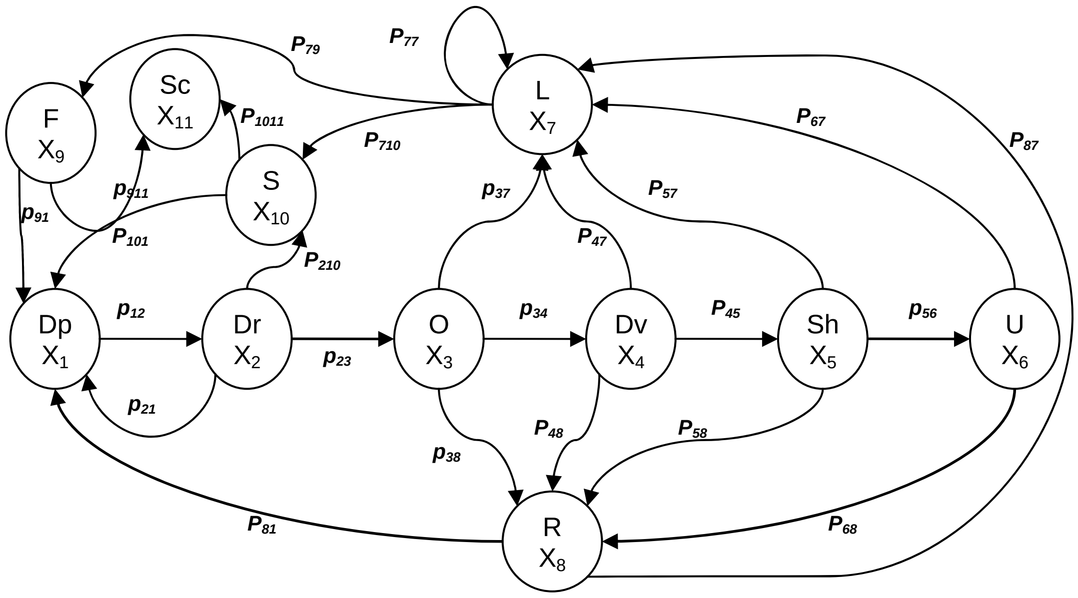

Fig. 1. Markov state space model capturing the sequence of events undertaken during AUV deployment and operation. A directional arrow from state i to state j means that the process can move from state i to state j.

The ‘abort to surface’ signal causes the vehicle to release the abort weight; this increases the buoyancy of the vehicle, forcing it to come to surface, where recovery is possible. The submerged, holding/test pattern (Sh) phase precedes the start of the mission proper.

Once the vehicle has completed the first dive, the practice with Autosub3 is to set the vehicle in a holding or test pattern, while submerged, near the deployment platform while the telemetry is assessed. If faults are indicated, the vehicle can be recovered, but with a risk of loss as discussed earlier. All well, the vehicle proceeds with its mission, transitioning to an underway state (U). The subsequent states are either recovery (R) or loss (L).

Based on mission experiences with AUVs by the community, the lost (L) state is either permanent, for example with Autosub2 under an ice shelf [19], or there is a probability that a lost vehicle may be salvaged (S), through a deliberate act, e.g. using an ROV [22], or the vehicle may be found (F), through serendipity. The latter may take place many months after the loss, as was the case in 2007 with a UK Royal Navy Remus100 found after 10 months at sea. The permanently lost state is shown by the link from L to itself. The recovery (R) may not proceed to plan as has been discussed, hence a link to Loss (L). Following either Salvage (S) or Found (F) the decision may be to scrap the vehicle as being beyond economic repair (Sc), or repair and then reuse, as happened with BAE Systems Merlin. The latter is captured with the links to the initial state Dp. The scrapped phase (Sc) is terminal. For each transition there is an associated sequence of events or conditions that must be met. A list of the conditions associated with each transition is presented in Table I.

TABLE I

TRANSITION PROBABILITIES NOTATION AND CONDITIONS FOR THE MARKOV MODEL PRESENTED IN FIGURE 1.

Notation	**Conditions for Transition**
p_{12}	Pre-deployment check passed
p_{21}	Pre-deployment check not passed
p_{23}	Deployment over-side OK
p_{210}	Salvaged after passing deployment check
p_{34}	Pre-mission diving mode entered
p_{37}	Lost when over-side
p_{38}	Recovery from over-side
p_{45}	Pre-mission Holding Pattern entered
p_{47}	Lost from Diving mode
p_{48}	Recovery from Diving mode
p_{56}	Underway mission started
p_{57}	Lost from Holding Pattern
p_{58}	Recovery from Holding Pattern
p_{67}	Lost when underway on mission
p_{68}	Recovery from underway on mission
p_{81}	Recovery to deck OK
p_{87}	Lost on recovery to deck
p_{77}	Vehicle remains lost
p_{79}	Vehicle found
p_{710}	Vehicle salvaged
p_{91}	Found vehicle repaired and reused
p_{1011}	Salvaged vehicle scrapped
p_{911}	Found vehicle scrapped
p_{101}	Salvaged vehicle repaired and reused

NOTE: The subscript *ij* signifies the succinct transition from state *i* to state *j*.

