Dufour and Soret Effects on Convective Heat and Mass Transfer in Non-Darcy Doubly Stratified Porous Media

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Abstract
This paper deals with the MHD convention non-Darcy flow with heat and mass transfer along the vertical surface in a fluid stratified non porous media in the presence of temperature gradients (Dufour effects) and concentration gradients (Soret effects). The differential equations of the governing flow are expand a in terms of perturbation functions as a result we get set of ordinary differential equations of zero,first and second order so the set of equations are solved by using the implicit finite difference scheme. A parametric study of the physical parameters involve in the problem is conducted and a representative set of numerical results are illustrated graphically

Key words: MHD, heat transfer , mass transfer double stratification , boundary layer, porous mediu, convection ,dufour number, soret number, finite difference method.

I. Introduction
Simultaneous heat and mass transfer from different geometries embedded in porous media has many engineering and geophysical applications such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed-bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. Cheng and Minkowycz [1] have presented similarity solutions for free thermal convection from a vertical plate in a fluid-saturated porous medium. The problem of combined thermal convection from a semi-infinite vertical plate in the presence or absence of a porous medium has been studied by many authors (see, for example [2–5]). Nakayama and Koyama [4] have studied pure, combined and forced convection in Darcian and non-Darcian porous media. Lai and Kulacki [6] has investigated coupled heat and mass transfer by mixed convection from an isothermal vertical plate in a porous medium. Hsieh et al. [5] has presented non-similar solutions for combined convection in porous media. Chamkha [7] has investigated hydro magnetic natural convection from a isothermal inclined surface adjacent to a thermally stratified porous medium. There has been a renewed interest in studying magneto hydrodynamic (MHD) flow and heat transfer in porous and non-porous media due to the effect of magnetic fields on the boundary layer flow control and on the performance of many systems using electrically conducting fluids. In addition, this type of flow has attracted the interest of many investigators in view of its applications in many engineering problems such as MHD generators, plasma studies, nuclear reactors, geothermal energy extractions. For example, Rapits et al. [8] have analyzed hydro magnetic free convection flow through a porous medium between two parallel plates. Aldoss et al. [9] have studied mixed convection from a vertical plate embedded in a porous medium in the presence of a magnetic field. Gribben [10] has studied boundary-layer flow over a semi-infinite plate with an aligned magnetic field in the presence of a pressure gradient. He has obtained solutions for large and small magnetic Prandtl numbers using the method of matched asymptotic expansion. Soudalekgar [11] obtained approximate solutions for two-dimensional flow of an incompressible, viscous fluid past an infinite porous vertical plate with constant suction velocity, the difference between the temperature of the plate and the free steam is moderately large causing the free convection currents. Raptis and Kafousias [12] have studied the influence of a magnetic field on steady free convection flow through a porous medium bounded by an infinite vertical plate with constant suction velocity. Raptis [13] has studied mathematically the case of time-varying two-dimensional natural convective flow of an incompressible, electrically conducting fluid along an infinite vertical porous plate embedded in a porous medium. Bian et al. [14] have reported on the effect of an electromagnetic field on natural convection in an inclined porous medium. Buoyancy-driven convection in a rectangular enclosure with a transverse magnetic field has been considered by Garandet et al. [15] and Khanafer and Chamkha [16].

It is well-known that in double-diffusive (e.g. thermohaline) convection the coupling between the transport of heat and mass takes place because the density \( q \) of the fluid mixture depends on both
temperature T and concentration C. For sufficiently small isobaric changes in temperature and concentration the mixture density q depends linearly on both T and C (Nield and Bejan [17]). In some circumstances there is direct coupling between T and C. This is when cross-diffusion (Soret and Dufour effects) is not negligible. The energy flux caused by a composition gradient was discovered in 1873 by Dufour and was correspondingly referred to the Dufour effect. It was also called the diffusion-thermo effect. On the other hand, mass flux can also be created by a temperature gradient, as was established by Soret. This is the thermal-diffusion effect. The Soret effect has been also used for isotope separation and in mixture between gases with very light molecular weight (H2; He) and of medium molecular weight (H2, air) (Postelnicu [18]). In many studies Soret and Dufour effects are neglected, on the basis that they are of a smaller order of magnitude than the effects described by Fourier’s and Fick’s laws. There are, however, exceptions. Eckert and Drake [19] have presented several cases when the Dufour effect cannot be neglected. Platten and Legros [20] state that in most liquid mixtures the Dufour effect is inoperative, but that this may not be the case in gases. Mojtabi and Charrier-Mojtabi [21] confirm this by noting that in liquids the Dufour coefficient is an order of magnitude smaller than the Soret effect. Benano-Molly et al. [22] have studied the problem of thermal diffusion in binary fluid mixture, lying within a porous medium and subjected to a horizontal thermal gradient and have shown that multiple convection-roll flow patterns can develop depending on the values of the Soret number. They conclude that for saturated porous media, the phenomenon of cross diffusion is further complicated because of the interaction between the fluid and the porous matrix and because accurate values of the cross-diffusion coefficients are not available. The onset of Soret-driven convection in an infinite cell filled with a porous medium saturated by a binary fluid was studied by Sovran et al. [23]. However, Soret and Dufour effects have been found to appreciably influence the flow field in free convection boundary layer over a vertical surface embedded in a fluid-saturated porous medium (Postelnicu [18] and Anghel et al. [24]).

