Description of the deformation process under thermo-mechanical fatigue

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ABSTRACT

Purpose: The main problem addressed in the paper is the description of a deformation process under the conditions of mechanical and thermal interactions.
Design/methodology/approach: The mathematical modelling has been used to describe the stress-strain behaviour of materials. The method of fatigue testing has been adopted to determine experimentally stress-strain characteristics. The method based on the long term own experience in thermo-mechanical investigations and new European Code-of-Practice for Thermo-Mechanical Fatigue Testing.
Findings: An appropriate model description has been developed. Fatigue examinations of the P91 steel that is used in power industry, were carried out. The validation of the model has been performed. So far, experimental verification of the usefulness of the model description to determine the stress-strain characteristics’ course for a selected value of the phase shift angle between the temperature and total strain cycles has been made. It has been found that the proposed model reflects the deformation process nature very well in variable temperature, strain and stress conditions. Hence, a conclusion seems to be justified that the approach presented in the paper could constitute the right basis for appropriate constitutive equations, which depict the material behaviour under thermo-mechanical conditions.
Research limitations/implications: The developed description should be useful in problems of fatigue behaviour predictions of materials under different mechanical and thermal loadings in industry practical applications and in research problems connected for instance with fatigue life criteria description and validation.
Originality/value: The new material characteristics have been shown in the work and the own new method of the material fatigue behaviour prediction. The work is addressed to researchers interested in problems of material testing and material behaviour prediction under different loadings that we can meet in the operation practice.
Keywords: Fatigue; Mechanical properties; Metallic alloys; Applied mechanics

1. Introduction

Thermo-mechanical fatigue (TMF) is one of important phenomena deciding upon the cracking processes in machine components and devices exposed to mechanical and thermal influences in the power, chemical and metallurgical industries, in aviation and transport [1-5]. In spite of its fatigue-like nature, this kind of material destruction is also connected with other processes stimulated by the long-term influence of increased temperature such as ageing, oxidation or creep, and in some cases the interdependence between the fatigue phenomena and the processes mentioned has to be taken into account. In order to identify the properties of materials used under cyclic temperature changes and mechanical load, it is also important to consider the interdependences mentioned as well as characteristics of individual processes consisting of conditions of use in individual elements.

This is because most frequently the application of properties indicated in tensile tests is then impossible. What constitutes a useful and practically applicable characteristics of the thermo-mechanical fatigue in a given material i.e. characteristics of cyclic deformation processes and strength understood as a number of
cycles to perform specimen fracture in agreed conditions of the fatigue tests. This data is necessary for designers when predicting behaviour and strength of the elements in case of the aforementioned types of materials application, and these are as important as creep and low-cycle fatigue characteristics, which are commonly used in practice when describing materials cracking processes taking place in elevated temperature [5-11].

Determining the characteristics of thermo-mechanical fatigue is frequently connected with a number of technical problems, and until the present moment no standard has been established to carry out this sort of fatigue tests. In the year 2000, the European Commission made the decision to fund a research project under the Framework Programme (acronym TMF – Standard), the aim of which was to establish guidelines to codify the procedures of thermo-mechanical fatigue tests performance. The project involved 20 industrial, research and scientific centres. It was completed in the year 2005 and its result at the current stage is a standard draft in the form of detailed procedure guidelines (Code-of-Practice) [2], referring to all practical aspects of the thermomechanical fatigue research (TMF) under controlled strain, such as adequate dynamic methods of measurement and temperature control, the effects caused by deviation in nominal temperature value, phase displacement between mechanical strain cycle and temperature cycle, test initiation method and its start after turning off, as well as admissible temperature gradients. The thermomechanical fatigue tests belong to the most complex mechanical tests. Briefly, it can be said that they consist in simultaneous temperature control and total (geometric) strain - \( \varepsilon \). The result of the two control signals (Fig. 1) is mechanical strain - \( \varepsilon_M \) being the difference between total strain-\( \varepsilon \) and thermal strain-\( \varepsilon_T \).

\[
\varepsilon_M = \varepsilon - \varepsilon_T.
\]  