The probability of the process going through a sequence of states is computed by manipulation of the transition matrix. The transition matrix that corresponds to the Markov chain presented in Figure 1 is presented in 3. Only those elements with probabilities set out in Table 1 are non-zero. The 1 for element p_{1111} signifies a terminal state.

$$P = \left\{ \begin{array}{ccccccccccc} p_{11} & p_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ p_{21} & p_{22} & p_{23} & 0 & 0 & 0 & 0 & 0 & 0 & p_{210} & 0 \\ 0 & 0 & p_{33} & p_{34} & 0 & 0 & p_{37} & p_{38} & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{44} & p_{45} & 0 & p_{47} & p_{48} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_{55} & p_{56} & p_{57} & p_{58} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{67} & p_{68} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_{77} & 0 & p_{79} & p_{710} & 0 \\ p_{81} & 0 & 0 & 0 & 0 & 0 & p_{87} & 0 & 0 & 0 & 0 \\ p_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{911} \\ p_{101} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{1011} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right\} \quad 3.$$

B. Incorporating transition probabilities

Having set out a topology for the Markov chain that represents the life cycle of AUV deployments, the next stage is to determine the transition probabilities. These could be established through hard evidence through the frequentist approach of logging the frequency of occurrence. However, in practice, this will rarely be the case and indeed may not necessarily be the best approach. Hard evidence is obtained easily for transition probabilities before the vehicle enters the water, such as p_{11}, and p_{12}. These are very likely to be probabilities determined overwhelmingly by the vehicle systems alone. In contrast, once overboard the transition probabilities are influenced by a large number of other factors. For example, the seamanship of the vessel's crew, experience of the deployment team, weather and sea conditions, the operational environment, whether it is coastal, open ocean, under sea ice, or under ice shelf. Hence, there is a fundamental choice of whether to populate the matrix with hard evidence probabilities applicable only to the set of circumstances appertaining to each event (fault or incident) or whether to generalise beyond the specific case. Such a generalisation can be achieved through the technique of eliciting expert judgment [23] for the probabilities. It is the approach taken here.

In a previous risk assessment exercise Brito et al. [16] developed an Autosub3 risk model based on its failure and incident history; a total of 63 faults were considered by a panel of experts. Ten successful missions were also used in their analysis. This section explains how a part of this data was used to populate the transition matrix 3. First, it was imperative to identify, where possible, which failures from this dataset were associated with which transition probabilities. We used the failure description to identify the phase of the mission in which the failure took place. The detailed result of our assessment is presented in Appendix A, and in aggregated form in Table II, where they are indicated as EJ (Group).

The transition probability p_{68} is calculated based on the extended Kaplan Meier statistical estimator as a function of mission distance as proposed by Griffiths et al. [18]. This is in order to model missions of different lengths, and provides a more realistic representation of risk than a fixed probability. This formulation for the survival function with range r is given in 4:

$$\hat{S}(r) = \prod_{r_i < r} 1 - \left(\frac{1}{n_i}\right) P(e_i) \qquad 4.$$

where $P(e_i)$ is the probability judgment of loss from the experts and n_i. is the failure index number. The $P(e_i)$ variable replaces the censor flag used by the original Kaplan Meier formulation [21]. The extended Kaplan Meier formulation was used for calculating the transition probability p_{68} and through the Markov condition p_{67}. Table II presents that survival data for open water environment The calculated survival distribution depicted in Figure 2 was obtained using the dataset of Table II.

TABLE II
PROBABILITIES OF LOSS USED FOR CALCULATING TRANSITION PROBABILITY P68 USING 4.

Index	**Code**	**Distance (km)**	**$P(e_i)$**	**Index**	**Code**	**Distance (km)**	**$P(e_i)$**
31	413	0.2	0.00000	**15**	391_1	31	0.02620
30	388_1	0.5	0.01730	**14**	391_2	31	0.03110
29	409_1	1.5	0.00678	**13**	391_3	31	0.00148
28	396_1	4	0.01220	**12**	404_3	75	0.02440
27	400	4	0.0000	**11**	404_4	75	0.00160
26	422	4	0.00000	**10**	417	80	0.00000
25	393_1	5	0.02610	**9**	419	82	0.00000
24	415_2	6	0.04600	**8**	406_1	104	0.01040
23	399	7	0.00000	**7**	406_2	104	0.00382
22	414	7	0.00000	**6**	406_4	104	0.02250
21	401_1	7.5	0.021800	**5**	403_2	140	0.00882
20	398_1	8	0.00845	**4**	403_3	140	0.02350
19	420	8	0.00000	**3**	407_2	204	0.03960
18	410_1	9	0.00695	**2**	402_2	274	0.04650
17	421	9	0.00000	**1**	402_3	274	0.02640
16	390	10	0.00000				

