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## MACHINING OPTIMISATION AND OPERATION ALLOCATION FOR NC LATHE MACHINES IN A JOB SHOP MANUFACTURING SYSTEM

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### Abstract

Numerical control (NC) machines in a job shop may not be cost and time effective if the assignment of cutting operations and optimisation of machining parameters are overlooked. In order to justify better utilisation and higher productivity of invested NC machine tools, it is necessary to determine the optimum machining parameters and realize effective assignment of cutting operations on machines. This paper presents two mathematical models for optimising machining parameters and effectively allocating turning operations on NC lathe machines in a job shop manufacturing system. The models are developed as non-linear programming problems and solved using a commercial LINGO software package. The results show that the decisions of machining optimisation and operation allocation on NC lathe machines can be simultaneously made while minimising both production cost and cycle time. In addition, the results indicate that production cost and cycle time can be minimised while significantly reducing or totally eliminating idle times among machines.

Keywords: Machining optimisation, Operation allocation, NC lathe, Job shop.

### 1. Introduction

Machining optimisation involves the determination of efficient machining parameters such as cutting speed, feed rate and depth of cut in process planning stage. It directly impacts the production economics of machining processes in terms of meeting the minimum production cost, minimum production time, maximum production rate, and maximum production profit objectives. Operation allocation is concerned with allocating machining operations among machines. It seeks to avoid some machines to become idle leaving others to be more occupied

<b>Nomenclatures</b>	
$C_o$	Operating cost, \$/min
$C_t$	Tool cost, \$/edge
$d_{i,j,k}^L, d_{i,j,k}^U$	Lower and upper allowed depth of cut for the last (finish) operation $j$ of feature $k$ on machine $i$ respectively, mm
$d_k^T$	Total amount of material to be removed from feature $k$ , mm
$d_{ijk}^U$	Upper allowed depth of cut for rough operations $j$ of feature $k$ on machine $i$ , mm
$F_{max}$	Maximum allowed cutting force, kg
$f_{ijk}^L, f_{ijk}^U$	Lower and upper allowed feed rate for operation $j$ of feature $k$ on machine $i$ respectively, mm/rev
$K_T, K_F, K_S, K_P$	Constants for tool life, cutting force, surface roughness, cutting power, respectively
$L_k$	Length of feature, mm
$P_{max}$	Maximum cutting power of the motor, kW
$S_{max}$	Maximum surface roughness for the feature, $\mu\text{m}$
$t_r$	Tool replacement time, min
$Z$	Cycle time, min
<i>Greek Symbols</i>	
$\alpha_T, \beta_T, \gamma_T$	Constants in tool life equation
$\alpha_S, \beta_S, \gamma_S$	Surface roughness constants
$\beta_F, \gamma_F$	Cutting force constants
$\eta_m$	Mechanical efficiency
<i>Decision Variables</i>	
$d_{ijk}$	Depth of cut, mm
$f_{ijk}$	Feed rate, mm/rev
$v_{ijk}$	Cutting speed, m/min
$X_{ijk}$	=1 if operation $j$ of feature $k$ is allocated to machine $i$ ; and = 0 otherwise
<i>Subscripts</i>	
$i, j, k$	Indices for machine $i=1, \dots, m$ ; operation $j=1, \dots, J_k$ ; and feature $k=1, \dots, K$

with machining operations. It equally influences the production economics in terms of effective machine utilisation within a manufacturing shop floor.

In today's manufacturing environment, the application of numerical control (NC) technology allows the machine tools to perform operations automatically. As such, the machining conditions governed by machining parameters can be easily controlled. Consequently, both utilisation and productivity can be improved with lower cost and time. However, the success or failure in achieving these goals

greatly depends on how the machining parameters are determined and whether the cutting operations are well allocated to machines.

Both machining optimisation and operation allocation problems have been extensively investigated. For example, Ermer and Kromodihardjo [1], Wang [2], Mustafa and Ali [3], Xueping et al. [4], and Mgwatu [5] have made significant efforts to optimise machining parameters while Stecke [6], Shanker and Tzen [7], Choudhary et al. [8], and Das et al. [9] devoted their time to study operation allocation problems. Further studies on the optimisation of NC turning operations have been reported in [10-13]. It is noted however that machining optimisation and operation allocation problems have been addressed separately. For the machining optimisation problems, researchers tend to study the single machine problems. In most machining activities, several machines are involved to perform similar or quite different operations on parts. In this case, the optimality of the solutions obtained in single machine problems cannot be guaranteed. On the other hand, the studies on operation allocation are often based on the assumption that machining parameters are well known in advance. This assumption may not be valid in many cases in that machining parameters for an operation cannot be specified without knowing the actual machine to be used to perform the operation. As a result, the operation allocation may not be feasible. To avoid the locality of solutions in the machining optimisation problems and the infeasibility decisions in operation allocation problems, this paper proposes an integrated approach to solve the two problems. The paper is therefore intended to determine the optimal machining parameters and effective allocation of operations on NC lathe machines with the objectives of minimising production cost and cycle time.

