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NUMERICAL SIMULATION OF WAVE PROPAGATION BY MODIFIED MILD-SLOPE EQUATION

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ABSTRACT: Variational principle is applied to derive a kind of modified mild-slope equation, which considers the first order derivative square term and the second order curvature effect of the topogr-aphy, this equation has higher precision in simulating wave propagation in the rapid changing sea-bed than original modified mild-slope equation. The capability of this model is validated by labor-atory experiment data; the results show that modified mild-slope equation can simulate wave prop-agation effectively in large-scale water area.

Keywords: Variational principle; mild-slope equation; numerical simulation; wave propagation;

INTRODUCTION

During the propagation, a series of complex phenomena named as wave transformation, such as shoaling, reflection, diffraction, energy dissipation caused by bottom friction and wave breaking, would happen as the result of the influence by complex topography, obstacles and currents. It is necessary and crucial for us to get an accuracy wave field after such transformation mentioned above take place.

Before 1970s, the scientific matter of wave propagation was divided into two parts, reflection and diffraction. Berkhoff(1972) developed a type of mild-slope equation, MSE for short, which considered both the reflection and diffraction problem. It could be simplified as shallow water equation when kh<<1, and could derive to Helmholtz equation when kh>>1 or kh is a constant. MSW reduced a 3D problem on potential wave theory to 2D one basing on the assumption that the slope of bottom was mild ($\mu =$ <<1), and that simplified the simulation. Booij(1983) discussed the accuracy performance of MSE on different slopes through a comparison between the result of them and 3D model。He pressed out that the MSE would have a satisfied precision if the bottom slide was less than 1/3.

The wave transformation could be usually found surround barrier on sandy coast if the cross seabed profile become steep, especially when the topography change was remarkable. Under that condition, the seafloor curvature ($\approx \nabla^2 h$) and the second order term of bottom slope ($(\nabla h)^2$) would affect the wave propagation, and such effects were ignored in the Berkhoff equation. Kirby(1986a, 1986b) defined the depth as slowly varying topography, which satisfied the assumption of MSE, and complex fluctuation one, which, accompanying with seafloor boundary condition, lead a series of enhanced MSE on time, and get the wave propagation equation on steep slope. Massel(1993) applied Galekin's method to setup governing equation while described the potential function as the sum of orthonormal function component $Z_n(x,y,z)$ that depends on depth, and he get the expanded MSE that picture the wave transformation after a series of complicated derivation as below:

$$\nabla^2\phi_0 + \frac{\nabla CC_g}{CC_g}\nabla\phi_0 + \{k^2 + \frac{1}{2ph^2}\frac{kh}{\tanh(kh)}[R_{00}^{(1)}(\nabla h)^2 + R_{00}^{(2)}\frac{\nabla^2 h}{\lambda}]\}\phi_0 = 0 \tag{1}$$

where the terms included in third parentheses considers the square of bottom slope and second order derivative of that. Chamberlain(1995) applied the variational principle and derived a type of MSE which consider the square and second order derivative of bottom slope. The modified MSE can be written as:

$$\nabla_h \cdot u_0 \nabla_h \varphi_0 + [k^2 u_0 + u_1(h)\nabla_h{}^2 h + u_2(h)(\nabla_h h)^2]\varphi_0 = 0 \tag{2}$$

$$u_1(h) = \frac{\mathrm{sech}^2(kh)}{4(K+\mathrm{sh}(K))}[\mathrm{sh}(K) - K\mathrm{ch}(K)] \tag{3}$$

$$o(\nabla h / kh) = o(\varepsilon)$$

$$u_2(h) = \frac{k\,\mathrm{sech}^2(kh)}{12(K+\mathrm{sh}(K))^3}[K^4 + 4K^3\mathrm{sh}(K) - 9\mathrm{sh}(K)\mathrm{sh}(2K) + 3K(K+2\mathrm{sh}(K))(\mathrm{ch}^2(K) - 2\mathrm{ch}(K) + 3)] \tag{4}$$

where K=2kh. Basing on of modified MSE established by Chamberlain, Porter(1995) took account of the continuity of mass flow on incontinuous submarine topography and improved the calculation accuracy after a further extend.