NOTE: The failure index number is given in columns 1 and 5. 21 failures and 10 successful missions are considered from when the vehicle was underway. The probability judgment corresponds to the linear pool aggregated judgment, EJ(Group). The probability judgments are for open water environment. Ten missions that were successful, with no failures, are also included in the calculation, $P(e_i)$ for these missions is 0. The Code expresses mission number and the index number of the fault on that mission, if a fault (see Appendix 1).

The distribution shows for example that for a distance of 80km the probability of survival is 0.97 therefore, $p_{67} = 0.03$ and $p_{68} = 0.97$. The shape of the survival distribution is similar to that obtained in previous analysis of Autosub3 failure history [16], which derived a statistical survival model for four operating scenarios: open water, coastal, sea ice and ice shelf. Two risk models were built for each environment, denoted as the optimistic and pessimistic. Considering the open water scenario, for a mission distance of 80km, the pessimistic model gave a probability of survival of 0.962, the optimistic model [16] gave a probability of survival of 0.992. The transition probability from the reduced dataset used here is between the optimistic and pessimistic estimates of previous analyses.

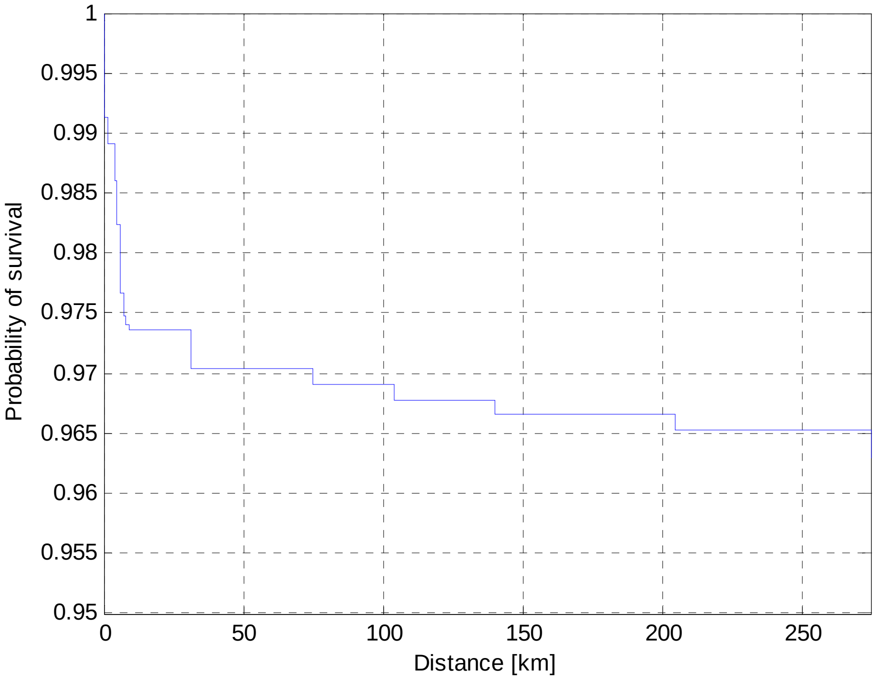

Fig. 2. Kaplan-Meier survival distribution. Source data is present in Table III.

C. Incorporating expert judgment

Some of the remaining transition probabilities were obtained by an expert judgment elicitation exercise that was conducted in private meetings with two senior Autosub3 engineers, each with more than 15 years AUV experience, and prior experience in assigning probabilities to support accident investigations [19], [20]. Their judgments were aggregated using the linear opinion pool. An equal weight was assigned to both experts [23], the aggregated probabilities are shown in Table III, indicated as EJ(McP-S). The group of experts (B-G) is formed by the authors of this paper. The first author has approximately 1.5 years experience in AUV risk analysis and the second author has approximately 22 years experience in AUV design, deployment and risk assessment.

