New Strip Theory Approach to Ship Motions Prediction

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Abstract

This paper describes the development of strip theory to predict quickly with sufficient accuracy ship motions in wave taking into account nonlinear effects caused by interactions ship-wave. Recently, many ship design tools are available, however a quick tool is still needed to use in preliminary design stage. The developed Strip Theory was applied to Ferry motions in heading wave. Here, the Ferry is assumed moving with velocity 6.2m/sec. in wave height 1m and 4m based on actual condition. The Ferry motions were computed in range wave length 0.5Lwl to 2.0Lwl. The research results in wave height 1m and 4m show the heaving amplitudes increase from 0.5Lwl to 0.6Lwl and then decrease to 0.7Lwl, furthermore they increase from 0.5Lwl to 2.0Lwl in increasing wave length, respectively. Moreover, pitching amplitude tends to decrease from 0.5Lwl to 0.6Lwl and then it increase until 2.0Lwl. The nondimensional heaving motions are greater than 1.0 from 1.6Lwl to 2.0Lwl. This means that the wave load affects heaving amplitude bigger than wave amplitude and this condition becomes a concern point whereas the pitching response is small caused by wave impact in the all wave length. In addition, the present results were compared with CFD results as well. Our present results indicate good agreement with CFD results. In our future works, the present results will be compared with experimental results as well.

Keywords: Strip Theory, Ship Motions, Heave Amplitude, Pitch Amplitude

1. Introduction

Ship accidents at the sea are still common happen. This accident usually causes financial loss, claims human lives, and leads to protracted legal wrangling. Therefore this accident takes more attentions many stakeholders. The poor design, navigation, bad weather, fire, and other human errors can lead to ship accident [1]. Study interaction ship-wave has been developed recently. As known that a strongly interaction between ship-wave can generate nonlinear effects suffered ship body [2]. Nonlinear effects are important consideration in ship design. Some phenomena due to nonlinear effects are experienced by a ship such as over rolling, slamming, water on deck, whipping and wave breaking. These can affect ship performance. Therefore, a ship should be designed properly by taking into account nonlinear effects to avoid an accident. In ship design, ship response is importantly predicted. Many ship motion predictions use linear and simple assumption. However, nonlinear effects in some prediction tools cannot be neglected and are rarely involved such as Slender-body theory [3], enhanced unified theory [4], 3D panel method [5] and Rankin panel method [6]. In additions, some computational fluid dynamics (CFD) tools are available. In our previous research, the ship motions in nonlinear wave were predicted by using CFD, hybrid Eulerian scheme with Lagrangian particle [2,7,8,9,10], which the tool can be used to predict ship motions taking into account all phenomena with high accuracy. However, almost the all of CFD tool needs time cost. Moreover, Ship design requires quick and accurate tool for making proper decision. Recently, it is possible to use ship motions programs based on the linear strip theory in ship design work. The designer needs quick and easy computational tools for convenient in use and accurate to interpret. Based on some reasons, our present research is focused on development of strip theory taking into account nonlinear effects caused by interaction between ship-wave with sufficient accuracy and quick.

2. Strip Theory Method

Strip theory, the forces on and motions of a three-dimensional floating body, can be determined by using results from two-dimensional hydromechanics coefficients and exciting wave loads. These values will be integrated over the ship length numerically. The ship is considered to be a rigid body. Strip theory considers a ship to be made up of a finite number of transverse two dimensional strips or cross sections, which are rigidly connected to each other. It is assumed that the problem of the motions of this floating body in waves is linear. Then, the differential equations will be solved to obtain the motions.

