Adaptive PID Controller with Parallel Feedforward Compensator for MIMO Systems

R. A. Fahmy¹, R. I. Badr², A. A. Abouelsoud³, F. A. Rahman¹

¹ Nuclear and Radiological Regulatory Authority (NRRA), Cairo, Egypt
² Electronics and Communications Dept. Faculty of Engineering, Cairo University, Giza, Egypt
Raniaafahmy@yahoo.com; Ragiabadr@yahoo.com; Aali711964@yahoo.com; farouk63@yahoo.com

Abstract

With the classical PID controller, good performance can be obtained, if all the model parameters and operating conditions are known. In case which some of the system parameters or operating conditions are uncertain or unknown, an adaptive PID (APID) learning controller (which consists of a set of learning rules for PID gain tuning) is used. To guarantee the stability of APID controller, the controlled system must be Almost Strict Positive Real (ASPR). Because not all the actual MIMO systems can satisfy that condition and to make the APID controller design applicable on more general systems, an auxiliary subsystem called parallel feedforward compensator (PFC) is added in parallel with the controlled system. Computer simulations are given to demonstrate the effectiveness of the overall system applying different shapes of input signals.

Keywords

Adaptive PID (APID); Tracking Problems; MIMO Systems; PFC; Almost Strict Positive Real (ASPR)

Introduction

The classical Proportional-Integral-Derivative (PID) controller is still the most common control algorithm used in many real applications due to its simplicity. The key idea of designing a PID controller is to determine the three gains (i.e., proportional gain $K_p$, integral gain $K_i$, and derivative gain $K_d$) of the controller. A good performance of the controller can be obtained, if all the model parameters and operating conditions are exactly known, if not, the adaptation of the PID gains must be considered. The APID control scheme is a mean to continuously adjust the controller parameters to maintain consistent optimal plant performance and guarantee the stability of the overall system.

In (Ghanadan, Reza and Blankenship, 1991), a systematic way to design an APID controller for nonlinear systems where the dominant dynamics are of second order has been developed. The controller in (Baek, Seung-Min and Kuc, 1997) consists of a feedback input from the APID controller and a feedforward input from the learning controller. There are two sets of on-line learning rules in the controller: one for APID gain tuning and the other for feedforward learning input. It is proved that all the error signals are bounded and the tracking error converges globally and asymptotically as the learning proceeds.

A self-tuning PID control system is proposed in (Ohnishi, Yamamoto and Shah, 2000) for multivariable systems with unknown parameters and time-delays. The controlled object is equipped with an internal model in order to compensate the time-delay as well as unstable zeros. An APID learning control scheme presented in (Kuc, Tae-Yong and Han, 2000) for the control of uncertain robot manipulator performing periodic tasks is composed of a fixed/adaptive PID feedback control scheme and a feedforward input learning scheme implementing the control strategy: linear PID feedback stabilization and feedforward learning for nonlinear compensation and tracking. In (Grassi, Tsakalis, Dash, Gaikwad and Stein, 2000), the tuning/adaptation of PID parameters with a loop-shaping objective is considered. A different update law that approximates the constrained minimization of the operator norm of the error system is designed.

The work in (Badreddine, Bader and Feng, 2001) proposes and analyzes a direct (APID) control scheme for off-line and on-line tuning of PID parameters. The adaptive backstepping-based PID presented in (Benaskeur and Desbiens, 2002) is improved in (Ranger and Desbiens, 2003) by incorporating the integral action through the definition of the first backstepping error, by allowing placing the regulation...
poles to any stable locations and by adding the switching sigma modification.

On the other hand, the neural networks have been recently used in control system design due to their powerful learning and adaptive abilities. Specifically, some PID control architectures, by using the neural networks, have been proposed as in (Chang, Hwang and Hsieh, 2003). The main idea is that output nodes of the fully connected neural networks are regarded as the gains $K_p$, $K_i$ and $K_d$ of PID controller, respectively. These gains adjusted on-line based on certain adaptation laws so as to achieve control objective.