TABLE III
TRANSITION PROBABILITIES FOR THE STATE SPACE MODEL

From state	**Transition Stimuli**	**To State**	**Source data and related information**	**Transition Prob.**
Dp	P_{11}	Dp	Markov condition	$1-p_{12}$
	P_{12}	Dr	EJ(McP-S)	0.875
Dr	P_{21}	Dp	EJ(McP-S)	0.055
	P_{23}	O	EJ(McP-S)	0.94
	P_{210}	S	EJ(McP-S)	0.005
O	P_{33}	O	EJ(McP-S)	0.0195
	P_{34}	D_v	EJ(McP-S)	0.97
	P_{37}	L	EJ(McP-S)	0.0015
	P_{38}	R	EJ(McP-S)	0.009
Dv	P_{45}	Sh	EJ(McP-S)	0.9565
	P_{47}	L	EJ(McP-S)	0.0085
	P_{48}	R	EJ(McP-S)	0.035
Sh	P_{56}	U	EJ(McP-S)	0.98
	P_{57}	L	EJ(McP-S)	0.001
	P_{58}	R	Markov condition	0.019
U	P_{67}	L	Markov condition	$1-p_{68}$
	P_{68}	R	EJ(Group)	$p_{survival}$
L	P_{77}	L	Markov condition	$1-(p_{79}+p_{710})$
	P_{79}	F	EJ(B-G)	0.33
	P_{710}	S	EJ(B-G)	0.33
R	P_{81}	Dp	EJ(McP-S)	0.998
	P_{87}	L	EJ(McP-S)	0.002
F	P_{91}	Dp	EJ(B-G)	0.75
	P_{911}	Sc	EJ(B-G)	0.25
S	P_{101}	Dp	EJ(B-G)	0.7
	P_{1011}	Sc	EJ(B-G)	0.3
Sc	P_{1111}	Sc	Absorbing state	1

NOTE: The Markov property states that the sum of the probabilities leaving any given state must equal to unity. If the sum of all transitions leaving a state is constant *c*, where *c* is lower than one, than the probability of the process remaining in the same state in the next transition is 1-*c*.

IV. TEST CASE – OPEN WATER MISSION

In this section we show how analysis of the Markov chain can address questions relevant to the risk quantification of the entire deployment or parts of a deployment. The results are used to identify possible risk mitigation activities. As a test case consider an 80km long mission in open water, with no risk mitigation activity carried out at the start of the mission, via the use of a monitoring distance.

A. Availability

Availability is a measure of what fraction of those occasions on which a system is needed it is in place and working. In the Markov topology set out here, availability is the probability of the operation running directly from state Dp to state U, $P(\mathrm{Dp} \rightarrow \mathrm{Dr} \rightarrow \mathrm{O} \rightarrow \mathrm{Dv} \rightarrow \mathrm{Sh} \rightarrow \mathrm{U}) = p_{12}\cdot p_{23}\cdot p_{34}\cdot p_{45}\cdot p_{56}$, is 0.75. As a result, the probability of not completing this sequence is 0.25. This figure is affected by a number of factors, such as the vehicle payload and the amount of testing prior to the start of the mission. Our experts took such factors in consideration when they assigned the probability transitions. In normal conditions the science manager would expect to have a high confidence in the success of this sequence of events. However, at present there is no process with the Autosub3 AUV to establish whether or not the probability of completing the sequence is sufficiently high to meet users' expectations.

In contrast, the US military Unmanned Air Vehicle community has set out availability targets for a range of vehicles, furthermore actual availability data is available [24]. These, together with the result for Autosub3 derived here are shown in Table IV.

TABLE IV
COMPARISON OF AUTOSUB3 AVAILABILITY WITH THAT OF THREE US MILITARY UAV SYSTEMS AT DIFFERENT STAGES OF THEIR DEVELOPMENT.

Vehicle	**Requirement**	**Actual**
Autosub3	n/a	0.75
Predator RQ-1A (concept demonstrator)	n/a	0.40
Predator RQ-1B (early production)	0.80	0.93
Pioneer RQ-2A (1990-1991)	0.93	0.74
Pioneer RQ-2B	0.93	0.78
Hunter RQ-5 (reliability enhanced 1996-2001)	0.85	0.98
Average UAV	0.88	0.77

The Autosub3 availability is slightly lower than the average of the US military UAVs listed; considerably below the 0.98 of the Hunter RQ-5, but almost twice that of the Predator RQ-1A when it was a concept demonstrator. As Autosub3 is a unique build a comparison with a concept demonstrator is not out of place. A reasonable availability target for a unique or low-volume build AUV would be 0.8 based on these results, which could be achieved through analysis of fault and incident data and remedial engineering work. From this analysis, the transition probability p_{12} is the lowest probability transition in the sequence Dp → Dr → O → Dv → Sh → U, and is consequently the area most likely to result in improvement through a more rigorous testing process An increase of 7% in p_{12} would raise P(Dp → Dr → O → Dv → Sh → U) to 0.80.

B. Recovering an AUV

Recovering the vehicle entails many risks, and is a mission phase for which some insurers require co-insurance [1]. Understanding and quantifying risk during this phase is therefore important. Once on the surface and in range, in the case of Autosub3, a line from the vehicle is grappled and the vehicle brought closer to the vessel. Collision with the vessel is one potential risk. The first failure on Autosub3 mission 403 gives another example of how problems can occur on recovery (Appendix I, Table VI). A high sea state can add complications to this phase. For a long range AUV it is not unusual for a mission to take 24 hours and for sea conditions to worsen during the mission.

