

# Speed Sensorless Robust Control of Permanent Magnet Synchronous Motor Based on Second-Order Sliding-Mode Observer

Amor Fezzani<sup>1</sup>, Said Drid<sup>1</sup>, Abdesslam Makouf<sup>1</sup>,  
Larbi Chrifi-Alaoui<sup>2</sup>, Mohamed Ouriagli<sup>3</sup>

**Abstract:** This paper is devoted to the study of the performances of a robust speed sensorless nonlinear control of permanent magnet synchronous machine. In the first part, the controllers are designed using two methods: the first one using the input output feedback linearization control and the second is a nonlinear control based on Lyapunov theory combined with sliding mode control. This second solution shows good robustness with respect to parameter variations, measurement errors and noises. In the second part, the high order sliding mode speed observer is used to overcome the occurring chattering phenomena. The super twisting algorithm is modified in order to design a speed and position observer for PMSM. Finally, simulation results are given to demonstrate the effectiveness and the good performance of the proposed control methods.

**Keywords:** Lyapunov function, Permanent magnet synchronous motors, Sensorless control, Second-order sliding modes, Robust nonlinear control.

## 1 Introduction

The PMSM is becoming more and more popular in servo systems because of its high power density, large torque to inertia ratio and high efficiency [1]-[2]. However, the PMSM model is nonlinear coupled and subject to parameter variations. It is described by a fifth-order nonlinear differential equation, where a part of states are not easily measurable, and often perturbed by an unknown load torque. Classical PI controller is a simple method used to control PMSM drives. However the main drawbacks of PI controller are the sensitivity of its performances to the system parameter variations and inadequate rejection of external disturbances and load changes. In order to, overcome these problems,

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<sup>1</sup>Laboratory, LSP-IE, Batna University, Rue Chahid M<sup>ed</sup> El-Hadi Boukhlof 05000, Algeria;  
E-mails: amorfezzani@yahoo.fr; s\_drid@yahoo.fr; a\_makouf@yahoo.fr;

<sup>2</sup>Laboratoire L.T.I, Université de Picardie Jules Verne, Dépt. GEII, Cuffies, Soissons, France;  
E-mail: larbi.alaoui@u.picardie.fr

<sup>3</sup>Laboratoire LIMA0, Faculté polydisciplinaire de Taza, Route d'Oujda – B.P. 1223, Taza, Maroc;  
E-mail: omenstz@yahoo.fr

many solutions have been proposed. Thus, extended state observers have been developed for motor control applications to compensate unmodeled dynamics and disturbances [3, 4], and to enable the use of active disturbance rejection control in passivity-based designs [5]. Nonlinear control strategies such as robust control [6], adaptive control [7, 8], Lyapunov based nonlinear control [9] and sliding mode control have been applied. Also to, decrease cost and size of the drive, reduce maintenance requirement and increase the reliability and robustness of the system, sensorless drives have received a wide attention. The basic idea for sensorless drive is to estimate motor speed and position through measured stator terminal quantities [13 – 17]. To do that, different approaches have been suggested, such as model reference adaptive system (MRAS), high frequency injection method [18, 19], observer based approach such as Extended Kalman Filter [20], nonlinear observer [21 – 24], adaptive interconnected observer [25], sliding mode observers [26 – 29], robust exact differentiators [30 – 32], and high order sliding mode observers [33 – 39].

In this work second-order sliding mode observers are used to estimate the rotor speed. These observers are widely used due to their, robustness with respect to unknown inputs, possibilities to use the values of the equivalent output injection for unknown inputs identification and finite time convergence to the reduced order manifold. To demonstrate the effectiveness and the good performance of the proposed control method versus input output feedback linearization control simulation investigations are performed.