In certain porous media applications such as those involving heat removal from nuclear fuel debris, underground disposal of radioactive waste material, storage of food stuffs, and exothermic and/or endothermic chemical reactions and associating fluids in packed-bed reactors, the working fluid heat generation (source) or absorption (sink) effects are important. Representative studies dealing with these effects have been reported by such authors as Acharya and Goldstein [25], Vajravelu and Nayfeh [26] and Chamkha [27,28]. In the combined heat and mass transfer studies in porous media, Murthy et al. [29] reported that the temperature and concentration became negative in the boundary layer depending on the relative intensity thermal and solutal stratification. These are a few articles to quote from the literature where the temperature/concentration or non dimensional heat/mass transfer coefficients becoming negative. Angirasa et al. [30] presented and analysis of combined heat and mass transfer in thermally stratified porous enclosure. Srinivasaschary et.al [31] studied the MHD and Radiation on Non-Darcy Mixed Convection, Ramakrishna and Hemalatha analyzed the Non-Darcy Mixed convection with Thermal Disipersion in a satrated porous medium. Rathi kumar and Shalini[32] presented the free convection heat and mass from a vertical wavy surface in a doubly stratified external porous layer.

The results presented in [29,33] have given the impetus for investigating free convection heat and mass transfer characteristics in a non-Darcian fluid saturated doubly stratified purpose medium .Unlike in [29] and [33], the medium is assumed to be stratified linearly with the height, which is more physically realistic case. As a result of this, mathematical model ceases to have a similar solution. In the light of the results obtained [34] Lakshmi Narayana and P.A.Murthy has extended the analysis be more significant given be clear from the section, “Results and Discussion.” This type of investigation is useful in understanding heat and mass transfer characteristics around a hot radioactive subsurface storage site or around a cooling magnetic intrusion, where the theory of convection heat and mass transport is involved. In this paper we have investigated the MHD effects on steady free convective heat and mass transfer from a vertical surface embedded in a doubly stratified non-Darcy porous medium by using the implicit finite difference
scheme. The results were compared with [35] when the Dufour and soureet number is absent.

**Governing Equations**

Consider the flow configuration shown in figure I. The flow is moderate (i.e., order of Reynolds number is greater than one), so pressure drop is proportional to the linear combination of fluid velocity and the square of the velocity (Forchheimer flow model). Viscous resistance due to the solid boundary is neglected under the assumption that the medium is with low permeability. A magnetic field is applied in the y-direction. The vertical wall temperature and concentration are assumed to be constant and the porous medium is assumed to be vertically linearly stratified with respect to both heat and concentration as indicated in Fig.I. The boundary layer equations along with the equation of continuity may be written as

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)

\[
\begin{align}
[1 + \frac{\sigma_B K}{\rho v}] \frac{\partial u}{\partial y} + \frac{c K}{v} \frac{\partial^2 u}{\partial y^2} &= \left( \frac{\sqrt{R} K}{v} \right) \frac{\partial T}{\partial y} + \left( \frac{\sqrt{R} K}{v} \right) \frac{\partial C}{\partial y} \\
\frac{\partial T}{\partial x} + \nu \frac{\partial^2 T}{\partial y^2} &= \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\nu u}{D_c} \frac{\partial^2 C}{\partial y^2} + \frac{\nu u}{D_c} \frac{\partial^2 T}{\partial y^2} \\
\frac{\partial C}{\partial x} + \nu \frac{\partial C}{\partial y} &= D_c \frac{\partial^2 C}{\partial y^2} + \frac{\nu u}{D_c} \frac{\partial^2 T}{\partial y^2}
\end{align}
\]  

