Embedding and NP-complete Problems for Some Equitable Labelings

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***Abstract*–A cordial labeling or an equitable labeling is a weaker version of Graceful and Harmonious labelings. At present many variants of equitable labeling are available. We discuss embedding and NP-complete problems in the context of some variants of equitable labeling such as E-cordial labeling, product cordial labeling, edge product cordial labeling, total product cordial labeling and prime cordial labeling. This work also rules out any possibility of forbidden subgraph characterizations for such labelings.**

***Keywords*: Graph labeling; equitable labeling; embedding of graphs; NP-complete problems.**

I. Introduction

We begin with simple, finite and undirected graph $G = (V(G), E(G))$. For all standard terminology and notations we follow Balakrishnan and Ranganathan [3]. We will give brief summary of definitions which are useful for the present investigations.

Definition 1.1: A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (edges) then the labeling is called a vertex (an edge) labeling.

In 1964, Ringel [10] conjectured that the graph K_{2n+1} can be decomposed into $2n+1$ isomorphic copies of a tree with n edges. In 1967, Rosa [11] introduced β - labelings as a tool to attack Ringel's Conjecture. This labeling was renamed as graceful labeling by Golomb [7] and it is now the popular term.

Definition 1.2: A graph $G = (V(G), E(G))$ of order p and size q is said to be *graceful* if there exists an injection $f : V(G) \to \{0,1,2,\ldots,q\}$ such that the induced function $f^* : E(G) \to \{1,2,\ldots,q\}$ defined by $f^*(e = uv) = |f(u) - f(v)|$ for each edge $e = uv$ is a bijection and f is said to be *graceful labeling* of G.

The famous Ringel-Kotzig tree conjecture and many illustrious works on graceful graphs brought a tide of different ways of labeling the elements of graph such as odd graceful labeling, harmonious labeling etc. Graham and Sloane [8] introduced harmonious labeling during their study on modular versions of additive bases problems stemming from error correcting codes.

Definition 1.3: A graph $G = (V(G), E(G))$ is said to be *harmonious* if there exists an injection $f : V(G) \to Z_q$ such that the induced function $f^* : E(G) \to Z_q$ defined by $f^*(e = uv) = (f(u) + f(v)) (\bmod q)$ for each edge $e = uv$ is a bijection and f is said to be *harmonious labeling* of G.

For an extensive survey and bibliographic references on graph labeling we refer to Gallian [5].

In 1987, Cahit [4] introduced cordial labeling as a weaker version of graceful labeling and harmonious labeling which is defined as follows.

Definition 1.4: For a graph G, a vertex labeling function $f : V(G) \to \{0,1\}$ induces an edge labeling function $f^* : E(G) \to \{0,1\}$ defined as $f^*(uv) = |f(u) - f(v)|$.

For $i = 0$ or 1, let us denote

$v_f(i)$ = number of vertices of G having label i under f

$e_f(i)$ = number of edges of G having label i under f^*

The function f is called *cordial labeling* of G if $|e_f(1) - e_f(0)| \le 1$ and $|v_f(1) - v_f(0)| \le 1$. A graph is called *cordial* if it admits cordial labeling.

In the same paper Cahit [4] proved that tree is cordial, K_n is cordial if and only if $n \le 3$.

After this some labelings like prime cordial labeling, A - cordial labeling, H - cordial labeling, product cordial labeling, etc. were also introduced as variants of cordial labeling. Such labelings are commonly referred as equitable labelings.

II. Some Equitable Labelings

Definition 2.1: For a graph G, an edge labeling function $f^* : E(G) \to \{0,1\}$ induces a vertex labeling function $f : V(G) \to \{0,1\}$ defined as $f(v) = \sum \{f^*(uv) / uv \in E(G)\}$ (mod 2). The function f^* is called *E-cordial labeling* of G

if $\left|e_f(1)-e_f(0)\right|\le 1$ and $\left|v_f(1)-v_f(0)\right|\le 1$. A graph is called *E-cordial* if it admits E-cordial labeling.

The E-cordial labeling was introduced by Yilmaz and Cahit [17].