## 2 The PMSM Model

Its dynamic model expressed in the rotor reference frame is given by voltage equations:

$$\begin{aligned} v_d &= R_s I_d + \frac{d\Phi_d}{dt} + p\Omega\Phi_q, \\ v_q &= R_s I_q + \frac{d\Phi_q}{dt} + p\Omega\Phi_d, \end{aligned} \quad (1)$$

where the fluxes expressions are given by

$$\Phi_d = L_d I_d + \Phi_f, \quad \Phi_q = L_q I_q.$$

Considering  $I_d$  and  $I_q$  as states variables, (1) can be written as:

$$\begin{aligned} \frac{dI_d}{dt} &= -\frac{R_s}{L_d} I_d + \frac{L_q}{L_d} p\Omega + \frac{v_d}{L_d}, \\ \frac{dI_q}{dt} &= -\frac{R_s}{L_q} I_q - \frac{L_d}{L_q} p\Omega I_d - \frac{\Phi_f}{L_q} p\Omega + \frac{v_q}{L_q}. \end{aligned} \quad (2)$$

The electromagnetic torque is given by

$$T_e = \frac{3}{2} p \left[ (L_d - L_q) I_d I_q + \Phi_f I_q \right] \quad (3)$$

and the associated equation of motion is

$$J_m \frac{d\Omega}{dt} = T_e - T_L - f_m \Omega. \quad (4)$$

From (2), (3) and (4), the state model is rewritten as:

$$u = U(t, x_4, x_3), \quad (5)$$

where

$$f_r(x) = \begin{bmatrix} a_{11}x_1 + a_{12}x_1x_2 \\ a_{21}x_2 + a_{22}x_1x_3 + a_{23}x_3 \\ a_{31}x_1x_3 + a_{32}x_2 + a_{33}x_3 + a_{34}T_r \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \\ f_3 \end{bmatrix}, \quad (6)$$

$$g_d(x) = \begin{pmatrix} \lambda_d \\ 0 \\ 0 \end{pmatrix}, \quad g_q(x) = \begin{pmatrix} 0 \\ \lambda_q \\ 0 \end{pmatrix};$$

$$v_s = [v_d \quad v_q]^T, \quad [x_1 \quad x_2 \quad x_3] = [I_d \quad I_q \quad \Omega]^T,$$

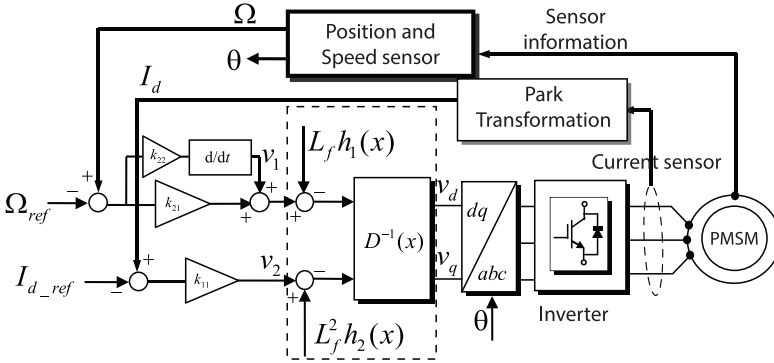
$$a_{11} = -\frac{R_s}{L_d}, \quad a_{12} = \frac{L_q}{L_d} p, \quad a_{21} = -\frac{R_s}{L_q}, \quad a_{22} = -\frac{L_d}{L_q} p, \quad a_{23} = -\frac{\Phi_f}{L_q} p,$$

$$a_{31} = \frac{3p}{2J_m} (L_d - L_q), \quad a_{32} = \frac{3p}{2J_m} \Phi_f, \quad a_{33} = -\frac{f_m}{J_m}, \quad a_{34} = -\frac{1}{J_m} C_r, \quad \lambda_q = 1/L_q,$$

where  $v_d$  and  $v_q$  are the stator voltages of the  $d-q$  axes;  $I_d$  and  $I_q$  are the stator currents of the  $d-q$  axes,  $\Phi_d$  and  $\Phi_q$  are flux linkages of the  $d-q$  axes,  $\Phi_f$  is the magnetic flux linkage,  $p$  is the number of poles pairs,  $T_L$  is the load torque;  $T_e$  is the electromagnetic torque,  $J_m$  is the moment of inertia,  $f_m$  is the viscous friction coefficient and  $\Omega$  is the rotor speed.

