Proceedings of the 7th International Conference on Asian and Pacific Coasts
(APAC 2013) Bali, Indonesia, September 24-26, 2013

NUMERICAL SIMULATION OF NONLINEAR NEARSHORE WATER WAVE

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ABSTRACT: The nonlinear parabolic mild-slope equation is a useful method to study the nearshore wave problem. The equation with the dispersive relation that Li had modified was numerically simulated over the complex field. The numerical method doesn't need transform the function to spectral-domain. This is different with that Lo used to simulate the MNLS equation with the nonlinear items. We simulated the surface water wave under the Berkhoff elliptic topography to verify the validity of the model, and the solution coincide quite well with the experimental data. The model is effective to simulate nonlinear water wave. The wave problem in Dachen island is studied with this model, and the distribution of wave height is predicted when the breakwater have been constructed. This method to solve the nonlinear items is very useful and easy to implement. The iteration and the limit of the periodic boundary are no longer necessary when solve the nonlinear parabolic mild-slope equation.

Keywords: Nonlinear; parabolic mild-slope equation; dispersion relation; water wave; numerical simulation, breakwater.

INTRODUCTION

The nonlinear parabolic mild-slope equation was derived by Kirby (1983) using a multiple-scale perturbation method, and the connection between the linearized version and a previously derived approximation of the linear mild slope equation is investigated. The equation is solved using the Crank-Nicolson difference method in the complex field. It can be used to predict the wave propagation on mildly varying topography. Wang (2006) improved a numerical model with this equation to study the distribution of wave direction. The nonlinear equation is linearized with some assumption when it is solved. The equation was divided to the real and imaginary parts by Lin (1998). Through, this avoids the difficulty of the calculation in the complex field, but there are two similar forms of nonlinear partial differential equations to solve. Due to the nonlinear terms of the equation, it always needs iteration to solve the equation numerically. This increases the the difficulty and computation in the discrete process.

Lo [3] proposed a split-step, pseudo-spectral method to solve the modified nonlinear Schrödinger equation (MNLS), which was further implemented by Karsten (1987) for the spatial MNLS equation. This method had been shown to effectively improve the numerical stability and accuracy without the employment of iterative procedure. However, the periodic boundary condition must be needed in the second computational step of the Discrete Fourier Transform (hereafter DFT), largely limiting its applications within wider range of practical conditions.

Dispersion relation plays an important role in the wave numerical calculation. Many scholars study the nonlinear dispersion relation, Li (2002, 2003) obtained a new formula of nonlinear dispersion relation, and compared with the dispersion relation obtained by Kirby. The accuracy of the nonlinear wave simulation is better with this nonlinear dispersion relation. Zhao (2009) based on hyperbolic mild slope equations and nonlinear dispersion relations to simulate the wave propagation.

In this paper, the numerical method mentioned above is improved without DFT process. Therefore the periodic boundary condition is no longer necessary and the new method is expected to be relatively more convenient and feasible. This improved method is then applied to solve the nonlinear parabolic mild-slope equation. And the modified nonlinear dispersion relation of Li is used in this nonlinear wave model. The simulating results of this model were compared with experimental data sets for wave propagation over Berkhoff's (1982) elliptical topography. The comparison between the numerical results and the experimental data reveals the validity of the model. Finally, the wave problem in Dachen island is studied with this model, and the distribution of wave height is predicted when the breakwater have been constructed.

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THE NONLINEAR PARABOLIC MILD-SLOPE EQUATION

The extended nonlinear parabolic mild-slope equation was derived by Kirby using a multiple-scale perturbation method.

$$2ikcc_g A_x + 2k(k-k_0)(cc_g)A + i(kcc_g)_x A + (cc_g A_y)_y - k(cc_g)K'|A|^2 A = 0 \tag{1}$$

$$K' = k^3\left(\frac{c}{c_g}\right)D \tag{2}$$

$$D = \frac{\cosh 4kh + 8 - 2\tanh^2 kh}{8\sinh^4 kh} \tag{3}$$

$$c = \frac{\omega}{k}, \quad c_g = \frac{\partial \omega}{\partial k} \tag{4}$$

Where x, y are horizontal coordinates, A is complex harmonic amplitude of wave surface elevation, ω is angular frequency, c is wave velocity, k is wave number and c_g is wave group velocity, $|A|$ is the module of A. Use the modified nonlinear dispersion relation by Li,

$$\omega^2 = gk(1+p\varepsilon^2)\tanh(kh+q\varepsilon) \tag{5}$$

$$p = \tanh(kh) \tag{6}$$

$$q = \left(\frac{kh}{\sinh(kh)}\right)^2 \tag{7}$$

NUMERICAL IMPLEMENTATION

Numerical method and Difference scheme

The split-step, pseudo-spectral method was used to simulate the MNLS equations by Lo.

