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# HEATING IN COLLISIONS OF SOLIDS: POSSIBLE APPLICATION TO IMPACT CRATERS

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**SUMMARY: Due to the importance of collisions and impacts in early phases of the evolution of the planetary system, it is interesting to estimate the heating of a solid target due to an impact on it. A physically simple calculation of the temperature to which a solid target heats up after the impact of a projectile with mass $m$ and speed $v$ is performed, and possibilities for the application of this result in planetology are pointed out.**

**Key words. planets and satellites: general**

## 1. INTRODUCTION

Colliding macroscopic bodies exchange momentum. In inellastic collisions, a part of the momentum is transformed into heat.

The aim of this paper is to analyze to some extent the thermal effects in a collision of solid bodies of different masses, and to estimate the temperature to which the more massive of the colliding bodies (the target) heats up. If the temperature of the target rises sufficiently, melting and ultimately vaporisation will occur. The melting temperature of a solid can be estimated by the so called Lindemann criterion. The value obtained by the Lindemann criterion will be compared with the temperature to which the target heats up in a collision. As the temperature to which the target heats up depends on the kinetic energy of the impactor this calculation will provide an estimate on whether or not the target melts in an impact, and, as a consequence, can it be analyzed by applying the solid state physics or not.

If the mass of the target is sufficiently bigger than the mass of the impactor, the impact will produce virtually no change in the momentum of the target, while the impactor will almost certainly be destroyed. The kinetic energy of the impactor will be spent on heating of the target and on introducing changes into its structure.

The following section contains the calculation of the temperature to which a target in a collision heats up in the case of an inellastic collision. The calculation is performed for two cases: for the "ideal" case, in which all of the kinetic energy of the impactor is converted into heat, and for a more realistic case, in which a part of the kinetic energy is used to form a crack in the target. The third part shows how to include the equation of state of the material of the target into the calculation in one particular example, and the last part of the paper points to possible planetological application of the calculation presented in the paper.

## 2. CALCULATIONS

### 2.1. The ideal case

The specific heat of a solid at low temperature is given by (Davydov 1980):

$$C_V = \frac{2\pi^2}{5}\frac{k_{\mathrm{B}}}{(\hbar\overline{V})^3}(k_{\mathrm{B}}T)^3 \quad (1)$$

where $k_{\mathrm{B}}$ is Boltzmann's constant, $\hbar$ is Planck's constant divided by $2\pi$, $T$ the absolute temperature and $\overline{V}$ the speed of sound waves in the material. It is known that:

$$\overline{V} = (\frac{\partial P}{\partial \rho})^{1/2}, \quad (2)$$

where $P$ and $\rho$ denote the pressure and density of the material, respectively. Inserting Eq. (2) into Eq. (1), it follows that:

$$C_V = \frac{2\pi^2}{5}\frac{k_{\mathrm{B}}}{(\hbar)^3}(\frac{\partial P}{\partial \rho})^{-3/2}(k_{\mathrm{B}}T)^3 \quad (3)$$

Heating up the volume $V$ of material of the target by one degree requires the amount of energy equal to $VC_V$. The temperature to which this volume will heat up when impacted by an object having mass $m$ and speed $v$, is in the ideal case, given by:

$$T_0 + \frac{1}{2}(\frac{mv^2}{VC_V}) = T_1, \quad (4)$$

where $T_0$ denotes the initial temperature of the target. Practically, the volume $V$ represents physically the volume of a crater resulting from the impact. The volume is obviously dependent on the shape of the crater. Assume a crater has shape of a half of a rotational ellipsoid with distinct semi axes, denoted by $a$, $b$ and $c$. Physically, $a$ and $b$ are the semi axes of the "opening" of the crater, and $c$ denotes its depth. The volume is $V = (2/3)\pi abc$, which leads to the final expression for the temperature to which this volume of the target heats up as a result of the impact:

$$T_1 = T_0 + \frac{15}{8\pi^3 k_{\mathrm{B}}}\frac{mv^2(\hbar\overline{V})^3}{abc}(k_{\mathrm{B}}T_0)^{-3}. \quad (5)$$