### 3 Input Output Feedback Linearization Control

Fig. 1 shows the control bloc diagram of a PMSM drive system using current and speed feedback control. The currents  $I_d$  and  $I_q$  can be calculated from  $i_a$  and  $i_b$  (which can be obtained from measurements) by Clarke and Park transformations. The terms  $L_f h_1(x)$  and  $L_f^2 h_2(x)$  are the first and second Lie derivatives.



**Fig. 1** – Bloc diagram of PMSM (IOC) scheme.

The outputs to be controlled are the motor speed  $\Omega$  and the stator current  $I_d$ . The function  $h(x)$  in (5) is defined as

$$h(x) = \begin{bmatrix} I_d \\ \Omega \end{bmatrix}. \quad (7)$$

The derivative of (7) is given by

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} L_f h_1(x) \\ L_f^2 h_2(x) \end{bmatrix} + D(x) \begin{bmatrix} v_d \\ v_q \end{bmatrix}. \quad (8)$$

The system has relative degree 1 for  $I_d$  and 2 for  $\Omega$ ,  $D(x)$  is the decoupling matrix defined by

$$D(x) = \begin{bmatrix} \lambda_d & 0 \\ \lambda_d a_{31} x_2 & \lambda_q (a_{32} + a_{31} x_1) \end{bmatrix}, \quad (9)$$

and

$$L_f h_1(x) = a_{11} x_1 + a_{12} x_2 x_3, \quad (10)$$

$$L_f^2 h_2(x) = a_{31} x_2 f_1(x) + (a_{32} + a_{31} x_1) f_2(x) + a_{33} f_3(x) + a_{34} \dot{T}_L + a_3 a_{34} T_L. \quad (11)$$

Since  $|D(x)| = \lambda_d \lambda_q (a_{32} + a_{31} x_1) \neq 0$ , then  $D(x)$  is not singular (machine with permanent magnets) and the MIMO system is input-output linearizable.

$$\begin{bmatrix} v_d \\ v_q \end{bmatrix} = D^{-1}(x) \begin{bmatrix} -L_f h_1(x) \\ -L_f^2 h_2(x) \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}, \quad (12)$$

where  $v = [v_1 \ v_2]^T$  is the new input vector.

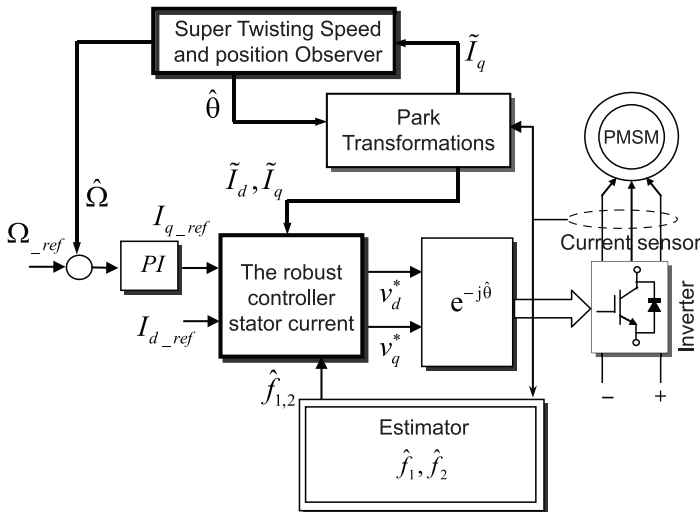
$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} k_{11}(I_d - I_{d\_ref}) \\ k_{21}(\Omega - \Omega_{ref}) + k_{22}(\dot{\Omega} - \dot{\Omega}_{ref}) \end{bmatrix} \quad (13)$$

The drawback of (12) is that it requires exact knowledge of the motor parameters and any variation in the parameters or the load torque will deteriorate the controller performances. In order to overcome this problem feedback nonlinear control based on Lyapunov theory is proposed.