Hong et al(2009). derived a class of mathematical wave propagation model, which was based on the

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nonlinear interaction between the surface gravity wave and long-wave, from inviscid and irrational fluid dynamics equations. This model cab be adequate for the wave propagation from deep water to shallow water accompanying with long-wave flow field and water level changes. The Control equation contains energy input, energy coefficient of friction and wave breaking loss , and the factor of local underwater topography. The terrain factor that describes the model submarine topography can be expressed as:

$$\tilde{\mathrm{F}}(x,y,\tilde{z},t)=\mathrm{ch}k(\tilde{\mathrm{h}}+\tilde{z})/\mathrm{ch}k\tilde{h}\,,\ \tilde{z}=\mathrm{z}\text{-}\eta_c\,,\ \ddot{h}=\mathrm{h}+\eta_c$$

$$(1+\frac{\tilde{\sigma}^2}{g}\eta)\frac{Dg\eta}{\mathrm{Dt}}+(\mathrm{g}\eta+\frac{1}{2}\tilde{\sigma}^2\eta^2)\nabla U_c+\nabla\{(\tilde{c}\tilde{c}_g A+g\eta)\nabla\Phi\}$$
$$+\{k^2\tilde{c}\tilde{c}_g+J\text{-}\tilde{\sigma}^2(1+\frac{\tilde{\sigma}^2}{g}\eta)\}\Phi=0 \tag{5}$$

$$g\eta+(\frac{D}{Dt}+W^*)\{(1+\frac{\tilde{\sigma}^2}{g}\eta)\}\Phi+\frac{1}{2}\{\nabla\Phi\nabla\Phi+(\frac{\tilde{\sigma}^2}{g}\Phi)^2=\mathrm{C(t)} \tag{6}$$

$$J=g\{\int_{\text{-h}}^{\eta}F\nabla^2Fdz+F\nabla F\nabla z|_{z=-h}^{z=\eta}\}$$
$$=g\{(\int_{\text{-h}}^{0}\tilde{F}\nabla^2\tilde{F}d\tilde{z}+\tilde{F}\nabla\tilde{F}\nabla\tilde{z}|_{z=-\tilde{h}})+\int_{0}^{\eta}\tilde{F}\nabla^2\tilde{F}d\tilde{z}+\tilde{F}\nabla\tilde{F}\nabla\tilde{z}|^{\tilde{z}=\eta}\} \tag{7}$$

Under some simplified conditions, Hong et al. ignore the last term of (7) when $\nabla\tilde{\mathrm{z}}=0$, and derived the expression of J as below, in which R1, R2 have a relationship with $k\tilde{h}$:

$$J=g\{\mathrm{R}_1\nabla^2\tilde{\mathrm{h}}+\mathrm{R}_2(\nabla\tilde{\mathrm{h}})^2\ \} \tag{8}$$

More scholars , such as Suh et al.(1997), Tsay et al.(1996),Zhang L et al.(1996),Lee et al.(2003) and Kyung et al.(2001), considered the more steeply type when improving and expanding the mild slope equation, and more and more analysis and numerical calculation of MSE on the complex terrain were done. The results show that, after considering the square of the first derivative of the water depth and the second-order derivative terms in MSE, the calculation accuracy of the wave field in complex terrain and less steep terrain (such as submarine corrugated terrain, Booij laboratory slope terrain, etc.) have a demonstrable improvement.

In order to better study the accuracy and numerical simulation of these models in complex terrain validity, Kyung (2003) held a special shallows terrain model test, of which the shallows depth of the value of the second derivative $\nabla^2 h$ is 3.55 and the maximum value of the square of the bottom slope $(\nabla h)^2$ is 0.64. The test measured the shallows near a number of different cross-profile of the wave height and provides a more detailed reference data for the numerical simulation study.

This paper, applying variational principle to the basic hydrodynamics equations, deduces an extended mild slope equation consider improvements in complex terrain. The potential function along the depth distribution is a function related to the unknown wave front. Applying the Taylor expansion to hyperbolic functions with free surface height （η） , omitting the higher order terms above O(η3), and using of the variational principle, a type of MSE containing the higher order terms of bottom slope and curvature items of that, which considers wave refraction and diffraction, is derived. The coefficients describe the sudden change of terrain $(\nabla h)^2$ and $\nabla^2 h$ in the model are only related with water depth, and thus the calculation is simplified and convenient. Due to the different methods of derivation and simplification used in the derivation, the model mentioned in this paper is different from the previous results of expressions. Exported equation, omitting bottom slope higher order terms, would be same with ones that deduced by Berkhoff linear assumption the model mentioned in this paper is a modified one of MSE.

THE DERIVATION OF EQUATIONS

In 1967, Luke extended variational method to the free surface of the fluid motion, and later many scholars applied variational principle method to study wave problem. To two-dimensional wave problems, wave energy density function can be considered as following:

$$L=\int_{-h}^{\eta}[\frac{1}{2}({\varphi_x}^2+{\varphi_y}^2)+\frac{1}{2}{\varphi_z}^2)+\varphi_t+gz]\mathrm{d}z \tag{9}$$

φ is a potential function, η is a free surface height function and h is water depth. Assuming:

$$\varphi=\tilde{\varphi}(x,y,t)f(z) \tag{10}$$

$$f(z)=\frac{chk(h+z)}{chk(h+\eta)} \tag{11}$$

Where $f(\eta)=1$ and k is an eigenvalue. Omitting η in (eq.11), the distribution of potential function is same with that under linear condition. The eigenvalue k represents wave number.

