

ONE DIMENSIONAL NUMERICAL MODELING OF POINT SOURCE POLLUTANT DISTRIBUTION IN ESTUARINE RIVER BASIN

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ABSTRACT: Pollutant is product that produced from every activities, both natural and human-made. In many case, mostly, pollutant brings a negative effect to the environment. It comes to the environment in many ways. This paper would be modelled a point source pollution in river in one dimensional numerical modelling to orient hydrological model by using explicit and implicit scheme. In the end, the concentration and spreadness in the river would be known. It would be modelled in three case condition which is has the same initial and boundary condition. Those cases give a different value of velocity of river and reaction term per day. The point source pollution located one of third of the river length. From the general form of the convective diffusion, this numerical modelling neglected the area because it assumed that the area is linear. To determine the constant of α , β , γ variables can be simplified in to the matrix simultaneous equations. The output gives the graph time and concentration. This paper brings the similar result between hypothesis and the numerical modelling.

Key words: Point source pollutant, estuarine, river basin, nutrient

INTRODUCTION

All activities on Earth, both natural processes and human-made processes, produce some type of byproduct from that activity. Under normal conditions these byproducts, some known as pollutants, are returned back into the environment. In fact, natural environmental processes have the ability to assimilate some pollutants and correct most imbalances if given enough time

Pollution originating from a single identifiable source, such as a discharge pipe from a factory or sewage plant is called point-source pollution. Liquid, solid, and airborne discharges from point sources may go either into surface water or into the ground (Biegel, 2005). The ability for these pollutants to reach surface water or groundwater is enhanced by the amount of water available from precipitation (rain) or irrigation (Arnold et al., 1998).

The raw materials and wastes may include pollutants such as solvents, petroleum products (such as oil and gasoline), or heavy metals. Point sources of pollution from agriculture may include animal feeding operations, animal waste treatment lagoons, or storage, handling, mixing, and cleaning areas for pesticides, fertilizers, and petroleum (Pohlert, et al., 2005).

The effects of point-source pollutants in river basin are temperature will increases and nutrients can result in excessive plant growth and subsequent decaying organic matter in water that depletes dissolved oxygen levels and consequently stressing or killing vulnerable aquatic life (Ganoulis et al., 2005). Microorganisms can be hazardous to both human health and aquatic life. Pesticides and other toxic substances can also be hazardous to both human health and aquatic life, but are

less commonly found in surface water because of high dilution rates.

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For all of these activities, hazardous materials may be included in the raw materials used in the process as well as in the waste stream for the facility. If the facility or operator does not handle, store, and dispose of the raw materials and wastes properly, these pollutants could end up in the water supply. This may occur through discharges at the end of a pipe to surface water, discharges on the ground that move through the ground with infiltrating rainwater, or direct discharges beneath the ground surface (Zhang, et al., 2002).

OBJECTIVE

In this paper, the objective is to model one dimensional numerical problem of point source pollution in river by using both explicit and implicit scheme and compare with hypothesis solution. It is a continuous time model which simulates both the water balance and the nutrient cycle with time step for the appropriate environmental management of a river basin. In this study a revised model was established by integrating point

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source into one-dimensional model on the basis of real-time hydrologic data

MODEL FORMULATION

Water quality models are based on the physical and biochemical mass balance of the water system considered. In this case, the river would be modelled in one dimensional base on the time step with the total length of the river is 750 m. The total length of the river would be divided by 150 grid (1 grid = 5 m) and the point source pollution is located 250 m (50 grid) from the upstream (Figure 1). The point source pollution is assumed constant source base on the time.

The general form of the convective diffusion describing the estuary situation:

$$\frac{\partial AC}{\partial \tau} = \frac{-\partial AUC}{\partial x} + \frac{\partial}{\partial x} \left(EA \frac{\partial C}{\partial x} \right) - KAC \quad (1)$$

With, initial condition $C(t, x) = C(0, 250) = C_0$,

Boundary condition

$$C(t, x) = C(t, 0) = C(t, 750) = 0$$

$$C(t, x) = C(t, 250) = C_0$$

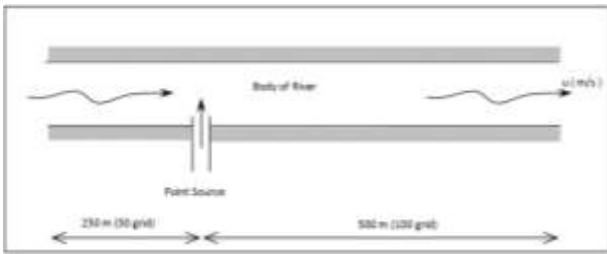


Figure 1. Model of case

Cases of simulation

In this modeling, the model would be simulated by three cases:

- i. $K > 0, u = 0$ m/s
- ii. $K > 0, u > 0$ m/s
- iii. $K = 0, u > 0$ m/s

where,

u = velocity of river flow (m/s)

K = reaction term (1/day)

According to the three case of simulation, the concentration of pollutant in river (depend u and k) could be estimated (Chapra, 2003), as in **Figure 2, Figure 3, Figure 4**:

- i. $K > 0, u = 0$ m/s

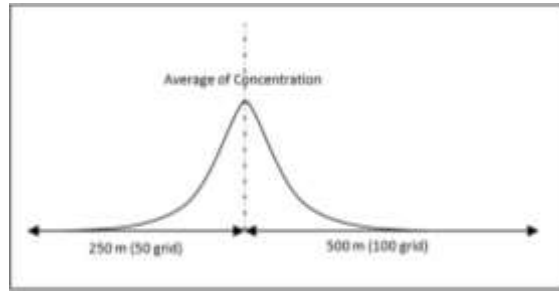


Figure 2. Hypothesis for case $K > 0, u = 0$ m/s

- ii. $K > 0, u > 0$ m/s

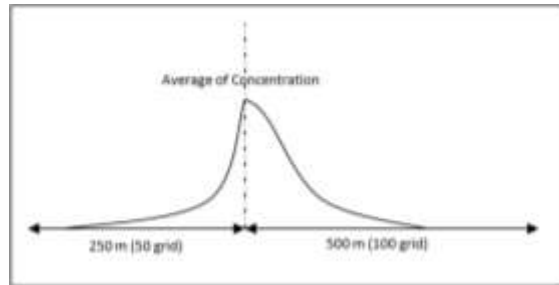


Figure 3. Hypothesis for case $K > 0, u > 0$ m/s

- iii. $K = 0, u > 0$ m/s

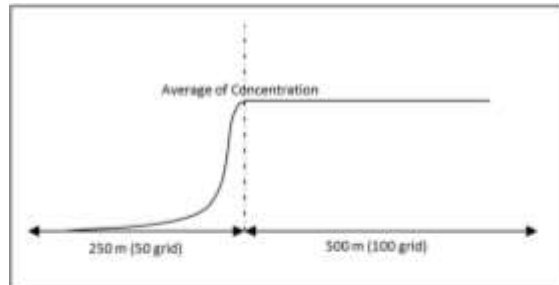


Figure 4. Hypothesis for case $K = 0, u > 0$ m/s

NUMERICAL METHOD

Explicit

The general form of the convective diffusion describing the estuary situation is

$$E \frac{\partial^2 C}{\partial x^2} - U \frac{\partial C}{\partial x} - KC - \frac{\partial C}{\partial t} = 0 \quad (2)$$

Each term in (2) is represented separately below by its difference approximation:

- a. Time derivative

$$\frac{\partial C}{\partial t} \approx \left(\frac{C_i^{j+1} - C_i^j}{k} \right) \quad (3)$$

- b. Advective term

$$U \frac{\partial C}{\partial x} \approx U \left(\frac{C_{i+1}^j - C_i^j}{h} \right) \quad (4)$$

- c. Dispersive term

$$E \frac{\partial^2 C}{\partial x^2} \cong E \left(\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{h^2} \right) \quad (5)$$

d. Reaction term

$$\kappa C \cong \kappa (C_i^j) \quad (6)$$

Re-arranging Equation (3) to (6) by substitution to Equation (2) will be produced with one unknown on right hand side at the j+1 time step and three known values on the left hand side at the j time step, i.e.

$$\begin{aligned} \frac{E}{h^2} k (C_{i-1}^j) + \left(\frac{uk}{h} - \frac{2Ek}{h^2} - \kappa k + 1 \right) C_i^j + \\ \left(\frac{Ek}{h^2} - \frac{uk}{h} \right) C_{i+1}^j = C_i^{j+1} \end{aligned} \quad (7)$$

In equation (7), the value of $\left(\frac{uk}{h} - \frac{2Ek}{h^2} - \kappa k + 1 \right)$ and $\left(\frac{Ek}{h^2} - \frac{uk}{h} \right)$ must be greater than 0.