In (Chang and Yan, 2005), based on the use of the sliding mode, a robust APID control tuning is newly proposed to deal with the control problem for a class of uncertain systems with external disturbance. Three gains of PID controller are regarded as on-line adjustable parameters. A self-tuning PID design methodology utilizing just-in-time learning (JITL) modelling technique is developed for nonlinear process control in (Kansha, Jia and Chiu, 2008). PID controller parameters are updated based on the information provided by the JITL and a self-tuning algorithm derived from the Lyapunov method.

In (Tamura and Ohmori, 2007), the APID control for output model tracking problem by constructing Lyapunov’s function based on feature of PID control in a class of MIMO system and the reference model is designed. The controlled MIMO system with $m$-input and $m$-output $n$-state $\{A, B, C\}$ is considered and the APID control with constant gain matrices and the time-varying gain matrices is proposed. However, the tuning laws of PID parameter matrices are derived by satisfying the Lyapunov’s stability theorem under some assumptions that the controlled system’s zero-dynamics is asymptotically stable, $(CB = 0$ and $CAB > 0)$ or $(CB > 0)$ and some rank conditions concerning system and reference model matrices.

However, the almost strict positive realness condition (ASPR) (i.e. $CB > 0$ and minimum phase of the system) is shown not to be satisfied in most controlled plants. To improve this situation, the so-called parallel feedforward compensator (PFC) is introduced in (Iwai, Mizumoto and Nakashima, 2006).

In (Iwai, Mizumoto, Liu, Shah and Jiang, 2006), a APID controller for SISO plant is proposed. PFC is constructed to achieve the ASPR of the controlled system. Thus the design always gives stable PID control system. Few years later, that controller has been modified to improve the windup phenomenon in (Mizumoto, Harada, Fujimoto and Iwai, 2011) and (Minami, Mizumoto and Iwai, 2010). Following the same methodology, (Mizumoto, Ikeda, Hirahata and Iwai, 2010) dealt with the design of discrete time APID controller with PFC.

The most well known tuning rules are still based on SISO plant because of the complexity and difficulty to handle the region of stability concerning PID parameters in MIMO system. There have been several methods of tuning MIMO PID controllers. Although most of these methods are simple extension of the SISO case, the determination of many PID controller parameters of multivariable systems is very complicated. In (Iwai, Mizumoto and Nakashima, 2006), a quite simple construction rule of multivariable PFC is proposed. It is determined based on the approximate model of the plant so that the stability of MIMO PID control system can easily be guaranteed as far as an approximate mathematical model of the MIMO plant can be obtained (Iwai, Mizumoto and Nakashima, 2006).

In this paper, the APID controller for output tracking problem of MIMO systems in (Tamura and Ohmori, 2007) is improved by adding an external subsystem called parallel feedforward compensator PFC following (Iwai, Mizumoto, Liu, Shah and Jiang, 2006), (Mizumoto, Harada, Fujimoto and Iwai, 2011) methodology. Although (Iwai, Mizumoto, Liu, Shah and Jiang, 2006) used a PFC for SISO systems only, yet it is shown in this paper that this method can ensure that the condition $CB > 0$ and minimum phase of the MIMO system is satisfied yielding the ASPR condition which in turn guarantees stability of the APID control for MIMO systems. Thus the MIMO APID controller can be applied to stable and square MIMO systems (Iwai, Mizumoto and Nakashima, 2006) for tracking problems. Simulation results in tracking the reference input signal demonstrate the effectiveness of that APID controller.