#### 4 Nonlinear Control Based on Lyapunov Theory for the PMSM

The suggested PMSM control scheme is shown in Fig. 2. We can see that only one PI speed controller is used and the currents are feedback-controlled in association with a sliding mode controller. We can also note the placement of the estimator block which evaluates the feedback function  $f_1$  and  $f_2$  given by (6). To determine the control feedback, we rewrite (2) as follow:

$$\begin{aligned} \frac{dI_d}{dt} &= \lambda_d v_d + f_1, \\ \frac{dI_q}{dt} &= \lambda_q v_q + f_2. \end{aligned} \quad (14)$$



**Fig. 2** – Bloc diagram of nonlinear control based on Lyapunov theory (NLC) with second-order sliding mode observer.

In a real situation, the nonlinear functions  $f_i$  involved in the state-space model (14) are strongly affected by the conventional effects of PMS motors,

such as temperature, saturation, skin effects and noise measurements. Then the design of a robust control law needs the exact knowledge of  $f_i$  functions.

Globally, we can write

$$K_{ii} \geq \beta_i, \quad (15)$$

where  $\hat{f}_i$  is the identified nonlinear feedback function,  $f_i$  is the effective function and  $\Delta f_i$  is the error of  $f_i$ . We assume that all  $\Delta f_i$  are bounded ( $|\Delta f_i| < \beta_i$ ).

Substitution of (15) into (14) yields:

$$\begin{aligned} \frac{dI_d}{dt} &= \lambda_d v_d + \hat{f}_1 + \Delta f_1, \\ \frac{dI_q}{dt} &= \lambda_q v_q + \hat{f}_2 + \Delta f_2. \end{aligned} \quad (16)$$

Let the candidate Lyapunov function related to the currents dynamics defined by:

$$V = \frac{1}{2}(I_d - I_{d\_ref})^2 + \frac{1}{2}(I_q - I_{q\_ref})^2 > 0. \quad (17)$$

This function is globally positive defined over the whole state space. Its derivative is given by

$$\dot{V} = (I_d - I_{d\_ref})(\dot{I}_d - \dot{I}_{d\_ref}) + (I_q - I_{q\_ref})(\dot{I}_q - \dot{I}_{q\_ref}). \quad (18)$$

Inserting (16) in (18) we obtain:

$$\begin{aligned} \dot{V} &= (I_d - I_{d\_ref})(\lambda_d v_d + \hat{f}_1 + \Delta f_1 - \dot{I}_{d\_ref}) + \\ &+ (I_q - I_{q\_ref})(\lambda_q v_q + \hat{f}_2 + \Delta f_2 - \dot{I}_{q\_ref}). \end{aligned} \quad (19)$$

Selecting the control law as

$$\begin{aligned} v_d &= \frac{1}{\lambda_d} \left( -\hat{f}_1 + \dot{I}_{d\_ref} - K_1(I_d - I_{d\_ref}) - K_{11} \text{sign}(I_d - I_{d\_ref}) \right), \\ v_q &= \frac{1}{\lambda_q} \left( -\hat{f}_2 + \dot{I}_{q\_ref} - K_2(I_q - I_{q\_ref}) - K_{22} \text{sign}(I_q - I_{q\_ref}) \right), \end{aligned} \quad (20)$$

where  $K_{ii} \geq \beta_i$ ,  $K_i > 0$  and  $i = 1, 2$ .

Inserting the control law (20) in (19), we obtain:

$$\begin{aligned} \dot{V} &= (I_d - I_{d\_ref}) \left( \Delta f_1 - K_{11} \text{sign}(I_d - I_{d\_ref}) \right) + \\ &+ (I_q - I_{q\_ref}) \left( \Delta f_2 - K_{22} \text{sign}(I_q - I_{q\_ref}) \right) + \dot{V} < 0, \end{aligned} \quad (21)$$

where  $\dot{V}$  is given by

$$\dot{V} = -K_1(I_d - I_{d\_ref})^2 - K_2(I_q - I_{q\_ref})^2 < 0. \quad (22)$$

Hence the  $\Delta f_i$  variations can be absorbed if we take

$$K_{11} > |\Delta f_1|, \quad K_{22} > |\Delta f_2|. \quad (23)$$

These inequalities are satisfied since  $K_i > 0$  and  $|\Delta f_i| < \beta_i < K_{ii}$ . Finally, we can write

$$\dot{V}_1 < \dot{V} < 0. \quad (24)$$

Hence, using Lyapunov theorem [2], we conclude that

$$\begin{aligned} \lim_{t \rightarrow \infty} (I_d - I_{d\_ref}) &= 0, \\ \lim_{t \rightarrow \infty} (I_q - I_{q\_ref}) &= 0. \end{aligned} \quad (25)$$

## 5 Observer Design

The proposed observer is based on a second-order sliding mode approach knowing to be robust versus parametric uncertainties, modeling errors and disturbances. The structure is identical to Fig. 2 except the sensor information which is replaced by the super twisting speed and position observer. The observer is based on the so-called broken super twisting algorithm presented in [33].

