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COUPLING OF BOUSSINESQ AND SEDIMENT TRANSPORT MODEL IN A WAVE FLUME

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Boussinesq type equation has been coupled with sediment transport model to simulate sediment transport in a wave flume. A new eddy viscosity model has been applied to calculate wave decay as well as suspended sediment concentration. A bed load transport formula based on an energetic transport of Bagnold-type model combined with suspended load model was validated under the condition of a spilling wave. The applicability of both ε_B and c_f has been investigated. The result indicated that two sets of parameters $c_f = 0.017$ with $\varepsilon_B = 0.21$ and $c_f = 0.003$ with $\varepsilon_B = 1.03$ calculated a similar bed level change. Comparison of calculated and measured bed level change is fairly good in offshore and near breaking point. However, the model cannot predict accretion in swash zone.

Key Words : *Bed-load transport, bed level change, suspended load transport.*

1. INTRODUCTION

Sediment transport in the surf zone is a complicated natural process. The wave-related transport is herein defined as the transport of sand particles by the oscillating (orbital) fluid motion due to high-frequency waves. Sediment transport occurs in two main modes: bed load and suspended load. The bed load is the part of the total load which is travelling immediately above the bed and is supported by inter-granular collisions rather than fluid turbulence, while the suspended load is moving in suspension without continuous contact with the bed as a result of agitation of fluid turbulence.

Advanced prediction of time-domain modeling of waves across the surf zone enabled us to obtain improved estimates of water velocities. Since this phenomenon influences the sediment transport, the accuracy of prediction is very important.

Van Rijn[1)] proposed a wave-averaged model for sediment transport in an oscillatory flow. However, this approach is not adequate due to the dominant role of the time-dependent oscillatory orbital motion near the sea bed, induced by the short wave[2)]. Time dependent sediment transport can be evaluated by considering the instantaneous velocity and sediment concentration.

Eddy viscosity is an important parameter to evaluate wave decay, water velocity as well as sediment concentration distribution. Rahman[3)] has proposed an eddy viscosity model to simulate these wave phenomena. This model has been evaluated to calculate sediment distribution in a wave flume. However, net sediment transport evaluated by two models (suspended load and bed-load) can not produce erosion in surf zone. In this study, further investigation on choosing the appropriate value of the important parameter will be discussed. A coupling of the Boussinesq model[4)] and a sediment transport model is developed in order to predict cross-shore sediment transport in a wave flume. The total sediment transport is obtained as the sum of the net bed load and suspended load. Morphological change due to suspended load will be evaluated by using an erosion-deposition mechanism.

2. MODEL DEVELOPMENT

(1) Wave model

Kirby[5] developed FUNWAVE model to simulate nonlinear wave propagation based upon a multi-layer approach. The extended Boussinesq equations[4], solved in this model are recovered by continuity and momentum equations. The fully nonlinear Boussinesq equations solve the surface elevation η and the velocity field **U** evaluated at some reference elevations. The formula of continuity and momentum equations is written as:

$$\eta_t + \nabla \cdot \mathbf{M} = 0 \tag{1}$$

$$\mathbf{U}_t = -g\nabla\eta - \frac{\delta}{2}\nabla\left(|\tilde{\mathbf{u}}|^2\right) + \mathbf{\Gamma}_1 + \mathbf{\Gamma}_2 \tag{2}$$

where: g is the gravitational acceleration, **M** is the depth-integrated volume flux, δ denotes a height-to-depth ratio characterizing nonlinear effects, $\tilde{\mathbf{u}}$ is the velocity vector obtained in the two Boussinesq reference levels, both $\mathbf{\Gamma}_1$ and $\mathbf{\Gamma}_2$ are the dispersive Boussinesq terms. The detail explanation of these variables can be found in Gobbi[4]. For the first step in the present study, the terms of $O(\mu^4)$ and higher are neglected.