The Markov formalism also allows us to compute the probability of having to recover the vehicle for all the sequences, in actuality, there are two subsets. The first subset comprises those instances where there is a need to recover the vehicle given it has reached the preceding state. These are simply single elements from Table III, thus the probabilities of needing to recover immediately following phase: Overboard = 0.009; Post-dive = 0.035; Holding/test pattern = 0.019; Underway = 0.97. The low probability of needing to recover immediately the vehicle is put overboard is understandable, as few, if any, further tests will have been made at that stage. Despite the fact that a higher number of tests take place during the holding pattern phase, history has shown that failures tend to manifest themselves during the diving phase, thus there is a higher probability that the vehicle will be recovered during the diving phase than during holding pattern phase. Recovery is most likely after the end of a mission.

The second subset examines the probability of needing to recover the vehicle over a span of two or more states, or via two or more routes. For example, in the question raised in the introduction, the operator is keen to know the probability of having to recover a vehicle once overboard if it is not able to set out on its mission. That is, what is the probability of reaching R via either of O, Dv or Sh?

1. Recovery from Overside? [Dp → Dr → O → R] = 0.0074;
2. Recovery from Dive? [Dp → Dr → O → D_v → R] = 0.0279;
3. Recovery from holding pattern? [Dp → Dr → O → D_v → S_h → R] = 0.0145;
4. The probability of having to recover the vehicle following phases O, Dv or Sh is 0.0498.

C. *Surviving the deployment*

Once the vehicle is underway the probability of losing the vehicle is calculated using distance-related survival statistics only [7]. However, a mission comprises several non-underway phases and the benefit of our approach is that it provides an estimate of the probability of the vehicle surviving each phase of the deployment. The successful completion of a mission (failure free mission) results in an increase in vehicle reliability, hence the transition probabilities need to be updated. Likewise if a mission presents one or more failures, these failures need to be added to the risk model and the transition probabilities must also be updated. However, the Markov chain model here presented is static, in that it allows us to estimate the risk associated with one mission only. A science campaign usually entails several missions [16], hence the model needs to be updated every time a new mission is entered.

A successful mission corresponds to a direct sequence Dp → Dr → O → Dv → Sh → U→R →Dr. Using the Markov assumption P (Dp → Dr → O → Dv → Sh → U→R →Dr) = $p_{12}{\cdot}p_{23}{\cdot}p_{34}$ $p_{45}{\cdot}p_{56}{\cdot}p_{68}{\cdot}p_{81}$ = 0.72. That is, prior to the start of the on-board testing, the operating calling on the use of Autosub3 can expect a 0.72 probability of recovery after a successful mission. This is the 'trouble-free' sequence. Of course, the overall probability of loss is not 1-0.72 or 0.38, as Availability (0.75) is a major factor.

The probability of losing the vehicle immediately following phase: Overboard = 0.0015; Post-dive = 0.0085; Holding/test pattern = 0.001; Underway = 0.03.

A key question is, "What is the path that is more likely to lead to the loss of the vehicle? The probability for each sequence is presented below:

1. Loss from Overside? [Dp → Dr → O → L] = 0.00123.
2. Loss from Dive? [Dp → Dr → O → Dv → L] = 0.00678.

3. Loss from Holding? [Dp → Dr → O → Dv → Sh → L] =0.000763.
4. Loss from Underway? [Dp → Dr → O → Dv → Sh → U→ L] = 0.0224 (for an 80km mission).

Apart from when the vehicle is underway, in open water operations the most significant risk of loss is when the vehicle is in diving mode.

D. Decommissioning the vehicle and estimated working life

Scrapping the vehicle is a possibility considered by the AUV operators following the salvage or finding of the vehicle. Scrapping or recycling can result in additional costs and the probability of this happening can be estimated using the Markov approach. This state can be reached from any of seven starting points, Table V.

TABLE V
PROBABILITIES OF NEEDING TO SCRAP THE AUV GIVEN THAT THE VEHICLE WAS PREVIOUSLY IN STATES DR, O, DV, SH, U, L OR R.

From state	P(Scrapped)
Dr	0.0050
O	$0.00012^{(S)} - 0.00015^{(F)}$
Dv	$0.00084^{(S)} - 0.00070^{(F)}$
Sh	$0.000545^{(S)} - 0.000453^{(F)}$
U	$0.00297^{(S)} - 0.00248^{(F)}$
L	$0.099^{(S)} - 0.0825^{(F)}$
R	$0.000198^{(S)} - 0.000165^{(F)}$

NOTE: The superscript (S) indicates a path following Salvage and superscript (F) indicates a path following vehicle Found.