Using (3) and (4) we obtain the following form:

$$\begin{aligned} \dot{x}_4 &= x_3, \\ \dot{x}_3 &= f_o(t, x_4, x_3, u) + \xi(t, x_4, x_3, u). \end{aligned} \quad (26)$$

where  $x_4$  and  $x_3$  are respectively  $\theta$  and  $\Omega$ ,  $u$  is the torque and  $\xi$  is the uncertainties.

$$\begin{aligned} \dot{\theta} &= \Omega, \\ \ddot{\theta} = \dot{\Omega} &= -\frac{f_m}{J_m} \Omega - \frac{1}{J_m} T_L + \frac{3}{2} p \Phi_f i_q + \xi. \end{aligned} \quad (27)$$

The super twisting second order sliding mode observer is designed as follows:

$$\begin{aligned} \dot{\hat{\theta}} &= \hat{\Omega} + z_1, \\ \ddot{\hat{\theta}} = \dot{\hat{\Omega}} &= -\frac{f_m}{J_m} \hat{\Omega} - \frac{1}{J_m} T_L + \frac{3}{2} p \Phi_f i_q + z_2, \end{aligned} \quad (28)$$

where  $\hat{\theta}$  and  $\hat{\Omega}$  are the states estimations and the correction variables  $z_1$  and  $z_2$  are output injections of the form:

$$\begin{aligned} z_1 &= \lambda |\Omega - \hat{\Omega}|^{1/2} \text{sign}(\Omega - \hat{\Omega}), \\ z_2 &= \alpha \text{sign}(\Omega - \hat{\Omega}). \end{aligned} \quad (29)$$

We consider initially that  $\hat{\theta} = \theta$  and  $\hat{\Omega} = 0$ . Taking into account  $e_\theta = \theta - \hat{\theta}$  and  $e_\Omega = \Omega - \hat{\Omega}$ , we obtain the following error equations

$$\begin{aligned} \dot{e}_\theta &= e_\Omega - \lambda |e_\Omega|^{1/2} \text{sign}(e_\Omega), \\ \dot{e}_\Omega &= -\frac{f_m}{J_m} e_\Omega - \alpha \text{sign}(e_\Omega). \end{aligned} \quad (30)$$

We assume that:

$$\left| -\frac{f_m}{J_m} e_\Omega + \xi \right| < f^+ \quad (31)$$

holds for any possible  $t, \theta, \Omega$  and  $\sup |\hat{\Omega}| \leq 2 \sup |\Omega|$ .

The use of this super twisting algorithm ensures the finite time convergence of  $\hat{\theta} \rightarrow \theta$  and  $\hat{\Omega} \rightarrow \Omega$ .