Implicit

Algorithm (named after Llewellyn Thomas), is a simplified form of Gaussian elimination that can be used to solve tridiagonal systems of equations. A tridiagonal system for n unknowns may be written as in numerical linear algebra, the tridiagonal matrix algorithm (TDMA), also known as the Thomas,

$$a_i x_{i-1} + b_i x_i + c_i x_{i+1} = d_i \quad (8)$$

Where $a_i = 0$ and $c_n = 0$. In matrix form, this system is written as

$$\begin{bmatrix} b_1 & c_1 & & & 0 \\ a_2 & b_2 & c_2 & & \\ & a_3 & b_3 & \ddots & \\ & & \ddots & \ddots & c_{n-1} \\ 0 & & & a_n & b_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \\ \vdots \\ d_n \end{bmatrix} \quad (9)$$

For such systems, the solution can be obtained in $O(n)$ operations instead of $O(n^3)$ required by Gaussian elimination. A first sweep eliminates the a_i 's, and then an (abbreviated) backward substitution produces the solution. Examples of such matrices commonly arise from the discretization of 1D Poisson equation (e.g., the 1D diffusion problem) and natural cubic spline interpolation.

The first step consists of modifying the coefficients as follows, denoting the new modified coefficients with primes:

$$c'_i = \begin{cases} \frac{c_1}{b_1} & ; i = 1 \\ \frac{c_i}{b_i - c'_{i-1} a_i} & ; i = 2, 3, \dots, n-1 \end{cases} \quad (10)$$

and

$$d'_i = \begin{cases} \frac{d_1}{b_1} & ; i = 1 \\ \frac{d_i - d'_{i-1} a_i}{b_i - c'_{i-1} a_i} & ; i = 2, 3, \dots, n. \end{cases} \quad (11)$$

This is the forward sweep. The solution is then obtained by back substitution:

$$x_n = d'_n \quad (12)$$

$$x_i = d'_i - c'_i x_{i+1} \quad ; i = n-1, n-2, \dots, 1. \quad (13)$$

Discretisation

The general form of the convective diffusion describing the estuary situation is

$$E \frac{\partial^2 C}{\partial x^2} - U \frac{\partial C}{\partial x} - KC - \frac{\partial C}{\partial t} = 0 \quad (14)$$

verify using finite difference techniques that

is solution for Equation (14).

Solution

Each term in (1) is represented separately below by its difference approximation :

a. Time derivative

$$\frac{\partial C}{\partial t} \cong \frac{1}{6} \left(\frac{C_{i-1}^{j+1} - C_{i-1}^j}{k} \right) + \frac{2}{3} \left(\frac{C_i^{j+1} - C_i^j}{k} \right) + \frac{1}{6} \left(\frac{C_{i+1}^{j+1} - C_{i+1}^j}{k} \right) \quad (15)$$

b. Advective term

$$U \frac{\partial C}{\partial x} \cong \frac{U}{2} \left(\frac{C_{i+1}^{j+1} - C_{i-1}^{j+1}}{2h} \right) + \frac{U}{2} \left(\frac{C_{i+1}^j - C_{i-1}^j}{2h} \right) \quad (16)$$

c. Dispersive term

$$E \frac{\partial^2 C}{\partial x^2} \cong \frac{E}{2} \left(\frac{C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}}{h^2} \right) + \frac{E}{2} \left(\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{h^2} \right) \quad (17)$$

d. Reaction term

$$\kappa C \cong \frac{\kappa}{2} (C_i^{j+1}) + \frac{\kappa}{2} (C_i^j) \quad (18)$$

Re-arranging equation (15) until (18) by combining terms an equation may be produced with three unknowns on left hand side at the j+1 time step and known values on the right hand side at the j time step, i.e.

$$\begin{aligned} \frac{E}{2} \left(\frac{C_{i+1}^{j+1} - 2C_i^{j+1} + C_{i-1}^{j+1}}{h^2} \right) - \frac{U}{2} \left(\frac{C_{i+1}^{j+1} - C_{i-1}^{j+1}}{2h} \right) - \frac{\kappa}{2} (C_i^{j+1}) - \\ \frac{1}{6} \left(\frac{C_{i-1}^{j+1}}{k} \right) - \frac{2}{3} \left(\frac{C_i^{j+1}}{k} \right) - \frac{1}{6} \left(\frac{C_{i+1}^{j+1}}{k} \right) = - \frac{E}{2} \left(\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{h^2} \right) + \\ \frac{U}{2} \left(\frac{C_{i+1}^j - C_{i-1}^j}{2h} \right) + \frac{\kappa}{2} (C_i^j) - \frac{1}{6} \left(\frac{C_{i-1}^j}{k} \right) - \frac{2}{3} \left(\frac{C_i^j}{k} \right) - \frac{1}{6} \left(\frac{C_{i+1}^j}{k} \right) \end{aligned} \quad (19)$$