There is 0.1815 (0.099 + 0.0825) probability of having to scrap the vehicle given that the vehicle has been lost. This comes as no surprise. More interestingly is that even though the path [Dr -> Sc] is shorter than the path [U -> Sc]. The probability of having to scrap the vehicle is lower if the deployment is at state Dr; the probability of having to scrap the vehicle given that the deployment is at state U is 0.00545 (0.00297 + 0.00248).

In probability theory, Markov chains are typically studied by considering special types of Markov chains, these are defined according to their topology. The node state Sc (vehicle scrapped) is denoted in the literature as an absorbing state. This is due to the fact that it is not possible to jump from state Sc to any other state, all arrows are converging to state Sc, there are no arrows leaving state Sc. A Markov chain that contains one or more absorbing nodes is called an absorbing Markov chain [14]. The probability of reaching an absorbing state will always increase with the number of transitions until the process is totally absorbed by this state. In practical terms, and with respect to the model presented in this paper, the number of transitions to total absorption is an indication of the likely working life of the vehicle. This estimate is presented in the fundamental matrix of the absorbing Markov chain. The process to follow to derive the fundamental matrix is presented in the literature [14]. The fundamental matrix is presented in 5, where values are approximated to the three most significant digits.

$$N = \begin{bmatrix} 94.5 & 82.7 & 79.3 & 76.9 & 73.6 & 72.1 & 4.8 & 74.7 & 1.6 & 2.0 \\ 93.4 & 82.7 & 79.3 & 76.9 & 73.6 & 72.1 & 4.8 & 74.7 & 1.6 & 2.0 \\ 93.5 & 81.8 & 79.4 & 77.0 & 73.7 & 72.2 & 4.8 & 74.8 & 1.6 & 2.0 \\ 93.5 & 81.8 & 78.4 & 77.1 & 73.7 & 72.3 & 4.8 & 74.8 & 1.6 & 2.0 \\ 93.7 & 82.0 & 78.6 & 76.2 & 73.9 & 72.4 & 4.8 & 75.0 & 1.6 & 2.0 \\ 93.7 & 82.0 & 78.6 & 76.2 & 72.9 & 72.5 & 4.8 & 75.0 & 1.6 & 2.0 \\ 68.5 & 60.0 & 57.5 & 55.7 & 53.3 & 52.3 & 5.0 & 54.1 & 1.6 & 2.0 \\ 94.5 & 82.7 & 79.3 & 76.9 & 73.5 & 72.0 & 4.8 & 75.7 & 1.6 & 2.0 \\ 70.9 & 62.0 & 59.5 & 57.7 & 55.2 & 54.1 & 3.6 & 56.0 & 2.2 & 1.5 \\ 66.2 & 57.9 & 55.5 & 53.8 & 51.5 & 50.5 & 3.4 & 52.3 & 1.1 & 2.4 \end{bmatrix} \quad 5.$$

The first line in the fundamental matrix informs us that assuming that the process starts in state 1, on average the process would pass 94.5 times in state 1, 82.7 times in state 2, 79.3 times in state 3 and so on for the remaining states, before being absorbed by state Sc. In this case, using the same vehicle, with no maintenance or replacement of components, the vehicle would be able to carry out 72 underway missions before it ended up scrapped. At first, this figure may seem pessimistic, after all Autosub2 was lost after 216 successful missions. To assess the significance of this estimate, one should consider Autosub3 fault history to date. The risk model presented in this paper has been developed based on operational history supplied from Autosub3 missions 384 to 422. Autosub3 has completed 12 more missions since mission 422, bringing the total number of missions to 50 (to February 2009). Half way through mission 431, Autosub3 collided with the ice shelf, whilst under the Pine Island Bay Glacier, Antarctica. This high impact fault caused structural damage to the vehicle. Autosub3 survived mission

431 and managed to complete two more missions under ice shelf. However following the incident that occurred during mission 431, a group of experts from NOCS, with a combined experience of 55 years, were asked to assign a probability of losing the vehicle given that the same fault emerges under the same conditions. The aggregated probability judgment was 0.58; Autosub3 was fortunate to have survived its 47th mission, mission 431 [25]. Thus, in light of this recent event, the estimate of 72 underway missions before being scrapped appears to be more plausible.

V. DISCUSSION

The proposed Markov model provides a useful approach for estimating the risks during phases in an AUV deployment. Its graphical structure injects transparency into the AUV deployment process that facilitates process criticism and improvement for each phase. In the motivating example presented, the model transition probabilities were based on expert judgments and statistical survival analysis of Autosub3 fault and incident history. Examples of how the model can be used to address questions concerning relative and absolute risk relevant to the owner, managers, and engineers have been presented. Analysis showed that the model produced plausible answers to all these queries.