(2)

and the boundary conditions are

\[y=0: \ u=0, \ T=T_w, \ C=C_w \]  

(3)

\[y=\infty: \ u=0, \ T=T_{w,\infty} + Ax, \ C=C_{w,\infty} + Bx\]  

(4)

Here \(x\) and \(y\) are the Cartesian coordinate, \(u\) and \(v\) are the averaged velocity components in \(x\) and \(y\) directions respectively, \(T\) and \(C\) are the dimensional temperature and concentration, respectively, \(\beta_T\) and \(\beta_C\) are the coefficients of thermal and solutal expansion, respectively, \(c\) is the Forchheimer constant, \(v\) is the kinematic viscosity of the fluid, \(K\) is the permeability, \(g\) is the acceleration due to gravity and \(\alpha\) are the thermal and solutal diffusivities of the medium. \(B\) is the strength of the magnetic field, \(\rho\) is the electrical conductivity of the fluid, \(\rho\) is the density, \(v\) is the kinematic viscosity. Here, \(A\) and \(B\) are constants varied to alter the intensity of stratification in the medium. Thicknesses of thermal and solutal boundary layers (\(\delta_T, \delta_C\)) which are shown in Fig. 1, are depending on the strength of the respective field and for all calculations we have taken the boundary layer thickness as the maximum of these values. The subscripts \(w, (\infty, 0)\) and \(\infty\) indicate the conditions at the wall, at some reference point in the medium, and at the outer edge of the boundary layer respectively. Making use of the following transformation which is derived using order magnitudes

\[H = \frac{R a_x}{x}, \ f(x, \eta) = \frac{\psi}{\alpha_k a_x}, \]  

(5)

\[\theta(x,\eta) = \frac{T-T_{w,\infty}}{T_{w,\infty}-T_w}, \]  

(6)

\[\varphi(x,\eta) = \frac{C-C_{w,\infty}}{C_{w,\infty}-C_w}\]  

The governing eqns.(1)-(4) become

\[(1+Ha^2) f'' + 2F f' + f' = \theta \square + N \varphi \square \]  

(7)

\[\theta^{1/2} + \frac{1}{2} \varphi^{1/2} + D_y \varphi = c f f' + \epsilon \left( f^2 \frac{\partial \varphi}{\partial \epsilon} - \varphi \frac{\partial f}{\partial \epsilon} \right) \]  

(8)

\[\varphi^{1/2} + \frac{1}{2} \epsilon \frac{\partial \varphi}{\partial \epsilon} + Le \text{ Sr} \theta^{1/2} = Le \text{ Sr} f f' + \epsilon \left( f^2 \frac{\partial \varphi}{\partial \epsilon} - \varphi \frac{\partial f}{\partial \epsilon} \right) \]  

(9)

And the boundary conditions (5) transform into

\[\eta=0: \ f(\epsilon, 0) = 0, \ \theta(0, 0) = 1, \ \varphi(0, 0) = 1 \]  

\[\eta=\infty: \ f(\epsilon, \infty) = 0, \ \theta(\infty, \infty) = 1, \ \varphi(\infty, \infty) = 0 \]  

(10)

The various parameters involved in the present study which influence the system are the Darcy-Rayleigh number \(Ra_x = [K \beta_T (T_w - T_{w,\infty})] / \alpha v\) which is defined with reference to the thermal conditions, the inertia parameter \(F\) which is defined with reference to the thermal Rayleigh number \(Ra = a/\alpha D\) and the buoyancy ratio \(N = [F (C_w - C_{w,\infty})] / [\beta_T (T_w - T_{w,\infty})]\). When the Forchheimer coefficient \(c\) is zero, \(F\) will become zero, the inertia effects are absent in the medium, and the study becomes double stratification in the porous medium. Thermal buoyancy acts always vertically upwards, the species buoyancy may act in either direction depending on the relative molecular weight. So \(N>0\) indicates aiding buoyancy where both the thermal and solutal buoyancies are in the same directions and \(N<0\) indicates opposing buoyancy where the solutal buoyancy is in the opposite direction to the thermal buoyancy. When \(N=0\), the flow is driven by thermal buoyancy alone. The thermal stratification parameter is given by \(\epsilon = Ax / (T_w - T_{w,\infty})\). Here we are defining a quantity called the stratification ratio \(S_r = B(T_w - T_{w,\infty}) / A(C_w - C_{w,\infty})\) which is a constant, it indicates the relative strengths of solutal and thermal stratification in the medium. As \((T_w - T_{w,\infty})\) and \((C_w - C_{w,\infty})\) are constants, when \(S_r>1\), solutal stratification is more than the thermal stratification in the medium and vice versa. The product \(S_r \epsilon = Bx / (C_w - C_{w,\infty})\) will be the solutal stratification parameter. \(Ha^2 = 1 + \frac{\sigma_B K}{\rho v}\) is the magnetic parameter.
Expanding the stream function, temperature distribution and concentration distribution in terms of perturbation functions