Substituting (eq.10) and (eq.11) into (eq.9) and taking into account of the following relationship:

$$\int_{-h}^{\eta}\varphi_t\mathrm{d}z=\tilde{\varphi}_t\frac{1}{k}\mathrm{th}k(h+\eta)-\tilde{\varphi}\eta_t[\mathrm{th}k(h+\eta)]^2$$
$$\int_{-h}^{\eta}(\nabla\varphi)^2\mathrm{d}z=\int_{-h}^{\eta}[(\nabla\tilde{\varphi})^2f^2+2f\nabla f\tilde{\varphi}\nabla\tilde{\varphi}+\tilde{\varphi}^2(\nabla f)^2]\mathrm{d}z \tag{12}$$

$$\int_{-h}^{\eta} f^2 \mathrm{d}z = \frac{1}{k}\mathrm{th}k(h+\eta)\frac{1}{2}[1+\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}] \tag{13}$$

$$\int_{-h}^{\eta} (\nabla f)^2 \mathrm{d}z = \frac{1}{2k}\frac{\mathrm{th}k(h+\eta)}{\mathrm{ch}^2 k(h+\eta)}[1-\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}][\nabla(kh)]^2$$
$$+\frac{1}{k^3\mathrm{ch}^2 k(h+\eta)}[-\frac{1}{4}kh-\frac{1}{6}(kh)^3+\frac{1}{8}\mathrm{sh}2k(h+\eta)](\nabla k)^2$$
$$+\frac{2}{k^2\mathrm{ch}^2 k(h+\eta)}[-\frac{1}{4}(kh)\mathrm{th}k(h+\eta)+\frac{1}{4}(kh)^2]\nabla k\nabla(kh)+O(k\eta)$$
$$\int_{-h}^{\eta} 2f\nabla f dz = -\frac{\mathrm{th}k(h+\eta)}{\mathrm{ch}^2 k(h+\eta)}h\nabla(kh)$$
$$+\frac{\mathrm{th}k(h+\eta)}{2k^2\mathrm{ch}k(h+\eta)}[\frac{2k(h+\eta)}{sh2k(h+\eta)}-1]\nabla k+O(k\eta)$$
$$\int_{-h}^{\eta} \varphi_z^{\,2}\mathrm{d}z = \tilde{\varphi}^2 k\mathrm{th}k(h+\eta)\frac{1}{2}[1-\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}] \tag{14}$$

$$\int_{-h}^{\eta} gz\mathrm{d}z = \frac{1}{2}g(\eta^2-h^2) \tag{15}$$

After complex calculations, following are obtained:

$$L=\frac{1}{2}g(\eta^2-h^2)+\tilde{\varphi}_t\frac{1}{k}\mathrm{th}k(h+\eta)-\tilde{\varphi}\eta_t[\mathrm{th}k(h+\eta)]^2$$
$$+\frac{1}{2}(\nabla\tilde{\varphi})^2\frac{1}{k}\mathrm{th}k(h+\eta)\frac{1}{2}[1+\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}]$$
$$+\frac{1}{2}\tilde{\varphi}^2\{\frac{1}{2k}\frac{thk(h+\eta)}{\mathrm{ch}^2k(h+\eta)}[1-\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}][\nabla(kh)]^2$$
$$+\frac{1}{k^3\mathrm{ch}^2k(h+\eta)}[-\frac{1}{4}kh-\frac{1}{6}(kh)^3+\frac{1}{8}\mathrm{sh}2k(h+\eta)](\nabla k)^2$$
$$+\frac{2}{k^2\mathrm{ch}^2k(h+\eta)}[-\frac{1}{4}(kh)\mathrm{th}k(h+\eta)+\frac{1}{4}(kh)^2]\nabla k\nabla(kh)\}$$
$$+\frac{1}{2}\nabla\tilde{\varphi}\tilde{\varphi}\{-\frac{\mathrm{th}k(h+\eta)}{\mathrm{ch}^2k(h+\eta)}h\nabla(kh)$$
$$+\frac{\mathrm{th}k(h+\eta)}{2k^2\mathrm{ch}k(h+\eta)}[\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}-1]\nabla k\}$$
$$+\frac{1}{2}(\tilde{\varphi})^2k\mathrm{th}k(h+\eta)\frac{1}{2}[1-\frac{2k(h+\eta)}{\mathrm{sh}2k(h+\eta)}]$$
$$+O(\eta^3) \tag{16}$$