This paper is organized as follows. In Section 2, the problem statement is presented. In Section 3, the parallel feedforward compensator (PFC) is introduced. An APID controller for MIMO system and its adaptation laws are introduced in section 4. The proposed technique is applied to a numerical example in section 5 and its results show the performance of the system in tracking different shapes of reference input signals using APID controller. Finally, in Section 6, the conclusion and some suggestions for further work are presented.
Problem Statement

Consider the following square MIMO stable system

\[ x(t) = Ax(t) + Bu(t) \]

\[ y(t) = Cx(t) \]

where \( x(t) \in \mathbb{R}^n \), \( u(t) \in \mathbb{R}^m \), \( y(t) \in \mathbb{R}^m \) are the state vector, the input vector, the output vector respectively, and \( A, B, C \) are constant matrices.

From equation (1), the \( m \times m \) transfer function matrix can be written as:

\[ G(s) = C(sI - A)^{-1}B \]  

(2)

To guarantee the stability of the resulting APID system (Tamura and Ohmori, 2007), the controlled system must be a ASPR system (i.e. \( CB > 0 \) and minimum phase). The following definitions have been mentioned in (Iwai, Mizumoto and Nakashima, 2006) to demonstrate the ASPR concept.

[Definition 1]

System in equation (1) is said to be ASPR if there exists a feedback gain matrix \( K_e \) such that the system defined by \((A - BK_eC, B, C)\) is strict positive real (SPR)

[Definition 2]

Transfer function matrix in equation (2) is ASPR if

1. High frequency gain matrix is positive definite (i.e. \( CB > 0 \)).
2. The system matrix (2) is minimum phase.

PFC Design

The PFC is applied in parallel with the controlled system \( G(s) \) in order to make the resultant system an ASPR one.

PFC can be defined as:

\[ \dot{x}_f = A_fx_f + B_fu \]

\[ y_f = C_fx_f \]

\[ C_fB_f > 0 \]

then the extended system becomes

\[ \dot{x}_a = A_ax_a + B_au \]

\[ y_a = C_ax_a + y + y_f \]  

(4)

Where

\[ A_a = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix}, B_a = \begin{bmatrix} B \\ 0 \end{bmatrix}, C_a = \begin{bmatrix} C & C_f \end{bmatrix} \]

The transfer function of the extended system will be as:

\[ G_a(s) = C_a(sI - A_a)^{-1}B_a \]  

(5)

then equation (4) can be rewritten as

\[ G_a(s) = G(s) + G_{PFC}(s) \]  

(6)

Moreover, an effective PFC design is proposed in (Iwai, Mizumoto, Liu, Shah and Jiang, 2006) based on the approximate plant model \( G^*(s) \) to realize ASPRness of the resultant plant \( G_a(s) \).

The PFC has been designed (Iwai, Mizumoto, Liu, Shah and Jiang, 2006) as

\[ G_{PFC} = G_{ASPR} - G^* \]  

(7)

where \( G_{ASPR} \) is an arbitrarily chosen ASPR transfer function matrix.

The resultant transfer function can be obtained by

\[ G_a(s) = G_{PFC}(s) + G(s) \]

\[ = G_{ASPR}(s) + G(s) - G^*(s) \]  

(8)

Where

\[ \Delta(s) = G_{ASPR}(s)(I(s)) \]

The extended system \( G^*(s) \) in equation (8) becomes ASPR if \( G(s) \) and \( G^*(s) \) are stable transfer function matrices and \( G^*(s) \) is a good approximation of the controlled system \( G(s) \). The details of the proof of the theorem exist in (Iwai, Mizumoto and Nakashima, 2006).

APID Controller

An APID controller is designed following the same methodology in (Tamura and Ohmori, 2007) but it is applied on extended system \( G_a(s) \) not on controlled system \( G(s) \) directly as shown in figure 1. APID controller is defined by

\[ u(t) = K_{i_0} \int_0^t e_a(\tau)d\tau + (K_{p_0} + K_{p_1}(t))e_a(t) \]

\[ + K_{p_1}(t)e_a(t) + K_{p_2}(t)y_a(t) \]

\[ + K_{d_2}(t)\dot{y}_a(t) \]  

(9)

Where

\[ e_a(t) = r(t) - y_a(t) \]
and $r(t)$ is the reference signal, the time-varying gain matrices $K_{p1}(t), K_{p2}(t)$, 
$K_{d1}(t), K_{d2}(t) \in \mathbb{R}^{m \times m}$ and the constant gain matrices $K_{p0}, K_{p0} \in \mathbb{R}^{m \times m}$.