\[ f(\varepsilon, \eta) = \sum_{m=0}^{\infty} (-1)^m \varepsilon^m f_m(\eta) \]
\[ \theta(\varepsilon, \eta) = \sum_{m=0}^{\infty} (-1)^m \varepsilon^m \theta_m(\eta) \]
\[ \varphi(\varepsilon, \eta) = \sum_{m=0}^{\infty} (-1)^m \varepsilon^m \varphi_m(\eta) \]  

(12)

and substituting Eq.(12) into Eqs.(8)-(11) and equating the coefficients of various powers of \( \varepsilon \) to zero, we have the following sets of ordinary differential equations for zeroth, first, and second order in \( \varepsilon \) as following:

\[ (1+Ha^2)f_0 + 2F_f f_0 \theta_0 + N \varphi_0 \]
\[ \theta_0 - \frac{1}{2} f_0 \theta_0 + D_f \varphi_0 = 0 \]  

(13)

Subject to the boundary conditions
\[ f_0(0) = 0, \quad \theta_0(0) = 1, \quad \varphi_0(0) = 1, \]
\[ f_0(\infty) = 0, \quad \theta_0(\infty) = 0, \quad \varphi_0(\infty) = 0. \]

(14)

\[ (1+Ha^2)f_1 + 2F_f(f_0 f_1 \theta_1 + f_1 f_0 \theta_0) = \theta_1 + N \varphi_1 \]
\[ \theta_1 + \frac{1}{2} (f_1 \theta_0 + f_0 \theta_1) + f_0 + (-f_0 \theta_0 + f_0 \theta_1) = 0 \]
\[ \varphi_1 + \frac{1}{2} Le (f_1 \varphi_0 + f_0 \varphi_1) + Le Sr f_0 + Le (-f_0 \varphi_1 + f_1 \varphi_0) + Le Sr f_0 = 0 \]

Subject to the boundary conditions
\[ f_1(0) = 0, \quad \theta_1(0) = 0, \quad \varphi_1(0) = 0, \]
\[ f_1(\infty) = 0, \quad \theta_1(\infty) = 0, \quad \varphi_1(\infty) = 0. \]

And

\[ (1+Ha^2)f_2 + 2F_f(f_0 f_2 \theta_2 + f_1 f_1 \theta_0 + f_2 f_0) = \theta_2 + N \varphi_2 \]
\[ \theta_2 + \frac{1}{2} f_1 \theta_0 + \frac{3}{2} f_2 \theta_0 - f_0 \theta_1 - 2f_0 \theta_2 + f_1 + D_f \varphi_2 = 0 \]
\[ \varphi_2 + \frac{1}{2} Le(f_2 \varphi_0 + f_1 \varphi_1 + f_0 \varphi_2) + Le Sr f_1 - Le (f_1 \varphi_1 + 2f_0 \varphi_2 - f_1 \varphi_1 - 2f_2 \varphi_0) + Le Sr f_0 = 0 \]

Subject to the boundary conditions
\[ f_2(0) = 0, \quad \theta_2(0) = 0, \quad \varphi_2(0) = 0, \]
\[ f_2(\infty) = 0, \quad \theta_2(\infty) = 0, \quad \varphi_2(\infty) = 0. \]

(17)

(18)

Results and discussion

In order to see the effects of various parameters on velocity \( f \), Temperature \( \theta \) and Concentration \( \varphi \) profiles, we display Fig1-6. The values of Nusselt number and the Sherwood number for various values of embedded parameters are also displayed in the figures.