A ship, co-ordinate systems as shown in Figure 1, advances in wave and in the positive x-direction with constant speed U. Regular waves with absolute frequency \( \omega_p \) and direction \( \beta \) are incident on the ship. The body frame is fixed to the ship and is described by three translations and rotations...
relative to the hydrodynamic frame. The second right-handed co-ordinate system $X(x_0,y_0,z_0)$ is fixed in space. The $(x_0,y_0)$-plane lies in the still water surface, $x_0$ is directed as the wave propagation direction $\beta$ and $z_0$ is directed upwards. Another right-handed co-ordinate system $G(x,y,z)$ is directed by $x$ in the direction of the forward ship speed $U$, $y$ in the lateral port side direction and $z$ vertically upwards. The origin $O$ lies vertically above or under the time-averaged position of the center of gravity $G$. The $(x,y)$-plane lies in the still water surface. Then, the third right-handed co-ordinate system $G(x_k,y_k,z_k)$ is connected to the ship with its origin at $G$ where $x_k$ in the longitudinal forward direction, $y_k$ in the lateral port side direction and $z_k$ upwards. In still water, the $(x_k,y_k)$-plane is parallel to the still water surface.

The equations of motions for six degrees of freedom of a ship in wave caused by any external loads are based on Newton’s second law $(F=m.a)$. Based on co-ordinate system as shown in Figure 1, the ship motion equations are given as follows:

![Figure 1 Co-ordinate system](image)

Here, a coupled equations of heave and pitch motions, ship motions in heading wave, are firstly distinguished. Then, the supposing the wave moves in the negative $x_0$ direction with an angle relative to the ship’s speed $U$, the relevant rigid body velocities with a constant forward velocity and heaving and pitching in head waves are given by:

$$\sum_{j=1}^{6} \left\{ (M_{kj} + A_{kj}) \ddot{x}_j + B_{kj} \dot{x}_j + C_{kj} x_j \right\} = F_k$$

For $k = 1,2,3,4,5,6$

where:
- $k=1,3,5$: coupled surge, heave and pitch
- $k=2,4,6$: coupled sway, roll and yaw
- $\ddot{x}_j$: Acceleration of harmonic oscillation in direction $j$
- $\dot{x}_j$: Velocity of harmonic oscillation in direction $j$
- $x_j$: Displacement of harmonic oscillation in direction $j$
- $F_k$: Harmonic exciting wave force or moment in direction $k$
- $M_{kj}$: Solid mass or inertia coefficient
- $A_{kj}$: Hydrodynamic mass or inertia coefficient
- $B_{kj}$: Hydrodynamic damping coefficient
- $C_{kj}$: Spring coefficient

### 2.1 Ship-body boundary condition

$$\vec{U} = U \vec{n}; \text{Forward speed}$$

$$\dot{\xi}_3(t) = \dot{\xi}_3(t) \dot{\vec{n}}; \text{Heave velocity}$$

$$\ddot{\xi}_5(t) = \ddot{\xi}_5(t); \text{Pitch angular velocity}$$

where, $\vec{n}(t)$ is the time-dependent unit vector to the instantaneous position of the ship-body defined by

$$\vec{n}(t)\dot{\vec{n}}_0 + \ddot{\xi}_5 \vec{n}_0 \text{ and } \vec{n}_0 \text{ is its value when the ship is at rest defined by } n_0 = n_1, n_2, n_3.$$ The body boundary condition is derived for small heave and pitch motions. The nonlinear boundary condition on the exact position of the ship hull is stated as follow:

$$\frac{\partial \Phi}{\partial n} = \vec{V} \cdot \vec{n}$$
where,
\[ \Phi = \Phi_3 + \Phi_5 \quad (5) \]

\[ V = U_{\frac{3}{4}} + K \left( X \right) \quad (6) \]

where, \( \Phi \) is unsteady velocity as the linear superposition of heave and pitch components. The total rigid-body velocity is the sum of vertical velocity due to heave and pitch.

The principal assumption of strip theory is certain components of the radiation and diffraction potentials.