Inserting Eq. (2) into Eq. (5), it finally follows that:

$$T_1 = T_0 + \frac{15}{8\pi^3 k_{\mathrm{B}}}\frac{mv^2\hbar^3(\frac{\partial P}{\partial \rho})^{3/2}}{abc}(k_{\mathrm{B}}T_0)^{-3}. \quad (6)$$

Is this heating sufficient or not to cause melting and even vaporisation of the material of the target? This can be determined by applying the so called Lindemann criterion which states that (Celebonovic 1993):

$$T_m = T_{m0}(\frac{\rho_0}{\rho})^{2/3}\exp(2\gamma_0(1-(\rho_0/\rho))). \quad (7)$$

In this expression $T_m$ denotes the melting temperature of a material at mass density $\rho$, $T_{m0}$ is the melting temperature at density $\rho_0$.The symbol $\gamma_0$ denotes the value of the Gruneisen parameter of the material at density $\rho_0$.The Gruneisen parameter is defined as

$$\gamma = \frac{\alpha K_T}{C_V \rho}, \quad (8)$$

where $\alpha$ is the thermal expansion coefficient and $K_T$ the isothermal bulk modulus of the material. Comparing Eqs. (6) and (7) it follows that $T_1 \leq T_m$ if:

$$\frac{E_{\mathrm{k}}}{abc}(\frac{\partial P}{\partial \rho})^{3/2}\hbar^3(k_{\mathrm{B}}T_0)^{-3} \leq \frac{4\pi^3}{15\hbar^3}T_{m0}(\frac{\rho_0}{\rho})^{2/3}\times$$
$$\exp(2\gamma_0(1-(\rho_0/\rho)))\times[1-\frac{T_0}{T_{m0}}$$
$$\times(\frac{\rho_0}{\rho})^{-2/3}\exp(-2\gamma_0(1-(\rho_0/\rho))], \quad (9)$$

where $E_{\mathrm{k}}$ denotes the kinetic energy of the impactor at the moment of impact.

In practical terms, the fulfillement of this condition means that the target heats up at the point of impact, but does not melt. A further implication is that such an impact can be analyzed by solid state physics. The analytical expression of the derivative $\frac{\partial P}{\partial \rho}$ depends on the form of the equation of state of the target material.

### 2.2. The "real" case

If the relative velocity of two colliding solid bodies is high enough, and if the material strength of the target is small enough, it can be expected that a fracture will occur in the target as a consequence of the collision. A standard result for the stress needed for a fracture to occur is given by (for example, Tilley 2004):

$$\sigma_C = \frac{1}{2}(\frac{E\chi\tau}{r_0 a})^{1/2} \quad (10)$$

where $E$ is Young's modulus of the material, $\chi$ is the surface energy, $\tau$ the radius of curvature of the crack, $r_0$ the interatomic spacing at which the stress becomes zero and $a$ denotes the length of the crack. A simple analysis shows that $\sigma_C$ has the dimensions of pressure, which means that the stress multiplied by a volume has the dimensions of energy.

This means that when a fracture forms in a target as a result of the impact, the energy ballance has the following form:

$$\frac{1}{2}mv^2 - \sigma_C V = C_V V(T_1 - T_0). \quad (11)$$

It follows that, in this case, the target heats up to a temperature given by:

$$T_1 = T_0 + \frac{1}{C_V}\times(\frac{E_{\mathrm{k}}}{V} - \sigma_C). \quad (12)$$

Inserting Eqs. (2), (3) and the expression for the volume into Eq. (12), simple algebra could give the result for the temperature $T_1$ to which the target heats up.