Coefficient of every term in the last-written contains a free-surface parameter η. In order to derive a linear mild slope equation, applying Taylor expansion on hyperbolic functions:

$$\mathrm{th}k(h+\eta)=\mathrm{th}(kh)+k\eta\{1-\mathrm{th}^2(kh)\}$$
$$+(k\eta)^2\{\mathrm{th}^3(kh)\text{-}\mathrm{th}(kh)\}+O(\eta^3)$$
$$=\sigma+k\eta(1-\sigma^2)+(k\eta)^2(\sigma^3\text{-}\sigma) \tag{17}$$

Substituting above to (eq.16) and omitting three-order above of $\tilde{\varphi}$ and η, the simplified one is

$$L=\frac{1}{2}g(\eta^2-h^2)+\frac{\sigma}{k}\tilde{\varphi}_t+(1-\sigma^2)\tilde{\varphi}_t\eta-\sigma^2\tilde{\varphi}\eta_t$$
$$+\frac{1}{2}(\nabla\tilde{\varphi})^2\frac{\sigma}{k}\frac{1}{2}(1+\frac{2kh}{\mathrm{sh}2kh})$$
$$+\frac{1}{2}\tilde{\varphi}^2\{\frac{1}{k}\sigma(1-\sigma^2)\frac{1}{2}(1-\frac{2kh}{\mathrm{sh}2kh})[\nabla(kh)]^2$$
$$+\frac{1}{k^3}(1-\sigma^2)[-\frac{1}{4}kh-\frac{1}{6}(kh)^3+\frac{1}{8}\mathrm{sh}2kh](\nabla k)^2$$
$$+\frac{1}{k^2}(1-\sigma^2)[-\frac{1}{2}\sigma(kh)+\frac{1}{2}(kh)^2]\nabla k\nabla(kh)\}$$
$$+\frac{1}{2}\nabla\tilde{\varphi}\tilde{\varphi}\{-\sigma(1-\sigma^2)h\nabla(kh)$$
$$+\frac{1}{2k^2}\sigma\sqrt{1-\sigma^2}[\frac{2kh}{\mathrm{sh}2kh}-1]\nabla k\}$$
$$+\frac{1}{2}(\nabla\tilde{\varphi})^2k\sigma\frac{1}{2}[1-\frac{2kh}{\mathrm{sh}2kh}]+O(\eta^3) \tag{18}$$

Here $\sigma=\mathrm{th}(kh)$. According to the following variational equation:

$$\frac{\partial L}{\partial F}-\frac{\partial}{\partial t}(\frac{\partial L}{\partial F_t})-\nabla(\frac{\partial L}{\partial\nabla F})=D_e$$

In the last-write, F represents η or $\tilde{\varphi}$, D_e means the energy loss caused by bottom friction. Applying variational basing on $\tilde{\varphi}$ and η, the following is deduced:

$$-\eta_t+(B+D-\frac{1}{2}\nabla C)\tilde{\varphi}-\nabla(A\nabla\tilde{\varphi})=0 \tag{19}$$

$$g\eta+\tilde{\varphi}_t=0 \tag{20}$$

Where

$$A=\frac{\sigma}{k}\frac{1}{2}(1+\frac{2kh}{\mathrm{sh}2kh})=\frac{1}{g}CC_g$$
$$B=\frac{1}{k}\sigma(1-\sigma^2)\frac{1}{2}[1-\frac{2kh}{\mathrm{sh}2kh}][\nabla(kh)]^2$$
$$+\frac{1}{k^3}(1-\sigma^2)[-\frac{1}{4}kh-\frac{1}{6}(kh)^3$$
$$+\frac{1}{8}\mathrm{sh}2kh](\nabla k)^2$$
$$+\frac{2}{k^2}(1-\sigma^2)[-\frac{1}{2}\sigma(kh)+\frac{1}{2}(kh)^2]\nabla k\nabla(kh)\}$$
$$=\frac{1}{k}\sigma(1-\sigma^2)(1-n)(k+k_hh)^2(\nabla h)^2$$
$$+\frac{1}{k^3}(1-\sigma^2)[-\frac{1}{4}kh-\frac{1}{6}(kh)^3+\frac{1}{8}\mathrm{sh}2kh]k_h^{\,2}(\nabla h)^2$$
$$+\frac{2}{k^2}(1-\sigma^2)[-\frac{1}{2}\sigma(kh)+\frac{1}{2}(kh)^2]k_h(k+k_hh)(\nabla h)^2$$
$$C=-\sigma(1-\sigma^2)(k+k_hh)h\nabla h$$
$$-\frac{1}{k^2}\sigma\sqrt{1-\sigma^2}(1-n)k_h\nabla h$$
$$D=k\sigma\frac{1}{2}[1-\frac{2kh}{\mathrm{sh}2kh}]=k\sigma(1-n)$$