Definition 2.2: For a graph G, a vertex labeling function $f:V(G)\to\{0,1\}$ induces an edge labeling function $f^*:E(G)\to\{0,1\}$ defined as $f^*(uv)=f(u)f(v)$. The function f is called *product cordial labeling* of G if $\left|e_f(1)-e_f(0)\right|\le 1$ and $\left|v_f(1)-v_f(0)\right|\le 1$. A graph is called *product cordial* if it admits product cordial labeling.

In 2004, Sundaram *et al.* [12] have introduced product cordial labeling and have investigated product cordial labeling for some standard graphs.

Definition 2.3: For a graph G, an edge labeling function $f^*:E(G)\to\{0,1\}$ induces a vertex labeling function $f:V(G)\to\{0,1\}$ defined as $f(v)=\Pi\{f^*(uv)/uv\in E(G)\}$. The function f^* is called *edge product cordial labeling* of G if $\left|e_f(1)-e_f(0)\right|\le 1$ and $\left|v_f(1)-v_f(0)\right|\le 1$. A graph is called *edge product cordial* if it admits edge product cordial labeling.

An edge analogue of product cordial labeling is recently introduced by Vaidya and Barasara [16] and they have investigated several results on this newly defined concept.

Definition 2.4: For a graph G, a vertex labeling function $f:V(G)\to\{0,1\}$ induces an edge labeling function $f^*:E(G)\to\{0,1\}$ defined as $f^*(uv)=f(u)f(v)$. The function f is called *total product cordial labeling* of G if $\left|\left(v_f(1)+e_f(1)\right)-\left(v_f(0)+e_f(0)\right)\right|\le 1$. A graph is called *total product cordial* if it admits total product cordial labeling.

In 2006, Sundaram *et al.* [14] have introduced total product cordial labeling and proved some general results related to total product cordial graphs.

The concept of prime cordial labeling was introduced by Sundaram *et al.* [13] which is defined as follows.

Definition 2.5: For a graph G, a vertex labeling function $f:V(G)\to\{1,2,\ldots,|V(G)|\}$ is bijective and induces an edge labeling function $f^*:E(G)\to\{0,1\}$ defined as $f^*(uv)=1$ if $\gcd(f(u),f(v))=1$ and $f^*(uv)=0$ if $\gcd(f(u),f(v))>1$. The function f is called *prime cordial labeling* of G if $\left|e_f(1)-e_f(0)\right|\le 1$. A graph is called *prime cordial* if it admits prime cordial labeling.

III. EMBEDDING OF EQUITABLE GRAPHS

Theorem 3.1: Any graph G can be embedded as an induced subgraph of an E-cordial graph.

Proof: It is always possible to label the edges of any graph G such that edge condition for E-cordial labeling is satisfied. Without loss of generality let us assume $0\le e_f(0)-e_f(1)\le 1$.

If no vertex with label 0 or label 1 is generated then add two vertices and attach them to any vertex of G. Label any one of the new edges with 0 and other with 1.

Let V_i be the set of vertices with label i and E_i be the set of edges with label i while $n(V_i)$ and $n(E_i)$ be the cardinality of set V_i and E_i respectively.

Case 1: When $n(V_1)-n(V_0)=r>1$.

The new graph H can be obtained by adding $r+4$ vertices, say $v_1,v_2,\ldots,v_{r+4}$ to the graph G. Join them to arbitrary member of V_0, say v. Assign label 1 to edges vv_1 and vv_{r+4} and label 0 to the remaining edges. Now add an edge between each v_i and v_{i+1} for $1\le i\le r+2$ and assign label 1 to these new edges. Also add an edge between v_{r+3} and v_{r+4} and assign label 0 to it. Consequently $r+2$ vertices will receive label 0 and 2 vertices will receive label 1.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right|=\left|n(V_1)+2-n(V_0)-r-2\right|=0$,

$\left|e_f(1)-e_f(0)\right|=\left|n(E_1)+r+4-n(E_0)-r-3\right|\le 1$.

Case 2: When $n(V_0)-n(V_1)=r>1$.

Sub Case 1: When r is even.

The new graph H can be obtained by adding r vertices say $v_1,v_2,\ldots,v_r$ to the graph G. Join them to arbitrary member of V_0 and assign label 1 to these new edges. Now add an edge between each v_i and v_{i+1} for $1\le i\le r-1$ and assign label 0 to these new edges. Consequently r vertices will receive label 1.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right|=\left|n(V_1)+r-n(V_0)\right|=0$,

$\left|e_f(1)-e_f(0)\right|=\left|n(E_1)+r-n(E_0)-r-1\right|\le 1$.