## 2. Theories of Production Cost and Cycle Time

The components of production cost to be used in this study are machining cost, tool cost, and tool replacement cost. The production cycle time includes machining time and tool replacement time. Both production cost and cycle time are explained as follows

### 2.1. Total production cost

Machining cost is the cost incurred during the actual cutting process that depends on machining time. Machining time is given as a function of spindle speed  $v$  (m/min) and feed rate  $f$  (mm/rev).

$$t_m = \frac{\pi D L}{1000 v f} \quad (1)$$

where  $D$  is the diameter of the workpiece (mm),  $L$  is the length of the workpiece (mm). The machining cost per piece is the product of machining time  $t_m$  (min) and operating cost  $C_o$  (\$) given as:

$$C_m = \frac{\pi D L C_o}{1000 v f} \quad (2)$$

Tool cost is the cost per cutting edge depending on tool life  $T_L$  (min) and machining parameters. The Taylor's tool life equation extended to deal with cutting speed  $v$  (m/min), feed rate  $f$  (mm/rev), and depth of cut  $d$  (mm) may be written as [2]:

$$T_L = \frac{K_T}{v^{\alpha_T} f^{\beta_T} d^{\gamma_T}} \quad (3)$$

where  $\alpha_T$ ,  $\beta_T$ ,  $\gamma_T$ , and  $K_T$  are constants and tool-workpiece dependent. Denoting  $C_t$  as tool cost per cutting edge and considering Eqs. (1) and (3), then the tool cost of machining a single part is given by:

$$C_e = \frac{\pi D L}{1000 K_T} v^{\alpha_T-1} f^{\beta_T-1} d^{\gamma_T} C_t \quad (4)$$

The tool replacement time distributed to each part is

$$t_w = \frac{\pi D L}{1000 K_T} v^{\alpha_T-1} f^{\beta_T-1} d^{\gamma_T} t_r \quad (5)$$

Tool replacement cost is the product of replacement time and operating cost and is given by the following expression

$$t_c = \frac{\pi D L}{1000 K_T} v^{\alpha_T-1} f^{\beta_T-1} d^{\gamma_T} C_o t_r \quad (6)$$

The total production cost per part for multi-operation turning process is the sum of machining cost, tool cost and tool replacement cost which is presented as:

$$C_p = \sum_{j=1}^J \left\{ \frac{\pi D_j L_j C_o}{1000 v_j f_j} + \frac{\pi D_j L_j}{1000 K_T} v_j^{\alpha_T-1} f_j^{\beta_T-1} d_j^{\gamma_T} (C_t + C_o t_r) \right\} \quad (7)$$

## 2.2. Production cycle time

The production cycle time is the maximum time allowed at each machine to complete all sets of operations. The total production time at each machine should always be less or equal to production cycle time. The total production time for multi-operation turning process is the sum of machining time and tool replacement time written as follows

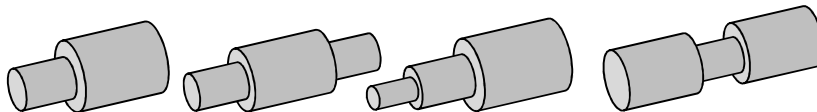
$$t_p = \sum_{j=1}^J \left( \frac{\pi D_j L_j}{1000 v_j f_j} + \frac{\pi D_j L_j}{1000 K_T} v_j^{\alpha_T-1} f_j^{\beta_T-1} d_j^{\gamma_T} t_r \right) \quad (8)$$

Equations (7) and (8) are total unit production cost and total unit production time for single-machine problem respectively. They will be used to develop models for multi-operation turning process in trying to solve machining optimisation and operation allocation problems jointly. Note that the set up time is not considered as the component of the total production time because it has no effects on the machining parameters.