Every spatial evolution equation can be written as the sum of its linear and nonlinear terms:

$$A_x = L(A) + N(A) \tag{8}$$

Where A is a complex function, x is horizontal coordinate, $A_x = \frac{\partial A}{\partial x}$, $L(.)$ and $N(.)$ are the linear and the nonlinear operators. This equation can be split into two equations:

$$A_x{}' = N(A) \tag{9}$$

$$A_x{}'' = L(A) \tag{10}$$

At each spatial step, both equations are solved independently, using the solution of the previous one as the initial condition for the next one. The first step is to solve the linear terms Eq. (9):

$$\tilde{A}(x+\frac{1}{2}\Delta x) = A(x) + \frac{1}{2}\Delta x N(A(x)) \tag{11}$$

$$\tilde{A}(x+\Delta x) = A(x) + \Delta x N(\tilde{A}(x+\frac{1}{2}\Delta x)) \tag{12}$$

The second step is to take the above solution to the linear terms by using the DFT.

$$A(x+\Delta x) = F^{-1}(e^{iP\Delta x}F(\tilde{A}(x+\Delta x))) \tag{13}$$

Where F, F^{-1} are the DFT and the inverse DFT, and P is determined by $F(L(A)) = PF(A)$.

As the function of the DFT must be periodic, the periodic boundary is limited in the simulation of surface water wave. But it is difficult to adjust the boundary to be periodic. So sometimes it can not simulate the water wave by the split-step, pseudo-spectral method. The second step can be modified by the finite-difference method as:

$$A(x+\Delta x) = A(x) + \Delta x L'(\tilde{A}(x+\Delta x)) \tag{14}$$

Where L' is the transformation of the linear term L by the finite-difference. In this modified step there is no limit of the periodic boundary. With the nonlinear term, the step should be stratify $\Delta x < O(\Delta y)^2$ to keep the method is numerically stable. It can simulate the nonlinear water wave equations generally. The modified method is applied to solve the nonlinear parabolic mild-slope equation.

Eq. (1) can be written into linear and nonlinear equations,

$$2iA_x - K'|A|^2 A = 0 \tag{15}$$

$$2ikcc_g A_x + 2k(k-k_0)(cc_g)A + i(kcc_g)_x A + (cc_g A_y)_y = 0 \tag{16}$$

Use the central difference scheme to discrete the nonlinear terms,

$$\tilde{A}_{m+\frac{1}{2},j} = \tilde{A}_{m,j} - \frac{i}{4}\Delta x K'_{m,j}|A_{m,j}|^2 A_{m,j} \tag{17}$$

$$\tilde{A}_{m+1,j} = \tilde{A}_{m,j} - \frac{i}{2}\Delta x K'_{m,j}|\tilde{A}_{m+\frac{1}{2},j}|^2 \tilde{A}_{m+\frac{1}{2},j} \tag{18}$$

Where m, j are the grid number in x and y coordinate.

And forward difference scheme is used for linear terms,

$$2i(kcc_g)\big|_{m,j}\frac{A_{m+1,j}-A_{m,j}}{\Delta x} + 2k(k-k_0)(cc_g)\big|_{m,j} A_{m,j} + i\frac{(kcc_g)\big|_{m+1,j} - (kcc_g)\big|_{m,j}}{\Delta x}A_{m,j} + \frac{(cc_g)\big|_{m,j+1} - (cc_g)\big|_{m,j}}{\Delta y}\frac{A_{m,j+1}-A_{m,j}}{\Delta y} + (cc_g)\big|_{m,j}\frac{A_{m,j+1}-2A_{m,j}+A_{m,j-1}}{(\Delta y)^2} = 0$$

(19)

Substituting $A_{m,j+1}, A_{m,j}, A_{m,j-1}$ with $\tilde{A}_{m+1,j+1}, \tilde{A}_{m+1,j}, \tilde{A}_{m+1,j-1}$ respectively, and then $A_{m+1,j}$ can be obtained.