The following expressions are obtained after elimination of η in eq.19 and eq.20:

$$\frac{1}{g}\tilde{\varphi}_{tt}+(B+D-\frac{1}{2}\nabla C)\tilde{\varphi}-\nabla(A\nabla\tilde{\varphi})=0 \tag{21}$$

Considering $\tilde{\varphi}_{tt}=-\omega^2\tilde{\varphi}$, eq.21 can be written as below:

$$\nabla(CC_g\nabla\tilde{\varphi})+(k^2CC_g-F)\tilde{\varphi}=0 \tag{22}$$

Where $F=g(B-\frac{1}{2}\nabla C)$, which contains the second derivative $\nabla^2 h$ and the first derivative squared term $(\nabla h)^2$ of the water depth. Writing F in the Form as $F=F_1\nabla^2 h+F_2(\nabla h)^2$, the expression of coefficients are:

$$F_1=g\{\frac{1}{2}\sigma(1-\sigma^2)(k+k_h h)h+\frac{k_h}{2k^2}\sigma(1-n)\sqrt{1-\sigma^2}\} \tag{23}$$

$$\begin{aligned}F_2=g\{&\frac{1}{k}\sigma(1-\sigma^2)(1-n)(k+k_h h)^2\\&+\frac{k_h{}^2}{k^3}(1-\sigma^2)[-\frac{1}{4}kh-\frac{1}{6}(kh)^3+\frac{1}{8}\mathrm{sh}(2kh)\\&+\frac{k_h}{k^2}(1-\sigma^2)(k+k_h h)[-\frac{1}{2}\sigma kh+\frac{1}{2}(kh)^2]\\&+\frac{1}{2}\sigma(1-\sigma^2)h(2k_h+k_{hh}h)\\&+\frac{1}{2}\sigma(1-\sigma^2)(k+k_h h)+\frac{1}{2}(1-\sigma^2)(1-3\sigma^2)h(k+k_h h)^2\\&+\frac{1}{2k^3}\sigma\sqrt{1-\sigma^2}(1-n)(kk_{hh}-2k_h{}^2)\\&+\frac{k_h}{2k^2}\sigma\sqrt{1-\sigma^2}[(1-\sigma^2)(1-n)(k+k_h h)\\&-\sigma(k+k_h h)\frac{\mathrm{sh}2kh-2kh\mathrm{ch}2kh}{\mathrm{sh}^2 2kh}]\\&-\frac{k_h}{2k^2}\sigma^2\sqrt{1-\sigma^2}(1-n)(k+k_h h)\}]\end{aligned} \tag{24}$$

Assuming $F_H=-F/k^2CC_g$, $\tilde{F}=1+F_H$, eq.22 could be written as :

$$\nabla(CC_g\nabla\tilde{\varphi})+k^2CC_g\tilde{F}\tilde{\varphi}=0 \tag{25}$$

The exported eq.25 is the Modified Mild-Slope Equation (MMSE as abbreviation), which considers complex terrain. Omitting the high order term of topographical change F_H , this equation has a same form of MSE. In the derivation above, Taylor expansion is used to the hyperbolic function with free surface height (η) , and thus the higher-order terms of the terrain parameters is improved comparing with Hong et al.

NUMERICAL CALCULATION OF MMSE

To solve eq.25 assume that:

$$\tilde{\varphi}(x,y,t)=\phi(x,y)\mathrm{e}^{-\mathrm{i}\omega t}=\phi(x,y,\bar{t})\mathrm{e}^{-\mathrm{i}\omega t}$$
$$\Phi=\phi(x,y)\sqrt{CC_g}$$

where $\bar{t}=\varepsilon t$ and it represents slowly varying time scales of the velocity potential amplitude changes. Substituting above two equations to eq.25 and considering $\tilde{\varphi}_{tt}=-\omega^2\tilde{\varphi}$, following deduction can be got:

$$\Phi_{\bar{t}\bar{t}}\varepsilon^2-2\mathrm{i}\omega\Phi_{\bar{t}}\varepsilon-k^2CC_gF\Phi-\nabla(CC_g\nabla\Phi)=0$$

Omitting the term of ε^2 and changing $\bar{t}$ to t , the equation above would turn to the following parabolic equation according to the time. So the time variable can be treated as iteration parameter and thus we can get:

$$-\frac{2\mathrm{i}\omega}{CC_g}\frac{\partial\phi}{\partial t}=\nabla^2\phi+k_c{}^2\phi \tag{26}$$

where.

$$k_c{}^2=k^2\tilde{F}-\frac{\nabla^2\sqrt{CC_g}}{\sqrt{CC_g}}$$

in Eq.26, which accompanies with appropriate boundary conditions, can be solved numerically.