It would be feasible to add time as a covariate in this analysis. Time is often at a premium at sea, and questions over how long vehicle tests may take, or how long might recovery take after a failed deployment, are not uncommon. Sojourn time at each state has not been modelled here, the data simply did not exist. Future work seeking to address this problem would need to draw upon more detailed record keeping. There is a case for a common form of structured record keeping for AUV deployments, informed by this type of phase topology, so that individual and comparative statistics can be obtained.

The integration of the Markov model with distance-based statistical survival models and the use of such models to estimate the risk of a science mission is novel, and as shown, this approach can provide more detailed risk estimates.

Verification of the absolute and relative probabilities is difficult, the first stage, though feedback of the results to those from whom judgments have been elicited, and subsequent revision to probability estimates, was undertaken in this work. A second stage might compare the results with one AUV to the results from another type. For those phases where a frequentist approach would be valid, direct comparison of judgments and recorded frequency would be informative. Arguably, the results obtained with a previous Autosub3 statistical model [16] cannot be used to validate the results produced by this Markov chain model. Only a fraction of the failures considered previously were incorporated for the underway aspect of the Markov model, furthermore additional expert judgments were needed to populate the non-underway transition probabilities.

Lastly, the vehicle's configuration is likely to introduce constraints on the reliability and subsequent changes to mission risk. Maturity of the vehicle's configuration will also influence mission risk. This phenomenon is not captured in the proposed approach. It is clear from UAV reliability analysis that risk may decrease markedly between a concept demonstrator and a production vehicle [24]. Future work should seek to model how different AUV configurations and maturity may influence the operational risk.

Appendix A

TABLE VI

TABLE OF MISSIONS, DISTANCES, FAULTS WITH COMMENTS AND STATE CLASSIFICATIONS

No.	Distance km	Fault/incident description	Transition
384	1.5	Mission aborted (to surface) due to network failure. (Much) later tests showed general problem with the harnesses (bad crimp joints).	P47
		Loop of recovery line came out from storage slot, long enough to tangle propeller.	P87
385	15.2	Autosub headed off in an uncontrolled way, due to a side effect of the removal of the upwards-looking ADCP.	P57
386	26	GPS antenna failed at end of mission.	P87
387	27.2	Homing failed, and the vehicle headed off in an uncontrolled direction. Mission was stopped by acoustic command. Problem was due to (a) the uncalibrated receiver array, and (b) a network message (“homing lost”) being lost on the network.	P87
388	0.5	Aborted after 4 minutes post dive, due to network failure. Logger data showed long gaps, up to 60s, across all data from all nodes, suggesting logger problem.	P67
		Depth control showed instability. +/- 1m oscillation due to incorrect configuration gain setting.	P87
389	3	Vehicle went into homing mode, just before dive and headed north. Vehicle mission stopped by acoustic command. It was fortunate that the ship-side acoustics configuration allowed the ship to steam at 9kt (faster rather than 6kt with the towfish) and catch the AUV.	P57
		Separately, homing mode not exited after 2 minutes, as expected. It will continue on last-determined heading indefinitely – a Mission Control configuration error.	P87
		Problem with deck side of acoustic telemetry receiver front end, unrelated to vehicle systems.	P87
391	31	ADCP down range limited to 360m, reduced accuracy of navigation.	P67
		GPS antenna flooded. No fix at end point of mission.	P67
		EM2000 swath sonar stopped logging during mission.	P67
392	32	As consequence of GPS failure on M391, AUV ended up 700m N and 250m E of expected end position.	P87
393	5	Acoustic telemetry giving poor ranges and no acoustic telemetry.	P67
394	3	Jack-in-the-box recovery float came out, wrapping its line around the propeller, jamming it, and stopping the mission. Caused severe problems in recovery, some damage to upper rudder frame, sub-frame and GPS antenna. Required boat to be launched.	P87
395	8	Jack-in-the-box line came out, wrapped around the propulsion motor and jammed.	P87
396	4	Current estimation did not work, because minimum time between fixes for current to be estimated had been set to 15min; leg time was only 10min. Mission stopped and restarted with configurable time set to 5min.	P67
397	4	Main lifting lines became loose, could have jammed motor.	P87
398	8	Operators ended mission prematurely, they believed the AUV was missing waypoints. In fact, a couple of waypoints had been positioned incorrectly.	P67

TABLE VI(cont.)