Due to the number of parameters deciding upon material behaviour in practical applications [1-4, 13-20], thermo-mechanical fatigue tests can show a large variety. Different sorts of temperature cycles used – cycles with constant strain and temperature rates, sinusoidal ones, trapezoidal or rectangular cycles. Dislocations in the strain cycle phase in relation to the temperature cycle are possible. The difference can also concern the values in maximal and minimal test parameters and the cycle period or its individual parts. Due to diversity of tests and their realisation period in laboratory conditions, it is possible to perform only selected types, relating the studies to the most frequent cases of fatigue in machine elements and devices. Among widely-used tests there are those using cycles characterised by constant heating and cooling rates and constant strain rate. However, the strain cycle in the phase can be dislocated compared to the temperature cycle [1, 2].

Most frequently this dislocation amounts to 180° (cycles with opposite phases) – out-of-phase test (OP), 0°, with the lack of phase displacement (phase compatible cycles) – in-phase test (IP), or 90° (−90°) (tests with rhombus-like characteristics) – diamond tests. Fig. 2, 3 represent results for the cases of two different test types – IP and OP. These are hysteresis loops determined for different angles in phase displacement, corresponding to different moments characterised in terms of the number of cycles. In order to illustrate the way of defining fatigue life, understood as a number of cycles until pre-arranged failure, Fig. 4 shows an example of the relationship between the number of cycles and the range of changes in stress.

The diagrams illustrate results of tests performed for the steel P91. The longstanding operation of this material has corroborated its usefulness for work at elevated temperatures. A still significant problem, however, is the cracking of the P91 steel, especially under variable temperature conditions.
Fig. 3. Test results for thermo-mechanical fatigue in the steel P91– the case of test in which the mechanical strain signal had opposite phase to the temperature signal – out-of phase test (OP)

Fig. 4. Test results for thermo-mechanical fatigue – the illustration of the method of defining fatigue life [2]

Taking into account a limited number of tests carried out in laboratory conditions, the problem to be solved is the description of materials characteristics in other cases, based on selected research results. The paper concentrates on the description of deformation characteristics (stress-strain characteristics) in the form of a hysteresis loop, working out their mathematical models based on low-cycle fatigue studies.

2. Algorithm of the procedure

One of the methods commonly applied in the analysis of a deformation process taking place in low-cycle fatigue conditions is an approach which refers to the steady state, which should be approximately characterised by stability of the characteristics in the form of a hysteresis loop for any selected strain range. Such an approach is correct only in some cases. Most often, the characteristics of a state known as the “steady” or “saturated” state depend on the load history and in the case when an assumption is made that a material shows the occurrence of a saturated state, what is left to be solved is the problem of an evaluation of the accuracy of description of the characteristics. Such evaluation is most frequently based on the results of laboratory research. In the case of thermo-mechanical fatigue, the problem becomes much more complex due to the influence of temperature on the cyclic deformation processes, which strengthen or weaken the material, and thus, on the saturated state characteristics. Taking into account the much higher degree of complexity of the description of material behaviour in thermo-mechanical fatigue conditions compared to the low-cycle fatigue at constant temperatures, at the stage of developing the basis for a model approach to the stress-strain characteristics in such conditions, it seems reasonable that an assumption is made about stability of the material characteristics.

A consequence of such an assumption is the possibility of presenting stress as a function of strain and temperature, without the necessity of taking into account the effects connected with the loading history. In this paper, the description of the deformation process under thermo-mechanical fatigue is based on the steady state characteristics for isothermal low-cycle fatigue. The method takes advantage of mathematical models of dependences describing the course of hysteresis loop branches, determined for individual constant temperatures of tests. The origin of the reference system was assumed to be found at the peak of a hysteresis loop, at minimal strain. In such case, the curves showed in Fig. 5 illustrated part of a hysteresis loops at increasing stress and strain.