Sub Case 2: When r is odd.

The new graph H can be obtained by adding $r-1$ vertices say $v_1,v_2,\ldots,v_{r-1}$ to the graph G. Join them to arbitrary member of V_0 and assign label 1 to these new edges. Now add an edge between each v_i and v_{i+1} for $1\le i\le r-2$ and assign label 0 to these new edges. Consequently $r-1$ vertices will receive label 1.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right|=\left|n(V_1)+r-1-n(V_0)\right|=1$,

$\left|e_f(1)-e_f(0)\right| = \left|n(E_1)+r-1-n(E_0)-r+2\right| \le 1$.

Thus in all the possibilities the constructed supergraph H satisfies the conditions for E-cordial graph. That is, any graph G can be embedded as an induced subgraph of an E-cordial graph.

Theorem 3.2: Any graph G can be embedded as an induced subgraph of a product cordial graph.

Proof: It is always possible to label the vertices of any graph G such that vertex condition for product cordial labeling is satisfied. Without loss of generality let us assume $0 \le v_f(1)-v_f(0) \le 1$.

Let V_i be the set of vertices with label i and E_i be the set of edges with label i while $n(V_i)$ and $n(E_i)$ be the cardinality of set V_i and E_i respectively.

Case 1: When $n(E_1)-n(E_0) = r > 1$.

The new graph H can be obtained by adding r vertices to the graph G. Consider a partition of r as $r = a+b$ with $0 \le a-b \le 1$.

Now out of r vertices assign label 0 to u_i number of vertices and label 1 to b number of vertices. i.e. label the vertices $v_1, v_2, \ldots, v_a$ with 0 and $u_1, u_2, \ldots, u_b$ with label 1. Then join all v_i's and u_i's to arbitrary element of V_0. Consequently r edges will receive label 0.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right| = \left|n(V_1)+b-n(V_0)-a\right| \le 1$,

$\left|e_f(1)-e_f(0)\right| = \left|n(E_1)-n(E_0)-r\right| = 0$.

Case 2: When $n(E_0)-n(E_1) = r > 1$.

The new graph H can be obtained by adding $2(r+1)$ vertices say $v_1, v_2, \ldots, v_{2(r+1)}$ to the graph G. Assign label 0 to the vertices $v_1, v_2, \ldots, v_{r+1}$ and join them to arbitrary member of V_0. Now assign label 1 to the vertices $v_{r+2}, v_{r+3}, \ldots, v_{2(r+1)}$ and join them to arbitrary member of V_1. Also add an edge between each v_i and v_{i+1} for $1 \le i \le r$. Consequently $r+1$ edges will receive label 0 and $2r+1$ edges will receive label 1.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right| = \left|n(V_1)+r+1-n(V_0)-r-1\right| \le 1$,

$\left|e_f(1)-e_f(0)\right| = \left|n(E_1)+2r+1-n(E_0)-r-1\right| = 0$.

Thus in all the possibilities the constructed supergraph H satisfies the conditions for product cordial graph. That is, any graph G can be embedded as an induced subgraph of a product cordial graph.

Theorem 3.3: Any graph G can be embedded as an induced subgraph of an edge product cordial graph.

Proof: It is always possible to label the vertices of any graph G such that edge condition for edge product cordial labeling is satisfied. Without loss of generality let us assume $0 \le e_f(1)-e_f(0) \le 1$.

If no vertex with label 0 or label 1 is generated then add two vertices and attach them to any vertex of G. Label any one of the new edges with 0 and other with 1.

Let V_i be the set of vertices with label i and E_i be the set of edges with label i while $n(V_i)$ and $n(E_i)$ be the cardinality of set V_i and E_i respectively.

Case 1: When $n(V_1)-n(V_0) = r > 1$.