This set of Boussinesq model does not include wave breaking phenomenon. To accommodate this, some researchers proposed their methods to incorporate wave breaking by adding breaking term R_b in momentum equation with a function of artificial viscosity (v_a). The relationship between artificial viscosity and eddy viscosity (v_t) has been proposed by Rahman[3], considering the Reynolds stress equation, by the following formula:

$$v_a = \left(\frac{L}{h}\right)^2 v_t \tag{3}$$

where h is the still water depth. The modified R_b equation is written as:

$$R_b = \frac{1}{h+\eta}\left(\left(\frac{L}{h}\right)^2 v_t\left((h+\eta)u_a\right)_x\right)_x \tag{4}$$

where L is the wave length and u_a is the depth averaged horizontal velocity.

Rahman[3] proposed v_t model by considering the definition of v_a proposed by Kennedy (2001). In Kennedy's model, v_a depends on the velocity of water surface fluctuation $\left(\frac{\partial \eta}{\partial t}\right)$. While in Rahman's model, the contribution of horizontal velocity has been included in the model. When run up process occurred, the horizontal velocity dominant to produce turbulence than the vertical velocity, so the eddy viscosity model was formulated as the following equation:

$$v_t = C\sqrt{\left[(d - d_{br})u\right]^2 + \left[d\, w\right]^2} \tag{5}$$

where C is a constant, d is the total water depth, d_{br} is total water depth at the initiation of wave breaking, u and w are the horizontal and vertical velocity at the water surface, respectively. Vertical velocity is calculated as follow[5]:

$$w(x,z,t) = -\mu^2\left(F_{21} + 2\xi F_{22}\right) \tag{6}$$

where ξ is the distance from the bottom and dispersion (μ) is obtained as follow:

$$\mu = k_0 h_0 \tag{7}$$

where k_0 and h_0 are wave number and still water depth in constant water depth, respectively. Two terms of F_{21} and F_{22} are variables of the second order accuracy $O(\mu^2)$ of the model. Detail explanation of F_{21} and F_{22} can be found in Gobbi[4].

(2) Sediment transport model

The mechanism of sediment transport can be divided into two components, wave-related transport rate and current-related transport rate. The former term is assumed as the form of suspended load transport due to the presence of wave. In the study of suspended load transport, the predominant factors should be considered to be the sediment diffusion, the settling velocity, the bed condition and so on. Relationship between some of these variables is presented in the classical convection-diffusion equation to compute the equilibrium concentration profile as follow[1]:

$$w_s c + \varepsilon \frac{dc}{sz} = 0 \tag{8}$$

where w_s is settling velocity, ε is sediment diffusion coefficient assumed to be the same as the flow eddy viscosity v_t[7] and c is the sediment concentration.

The second mode of transport is due to wave current. In this study we apply the Bagnold-type sediment transport model developed by Long[8] with the following formula:

$$\mathbf{q}_B = \frac{\rho}{g(\rho_s - \rho)} c_f \frac{\varepsilon_B}{\tan\phi}\left[|\mathbf{u}_b|^2\overline{\mathbf{u}}_b - \frac{\tan\beta}{\tan\phi}|\mathbf{u}_b|^3\right] + \frac{\rho}{g(\rho_s - \rho)} c_f \frac{\varepsilon_S}{w_s}\left[|\mathbf{u}_b|^3\overline{\mathbf{u}}_b - \frac{\varepsilon_S \tan\beta}{w_s}|\mathbf{u}_b|^5\right] \tag{9}$$

where ρ is the water density, c_f is the bottom friction coefficient, ε_B is bedload efficiency, ε_S is suspended load efficiency, $\mathbf{u}_b$ is the instantaneous bottom velocity, the over bar means time averaged, tan β is the bottom slope, and ϕ is the particle friction angle.

Table 1 shows proposed value of c_f and ε_B by some researchers. Bagnold[10] suggested that ε_B should be less than one. However, Drake[11] found this value exceeds one. Van der Molen[12] replaced c_f

Table 1 Important parameter of bed load transport.

Author	c_f	tan ϕ	ε_B
Bailard[9]	0.012~0.017	0.63	0.21
van der Molen[12]	-	0.63	0.1
Gallagher[15]	0.003	0.63	0.135
Drake[11]	0.003	0.47	1.03

with a friction factor for combined currents and wave.