2.2 Radiation problem

The ship is forced to oscillate in heave and pitch in advancing at a speed \( U \). The variation of the flow in the \( x \)-direction is more gradual than its variation around a ship section and then expressed by:

\[ \frac{\partial^2 \phi}{\partial x^2} \ll \frac{\partial \phi}{\partial t} \quad (7) \]

The 2D equation applies for each strip location at station \( X \). The heave and pitch potentials are given as follows:

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \Phi_j = 0, \text{ in fluid } j = 3, 5 \quad (8) \]

Then, ship-hull condition at station-\( X \) for the heave and pitch potentials are given as follows:

\[ \Phi = Re\{pe^{i\omega t} \} \quad (9) \]

\[ \phi = \Pi_3 X_3 + \Pi_5 X_5 \quad (10) \]

\[ \frac{\partial X_3}{\partial n} = i \omega n_3; \text{ on } S_B \quad (11) \]

\[ \frac{\partial X_5}{\partial n} = -i \omega n_3 + U n_3; \text{ on } S_B \quad (12) \]

where,

\[ \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) X_j = 0, \text{ in fluid domain} \quad (13) \]

Then, it is defined by the normalized potential \( \psi_3 \) as follows:

\[ \frac{\partial \psi_3}{\partial n} = n_3; \text{ on } S_B \quad (14) \]

\[ \left( \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \psi_j = 0, \text{ in fluid domain} \quad (15) \]

In order to obtained the solution of the 2 two dimension (2D) along a ship which consists of several sections used to describe hull form can be carried out 2D panel method. Moreover, the heave and pitch potentials as given by the equations as follows:

\[ \phi = \Pi_3 X_3 + \Pi_5 X_5 \]

\[ X_3 = i \omega n_3 \quad (16) \]

\[ X_5 = (-i \omega X + U) \psi_3 \quad (17) \]

\[ \Phi = Re\{pe^{i\omega t} \} = \Phi_3 + \Phi_5 \quad (18) \]

The 2D heave added-mass and damping coefficients due to a section oscillation are expressed by:

\[ a_{33}(\omega) - \frac{i}{\omega} b_{33}(\omega) = \rho \int_{\Gamma_X} \psi_3 n_3 dl \quad (19) \]

where the encounter frequency is given by:

\[ \omega = \left| \omega_0 - U \frac{\omega_0 \cos \beta}{\beta} \right| \quad (20) \]

Based on integration along a ship and over each cross section at station \( -X \) the 3D added mass and damping coefficients for heave and pitch are expressed by as follows:

\[ A_{33} = \int_X dX a_{33}(X) \quad (21) \]
\[ B_{33} = \int_L dX b_{33}(X) \]  
\[ A_{35} = -\int_L dX a_{33} - \frac{v}{\omega^2} B_{33} \]  
\[ A_{53} = -\int_L dX a_{33} + \frac{v}{\omega^2} B_{33} \]  
\[ B_{35} = -\int_L dX b_{33} + \frac{v}{\omega^2} A_{33} \]  
\[ B_{53} = -\int_L dX b_{33} - \frac{v}{\omega^2} A_{33} \]  
\[ A_{55} = \int_L dXX^2 a_{33} + \frac{v^2}{\omega^2} A_{33} \]  
\[ B_{55} = \int_L dXX^2 b_{33} - \frac{v^2}{\omega^2} B_{33} \]  

2.3 Diffraction problem

The total potential is relatively to the ship frame and it can be written as follow:
\[ \Phi = \Phi_I + \Phi_D = \text{Re} \{ (\phi_I + \phi_D) e^{i\omega t} \} \]
where,
\[ \phi_I = i g \omega \int_{-\infty}^{\infty} e^{iKZ - iKX \cos \beta - iKY \sin \beta} d\theta \]

Then, the diffraction potential is defined by:
\[ \phi_0 = i g \omega \int_{-\infty}^{\infty} e^{-iKX \cos \beta}; \psi_7 = (X,Y,Z) \]

The 3D linear free surface condition for \( \phi_D \) can be written as follows:
\[ \left(i \omega - \frac{\partial}{\partial X} \right)^2 \phi_D + \frac{g}{\omega^2} \left( \frac{\partial \phi_D}{\partial X} \right) = 0; Z = 0 \]

The same condition is satisfied by \( \phi_I \)
\[ \left(i \omega - \frac{\partial}{\partial X} \right)^2 \phi_I + g \frac{\partial \phi_I}{\partial X} = 0; Z = 0 \]