HYDRAULIC MODEL VALIDATION

To verify wave transformation on the steep slope, a hydraulic model test was held by Kyung in the Coastal Engineering Laboratory of South Korea National University. The test basin size was 23m by 11m by 1m (length by breadth by depth). Test topographic contained a circular shallows placed on frying terrain, as shown in Figure 1. Waves formed from the left boundary (X =-6m), and the right downstream boundary (X = 10.75m) was treated as absorbing boundary. The distance of center Round Shoal to wave boundary was 6m, and the shallows radius R was 0.45m. The depth of beach face away from the shallows center is:

$$h=h_0-b[1-(\frac{r}{R})^2] \tag{27}$$

where h_0 is the depth on flat, and h_0= 0.3m. The depth at shallow center b=0.18m. Wave height measurements were held at five lateral cross-sections (see section X0 to X4) and a centerline cross-section (see section Y0).

Tests were under regular wave condition. The incident height was 3cm and three wave period were 1.259s, 0.791s and 0.636s, corresponding k0h0 = 1, 2, 3.

k0 was deep water wave number and h0 was the depth of the water.

There is square term in the shallow depth function. The second-order partial derivative of beach depth $\nabla^2 h$ is a constant. $4b/R^2 = 3.55$. The value of the square of first derivative of depth $(\nabla h)^2$ is $(4b^2/R^2)(r^2/R^2)$, which equals 0 at r=0 and 0.64 at r=R. Eq.25 can be written in the following form:

$$\nabla(CC_g \nabla\tilde{\varphi}) + k^2 CC_g \left(1 - \frac{F_1 \nabla^2 h + F_2 (\nabla h)^2}{k^2 CC_g}\right)\tilde{\varphi} = 0 \qquad (28)$$

To analyze the impact of $\nabla^2 h$ and $(\nabla h)^2$ in the MMSE, examination of the relationships of $M_1 = -F1/k^2 CC_g$ changes in the and with depth and that of $M_2 = -F2/k^2 CC_g$ is available. Fig. 2 shows the change of M1 and M2 relative to kh/π when T = 1.259s. As can be seen from the figure, M1 and M2 close to 0 at larger relative depth, which the impact of $\nabla^2 h$ and $(\nabla h)^2$ are small.

Under the condition of smaller relative depth (kh/π<0.1), M2 close to 0, which the impact of $(\nabla h)^2$ is less, and M1 close to -0.167, which $\nabla^2 h$ cannot be ignored. In the case of medium depth, M1 and M2 are expressed as the impossibility ignored, and the value of M1 is between -0.167 to 0.047, while M2 is between -0.043 to 0.000.

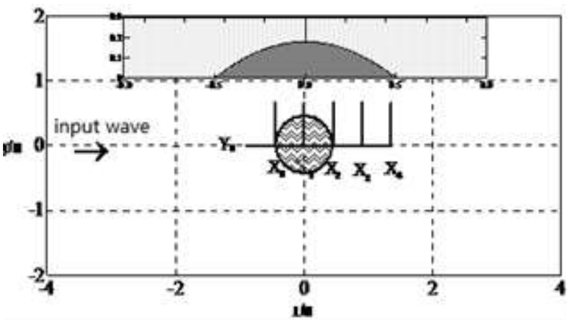


Fig. 1 Model layout and section location

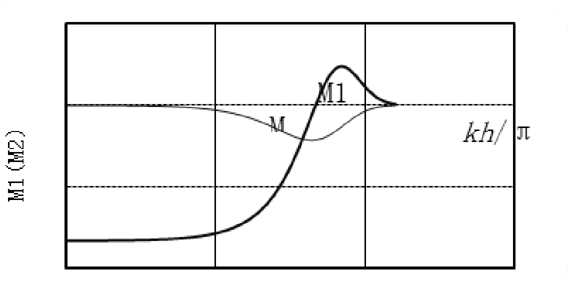


Fig. 2 M1、M2- *kh* curve

MMSE model was applied to the numerical calculation of wave propagation on Kyung terrain. Space step in the calculation is 0.025m, and time step is 0.002s.The three calculated wave period is 1.259s, 0.791s and 0.636s. Wave height contour was shown in Figure 3. In Figure the shoal border is presented in the form of thick dashed line, the shallow center coordinates is (0.0m, 0.0m), and the radius of it is 0.45m.