The constant PID parameter matrices and the adaptive tuning laws of the time-varying PID parameters matrices in equation (9) are derived by satisfying the Lyapunov’s stability theorem under some assumptions (i.e. the controlled system’s zero-dynamics is asymptotically stable and the high frequency gain matrix is greater than zero) to make the closed loop system stable and the output error converges to zero (i.e. $e_a(t) \to 0$) (Tamura and Ohmori, 2007).

The adaptive tuning gain design in (Tamura and Ohmori, 2007) sets the constant gain matrix $K_{p0}, K_{p0}$ to

$$K_{p0} = \gamma_1 H_1, \quad K_{p0} = \gamma_2 H_2 \quad (10)$$

and the adaptive tuning laws of the time-varying gain matrix $K_{p1}(t), K_{p2}(t), i = 1, 2$ are defined as

$$\dot{K}_{p1}(t) = \Gamma_{p1}(t)e_i(t)^T + \Gamma_{p2}(t)r(t)^T \quad (11-a)$$

$$\dot{K}_{p2}(t) = \Gamma_{p2}(t)e_i(t)^T + \Gamma_{d1}(t)r(t)^T \quad (11-b)$$

$$\dot{K}_{d1}(t) = \Gamma_{d1}(t)e_i(t)^T \quad (11-c)$$

$$\dot{K}_{d2}(t) = \Gamma_{d2}(t)e_i(t)^T \quad (11-d)$$

where

$H_1 = \text{diag}\{h_{11}, \ldots, h_{1m}\}, \quad H_2 = \text{diag}\{h_{21}, \ldots, h_{2m}\}$,

$h_{1j}, h_{2j} > 0, \quad j = 1, \ldots, m$ and $\Gamma_\rho, \Gamma_{p1}, \Gamma_{p2}, \Gamma_{d1}, \Gamma_{d2} \in \mathbb{R}^{m \times m}$ are arbitrary positive definite matrices and $\gamma_i$ is arbitrary positive scalar. Also the detailed proof of the above APID controller is described in (Tamura and Ohmori, 2007).

**Numerical Example**

In the following, the proposed overall system in Figure 1 is examined through a numerical example with different input signals.

Consider the system used in (Iwai, Mizumoto and Nakashima, 2006):

$$G(s) = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix}$$

$$G_i(s) = \frac{k_i}{(s+1)(T_i s+1)(T_j s+1)(T_k s+1)(T_l s+1)} \quad (13-a)$$

where

$$G_{11}(s) = \frac{2}{(s+1)(0.3s+1)(0.04s+1)(0.008s+1)}$$

$$G_{12}(s) = \frac{1}{(s+1)(0.6s+1)(0.05s+1)} \quad (13-b)$$

$$G_{21}(s) = \frac{1}{(2s+1)(0.2s+1)(0.02s+1)}$$

$$G_{22}(s) = \frac{2}{(s+1)(0.7s+1)(0.02s+1)(0.008s+1)}$$

Since this system is not ASPR, the proposed design of APID for MIMO systems consists of two stages. The first stage is to design PFC then the second stage is to obtain the APID gains.