A physically more interesting approach to the same problem is given by dynamic quantized fracture mechanics ($DQFM$) (for example Pugno 2006). The basic difference between $DQFM$ and the usual approach used in material science is that $DQFM$ introduces geometry in studies of scaling laws in material science (for example Carpinteri and Pugno 2005). Considering that the occurence of a fracture in a material is a sign of its failure, it can be shown in $DQFM$ that the stress needed for the occurence of a failure is given by (Pugno 2006):

$$\sigma_f = K_{Ic}[\frac{1+(\frac{\rho_0}{2q})}{\pi(l_0+(q/2))}]^{1/2}, \quad (13)$$

where $K_{Ic}$ denotes the fracture toughness, $\rho_0$ is the radius of the crack of length $l_0$ and $q$ is the length of the so called fracture quantum. In this case, the energy balance is determined by Eq. (11), with $\sigma_f$ instead of $\sigma_C$, and the temperature to which the target heats up is given by Eq. (12) with the same replacement.

The final result is:

$$T_1 = T_0 + \frac{3\hbar^3}{2k_B}(\frac{\partial P}{\partial \rho})^{3/2}\frac{1}{(k_BT_0)^3}\left\{(\frac{3E_k}{2\pi abc})- K_{Ic}[\frac{1+(\frac{\rho_0}{2q})}{\pi(l_0+(q/2))}]^{1/2}\right\}. \quad (14)$$

In this way we have obtained an expression for the temperature to which a target heats up after the impact in the case a fracture forms. Whether or not it melts as a result of the impact can again be determined by comparing $T_1$ with the result of Lindemann's criterion, expressed by Eq. (7). It turns out that the condition for "non-melting" is given by:

$$\frac{E_k}{V} - \sigma_f \leq C_V T_{m0}(\frac{\rho_0}{\rho})^{2/3}\exp(2\gamma_0(1-(\rho_0/\rho))) \times[1-\frac{T_0}{T_{m0}}\times(\frac{\rho_0}{\rho})^{-2/3}\exp(-2\gamma_0(1-(\rho_0/\rho))]. \quad (15)$$

## 3. THE INFLUENCE OF THE EQUATION OF STATE

Several equations in the preceeding section contain the derivative $\frac{\partial P}{\partial \rho}$, which implies that their application demands the knowledge of the equation of state ($EOS$) of the material of the target. Generally speaking, the $EOS$ is an equation of the form $P = f(\rho, T)$ where all the symbols have their standard meanings, and $f$ is some function. The choice of the $EOS$ appropriate for a given material is a complex task.

As an example, the so called Birch-Murnaghan $EOS$ will be used. This equation has the form (Stacey 2005):

$$P(V) = \frac{3B_0}{2}\left[(\frac{V_0}{V})^{7/3} - \frac{V_0}{V})^{5/3}\right]\times \left\{1+\frac{3}{4}(B_0'-4)\left[(\frac{V_0}{V})^{2/3}-1\right]\right\}, \quad (16)$$

where $B_0 = -V(\frac{\partial P}{\partial V})_T$ is the bulk modulus of the material and $B_0' = (\frac{\partial B}{\partial P})_T$ is its pressure derivative. The symbols $V_0$ and $V$ denote the volume of a specimen under consideration at the initial value of the pressure $P_0$ and at some arbitrary value $P$. It can be shown that:

$$\frac{\partial P}{\partial \rho} = \frac{B_0}{8\rho_0^3}[27(B_0'-4)\rho^2 + 14(14- 3B_0')\rho_0\rho(\frac{\rho}{\rho_0})^{1/3} + 5(3B_0'-16)\rho_0^2(\frac{\rho}{\rho_0})^{2/3}] \quad (17)$$

and:

$$(\frac{\partial P}{\partial \rho})^{3/2} = \frac{1}{16\sqrt{2}}(\frac{B_0}{\rho_0^3})^{3/2}[27(B_0'-4)\rho^2 + 14(14- 3B_0')\rho_0\rho(\frac{\rho}{\rho_0})^{1/3} + 5(3B_0'-16)\rho_0^2(\frac{\rho}{\rho_0})^{2/3}]^{3/2}. \quad (18)$$