It is shown from the figure that wave energy concentration on shoals or after the shallows, the refraction and diffraction on both sides of the shoal, and the wave reflection on the shallows can be clearly performed in the numerical simulation. It is also seen that wave focal point cycle decreases with wave moves downstream when the wave period decreases. The possible reason of that may be due to the wave refraction weakened in the shallows when the wave period decreases.

Figure 4 shows the wave height result comparison on shallows centerline cross section (Figure 1 Y0) among the MMSE mode, the traditional mode and the measured data. Tests were held according to three different periods. MMSE mode and MSE of FIG mode calculation results are respectively represented by a solid line and the dotted lines while solid circles represent experimental results.

The figures tell that the calculation results of MMSE mode have fairly good agreement with the experimental results; especially the maximum wave height and its position near the shallows meet the experimental results quite good, while the MSE mode is a serious deviation from the test results. From Figure 4 (a), a downstream drift of the maximum wave height position can be found in MSE mode calculation when T = 1.259s.

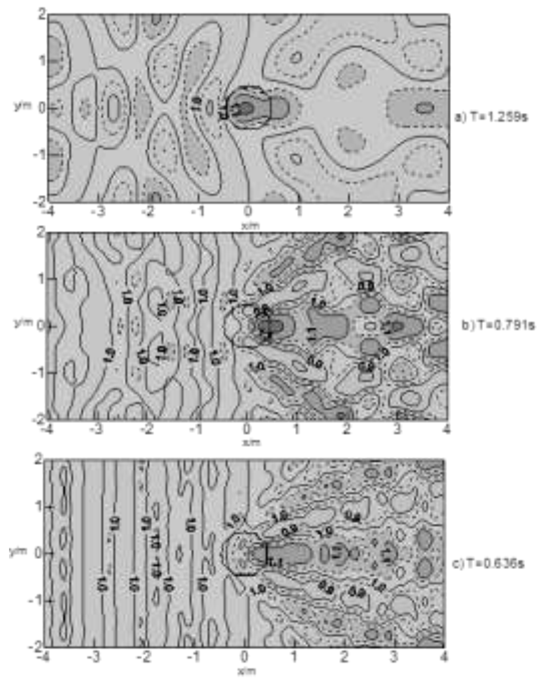
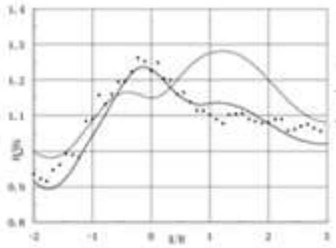
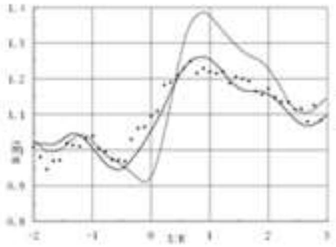


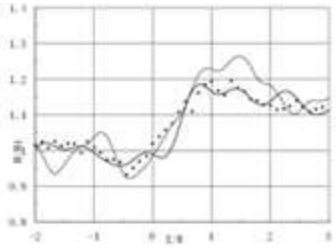
Fig. 3 wave height contour



a) T=1.259s



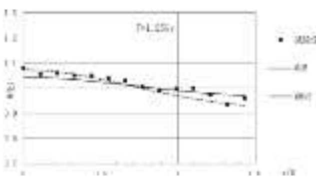
b) T=0.791s



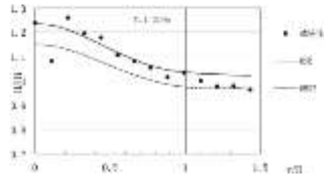
c) T=0.636s

Fig. 4 Comparison of simulated and tested results at center section(Y_0)

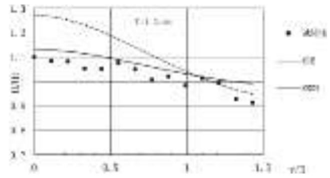
Besides, Kyung terrain lateral cross-section of the wave height calculated results were compared. Figures 5 to 7 are comparison of the calculated values with the experimental results at five sections (see Figure 1 in X0, X1, X2, X3, and X4) when the wave period is 1.259s, 0.791s, and 0.636s. It can be seen from the figure that wave height of each section of the MMSE model agree well with the experimental results while that of MSE model shows obvious deviation with data, especially in the long-period wave bottom slope effect. Under such condition result of MSE mode is too large at the location where wave energy centralized, and it is too small in the wave divergence area. Waves lateral height varied little at the beginning of the shallows (X =-R), and it varies strongly when spread to the shallows, which influence extends directly to the downstream of shallows.