**Design the PFC**

An approximate model can be obtained by using 0/1 Pade approximation in (Seborg, Edgar and Mellichamp, 2004),

$$\frac{K}{(T+1)(L+1)(T+1)(T+1)} \approx \frac{K}{(Ts+1)(Ls+1)} \quad (14)$$

Where $T = T_1, \quad L = T_2 + T_3 + T_4, \quad T_i >> T_2, T_3, T_4$

Thus, a second-order approximation of the system in equation (12) can be obtained such as

$$G'(s) = \begin{bmatrix} 2/(s+1)(0.35s+1) & 1/(s+1)(0.65s+1) \\ 1/(2s+1)(0.22s+1) & 2/(s+1)(0.73s+1) \end{bmatrix} \quad (15)$$

Following (Iwai, Mizumoto and Nakashima, 2006), the ideal ASPR model $G_{ASPR}(s)$ can be set as
\[
G_{\text{ASPR}}(s) = \begin{bmatrix}
\frac{2}{s+1} & \frac{1}{1.5s+1}
\frac{1}{s+1} & \frac{2}{1.5s+1}
\end{bmatrix}
\] (16)

By using the same parameters in (Iwai, Mizumoto and Nakashima, 2006) and equation (7) to design PFC, thus \(G_{\text{PFC}}(s)\) can be written as

\[
G_{\text{PFC}}(s) = G_{\text{ASPR}} - G^* = \begin{bmatrix}
G_{p11}(s) & G_{p12}(s)
G_{p21}(s) & G_{p22}(s)
\end{bmatrix}
\] (17)

where

\[
G_{p11}(s) = \frac{0.7s^2 + 0.7s}{0.35s^3 + 1.7s^2 + 2.35s + 1}
G_{p12}(s) = \frac{0.65s^3 + 0.15s}{0.975s^3 + 3.125s^2 + 3.15s + 1}
G_{p21}(s) = \frac{0.44s^3 + 0.72s}{0.66s^3 + 3.77s^2 + 3.72s + 1}
G_{p22}(s) = \frac{1.46s^3 + 1.46s}{0.73s^3 + 2.46s^2 + 2.73s + 1}
\]

**Design The APID Gains**

For the extended system \(G_a\), an APID controller is designed. Next, different input signals are applied to the system as in figure 1 to examine the performance of the APID controller with PFC for MIMO system in equation (13).

**1) For Step Reference Input:**

A step reference signal is applied as input to the overall system as in figure 1. The step reference signal can take the following form

\[
r(t) = \begin{cases}
1, & t \geq 0 \\
0.6, & \end{cases}
\] (18)

The parameters \(H_1, H_2\) in equation (12) are set as in (Tamura and Ohmori, 2007)

\[
H_1 = \begin{bmatrix}
0.5 & 0 \\
0 & 0.5
\end{bmatrix}, \quad H_2 = \begin{bmatrix}
0 & 0 \\
0 & 0
\end{bmatrix}
\] (19)

and let \(\Gamma_{p1} = \Gamma_{p2} = \Gamma_{D1} = \Gamma_{D2} = I_m\) and \(\gamma_1 = I\).

The initial values are

\[
x(0) = 0, \quad K_{p1}(0) = K_{p2}(0) = 0, \quad i = 1, 2.
\]

It is observed from the simulation results in Figures 2, 3 that the system manages to track the reference step input and they are compared with the fixed PID gains controller in (Tamura and Ohmori, 2007).
For Pulse Reference Input:

A pulse reference signal is applied as input to the overall system as in figure 1. This pulse input signal can be arbitrary chosen as

\[
    r(t) = \begin{bmatrix} \tau_1(t) \\ \tau_2(t) \end{bmatrix}
\]

(20)

where

\[
    \tau_1(t) = \begin{cases} 
        1 & 5 \leq t \leq 25 \\
        0 & \text{otherwise}
    \end{cases}
\]

and

\[
    \tau_2(t) = \begin{cases} 
        0.6 & 10 \leq t \leq 45 \\
        0 & \text{otherwise}
    \end{cases}
\]
The parameters $H_1, H_2, \Gamma_{D1}, \Gamma_{D2}, \Gamma_{P1}, \Gamma_{P2}$ and $\gamma_1$ are chosen as in step reference input case. In addition, the initial values $x(0)$ and $K_{Di}(0), K_{Di}(0), i = 1, 2$ set as in above case. However, $\Gamma_{P1}, \Gamma_{P2}$ are changed to $\Gamma_{P1} = \Gamma_{P2} = 50 \times I_m$ to reduce the rise time of the output signal. Figure 10 shows the effectiveness of the proposed controller in case of pulse input signal. In figures 11,12 the change in plant parameters (i.e. $T_{ij}, K_{ij}$ are parameters of $G_{ij}(s)$) is examined.