In Fig. 6 it will be the upper section between points of coordinates $(\varepsilon_0, \sigma_0)$ and $(\varepsilon_c, \sigma_c)$. In the proposed method a mathematical model of the characteristics shown in the figure 5 was adopted, with its general form as follows:

$$\sigma' = f(\varepsilon', T), \quad \varepsilon' \in (0, \Delta \varepsilon) \quad (2)$$

Performing transformation of the function (1) and taking into account initial conditions for a part of the hysteresis loop illustrating the course of deformation with increasing strain, the following dependency is obtained:

$$\sigma = \int \left[ \varepsilon - \varepsilon_R \right]_T + \sigma_R \quad (3)$$

A similar transformation for a part of the cycle with decreasing strain gives:

$$\sigma = -\int \left[ \varepsilon - \varepsilon_C \right]_T + \sigma_C \quad (4)$$

Equations (3) and (4) include the values being the coordinates of the hysteresis loop peaks. These coordinates have constant values for stabilized hysteresis loops for fatigue in isothermal conditions. The issue of determining the course of hysteresis loop branches with increasing and decreasing strain under the assumption that the course of both hysteresis loop branches can be described by the same dependency (2), it therefore comes down to its various transformations.

Determining the course of dependences between mechanical strain, stress and temperature under thermo-mechanical fatigue becomes a considerably complex issue. The dependences between stress and strain will be influenced by the temperature which is an independent variable in function $f(\varepsilon', T)$ and, at the same time, they will be depended on the values of $\sigma_0$ and $\sigma_c$. What appears
as a result is the problem of determining the coordinates \((\varepsilon_R, \sigma_R)\) and \((\varepsilon_C, \sigma_C)\) (Fig. 7), which can not be determined from simple dependences between stress and strain and temperature for maximal and minimal strain in thermo-mechanical cycle. The values of \(\sigma_R\) and \(\sigma_C\) should be treated in this case as temperature functions and they will change during deformation with temperature and mechanical strain.

\[
\sigma_R = \sigma_{R0} + \phi(\varepsilon, T) \tag{5}
\]

In the case of the part of the loop which corresponds to decreasing strain, the following dependence is assumed:

\[
\sigma_C = \sigma_{C0} + \phi'(\varepsilon, T) \tag{6}
\]

The influence of temperature on the values of \(\sigma_R\) and \(\sigma_C\) can be considered in equations (3) and (4) by introducing additional functions \(\phi(\varepsilon, T)\) or \(\phi'(\varepsilon, T)\), taking into account the effect of the material memorizing the value of initial deformation and strain. In this way, for the part of hysteresis loop corresponding to increasing strain, the following equation is obtained:

\[
\sigma_R = \sigma_{R0} + \phi(\varepsilon, T)
\]

At the present
\[ \varphi(\varepsilon, T) = \frac{1}{2} \left[ f(\varepsilon - \varepsilon_c, T) - f(\varepsilon - \varepsilon_c, T) \right] \]  
\[ \varphi'(\varepsilon, T) = \frac{1}{2} \left[ f(\varepsilon - \varepsilon_c, T) - f(\varepsilon - \varepsilon_c, T) \right] \]

The graphic interpretation of the function \( \varphi(\varepsilon, T) \) is presented in the Fig. 8, 9, which presented how the abrupt changes of temperature will influence the shape of deformation curve in the part of hysteresis loop at increasing strain. The figure 8 represents the case \( T_1 < T_2 \), the figure 9 illustrates the change in deformation characteristics, if \( T_1 > T_2 \).

Under thermo-mechanical fatigue \( \sigma_R \) and \( \sigma_c \) will vary continuously, according to the dependences (5) – (8).

\[ \sigma_R(\varepsilon, T) \]

\[ \varphi \]

\[ \sigma \]

\[ \varepsilon' \]

\[ \varepsilon \]

\[ \sigma_0 \]

\[ \varepsilon_0 \]

Fig. 9. Graphic representation of the influence the abrupt changes of temperature exert on the course of deformation characteristics in the case of hysteresis loop branches with increasing strain – increase of temperature