### 3. Modeling of Multiple Turning Operations for Parts with Multiple Features in Job Shop Layout

In general, a multi-stage manufacturing system involves several machines. The machines may be arranged in a job shop, flow shop or cellular layout. In job shop environment, these machines are grouped together to perform similar operations for different parts. For example, several general-purpose NC lathe machines may form a turning work center. In flow shop, the machines are arranged together according to the process sequences. In cellular systems, the machines are grouped according to the process needed for a family of parts. The main advantage of the job shop layout is its flexibility where there is less restriction on part movements among machines therefore allowing alternative part routings. However, the negative effects of job shop arrangement including longer production time, high degree of idle time, and inherent in-process inventory have necessitated the formulation of dedicated production planning and scheduling methods where the goal is to run the job shop systems as efficiently as possible. Moreover, NC machine tools installed in a job shop should be effectively utilised in order to payback the committed investment as quickly as possible.

Normally, a workpiece processed in a job shop layout travels from one area of similar machines to another according to the established sequence of operations. However, in special cases, the workpiece may also need to travel within one area of similar machines for processing to the finished features. Suppose turning operations are performed using NC lathe machines arranged in the job shop manufacturing system to transform a raw material stock to a finished part with different features. NC lathe machines allow automatic tool changing between cutting operations thus reducing non-productive time. Typical cylindrical parts with several features are shown in Fig. 1. The problem is to assign cutting operations and select optimal cutting parameters for the available machines in order to obtain different part features so that desired dimensions of part features can be obtained at minimum production cost and cycle time. Common external cutting operations for rotational parts include rough turning, semi finish turning and finish turning operations. In this section, two models associated with different objectives are formulated. The first model is formulated with the objective of minimising the total production cost and the second model is formulated with the objective of minimising the cycle time while achieving final dimensional requirements of part features.



**Fig. 1. Typical Parts with Several Features Produced Using Turning Process.**

#### 3.1. Production cycle time

The first model is developed to minimise the total production cost by assigning part features and cutting operations to individual NC lathe machines and properly selecting machining parameters. The cutting operations in consideration are rough

and finish turning. Solving this model will simultaneously provide effective workload for each machine and optimum machining parameters for all machine-operation-feature combinations. The model is formulated as follows.

Minimise

$$\sum_{i=1}^m \sum_{j=1}^{J_k} \sum_{k=1}^K \left\{ \frac{\pi D_{(j-1)k} L_k C_o}{1000 v_{ijk} f_{ijk}} + \frac{\pi D_{(j-1)k} L_k}{1000 K_T} v_{ijk}^{\alpha_T-1} f_{ijk}^{\beta_T-1} d_{ijk}^{\gamma_T} (C_t + C_o t_r) \right\} X_{ijk} \quad (9)$$

Subject to:

$$\sum_{j=1}^{J_k} \sum_{k=1}^K \left( \frac{\pi D_{(j-1)k} L_k}{1000 v_{ijk} f_{ijk}} + \frac{\pi D_{(j-1)k} L_k}{1000 K_T} v_{ijk}^{\alpha_T-1} f_{ijk}^{\beta_T-1} d_{ijk}^{\gamma_T} t_r \right) X_{ijk} \leq Z, \quad \forall i \quad (10)$$

$$\sum_{i=1}^m \sum_{j=1}^{J_k} d_{ijk} = d_k^T, \quad \forall k \quad (11)$$

$$\sum_{i=1}^m \sum_{j=1}^{J_k} d_{ijk} X_{ijk} = d_k^T, \quad \forall k \quad (12)$$

$$\sum_{i=1}^m X_{ijk} = 1, \quad \forall (j, k) \quad (13)$$

$$v_{ijk}^L \leq v_{ijk} \leq v_{ijk}^U, \quad \forall (i, j, k) \quad (14)$$

$$f_{ijk}^L \leq f_{ijk} \leq f_{ijk}^U, \quad \forall (i, j, k) \quad (15)$$

$$d_{ijk} \leq d_{ijk}^U, \quad \forall (i, j, k) \quad (16)$$

$$d_{i,j,k}^L \leq d_{i,j,k} \leq d_{i,j,k}^U, \quad \forall (i, j, k) \quad (17)$$

$$K_F f_{ijk}^{\beta_F} d_{ijk}^{\lambda_F} \leq F_{\max}, \quad \forall (i, j, k) \quad (18)$$

$$K_P v_{ijk} f_{ijk}^{\beta_P} d_{ijk}^{\lambda_P} \leq P_{\max}, \quad \forall (i, j, k) \quad (19)$$

$$K_S v_{i,j,k}^{\alpha_S} f_{i,j,k}^{\beta_S} d_{i,j,k}^{\lambda_S} \leq S_{\max}, \quad \forall (i, k) \quad (20)$$

where

$$D_{(j-1)k} = D_{0k} - 2 \sum_{q=1}^{j-1} d_{iqk}, \quad \forall (i, k) \quad (21)$$

and,  $D_{0k}$  is the original diameter of feature  $k$  (mm).