## 6 Simulation Results

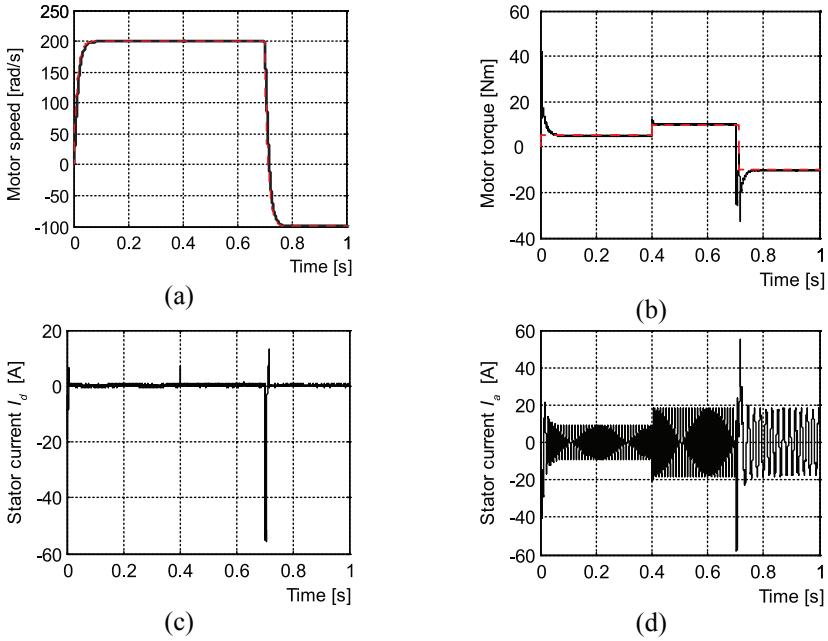
To demonstrate the efficiency of the proposed composite control method, simulations on a PMSM servo system have been performed. Three control methods have been applied: input output feedback linearization Control (IOC), nonlinear control based on Lyapunov theory (NLC), and speed sensorless control of PMSM based on second-order sliding mode observer (super Twisting algorithm observer).

The parameters of the PMSM used in the simulation are:

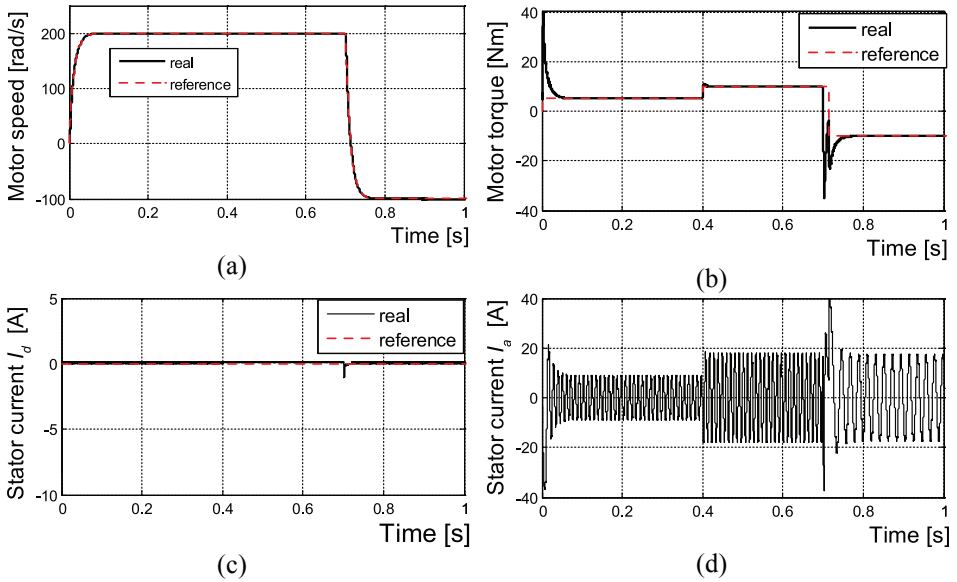
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| rated voltage       | $V = 511 \text{ V}$                                  |
| number of poles     | $p = 3$  |
| armature resistance | $R_s = 1.4 \Omega$                                   |
| stator inductances  | $L_d = 0.0066 \text{ H}$<br>$L_q = 0.0058 \text{ H}$ |
| viscous damping     | $f_m = 0.00038 \text{ Nms/rad}$                      |
| moment of inertia   | $J_m = 0.0016 \text{ kgm}^2$                         |
| rotor flux          | $\Phi_f = 0.1546 \text{ Wb}$                         |
| rated torque        | $T_n = 10 \text{ Nm}$                                |



## Speed Senseless Robust Control of PMSM Based on Second-Order SM Observer



**Fig. 3** – Simulation results of feedback linearization control (IOC) :  
 (a) motor speed; (b) motor torque; (c) stator current  $I_d$ ; (d) stator current  $I_a$  [A].



**Fig. 4** – Simulation results of nonlinear control based on Lyapunov theory (NLC):  
 (a) motor speed; (b) motor torque; (c) stator current  $I_d$ ; (d) stator current  $I_a$  [A].