The new graph H can be obtained by adding $3r+2$ vertices, say $v_1, v_2, \ldots, v_{3r+2}$ to the graph G. Join $v_1, v_2, \ldots, v_{2r+1}$ to arbitrary member of V_0 and assign label 0 to these new edges. Now join $v_{2r+2}, v_{2r+3}, \ldots, v_{3r+2}$ to arbitrary member of V_1 and assign label 1 to these new edges. Also add an edge between each v_i and v_{i+1} for $2r+2 \le i \le 3r+1$ and assign label 1 to these new edges. Consequently $2r+1$ vertices will receive label 0 and $r+1$ vertices will receive label 1.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right| = \left|n(V_1)+r+1-n(V_0)-2r-1\right| = 0$,

$\left|e_f(1)-e_f(0)\right| = \left|n(E_1)+2r+1-n(E_0)-2r-1\right| \le 1$.

Case 2: When $n(V_0)-n(V_1) = r > 1$.

The new graph H can be obtained by adding $3r+2$ vertices, say $v_1, v_2, \ldots, v_{3r+2}$ to the graph G. Join $v_1, v_2, \ldots, v_{2r+1}$ to arbitrary member of V_1 and assign label 1 to these new edges. Now join $v_{2r+2}, v_{2r+3}, \ldots, v_{3r+2}$ to arbitrary member of V_0 and assign label 0 to these new edges. Also add an edge between each v_i and v_{i+1} for $2r+2 \le i \le 3r+1$ and assign label 0 to these new edges. Consequently $2r+1$ vertices will receive label 1 and $r+1$ vertices will receive label 0.

As a result of the above procedure we have the following:

$\left|v_f(1)-v_f(0)\right| = \left|n(V_1)+2r+1-n(V_0)-r-1\right| = 0$,

$\left|e_f(1)-e_f(0)\right| = \left|n(E_1)+2r+1-n(E_0)-2r-1\right| \le 1$.

Thus in all the possibilities the constructed supergraph H satisfies the conditions for edge product cordial graph. That is, any graph G can be embedded as an induced subgraph of an edge product cordial graph.

Theorem 3.4: Any graph G can be embedded as an induced subgraph of a total product cordial graph.

Proof: Label the vertices of graph G in such a way that $0 \le v_f(0)-v_f(1) \le 1$.

Let V_i be the set of vertices with label i and E_i be the set of edges with label i while $n(V_i)$ and $n(E_i)$ be the cardinality of set V_i and E_i respectively.

Case 1: When $\left(\left(n(V_1)+n(E_1)\right)-\left(n(V_0)+n(E_0)\right)\right)=r>1$.

The new graph H can be obtained by adding $\left\lceil \frac{r}{2} \right\rceil$ vertices to the graph G. Assign label 0 to these vertices and join them to arbitrary member of V_0. Consequently $\left\lceil \frac{r}{2} \right\rceil$ edges will receive label 0.

As a result of the above procedure we have the following:

$$\left|\left(v_f(1)+e_f(1)\right)-\left(v_f(0)+e_f(0)\right)\right|$$

$$=\left|\left(n(V_1)+n(E_1)\right)-\left(n(V_0)+n(E_0)+2\left\lceil \frac{r}{2} \right\rceil\right)\right|\leq 1$$

Case 2: When $\left(\left(n(V_0)+n(E_0)\right)-\left(n(V_1)+n(E_1)\right)\right)=r>1$.

The new graph H can be obtained by adding $\left\lceil \frac{r}{2} \right\rceil$ vertices to the graph G. Assign label 1 to these vertices and join them to arbitrary member of V_1. Consequently $\left\lceil \frac{r}{2} \right\rceil$ edges will receive label 1.

As a result of the above procedure we have the following:

$$\left|\left(v_f(1)+e_f(1)\right)-\left(v_f(0)+e_f(0)\right)\right|$$

$$=\left|\left(n(V_1)+n(E_1)+2\left\lceil \frac{r}{2} \right\rceil\right)-\left(n(V_0)+n(E_0)\right)\right|\leq 1$$

Thus in all the possibilities the constructed supergraph H satisfies the conditions for total product cordial graph. That is, any graph G can be embedded as an induced subgraph of a total product cordial graph.

Theorem 3.5: Any graph G can be embedded as an induced subgraph of a prime cordial graph.

Proof: Label the vertices of graph G with $1,2,\ldots,p$.

Let E_i be the set of edges with label i and $n(E_i)$ be the cardinality of set E_i.

Case 1: When $n(E_1)-n(E_0)=r>1$.