Initial conditions

The incoming wave boundary condition ($x = 0$) is,

$$A_{1,j} = A_0 e^{ikj\Delta y \sin\theta} \quad (1 \le j \le N) \qquad (20)$$

Where, A_0 is initial amplitude of water wave, θ is the wave direction to x direction, N is grid points number in y direction.

Boundary conditions

At the lateral boundary ($y = 0, N\Delta y$), the total reflection boundary condition is $\partial A / \partial y = 0$. Here this boundary condition is approximated as that Kirby had done.

$$A_{m,1} = A_{m,2}, A_{m,N} = A_{m,N-1} \; (1 \le m \le M) \quad (21)$$

Where, M is grid point number in x direction.

Verification results

The calculation topography and the experimental cross-section used in Berkhoff verification test are shown in Fig.1. In the verification test, incoming wave direction along x and the amplitude A_0 is 0.0232m. Fig.2 shows that the numerical results are in fair agreement with the experimental data. This shows that the model is available to simulate the wave propagation.

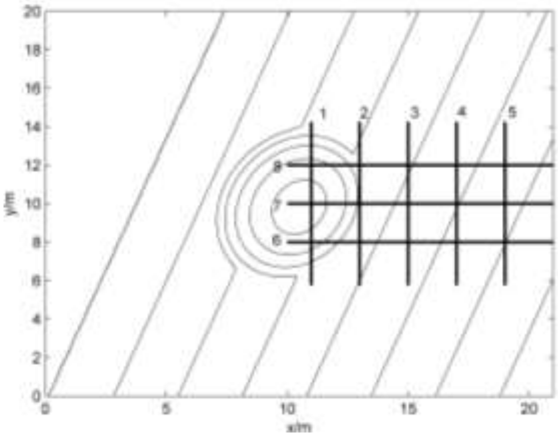

Fig.1 The graph of calculation topography and experimental cross-section

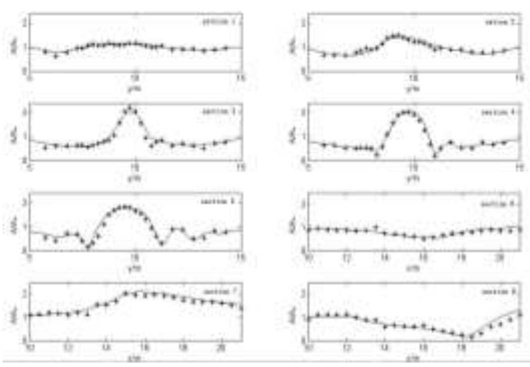

Fig.2 Verification of section 1-8, *, the testing data; -------, calculating data

THE WAVE SIMULATION OF DACHEN ISLAND

Regional situation

Dachen island is located in Chinese Taizhou bay and 55km away from the Jiaojiang Haimen port. There are two islands: Shan Dachen island and Xia Dachen island. The Shan Dachen island area is 74.89km^2. It's surrounded by mountains, and the west side faces sea. There will construct a harbour of refuge and build a breakwater at the entrance of the port. Fig.2 shows the location of the project. Fig.3 shows the calculation region.

Fig.3 Project location

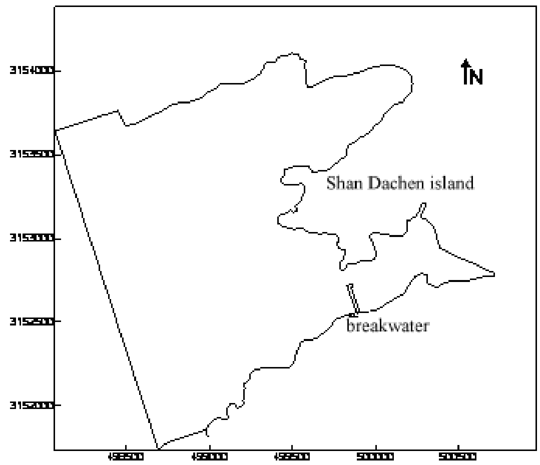

Fig.4 Calculation region

Simulation result

The grid step is 10m in the horizontal direction. We consider two incoming wave conditions that are 1:50 year wave events according to the following specifications:

Table.1 The parameters of the incoming wave

test	wave height (m)	wave period(s)	wave direction(degree)
1	2.68	9	90
2	2.68	9	60

Fig.5~Fig.8 show the wave height distribution. The wave height is above 1.5m at somewhere of the port in test 1 without breakwater. After the breakwater has been built, the wave height is smaller as the covering of the breakwater. It's about 0.5m. In test 2, as the impact of coastline and the wave direction, the wave height is smaller and about 0.5m. And the breakwater plays a small role in covering the incoming wave of test 2.

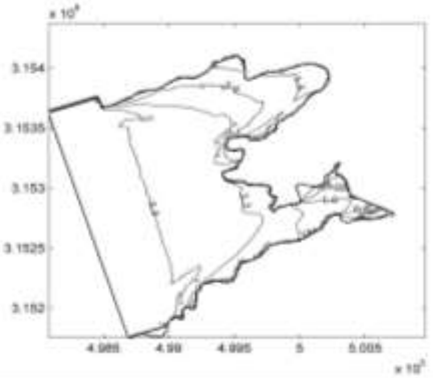

Fig.5 Wave height distribution without breakwater (test 1)

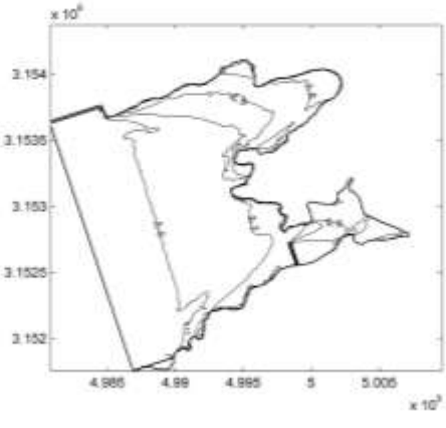

Fig.6 Wave height distribution with breakwater (test 1)

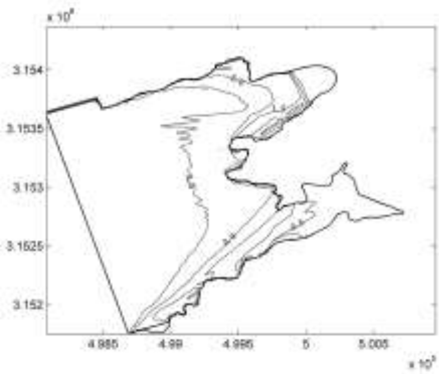

Fig.7 Wave height distribution without breakwater (test 2)

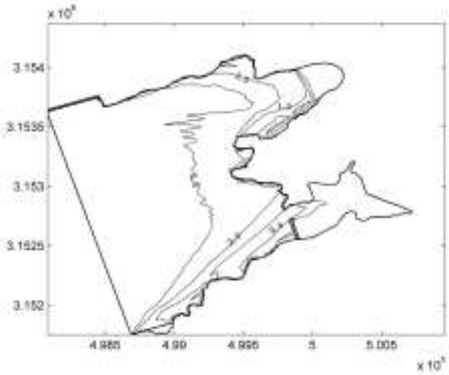

Fig.8 Wave height distribution with breakwater (test 2)

CONCLUSIONS

The nonlinear parabolic mild-slope equation is numerically solved by the modified split-step, pseudo-spectral method. As the modification in the second step, the improved method doesn't need transform the function to spectral-domain. The periodic boundary condition is no longer necessary and the new method is more convenient to solve the nonlinear wave equations. And the modified dispersion relation is used with the nonlinear parabolic mild-slope equation.

The surface water wave under the Berkhoff elliptic topography is simulated to verify the validity of the model, and the solution coincide quite well with the experimental data.The wave problem in Dachen island is studied with this model, and the distribution of wave height is predicted when the breakwater have been constructed. The breakwater can decrease the wave height in the port when the wave direction is along the bay.

With this method the nonlinear terms can be solved without iterating, and it is very useful and easy to implement.It also should be used in other nonlinear water wave equations in future.

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