**3. Validation**

Function \( f(\varepsilon', T) \) can be assumed in a different form, since it is a mathematical approximation of experimentally determined deformation characteristics in steady state conditions, at fatigue in a range of low-cycle fatigue at constant temperatures. In the case of the first approximation in the paper, the following form has been proposed for the function \( f(\varepsilon', T) \):

\[ f(\varepsilon', T) = (A - CT') \arctan(D \varepsilon') \]  
\( \varepsilon' = \varepsilon - \varepsilon_R \) should be then substituted for the hysteresis loop branch at growing strain and \( \varepsilon' = |\varepsilon - \varepsilon_R| \) for the hysteresis loop branch at a decreasing strain. Being given dependence (9), for which constants A, C and D have been determined, one can determine based on it the hysteresis loops for different temperatures and ranges of strain in low-cycle fatigue conditions. An example is presented in Fig. 11.

Fig. 10. Graphic form of dependence (9)

Fig. 11. Example of the course of a hysteresis loop determined using equation (9)

Insignificant discrepancies between the research results and their mathematical representation testify to accuracy of the model. It should be noted here that dependence (9) is only one of possible representations of function \( f(\varepsilon', T) \). It also seems reasonable to seek a physical interpretation of function \( f(\varepsilon', T) \). At the present stage, its simplest possible form has been assumed in an attempt to present the modelling procedure itself as well as to evaluate the ability of its application in practice. The developed approach was
then subject to validation, including a comparison of calculation results with the test results. The first stage of the validation process consisted in the determination, based on the model approach, of deformation characteristics for different mechanical strain and temperature cycles. One of the cases investigated was a test with an abruptly changing temperature and a mechanical strain cycle of a constant strain rate. Dependences between strains and stresses, appropriate for a test of this type, are presented in Fig. 12, 13.

Fig. 12. Courses of deformation characteristics for cases of abrupt temperature changes during one loading cycle - increase of temperature during increasing strain and decrease of temperature during decreasing strain

Fig. 13. Courses of deformation characteristics for case of abrupt temperature changes during one loading cycle - decrease of temperature during increasing strain and increase of temperature during decreasing strain

Fig. 14. Courses of deformation characteristics for a test with mechanical strain consistent in phase with the temperature (in phase - IP test)

Fig. 15. Courses of deformation characteristics for a test where mechanical strain is shifted in phase, in relation to the temperature, by an 180° angle (OP test)

The consecutive figures, 14 and 15, illustrate the results of tests carried out with the application of mechanical strain cycles with phases consistent with (IP) or opposite (OP) to the temperature cycles. Figures 16 and 17 illustrate tests with the use of mechanical strain courses shifted in phase by a 90° angle: a rhomboidal test with a clockwise or an anticlockwise course. Figures 12-17 reflect only the nature of the courses of the stress-strain dependence.

In this case, correctness of the mathematical approach can be referred in qualitative terms to the literature data [1,4].

The study also encompasses verification of the model developed by referring the characteristics determined based on the model, the latter having been based on the low-cycle fatigue tests' results, to the results obtained in thermo-mechanical fatigue tests. The tests were carried out for the P91 steel.

Figure 18 illustrates stress-strain characteristics, based on the model, for the test with cycles consistent in phase. It also shows the results in the form of experimental points from the test, in the range from the hysteresis loop stabilisation and period until failure.
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The next example shows the other type of more sophisticated experiments. Figures 19-21 present the results of verification obtained for the test made at a shift in phase of the total strain cycle by a 90° angle in relation to the temperature cycle.

Consecutive figures show the dependences of thermal and mechanical strain on time (Fig. 19), the dependences of mechanical strain on temperature (Fig. 20) and characteristics of stress as a function of mechanical strain (Fig. 21), determined experimentally for the period of a stabilised hysteresis loop course and based on calculations.

In Fig. 19-21 have been shown results of the particular rarely used type of the TMF test. This type of test has been chosen taking into account the possibility of model validation in different, also special conditions of thermo-mechanical fatigue.
The curves shown in the Fig. 21 have similar shape although we can see some discrepancies between results obtained from the model and experiments. The main source of these discrepancies is, as we can suppose, the simple model of the function $f(e', T)$ presented in the form of equation (9). But it is apparent from the tests, which chosen results have been shown in Fig. 18 and 21, that the discussed method of the stress-strain behaviour description gives the correct results concerning to the character of the process.