No.	Distance km	Fault/incident description	Transition
401	7.5	Configuration mistake; ADCP up configured as down- looking ADCP causing navigation problems through tracking sea surface as reference. This data was very noisy and put vehicle navigation out by a factor of 1.5.	P67
		Damaged on recovery, "moderately serious" to sternplane, shaft bent.	P87
402	274	Stern Plane stuck up during attempt to dive, 2d 20h into mission. Stern plane actuator had flooded.	P37
		Abort due to network failure. Abort release could not communicate with depth control node for 403s. Possibly side-effect of actuator or motor problems.	P67
		Motor windings had resistance of 330 ohm to case. Propeller speed dropping off gradually during a dive	P67
		Only one position fix from tail mounted ARGOS transmitter.	P87
		GPS antenna damaged on recovery.	P87
403	140	Recovery light line was wrapped around the propeller on surface. Flaps covering the main recovery lines (and where the light line was towed) were open.	P87
		Took over 1 hour to get GPS fix at final waypoint.	P67
		Propeller speed showed same problem as m402. Subsequent testing of motor with Megger showed resistance of a few kohm between windings.	P67
404	75	Pre-launch, abort weight could not be loaded successfully due to distorted keeper. "If not spotted, could have dropped out during mission", considered low probability of distortion and not checked.	P21
		Pre-launch, potential short circuit in motor controller that could stop motor.	P21
		Propeller speed showed same problem as on m402 and 403.	P21
		CTD drop-out of 1 hour (shorter drop-outs noted in previous missions).	P67
		M404 recovery was complicated when lifting lines and streaming line became trapped on the rudder (probably stuck on the Bolen where the two were attached). Recovery from the situation required the trapped lifting lines grappled astern of the ship, attached to the gantry lines, and the caught end cut.	P87
		The forward sternplane was lost due to lifting line trapping between the fin and its flap on recovery.	P87
		The acoustic telemetry nose transducer was damaged due to collision with the ship.	P87
405	2.5	Fault found pre-launch, LXT tracking transducer had leaked water – replaced.	P21
		Fault found pre-launch, starboard lower rudder and sternplane loose.	P21
406	104	AUV ran slower than expected and speed dropped off during mission, due to motor problem.	P67
		Current spikes of 3A and voltage drops in first part of mission.	P67
		Propulsion motor failed 500V Megger on recovery on windings to case.	P87
		One battery pack out of four showed intermittent connection.	P67
		Acosutci telemetry unit gave no replies.	P87
		On surfacing first GPS fix was 1.2km out.	P87
		Spikes in indicated motor rpm	P87
407	204	Acoustic telemetry unit gave no replies at all – no tracking or telemetry.	P57
		Noise spikes on both channels of turbulence probe data.	P67

Table VI(cont.)

No.	Distance km	Fault/incident description	Transition
408	302.5	Propulsion motor felt rough when turned by hand – bearings replaced before deployment.	P21
		Aborted at 50m due to overdepth as no depth mode commanded. Unless compounded by another problem, this would show itself immediately on first dive.	P36
		No telemetry from Acoustic telemetry unit.	P57
		Difficulty stopping Autosub on surface via radio command. Separate problems with the two WiFi access points.	P57
		Still spikes on motor rpm that need investigating.	P57
409	1.5	No acoustic telemetry or transponding. LXT ship side USBL receiver had leaked during mission giving poor bearings to sub, replaced with spare.	P57
410	9	No acoustic telemetry or transponding.	P57
411	128	No GPS fix at the end of the mission. GPS antenna bulkhead had water inside and had flooded.	P87
412	270	No GPS fix at end of mission. After next mission, GPS fixes started coming in after vehicle power up/power down; perhaps problem was due to initialisation with receiver – and not this time the antenna.	P87
		Problem at start for holding pattern. Holding pattern timed out due to programming mistake.	P57
415	6	Prior to dive, checks showed reduced torque on rudder actuator. Actuator replaced with new one - first use for this new design of actuator motor and gearbox. However, AUV spent most of mission “stuck” going around in circles at depth due to rudder actuator fault. The new actuator overheated, melting wires internally, the motor seized, and internal to the main pressure case, the power filter overheated. Some of the damage may have been caused by an excessive current limit (3A); correct setting was 0.3A. But this does not explain high motor current. Possible damage during testing when motor stalled on end stop? Compounded by wiring to motor held tightly to case with cable ties, and worse, covered with tape (acting as an insulator). Wires were not high temperature rated.	P57
		Three harness connectors failed due to leakage, affecting payload systems: EM2000 tube, ADCP_down, and Seabird CTD. Despite connector problems the system worked without glitches and failed only when the power pins had burned completely through on the connector feeding power to the abort system	P67
		Although it worked properly at the start of the mission at a range of 1200m, the acoustic telemetry stopped working at the end of mission. Hence could not stop the mission acoustically when needed.	P57
416	18	Not possible to communicate with vehicle at 1180m depth; holding pattern caused a timeout, and AUV surfaced. Acoustic telemetry max range was 500m for digital data.	P57
418	15	When homing was stopped deliberately after 10 min, the AUV did not go into a “stay here” mode. Rather it continued on the same heading; stopped by acoustic command 500m from shore. Cause was incorrect configuration of mission exception for homing. Default in campaign configuration script was not set due to inexperience with new configuration tools.	P87

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