Equation (19) may be written as

$$\alpha_i^{j+1}C_{i-1}^{j+1} + \beta_i^{j+1}C_i^{j+1} + \gamma_i^{j+1}C_{i+1}^{j+1} = \delta_i^j(20)$$

Where :

$$\alpha_i^{j+1} = \frac{E}{2h^2} + \frac{U}{4h} - \frac{1}{6k}$$

$$\beta_i^{j+1} = -2\frac{E}{2h^2} - \frac{\kappa}{2} - \frac{2}{3k}$$

$$\gamma_i^{j+1} = \frac{E}{2h^2} - \frac{U}{4h} - \frac{1}{6k}$$

$$\delta_i^j = -\frac{E}{2}\left(\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{h^2}\right) + \frac{U}{2}\left(\frac{C_{i+1}^j - C_{i-1}^j}{2h}\right) + \frac{\kappa}{2}(C_i^j)$$

If j=1, Equation (20) becomes for i=2

$$\beta_2^2 C_2^2 + \gamma_2^2 C_3^2 = \delta_2^1 - \alpha_2^2 C_1^2 \quad (21)$$

for i=3

$$\alpha_3^2 C_2^2 + \beta_3^2 C_3^2 + \gamma_3^2 C_4^2 = \delta_3^1 \quad (22)$$

for i=n-1

$$\alpha_{n-1}^2 C_{n-2}^2 + \beta_{n-1}^2 C_{n-1}^2 + \gamma_{n-1}^2 C_n^2 = \delta_{n-1}^1 \quad (23)$$

for i=n

$$\alpha_n^2 C_{n-1}^2 + \beta_n^2 C_n^2 = \delta_n^1 - \gamma_n^2 C_{n+1}^2 \quad (24)$$

Equation (20) to (24) may be written as a system of n-1 linier algebraic equations where $1 < i < n-1$ and C_1 and C_{n+1} are known from boundary condition, i.e.

$$\begin{bmatrix} \alpha_2 & \beta_2 & \gamma_2 & & & & \\ & \alpha_3 & \beta_3 & \gamma_3 & & & \\ & & \dots & \dots & \dots & & \\ & & & \alpha_{n-1} & \beta_{n-1} & \gamma_{n-1} & \\ & & & & \alpha_n & \beta_n & \gamma_n \end{bmatrix} \begin{bmatrix} C_2 \\ C_3 \\ \vdots \\ C_{n-1} \\ C_n \end{bmatrix} = \begin{bmatrix} \delta_2 \\ \delta_3 \\ \vdots \\ \delta_{n-1} \\ \delta_n \end{bmatrix} \quad (25)$$

The left hand side of Equation (25) is a matrix for unknown values at j+1 time step and the right hand side is known values at j time step, so Equation (25) is numerical solution for equation (2).

ALGORITHM

The algorithm for solving the solution both explicit method and implicit method are show in Fig 5.