In the first part of this section, two schemes have been simulated: input output feedback linearization (IOC) and the proposed nonlinear control based on Lyapunov theory scheme (NLC), to analyze and compare the performance of the PMSM in terms of accuracy, dynamic performance and load disturbance rejection.

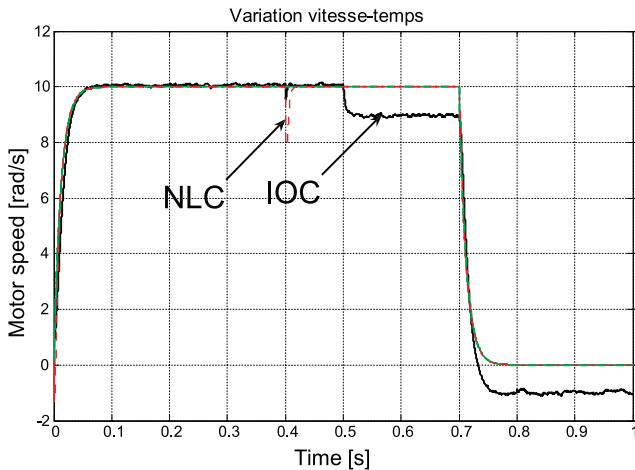
Figs. 3 and 4 show the PMSM response to square-wave speed reference 200 rad/s, using the IOC and NLC. The NLC PMSM drive speed trajectory is characterized by zero steady-state error and very fast dynamic response.

To test the robustness of the two controls with respect to motor parameters variations, the following profile of speed reference is applied:

The PMSM started with a constant acceleration after 0.1 s, the speed was maintained to 10 rad/s, while the motor is loaded with a constant torque of 5 Nm at starting. Then the motor is loaded with a constant torque of 10 Nm at  $t = 0.4$  s. At  $t = 0.7$  s, the speed change from 10 rad/s to 0 rad/s with same constant load torque. Maintaining a reference current  $I_d$  to zero. Two sets of simulation tests are carried out.

The first set is carried out with stator resistance having a mismatch of 100% at  $t = 0.5$  s using the control law, the results of this test set are shown in Fig. 5 (IOC). It is clear that when considering stator resistance uncertainty, a very large steady state error occurred in motor speed.

Finally the motor having a  $f_i$  (NLFF) mismatch of 300% at  $t = 0.5$  s and in the presence of noise is simulated using the proposed control. The results are shown in Fig. 5 (NLC). The control shows better speed response in the presence of parameter uncertainty and measurement noises.



**Fig. 5** – Comparison between IOC and NLC speed transient evolutions with parameter uncertainty and measurement noises.

In the second part of this section, we illustrate the performance of the proposed Sensorless Control. The typical step references of the speed and load torque are given in Fig. 6a and Fig. 6b, respectively.

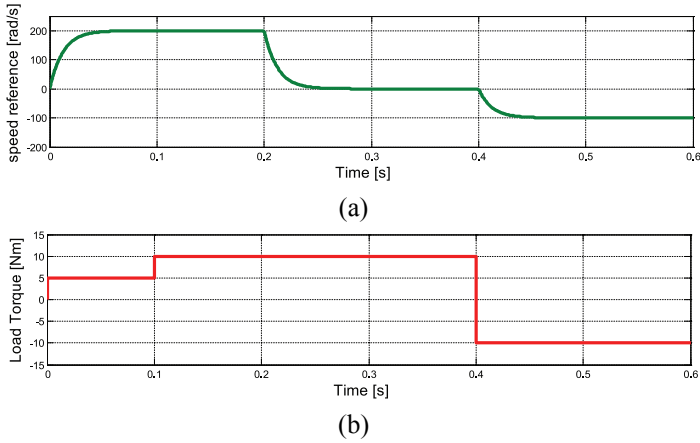


Fig. 6 – The typical step references.