The model is solved for effective assignment of cutting operations for each machine and optimal cutting speed, feed rate, depth of cut for all rough and finish operations for different part features. The objective function (9) minimises the total production cost. Constraint (10) forces the production time of the machines not to exceed the cycle time. If the production time is less than the cycle time, then slack time is allowed on machines. Constraint (11) indicates that the sum of depths of cut of a part feature should be equal to total stock of material to be removed from that feature. Constraint (12) means that each operation of a feature has to be processed by only one machine. Constraints (11), (12) and (13) will jointly guarantee the value of  $X_{ijk}$  to binary, either 0 or 1. Constraints (14) through (16) give the lower and upper bounds for cutting speed, feed rate, and depth of cut respectively. Constraint (17) restricts the depth of cut for the last or finish operation of each feature to be controlled in the range specified by the lower and upper bounds in order to meet

surface finish requirements. The restrictions for cutting force and cutting power are respectively presented in Constraints (18) and (19). Finally, the surface finish limit for last operation is imposed by Constraint (20).

### 3.2. Minimisation of cycle time

In practice, the cycle time may become a more important concern than the production cost. In this case, a second model is required to minimize cycle time. The model is formulated in the as follows.

$$\text{Minimise } Z^* \tag{22}$$

Subject to:

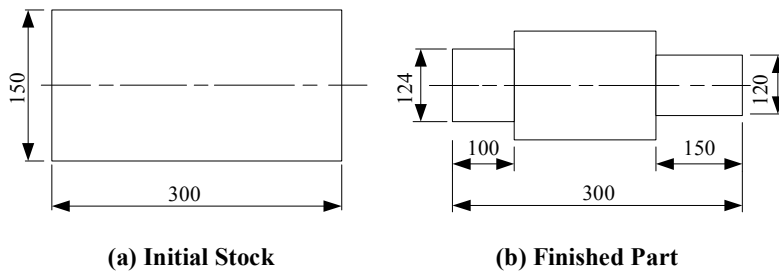
$$\sum_{j=1}^{J_k} \sum_{k=1}^K \left( \frac{\pi D_{(j-1)k} L_k}{1000 v_{ijk} f_{ijk}} + \frac{\pi D_{(j-1)k} L_k}{1000 K_T} v_{ijk}^{\alpha_T-1} f_{ijk}^{\beta_T-1} d_{ijk}^{\gamma_T} t_r \right) X_{ijk} = Z^*, \quad \forall i \tag{23}$$

and, Constraint (11) through Constraint (20).

The main goal of the second model is to assign cutting operations to machines in a job manufacturing system with the objective of minimising the cycle time. According to Agapiou [14], the production time equality constraint tends to reduce or eliminate the slack time on all machines with a remarkable reduction in cycle time. Constraint (23) replaces Constraint (10) and specifies the total production time to be equal to the cycle time thus eliminating the slack times among the machines. Other constraints in this model remain the same as those used in the first model. Solving this second model will result in effective cycle time and workload assignment, and optimal cutting speed, feed rate, and depth of cut for each machine-operation-feature combination.

### 4. Results and Discussion

This section presents computational analyses using a numerical example to test the feasibility of the two models. Consider a low carbon steel shaft (Fig. 2) with three features is to be processed using HSS tools. The cutting operations will be allocated on three identical NC lathe machines. Each feature has to undergo three cutting operations namely first rough turning operation, second rough turning operation and finish turning operation.



**Fig. 2. Initial Stock Transformed to Finished Part through Rough and Finish Turning Operations (dimensions in mm).**

The first feature is produced from the initial stock with diameter of 150 mm and length 300 mm where the total depth of cut to be removed from the stock is 5.0 mm. From the resulting diameter, the second and third features are produced. The second feature is produced by removing the total depth of cut of 8.0 mm at a length of 100 mm and the third feature is produced by removing the total depth of cut of 10.0 mm at a length 150 mm. The surface roughness limits for all the features are 1.6  $\mu\text{m}$ . More input data including constants for tool life, cutting force and surface roughness equations were obtained from [2] and are listed in Table 1. The power consumption equation is given according to the following relationship:

$$P_c = \frac{F_c v}{60 \eta_m} \quad (24)$$

where  $F_c$  is the cutting force (N),  $v$  is the cutting speed (m/min) and  $\eta_m$  is the mechanical efficiency.