(3) Bed level elevation

Since sediment transport is divided into two transport rate terms, suspended load and bed load, here bed level changes are also split into two terms. Bed level (z_b) change due to suspended load transport is calculated by using erosion (E_a) and deposition rate (D_a) as the following equation[13]:

$$\left(\frac{\partial z_b}{\partial t}\right)_s = -\frac{1}{1-n_p}\left(D_a - E_a\right) \tag{10}$$

where n_p is the bed porosity. E_a and D_a are influenced by settling velocity (w_s) by the equations:

$$E_a = C_a w_s \tag{11}$$

$$D_a = C_b w_s \tag{12}$$

where C_a is the sediment concentration at the reference level[1], C_b is the sediment concentration at the bottom evaluated by using convection-diffusion sediment continuity equation.

Bed level change due to bed load transport rate is given by[14]:

$$\left(\frac{\partial z_b}{\partial t}\right)_b = -\frac{1}{1-n_p}\nabla \cdot \mathbf{q}_B \tag{13}$$

3. COMPUTATIONAL MODEL SETUP

We shall now test our model discussed in the previous section. A series of laboratory experiment data is used to test the applicability of the eddy viscosity model. Beaches with uniform slope (1 : m) are connected to a region with constant depth (h). Some important parameters are tabulated in **Table 2**. Wave breaking criteria, spilling and plunging wave conditions (case 1 and 2) collected by Ting and Kirby[16], were used to validate the eddy viscosity model. While case 3, collected by Ikeno[18], was used to validate the sediment transport model.

The origin is set on the slope where the still water depth over toe of bottom slopes as show in **Fig.1**. The internal wave-maker is located two wavelength

Table 2 Selected experiment data.

Case	h (m)	H_0 (m)	T (s)	1 : m	Breaker type
1	0.4	0.125	2.0	1:35	Spilling
2	0.4	0.128	5.0	1:35	Plunging
3	4.0	1.0	5.0	1:20	Spilling

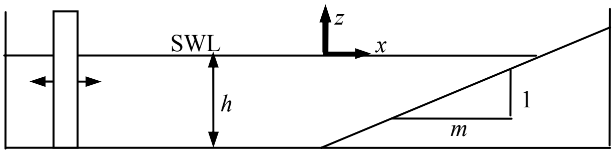

Fig.1 Computation domain

in front of the bottom slope. The left boundary is made to be a radiation boundary that is behind an artificial sponge layer with a length of 1.5L. Grid size (Δx) and time step (Δt) are set to meet stability computation condition with the Courant number not exceed 0.4. The simulations are set to run for 40 wave cycles with the assumption that the wave has reached quasi-steady state motion.

4. RESULT AND DISCUSSION

In this section we examine the applicability of a new eddy viscosity model proposed by Rahman[3] under the condition of spilling and plunging breaking. Eddy viscosity cannot be observed directly in laboratory; therefore validation is conducted by comparing the data of RANS-VOF model[19] for the case 1 and 2 in **Table 2**.

Further discussion will deal with the performance of combined suspended load and bed load transport formula.

(1) Model applicability for spilling breaker case

Fig.4 shows the Evaluation of eddy viscosity. This figure indicates that the calculated eddy viscosity is comparable to the RANS-VOF data in surf zone. However, the discrepancy is occurred when approaching the shore line until the swash zone.

Fig.5 shows the comparison of cross-shore variations wave height. New model predicted the breaking point later than laboratory data. However, the wave height is reasonable accepted from the offshore until the measured breaking point. The discrepancy of wave height in surf zone due to the under predicted of eddy viscosity in this area.

(2) Model applicability for plunging breaker case

Fig.6 plots the calculated time-averaged eddy viscosity compared to RANS-VOF data. Present model over estimated eddy viscosity in two locations, after the measured breaking point and along the swash zone. Vertical profile of velocity is underestimated by proposed model. Therefore, higher velocity in bore cannot be produced well. Since eddy viscosity depends on water particle velocity, the eddy viscosity should be underestimated on this condition.