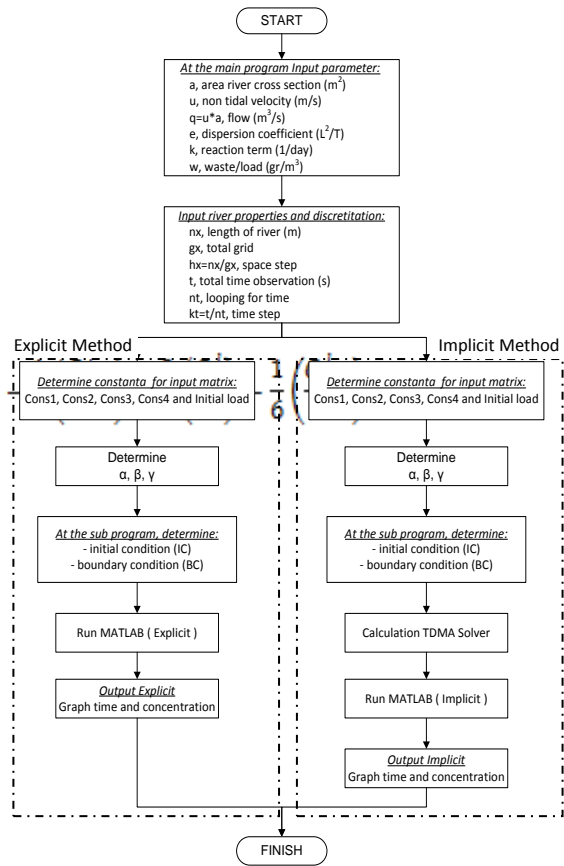


Fig 5. Algorithm For Explicit and Implicit Method

INPUT DATA

Input data for MATLAB program either in case 1, case 2, case 3, were given in **Table 1, Table 2, Table 3, Table 4, Table 5, and Table 6.**

Case 1 ($K > 0, u = 0$ m/s)

Table 1 Parameter Input for Case 1

parameter input	value	unit
a	150	m ²
u	0.0001	m/s
e	5	L ² /T
q	0.015	m ³ /s
k	0.1	1/day
w	100	gr/m ³

Table 2 River Properties for case 1

input river properties and discretitation	value
nx	250
gx	50
hx	5
t	50
nt	50
kt	1

Case 2 ($K > 0, u > 0$ m/s)

Table 3 Parameter Input for Case 2

parameter input	value	unit
a	150	m ²
u	1	m/s
e	5	L ² /T
q	150	m ³ /s
k	0.1	1/day
w	100	gr/m ³

Table 4 River Properties for case 2

input river properties and discretitation	value
nx	250
gx	50
hx	5
t	50
nt	50
kt	1

Case 3 ($K = 0, u > 0$ m/s)

Table 5 Parameter Input for Case 3

Table 6 River Properties for case 3

parameter input			input river properties and discretitation	
a	150	m ²	nx	250
u	1	m/s	gx	50
e	5	L ² /T	hx	5
q	150	m ³ /s	t	50
k	0	1/day	nt	50
w	100	gr/m ³	kt	1

Where,

- a, area river cross section (m²)
- u, non tidal velocity (m/s)
- q = u*a, flow (m³/s)
- e, dispersion coefficient (L²/T)
- k, reaction term (1/day)
- w, waste/load (gr/m³)
- nx, length of river (m)
- gx, total grid
- hx = nx/gx, space step
- t, total time observation (s)
- nt, looping for time
- kt = t/nt, time step
- $initex = \frac{w}{q \cdot \left(1 + \frac{4 \cdot k \cdot e}{u^2}\right)}$, initial load

OUTPUT

Case 1 (K > 0 , u = 0 m/s)

Comparison output for both scheme methods in 3-D and 2-D at t=51s by using MATLAB software in case 1 show in Fig 6, Fig 7, Fig 8, and Fig 9.