**Table 1. Input Data for Developed Models.**

Symbol	Value
$C_o, C_t$	\$3/min, \$5.5/edge, respectively
$t_r$	0.5 min
$\eta_m$	0.8
$\alpha_T, \beta_T, \gamma_T, K_T$	1.7, 1.55, 1.22, 1570000, respectively
$\beta_F, \gamma_F, K_F$	1.18, 1.26, 1.38, respectively
$\alpha_S, \beta_S, \gamma_S, K_S$	-0.25, 0.72, 0.23, 1.17, respectively
$F_{max}, P_{max}$	20 kg, 2 kW, respectively
$v_{ijk}^L, v_{ijk}^U$	90, 168 m/min for rough turning, 120, 210 m/min for finish turning, respectively
$f_{ijk}^L, f_{ijk}^U$	0.8, 0.13 mm/rev for rough turning, 2.0, 0.5 mm/rev for finish turning, respectively
$d_{ijk}^U$	5.0 mm
$d_{ijk}^L, d_{ijk}^U$	0.3, 1.0 mm, respectively

The two models are solved in LINGO nonlinear software package [15]. The computational results for the first model are shown in Table 2. The decisions of machining parameters and operation allocation can be made concurrently based on these results. For example, the first rough turning operations of feature 1 is allocated to machine 2 while the second rough and finish turning operations of the same feature is allocated to machine 1. The effective machining parameters for the finish turning operation on feature 1 at machine 1 are: 240 m/min (cutting speed), 0.5 mm/rev (feed rate) and 0.5 mm (depth of cut). The total production cost is \$ 20.4 with a cycle time of 2.24 min. If production time or due date is a more concern, cycle time may be used as the objective and to this end, the second model of minimising cycle time can be used. On solving the second model, the effective workload assignment and optimal machining parameters for all machine-operation-feature combinations were achieved as presented in Table 3. The minimum cycle time is 1.96 min which is shorter than 2.24 min obtained in the first model. This is about 12.5% reduction in cycle time. The first model of minimising production cost is solved again with the minimum cycle time  $Z^*$  obtained in the second model being treated as  $Z$  in Constraint (10). The intention was to reduce the total production cost as much as possible by adjusting the machining parameters and take advantage of operation re-allocation. The refined



results of the first model are summarised in Table 4. The total production time was computed to \$17.84 representing a 12.5% reduction of total production cost.

**Table 2. Minimum Production cost = \$ 20.4 with a Cycle Time = 2.24 min.**

Machine-operation-feature combination	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)
1-2-1	160	1.5	4.0
1-3-1	240	0.5	0.5
1-1-2	100	2.5	2.5
1-1-3	100	1.93	4.6
2-1-1	69.6	2.5	0.5
2-2-2	100	0.5	4.0
2-3-2	235.7	0.32	1.5
3-2-3	100	0.5	4.0
3-3-2	160	0.42	1.5

**Table 3. Minimum Cycle Time = 1.96 min.**

Machine-operation-feature combination	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)
1-3-1	240	0.5	0.5
1-1-2	100	2.5	2.5
1-1-3	100	1.93	4.5
1-2-3	100	1.5	4.0
2-3-2	160	0.14	1.5
3-1-1	100	2.4	3.68
3-2-1	160	1.5	0.82
3-2-2	117.2	1.5	4.0
3-3-3	240	0.5	1.5

**Table 4. Minimum Production Cost = \$17.84 at Cycle Time = 1.96 min.**

Machine-operation-feature combination	Cutting speed (m/min)	Feed rate (mm/rev)	Depth of cut (mm)
1-1-1	68.97	2.5	2.89
1-3-1	240	0.5	1.5
2-2-1	160	1.5	0.61
2-3-2	233.25	0.5	1.32
2-2-3	100	0.67	4.0
3-1-2	60	1.5	5.71
3-2-2	122.56	1.5	0.97
3-1-3	60	1.5	4.86
3-3-3	240	0.5	1.14

## 5. Conclusions

Two models have been developed for machining optimisation and operation allocation decisions in rough and finish turning environment in an attempt to justify

the effective utilisation of highly invested NC lathe machines. Numerical examples have been solved using LINGO nonlinear software to test the feasibility of the developed models. The computational results show that decisions of machining optimisation and operation allocation can be concurrently made while minimising production cost and cycle time using these models. The study has shown that if the cycle time is minimised with the production time equal to cycle time, then slack times among machines can totally be eliminated. The study has also confirmed that if the minimised cycle time is treated as the constraint in the production cost model, then the production cost can further be reduced.

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