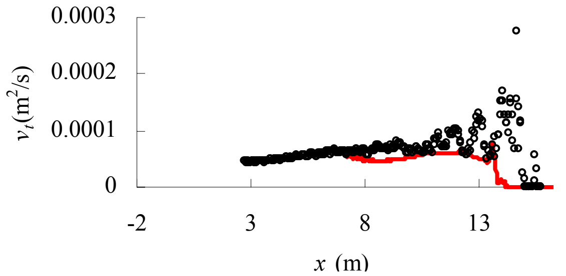

Fig.4 Time-averaged of eddy viscosity for spilling breaker. Line: calculated; circle: RANS-VOF model[19)]

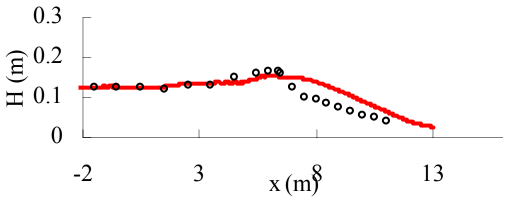

Fig.5 Wave decay in surf zone for spilling breaker Black line: calculated; gray line: measured[16)]

The initiation of breaking wave is earlier than measured data, as shown in **Fig.7**. However, calculated wave height in surf zone is comparable to the measured data.

(3) Sediment transport

Validation of sediment transport model is conducted against laboratory experiment data, case 3 in **Table 2**. Comparison of calculated and measured wave height is shown in **Fig. 8**. This figure also shows location of measurement points. This study investigated the sediment transport in a large wave flume with the grain size (d_{50}) equal to 1.0 mm. Settling velocity is calculated by using formula proposed by Ahrens[20)].

Comparison of vertical distribution of both velocity and sediment concentration have been

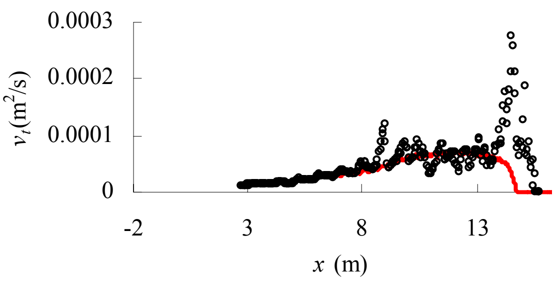

Fig.6 Time-averaged of eddy viscosity for plunging breaker. Red line: calculated; circle: RANS-VOF[19)]

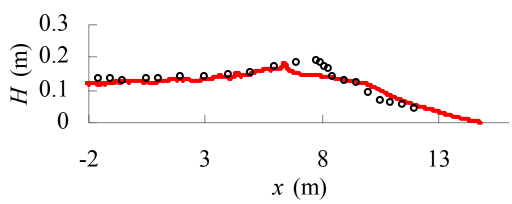

Fig.7 Wave decay in surf zone for plunging breaker. Red line: calculated; circle: measured[16)]

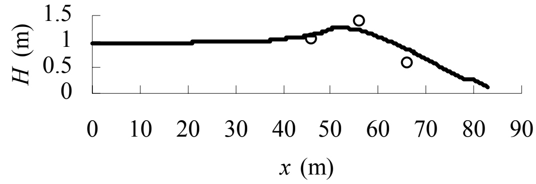

Fig.8 Wave height distribution. Line: calculated; circle: measured[18)]

discussed in Rahman[21)]. It has been concluded that the proposed eddy viscosity model is applicable to calculate sediment concentration over the water depth. These two distributions are used to evaluate vertical distribution of sediment flux in corresponding point as shown in **Fig.9**. The higher sediment flux is occurred at x = 56 m. This is due to the high turbulence produces by wave breaking. Both measured and calculated show that directions of sediment fluxes near bottom are seaward, produce by undertow at these three points as shown in **Fig. 10**.

Four sets of bed load transport parameters shown in **Table 1** have been evaluated to calculate bed level change. **Fig. 11** shows that although ε_B exceeds one[11)], it can produce bed level change similar to that by using the parameters proposed by Bailard[9)]. Parameters proposed by van der Molen[12)] calculated very high bed level change, while Gallagher's[15)] parameter produced relatively small bed level change.