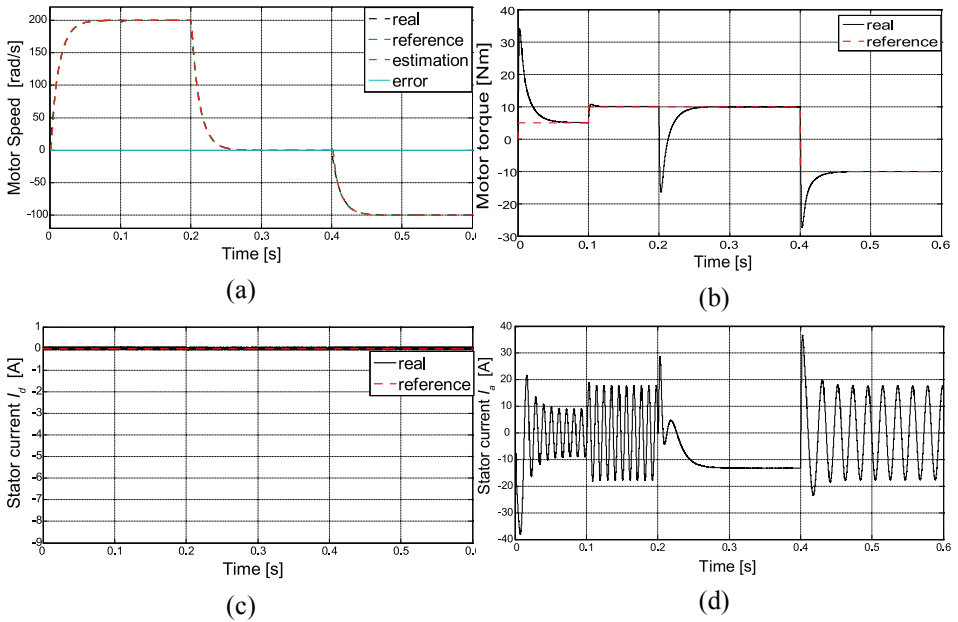
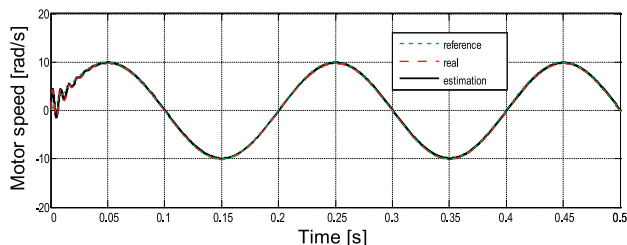
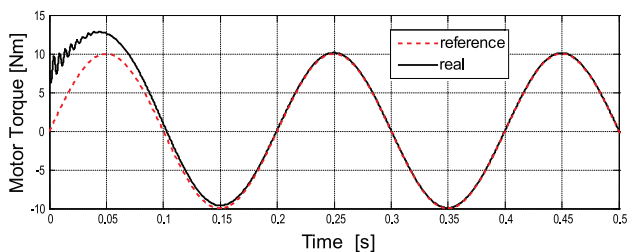


Fig. 7 – Simulation results of NLC\_super twisting observer:  
 (a) motor reference, actual and estimated speed; (b) motor torque;  
 (c) stator current  $I_d$ ; (d) stator current  $I_a$  [A].

Fig. 7 shows the simulation result. The actual speed is compared with the estimated one. It can be seen that very good performances are obtained (Fig. 8). Another test with 5 Hz sinusoidal reference speed of 10 rad/s peak value is realized confirm the above results. The comparison between the actual speed and the estimated one shown by Fig. 8a demonstrate the effectiveness of the method. To track a reference torque, a 5 Hz sinusoidal torque reference with magnitude 10 Nm is applied to the machine where the speed is maintained at 200 rad/s. Good performances are obtained Fig. 8b.



(a)



(b)

**Fig. 8** – Behavior at low motor speed: (a) sinusoidal reference, actual and estimated speed; (b) torque-tracking response reference and actual.

## 7 Conclusion

In this paper, a nonlinear control based on Lyapunov theory scheme combined with sliding mode observer was applied for robust speed sensorless control of PMSM. The theoretical study of this nonlinear control (NLC) has been discussed and control stability verified via Lyapunov stability analysis.

A second-order sliding mode observer based on an exact differentiator (super-Twisting algorithm) was used for two main reasons: the finite time convergence and the ability to take into account the variable nature of the system structure.

The simulation results demonstrate the effectiveness and the good performance of the proposed control methods.

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