The last expression can be simplified by assuming $B_0' = 0$. It follows that in this case the temperature to which the target gets heated as a result of an impact with a fracture forming in it is given by:

$$T_1 = T_0 + \frac{3\hbar^3}{32k_B\sqrt{2}}(\frac{B_0}{\rho_0^3})^{3/2} \times[196\rho_0\rho(\frac{\rho}{\rho_0})^{1/3} - 108\rho^2 - 80\rho_0^2(\frac{\rho}{\rho_0})^{2/3}]^{3/2} \times\frac{1}{(k_BT_0)^3}\times[\frac{3E_k}{2\pi abc} - K_{Ic}[\frac{1+(\frac{\rho_0}{2q})}{\pi(l_0+(q/2))}]^{1/2}]. \quad (19)$$

A similar result could be obtained for any other form of the $EOS$.

## 4. DISCUSSION AND CONCLUSIONS

The calculations discussed in this paper are mathematically simple, but physically they are of considerable interest in planetary science. The main results expressing the temperature to which a massive target heats up as a result of impact of a projectile of smaller mass are given by Eqs. (6) and (16). The calculations have been performed for two distinct cases: when the impact has no consequences on the structure of the target, and when it is so strong that a fracture occurs in the body of the target.

There is a distinct difference in the behaviour of the results obtained in the two cases. In what we have called "the ideal case", the temperature of the target always changes as a result of the impact, as $\partial P/\partial \rho$ is always different from zero. No heating effect would occur only in a special case $\partial P/\partial \rho = 0$ which is physically unrealistic because a material in which pressure and density are not related in some way does not exist.

An interesting result has been obtained in the "real case". Namely, using standard material science would have given a result which would have taken into account only the parameters of the material of the target. Instead of such an approach, we have used the *DQFM* approach, which takes into consideration the material parameters, but also the geometry of the problem. Note that the result contains the length and radius of the crack, as well as the length of the fracture quantum.

A possible extension of the calculations discussed here could be the inclusion of magnetic fields. Recent experiments in material science have shown that the presence of low magnetic fields can lead to strenghtening of materials (Erb et al. 2012). This means that their failure strength is increased.

In this case the heating effect does not always occur. It is clear that for the particular value of the kinetic energy of the impactor, given by:

$$E_{\mathrm{k}} = \sigma_f V \tag{20}$$

$T_1 = T_0$ in Eq. (12), which means that in that case the target does not heat up as a consequence of the impact. Such a result may seem strange, but it is a consequence of the partition of the kinetic energy of the impactor on the heating of the target and formation of a fracture in it.

Results of this paper have obvious applications in planetary science. It is known that impacts and collisions between various bodies have been occurring throughout the history of our planetary system. One of the consequences of these events are the impact craters on the Earth, Moon, Mars, (at least) some asteroids (for example on Eros, see Veverka et al. (2001)) and satellites. It is expectable that the spots on the surfaces of these objects in which impacts occur get heated. The question which arises is whether or not such events can be analyzed by using solid state physics. Namely, if the material of the target(s) heats up but *does not* melt or vaporize, an impact can be analyzed by using laws of solid state physics. However, if melting occurs, solid state physics is applicable only up to the temperature of melting. Details will be discussed elsewhere.

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# ЗАГРЕВАЊЕ ЧВРСТИХ ТЕЛА У СУДАРИМА: МОГУЋА ПРИМЕНА НА УДАРНЕ КРАТЕРЕ


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*Оригинални научни рад*


Познато је да су удари и судари били веома важни у раним периодима развоја планетног система. Услед тога, природно се поставља питање загревања чврсте мете при удару пројектила у њу. У раду је дискутовано физички једноставно израчунавање температуре до које се загреје чврста мета при удару у њу пројектила масе $m$ и брзине $v$. Указано је на могуће примене добијених резултата у планетологији.