Fig.12 shows the proportion of 14 hours bed level change due to suspended load transport rate, bed load transport rate using parameters proposed by Bailard[9)] and total sediment transport rate. Suspended load component is the dominant factor around the breaking point eroding an accumulation of sand bed to the shoreward direction. While in surf zone to swash zone, bedload component erodes sand material seaward. In surf zone, bedload transport becomes the dominant transport. However, it is not adequate to transport sediment landward in swash zone. The discrepancy in this area because of the infiltration-exfiltration during run up and rundown processes haven't been considered in this model. The infiltration can make the settling velocity higher producing accretion in swash zone as observed in laboratory shown in **Fig.12**. Turner[22)] found that swash infiltration-exfiltration across a beach face enhances the net upslope transport of sediment. By introducing the acceleration during uprush dan backwash into the Bagnold-type sediment transport model, Puleo found that the modified model predicted essentially maximum suspended sediment transport rate during uprush. If the model can predict accretion in swash zone well, more erosion in surf zone can be expected. **Fig.12** also shows that for bed load transport, the effect of return flow play an important role to transport sand bed seaward direction.

5. CONCLUSIONS

A new eddy viscosity model has been applied to calculate wave decay in surf zone. Generally, the model calculated eddy viscosity with a reasonable value compare to RANS-VOF model output data in surf zone for spilling and plunging breaker case. The discrepancy may be improved by improving vertical profile of particle velocity. Calculated wave heights are fairly good for both spilling and plunging breaker before wave breaking, but still overestimated in surf zone, especially for spilling breaker.

A coupling of wave model and sediment transport model has been conducted to assess bed level change in a wave flume. The applicability of both ε_B and c_f has been investigated. The result indicated that two sets of parameters $c_f = 0.017$ with $\varepsilon_B = 0.21$ and $c_f = 0.003$ with $\varepsilon_B = 1.03$ calculated a similar bed level change. Very small bed level change was obtained for $c_f = 0.003$ with $\varepsilon_B = 0.135$. While formulated c_f with $\varepsilon_B = 0.1$ obtained very high sediment erosion in surf zone and accretion in offshore. Sediment transport due to wave motion is the dominant factor near the breaking point, while bed load transport dominant in surf zone. Proposed model produced significant erosion before and after breaking point. The discrepancy in swash zone may be reduced by including the effect of infiltration-exfiltration in this area.

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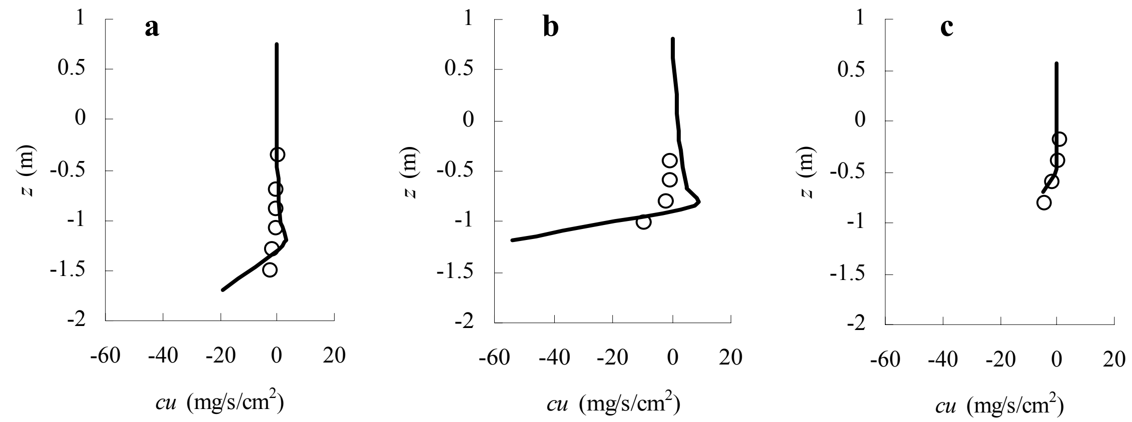