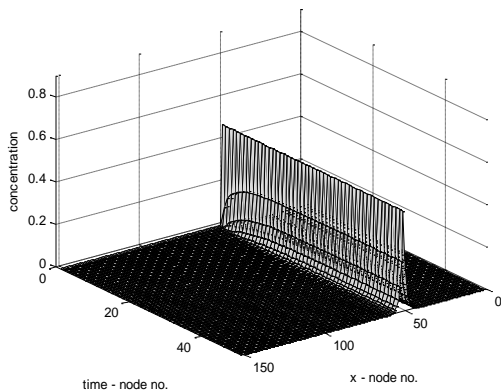


Fig 6. 3-D Point source pollution at t=51 s for explicit method

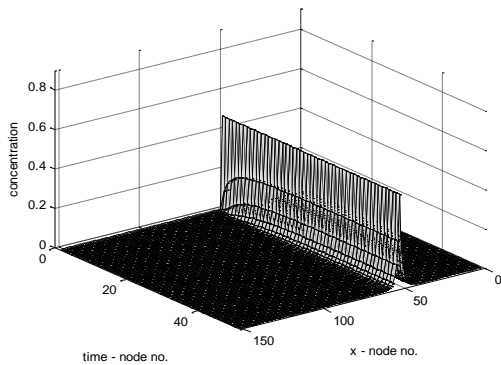


Fig 7. 3-D Point source pollution at t=51 s for implicit method

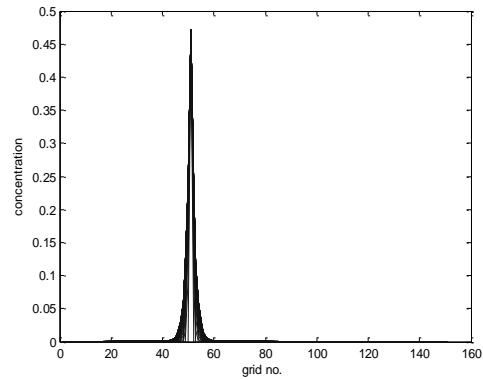


Fig 8.2-D Point source pollution at t=51 s for explicit method

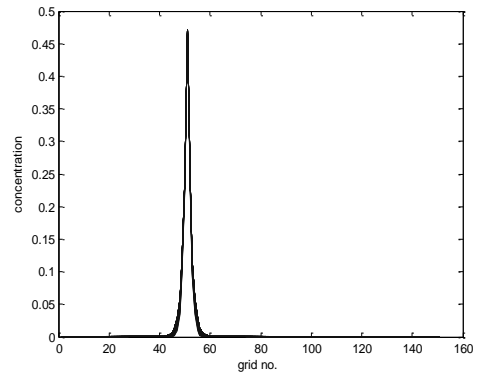


Fig 9.2-D Point source pollution at t=51 s for implicit method

Through Figure 7 and Figure 8, the concentration both explicit and implicit method shows the similar result correspond to hypothesis solution. In fact according Figure 9 and 10 the solution for implicit seem more stable than explicit

Case 2 (K > 0 , u > 0 m/s)

Comparison output for both scheme methods in 3-D and 2-D at t=51s by using MATLAB software in case 2 show in Fig 10, Fig 11, Fig 12, and Fig 13..