3) For Sinusoidal Reference Input:

The input signal which is applied to the overall system as in figure 1 can be chosen in the sinusoidal form

$$r(t) = \begin{bmatrix} 0.5 \cos(t) \\ \sin(0.5t) \end{bmatrix}$$

(21)

The parameters $H_1, H_2, \Gamma_{D1}, \Gamma_{D2}, \Gamma_{P1}, \Gamma_{P2}$ and $\gamma_1$ are chosen as in step reference input case. Further, the initial values $x(0)$ and $K_{Pi}(0), K_{Di}(0), i = 1, 2$ set as in above case. The output of the system in case of sinusoidal input signal is shown in Figure 13 which confirmed the effectiveness of the proposed technique. In figures 14, 15 the robustness of the proposed method is examined against the change in plant parameters.
Figure 16 shows that the errors reach zero steady state values with the proposed approach.

**Conclusions**

In this paper, a new technique is proposed to design APID controller by adding a Parallel feedforward compensator to generalize the application of APID controllers on any stable square MIMO systems not necessarily ASPR systems. The response of the proposed method is examined through several simulation cases. The simulation results confirm the effectiveness of the proposed technique that combine the APID controller technique with PFC and its robustness against any variation in plant parameters. Even with different shapes of reference signals (namely step, pulse and sinusoidal), the output still tracks the input reference signal even when the controlled plant is not an ASPR system. Further extension of this work involves the ability of applying the proposed technique on unstable plant or adding a disturbance signal to the system. The improved APID may be examined for more shapes of reference input to guarantee the tracking perfection and the stability of the closed loop system. This technique thus presents more general design of APID controllers that can be widely applied on several MIMO systems.

**REFERENCES**


R. A. Fahmy received B.Sc., in Electronics and Communications from Faculty of Engineering, in 2002 and M.Sc. degree in Systems and Control Engineering in 2009 from Faculty of Engineering, both from Cairo University, Egypt. She is currently an assistant lecturer in the Nuclear and Radiological regulatory Authority (NRRA), Egypt. Her research interests include Adaptive Control and the Control of Fusion Reactors.

A. Abouelsoud was born in Cairo, Egypt on January 8, 1964. He received B.Sc., in electrical engineering in 1986, the M. Sc. in control engineering in 1990, and the Ph.D. in control engineering in 1995 all from Cairo University, Egypt. He is a member of IEEE automatic control society. He has published more than 24 papers in automatic control, and currently, professor of control at Cairo University. His research interests are in nonlinear and adaptive control and robotics.

R. I. Badr obtained her B.Sc. degree with distinction and honors in 1979 from the Electronics and Communications Department, Faculty of Engineering, Cairo University, M.Sc. degree in Systems and Control Engineering in 1982 and Ph.D. in Systems and Control Engineering in 1987 from Cairo University. In 1987 she was appointed as an assistant professor in the Electronics and Communications Department, Faculty of Engineering, Cairo University. She is currently Professor at the same department and her research areas include Robust Control, Neuro-Fuzzy Control, and Adaptive-Non-Linear Control.

F. A. Rahman holds B.Sc. degree in 1970, M.Sc. degree in 1973 and PhD in 1977 in Nuclear Engineering from Faculty of Engineering, Alexandria University, as well as a Bachelor of Laws degree in 1991 from the Faculty of Law and Bachelor of Commercial Science in 1987 from Faculty of Commerce, Ain-Shams University. He is Head of the Nuclear Reactor operation Licensing Group, and member of the IAEA Transport Safety Standards Committee (TRNSSC). His research interests are in the safe operation, quality assurance and regularity inspections of nuclear research reactors.