Fig.9 Time-averaged sediment flux at x = 46 m (a), x = 56 m (b) and x = 66 m (c).
Line: present model; circle: calculated[18)]

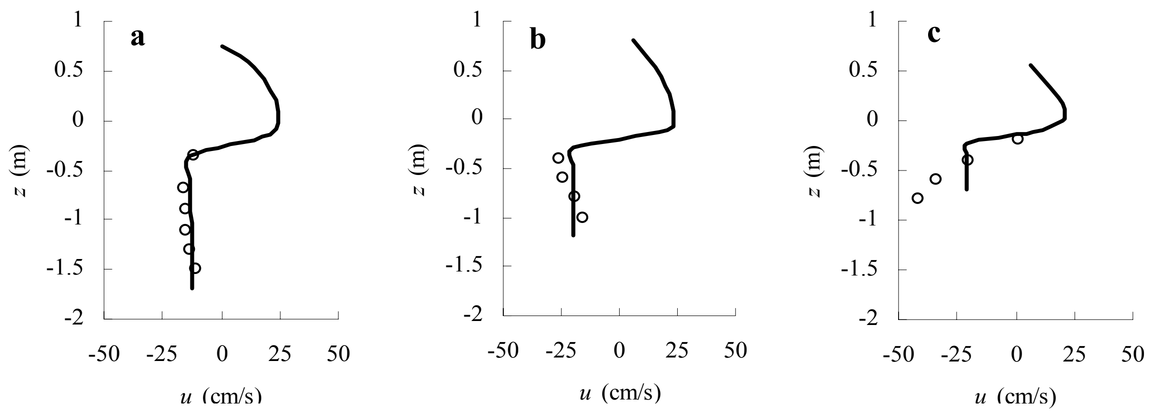

Fig.10 Time-averaged horizontal velocity at x = 46 m (a), x = 56 m (b) and x = 66 m (c).
Line: present model; circle: measured[18)]

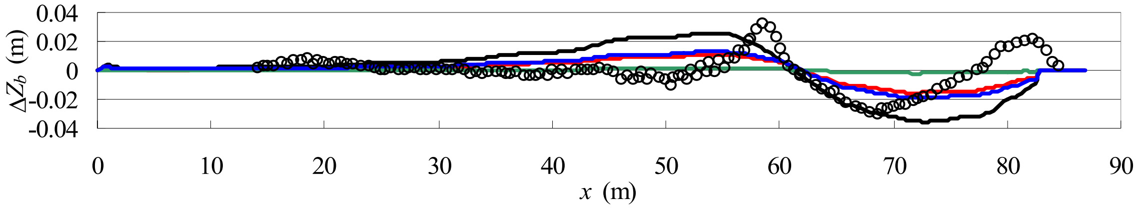

Fig.11 Comparison of the performance of bed load transport parameters after 1 hour simulation. Black line: van der Molen[12)]; Blue line: Drake[11)]; red line: Bailard[9)]; green line: Gallagher[15)]; circle: measured[18)].

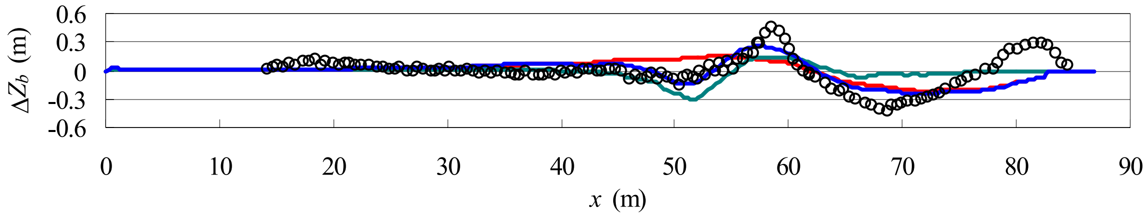

Fig.12 Calculated bed level change due to bed load (red line), suspended load (green line), total sediment transport (blue line) and measured[18)] (circle) after 14.0 hours.

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