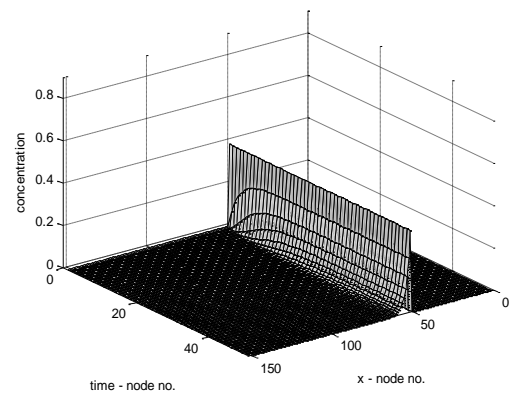


Fig 10.3-D point source pollution at t=51 s for explicit method

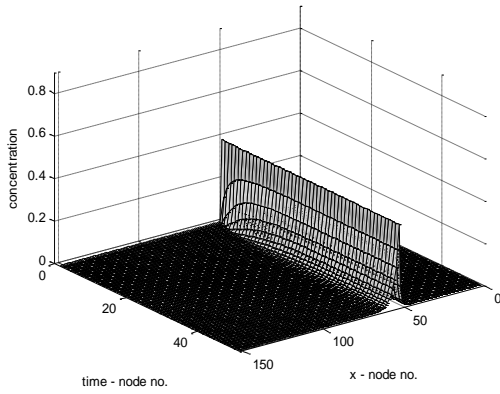


Fig11.3-D point source pollution at t=51 s for implicit method

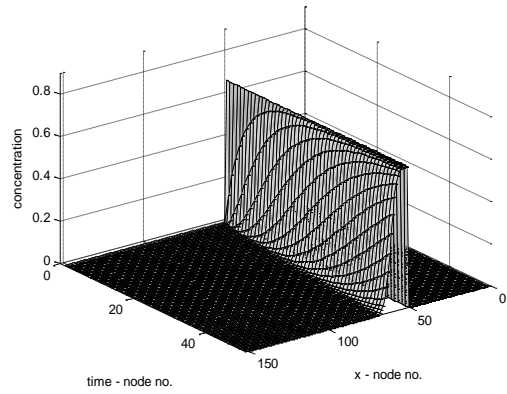


Fig 14.3-D point source pollution at t=51 s for explicit method

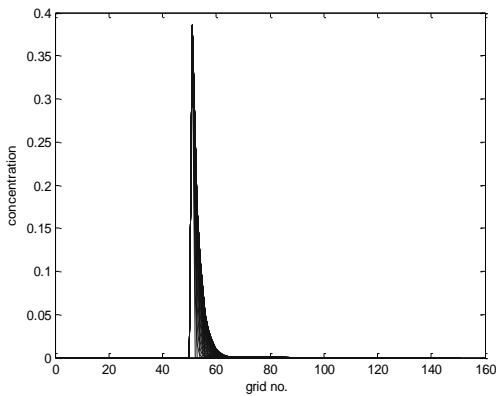


Fig 12.2-D point source pollution at t=51 s for explicit method

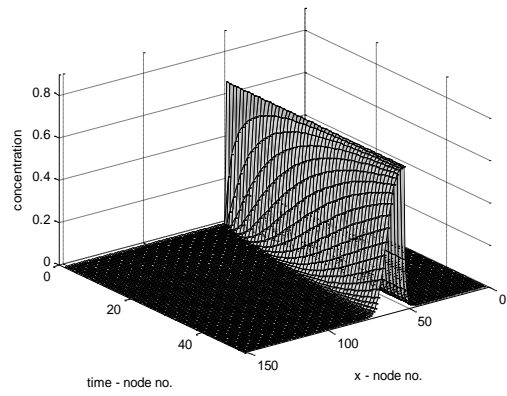


Fig 15.3-D point source pollution at t=51 s for implicit method

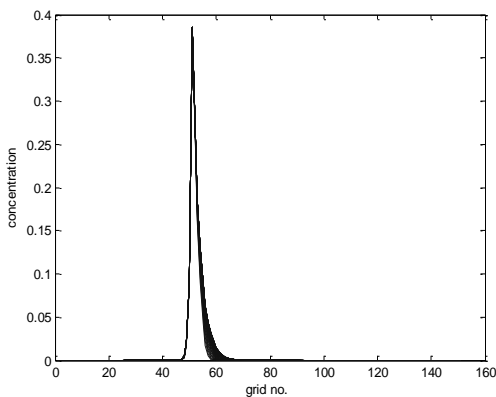


Fig 13.2-D point source pollution at t=51 s for implicit method

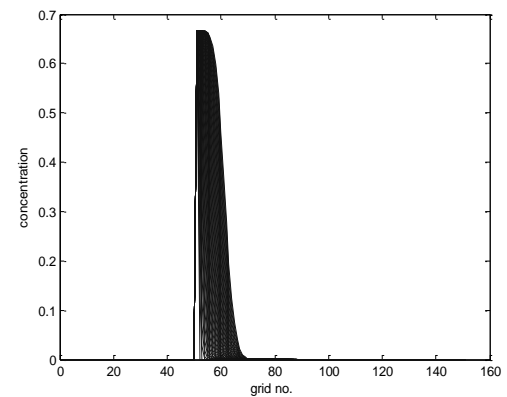


Fig 16.2-D Point source pollution at t=51 s for explicit method

Through Figure 10 and Figure 11 the concentration both explicit and implicit method shows the similar result correspond to hypothesis solution. According Figure 12 and 13 the solution for implicit seem more stable than explicit.

Case 3 ($K = 0$, $u > 0$ m/s)

Comparison output for both scheme methods in 3-D and 2-D at t=51s by using MATLAB software in case 3 show in Fig 14, Fig 15, Fig 16 and Fig 17.

Similar with previous cases, both explicit and implicit method shows the similar result and correspond to hypothesis solution. In fact the solution for implicit seem more stable than explicit.

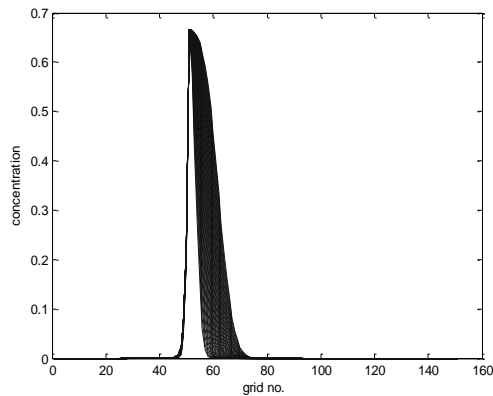


Fig 17.2-D point source pollution at t=51 s for implicit method

CONCLUSION

The conclusion in this paper is, both of the methods (explicit and implicit) give the same result for the modelling of one dimensional point source pollution in river, either in case 1, case 2 or case 3. According to the hypothesis, both result explicit and implicit methods are also match with the hypothesis.

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