Such approach would be useful for instance in a situation, when it is necessary to describe stresses changeable in time for the particular temperature and strain cycles and phase shift between these variables. Application of the proposed method of stress-strain behaviour description enables to determine characteristics of the stress cycles in different, possible in practice, cases and it could be useful for the local behaviour prediction of components under mechanical and thermal loading.

4. Conclusions

The presented approach to the cyclic deformation process in the form of mathematical dependences describing the relations between stress, strain and temperature in a uniaxial state of stress was subjected to verification both with reference to the nature of the diagrams obtained based on the approach. The verification was also made as regards the quantitative approach, by referring the characteristics determined from mathematical models to the test results.

It has been found that the proposed model reflects the deformation process nature very well in variable temperature, strain and stress conditions. This statement refers to the author’s own test results and to literature data, which were taken into account when analysing the courses of $\sigma(e, T)$ for different types of thermo-mechanical cycles [2,4]. Hence, a conclusion seems to be justified that the approach presented in the paper could constitute the right basis for appropriate constitutive equations, which depict the material behaviour under thermo-mechanical conditions.
The dependences included in the paper are based on an assumption of cyclic stability of the material. Thus, the results obtained on such basis can be referred to the state of saturation. For the investigated materials, the above approach seems to be reasonable, since the period of a stabilised range of stress encompasses a prevailing part of their characteristics $\Delta \sigma(N)$ (Fig. 4). The form of the mathematical dependences presented also ensures the possibility of taking into account the effects of cycling strengthening or weakening of a material in future research.

The constants of the mathematical model have been determined based on the results of low-cycle fatigue investigations at constant temperatures, by referring the obtained model to the tests conducted at a variable temperature. Such procedure entails an implicated assumption that the state of saturation, i.e. stabilisation of the characteristics of $\Delta \sigma(N)$, does not depend on the “path” which we take to achieve such state. This assumption comes true only in an approximation, whilst its correctness depends on the type of material.

Although the proposed approach reflects, in a way satisfactory to engineering applications, the course of characteristics in both quantitative and qualitative terms, the problems that remain unsolved are approximation of the low-cycle research results by means of function $f(\varepsilon,T)$ and physical interpretation of this function. Yet another significant problem is that the effects of cyclic strengthening or weakening, the influence of deformation history and the determination of relations between the constants present in the models and their types and material structures, should be taken into account in the description presented.

In such terms, the presented approach constitutes a useful at the current stage approximation which will be further developed so as to take account of a larger number of the phenomena that take place during cyclic elastic-plastic deformation as well as a larger number of factors which determine those phenomena. This time it seems to be important to discuss about the TMF problems together with the discussion concerning the fracture of component under mechanical and thermal loading [5-11] that is one of the main phenomena deciding for instance on power industry component durability.

Despite the long term experience in thermo-mechanical testing [1-4, 13-20] regarding to many aspects of this type of a material damage it is still one of the main, but not well known and described, problem of component durability under mechanical and thermal loadings. It is still necessary to develop the methods of material testing, methods of material behaviour and life prediction and method of implementation the laboratory test result in problems of a component durability assessment.

The work has mainly methodological character and shows the own common algorithm of the stress-strain characteristic description. The verification on this stage was made for one of steels used mainly in power industry. This choice was performed mainly due to the importance of the problems of the life assessment of components which work under mechanical and thermal loading in power industry.

We can find many papers and books concerning the problems of creep, fracture of elements with cracks and concerning low cycle fatigue, but the number of works about the thermo-mechanical fatigue of this type of components and their materials is still limited. It is of course necessary to make verification for the wider group of materials in this particular case of deformation process description. But more common problem is how to find methods of implementation TMF characteristics in procedures of design and assessment like for instance SINTAP or FITNET methods developed in European Projects financed by European Commission under the 5th and 6th Framework Programmes [21-23]. It is particularly important because, despite the conviction about the importance of the thermo-mechanical process in problems of power plant components cracking, the methods of design and assessment rarely contain rules and characteristics based on the TMF analysis.

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