Therefore, the heave and pitch exciting forces follow by simply pressure integration as follows:
\[ X_i(t) = \text{Re} \left[ \sum_{i=3,5} \int_{C_i} \phi_i \right] \]
\[ X_3 = \rho g \int_{L_w} dX e^{-iKX \cos \beta} \int_{C_3} (\psi_i + \psi_J) n_3 d\ell \]
\[ X_5 = \rho g \int_{L_w} dX (-X) e^{-iKX \cos \beta} \int_{C_3} (\psi_i + \psi_J) n_3 d\ell \]
where the approximation \( n_5 = -X n_3 \)

Finally the coupled heave and pitch equation can be expressed by:
\[ \sum_{j=3,5} \left( (M_{kj} + M_{kj}) \ddot{x}_j + B_{kj} \dot{x}_j + C_{kj} x_j \right) = F_k \]

3. Result And Discussion

Here, the new Strip Theory, the developed method, was applied to heave and pitch motions of the Ferry in heading waves. The main dimensions and body lines plan are presented in Table 1 and Figure 2, respectively. For the wave conditions, the ship is simulated into two wave height cases and 16 wave length cases. The wave heights \( H_w \) are 1m and 4m based on actual conditions where the conditions were experienced by the ship. Then, the wave length \( \lambda \) is defined by the water line length of the ship \( L_w \) because it is consider that the wave length is relatively to the ship length. Therefore, the wave lengths are from 0.5\( L_w \) to 2.0\( L_w \). The ship hull is divided into 20 sections.
acceptable displacement because by referring some research results, small wave length impact generates low motion response. The overall amplitude were obtained and then used to analyze to interpret the ship behavior using nondimensional parameter.

In the field of ship design, nondimensional parameter of ship response is used to determine the behavior of a ship when moving at sea state. In addition, this parameter can be used to interpret safety reason to allow modify ship design. Nondimensional heave motion $\xi_3^*$ and pitch motion $\xi_5^*$ are defined by motion amplitude and wave height. The equations are given as follows:

$$\xi_3^* = \frac{\xi_3}{H_w} \quad (39)$$
$$\xi_5^* = \frac{\xi_5}{(H_w k)} \quad (40)$$

where, $k$ is wave number.

<table>
<thead>
<tr>
<th>Table 1. The Ferry’s main dimensions</th>
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<tbody>
<tr>
<td>Loa (m)</td>
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<td>Lwl (m)</td>
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<td>B (m)</td>
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<tr>
<td>H (m)</td>
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<td>D (m)</td>
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<tr>
<td>U (m/s)</td>
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Figures 5a,b show the nondimensional heave motions over the wave length in both wave height cases. From those figures, the heave motions increase initially when $0.5Lwl$ and then decrease until $0.8Lwl$. The heave motions gradually increase when wave length greater than $0.9Lwl$ in increasing wave length. The tendency is similar for the both cases and the response displacement increases linear from 1m to 4m wave length. This means that any developed Strip Theory produces linear result. The maximum heave amplitude is in $2.0Lwl$ that are 0.5628m in 1m wave length and 2.2517m in 4m.

Fig. 3 Time history of heave amplitude
In additions, figures 6a,b show nondimensional pitch motions. The tendencies of pitch motions are the same with heave motions for both cases. However, the pitch motions decrease small in $0.5L_w$ and then increase gradually when greater than $0.6L_w$ in increasing wave length. The maximum heave amplitude is given in $2.0L_w$ where $0.0842\text{rad.}$ for 1m wave length and $0.3187\text{rad.}$ for 4m.

Furthermore, the present results are compared with CFD results.
4. Conclusion

The strip theory is developed by considering nonlinear wave effects. It is capable to compute in predicting heave and pitch motions of a ship. The developed strip theory was applied successfully to the Ferry. The present results indicate acceptable and quite accurate. In order to increase the useful of this developed strip theory it would be applied in many kinds of ship types. Then, the developed strip will be enhanced to handle other ship motions 6DOF. However, this developed method will be modified perfectly more to become robust tool. The comparison results between the present results and CFD results show quite a little different. Therefore, we have some efforts to perform experiment in our future works to validate the present results.

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