The new graph H can be obtained by adding $3r$ vertices to the graph G label them with $p+1, p+2,\ldots,p+1+3r$. From new vertices join all even label vertices to vertex with label 2, from the remaining vertices join all vertices with label divisible by 3 to the vertex with label 3 after that join all remaining vertices to vertex with label 1. Consequently $2r$ edges will receive label 0 and r edges will receive label 1.

As a result of the above procedure we have the following:

$$\left|e_f(1)-e_f(0)\right|=\left|n(E_1)+r-n(E_0)-2r\right|=0.$$

Case 2: When $n(E_0)-n(E_1)=r>1$.

The new graph H can be obtained by adding r vertices to the graph G and label them with $p+1, p+2,\ldots,p+1+r$. Join all these new vertices to vertex with label 1. Consequently r edges will receive label 1.

As a result of the above procedure we have the following:

$$\left|e_f(1)-e_f(0)\right|=\left|n(E_1)+r-n(E_0)\right|=0.$$

Thus in all the possibilities the constructed supergraph H satisfies the conditions for prime cordial graph. That is, any graph G can be embedded as an induced subgraph of a prime cordial graph.

IV. NP-Complete Problems

The NP-complete problems in the context of various labelings are discussed in Acharya [1], Princy [9], Acharya *et al.* [2] and Vaidya and Vihol [15].

We explore the NP-complete problems in the context of various equitable labelings.

Theorem 4.1: Any planar graph G can be embedded as an induced subgraph of a planar E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graph.

Proof: If G is planar graph. Then the supergraph H constructed in Theorem 3.1 (corresponding theorems 3.2, 3.3, 3.4 and 3.5) is a planar graph. Hence the result.

Theorem 4.2: Any connected graph G can be embedded as an induced subgraph of a connected E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graph.

Proof: If G is connected graph. Then the supergraph H constructed in Theorem 3.1 (corresponding theorems 3.2, 3.3, 3.4 and 3.5) is a connected graph. Hence the result.

Theorem 4.3: The problem of deciding whether the chromatic number $\chi(G)\leq k$, where $k\geq 3$, is NP-complete even for E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graphs.

Proof: Let G be a graph with chromatic number $\chi(G)\geq 3$. Let supergraph H constructed in Theorem 3.1 is E-cordial (corresponding theorems 3.2, 3.3, 3.4 and 3.5), which contains G as an induced subgraph. Then obviously we have $\chi(H)\geq\chi(G)$. Since the problem of deciding whether the chromatic number $\chi(G)\leq k$, where $k\geq 3$, is NP-complete by [6]. It follows that deciding whether the chromatic number $\chi(H)\leq k$, where $k\geq 3$, is NP-complete even for E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graphs. Hence the result.

Theorem 4.4: The problem of deciding whether the clique number $\omega(G)\geq k$ is NP-complete even when restricted to E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graphs.

Proof: Since the problem of deciding whether the clique number of a graph $\omega(G) \geq k$ is NP-complete by [6] and $\omega(H) \geq \omega(G)$ for the supergraph H constructed in Theorem 3.1 is E-cordial (corresponding theorems 3.2, 3.3, 3.4 and 3.5). Hence the result.

Theorem 4.5: The problem of deciding whether the domination number (total domination number) is less than or equal to k is NP-complete even when restricted to E-cordial (product cordial, edge product cordial, total product cordial or prime cordial) graphs.

Proof: Since the problem of deciding whether the domination number (total domination number) of a graph G is less than or equal to k is NP-complete by [6] and the supergraph H constructed in Theorem 3.1 is E-cordial (corresponding theorems 3.2, 3.3, 3.4 and 3.5) has domination number greater than or equal to domination number of G. Hence the result.

V. Concluding Remarks

A very general problem is considered:

Given a graph G having the property P, is it possible to embed G as an induced subgraph of an equitable graph H having the property P? We present here an affirmative answer to the above posed problem for planner graphs, connected graphs, graphs with given chromatic number, graphs with given clique number, graphs with given domination number and graphs with given total domination number. As a consequence we deduce that deciding whether the chromatic number is less than or equal to k, where $k \geq 3$ is NP - complete for all the labeling considered in this paper. We obtain analogous results for clique number, domination number and total domination number.

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