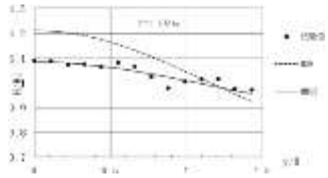
(a) sectionX_0（X=-R）



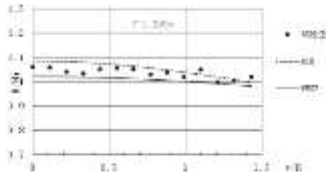
(b) sectionX_1（X=0）



(c) sectionX_2（X=R）

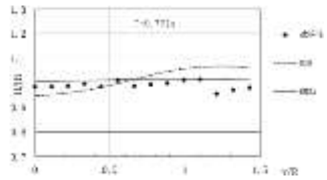


(d) sectionX_3（X=2R）

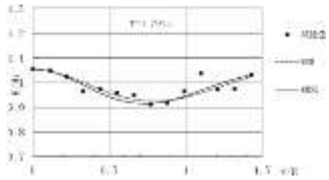


(e) sectionX_4（X=3R）

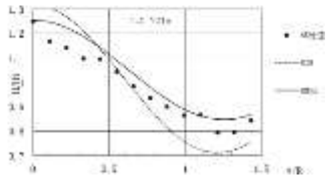
Fig. 5 Comparison of simulated and tested results at side section(T=1.259s)



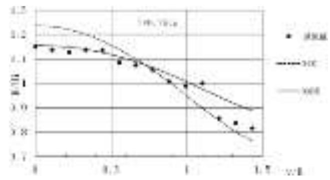
(a) sectionX_0（X=-R）



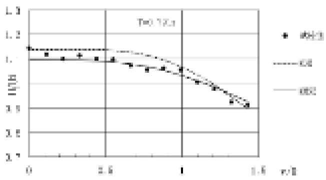
(b) sectionX_1（X=0）



(c) sectionX_2（X=R）

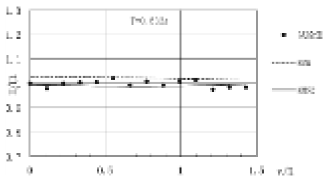


(d) sectionX_3（X=2R）

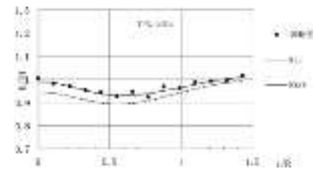


(e) sectionX_4（X=3R）

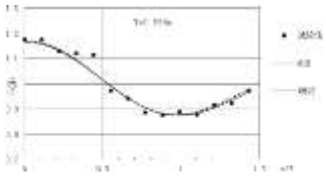
Fig.6 Comparison of simulated and tested results at side section(T=0.791s)



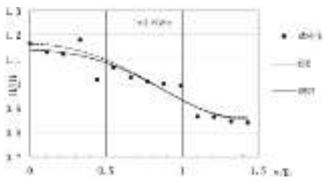
(a) sectionX_0（X=-R）



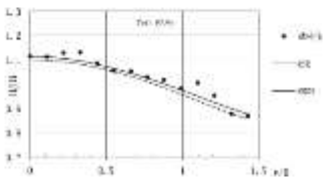
(b) sectionX_1（X=0）



(c) sectionX_2（X=R）



(d) sectionX_3（X=3R）



(e) sectionX_4（X=3R）

Fig. 7 Comparison of simulated and tested results at side section(T=0.636s)

CONCLUSION

This paper, basing on theoretical analysis from the basic equations of hydrodynamics, uses the variational principle to derive the mathematical model of improvements mild slope equation which describes wave propagation on complex terrain. The main conclusions are as follows:

1) Consider the potential function along the depth distribution as a function of the unknown wave front, apply Taylor expansion to hyperbolic function with free surface height （η） , omit the higher order terms than O(η3), and use the variational principle to get the wave propagation equation includes the slope higher order terms and the curvature of the bottom slope.

2) Verification contrast with laboratory measured data on Complex terrain show that, the MMSE model can clearly present wave energy focus on shoal or behind shallow, refraction and diffraction on both sides of the shoal, and wave reflection on the shallows. The calculation of wave height value on each section is in good agreement with the experimental results and this model can accurately present the maximum wave height and its location near the shallows.

3) The wave height lateral variational at the beginning of the shallows is small. It turns to strong when the waves spread to the shallows, and its influence extends directly to the downstream. The MMSE mode can significantly improve the simulation accuracy. The improved model can get a more accuracy result of the wave field in complex terrain.

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