Fig.1 shows the effect of Thermal stratification parameter \( \varepsilon \) on velocity, temperature and concentration profiles. In the figure 1(i), variations of non-dimensional velocity profiles are present. From the figure it is seen that the increase in Thermal stratification parameter \( \varepsilon \), the velocity profiles decreases up to some value of \( \eta \) inside boundary layer \((0<\eta<5)\) after that the reverse phenomenon is observed for both the cases of \( Fc=0 \), \( Fc=1 \).

Fig.1(ii). Illustrates the effect of Thermal stratification parameter \( \varepsilon \) on the Temperature profiles, it is noticed that as Thermal stratification parameter \( \varepsilon \) increases, the temperature field decreases, as seen in the earlier cases for away from the plate the effect is not that much significant when \( Fc=0 \) and \( Fc=1 \). The effect of the Thermal stratification parameter \( \varepsilon \) on the concentration profiles is illustrate in fig.1.(iii). It is observed that as the Thermal stratification parameter \( \varepsilon \) increases the concentration appear to be decreasing not much of effect in the concentration is noticed in the boundary layer region and in the area far away from the plate. However significant variation is observed at a moderate distance from the plate when \( Fc=0 \) and \( Fc=1 \).
Fig. 2. shows the effects of Diffusivity ratio $Le$ on velocity($f$), Temperature($\theta$) and Concentration($\phi$). Fig. 2(i). illustrates that with the increase of diffusivity ratio $Le$, the velocity profiles decreases up to some value of $\eta$ after that the reverse phenomenon is observe for both of the cases of $Fc=0$ and $Fc=1$.

Fig. 2(ii). Shows the effects of Diffusivity ratio $Le$ for the various values of $Fc=0$ and $Fc=1$ on temperature profiles. Note that the temperature filed and the boundary layer thickness increases with Diffusivity ratio $Le$ increases when $Fc=0$. It is seen in the Fig. 2(ii)(a). and the reverse phenomenon we observe in the fig.2(ii)(b). when $Fc=1$.

Fig. 2.iii. displays the concentration profiles from this figure we seen that the concentration and the boundary layer thickness increases with Diffusivity ratio $Le$ increases when $Fc=0$ and $Fc=1$.
Fig. 3 illustrates for the different values of Buoyancy ratio $N$ effects on velocity($f$), Temperature($\theta$) and Concentration($\phi$). In Fig. 3(i), the effect of Buoyancy ratio $N$ for aiding Buoyancy ($N>0$) is to increase the velocity profiles, whereas the effect of opposing buoyancy ($N<0$) is to decrease the velocity profiles. The effect of Buoyancy ratio $N$ on Temperature profiles from Fig. 3(ii) observed that when $N$ increases the temperature profiles will decrease for $Fc=0$ and $Fc=1$, same phenomenon we observe in the concentration profiles is evident from Fig. 3(iii).
In fig 4 it is observed that the Duffer number Df effect is slightly reduces the velocity profiles is evident from fig.4.i. we have seen in the fig.4.(ii). The effect of Duffer number Df on the Temperature profiles it is noticed that as the Df increases a increasing trend in the temperature filed is noticed . not much of significant contribution of Df is noticed as we move far away from the plate when Fc=0 and Fc=1. The reverse phenomenon we observe in fig.4.iii. of concentration profiles.
In the figure 5, the velocity($f$), Temperature($\theta$) and Concentration($\phi$) graphs are shown graphically. It is observed that from the figure the effect of stratification number $Sr$, is to increases the velocity profiles. The reverse phenomenon observes after the some number $\eta$.

Fig.5.ii. shows that the effect of $Sr$ on temperature profiles. We observed that when $Fc=0$, the temperature profile decreases when $Sr$=1 increase when $Sr$=3. Compare with $Sr$=0 but $Fc=1$, in this case temperature profiles decreasing with increasing of $Sr$. It is clear from the figure 5.iii. the effect of $Sr$ is to increases the concentration profiles.

The effect of magnetic parameter on on $f, \theta$ and $\phi$ explains graphically from the fig.6. we observe in these fig. the magnetic parameter increases the velocity profiles but decrease the temperature profiles and concentration profiles. It explains fig.6.i.,fig.6.ii.,fig.6.iii. respectively.

In the case of aiding buoyancy the heat transfer coefficient increases with the increases of $Sr$ is observed from figure. the mass transfer coefficient is shown in fig.7 for different values of Le and $Sr$ the mass transform coefficient become negative and increases with the increasing value of $Sr$ and it decreases of with the increases of Lowies number Le. From fig.8 it is evideal that with the effect of magnetic parameter the concentration profiles increases.
References


