

## **RANDOM WAVE LOADS ON A LONG DETACHED BREAKWATER CONSIDERING DIFFRACTION**

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**ABSTRACT:** Battjes (1982) found the loads of short-crested random waves on a long structure decrease with the structure length and also with the obliqueness of wave incidence. These decreases come from the spatial phase difference along the structure. Lee et al. (2010) found that obliquely incident random waves in a nearshore area become directionally asymmetric due to refraction. They also found the asymmetry becomes more significant in shallower waters. Recently, Jung et al. (2011) studied random wave loads on a long structure considering diffraction and directional asymmetry. In this study, we further study random wave loads on a detached breakwater considering diffraction of waves which propagate at both ends of the breakwater. We also consider directional asymmetry. The structure may be placed along the bottom contours in order to protect on-shore incoming waves. In that case, refraction-induced random waves may become asymmetric, i.e., on-shore components are more dominant than along-shore ones. Therefore, directional obliqueness on the structure becomes less and thus the wave loads decrease in less degree than the symmetric waves. When waves are obliquely incident on a long structure, the diffracting waves give forces on the lee side of the structure. The diffracting wave has a spatial phase variation along the lee side which is different from that the obliquely incident wave has on the front side. Thus, the wave loads decrease with the existence of diffracting waves and also the phase difference between the incident and diffracting waves.

**Keywords:** Random wave loads, long detached breakwater, diffraction, directional asymmetry

### **INTRODUCTION**

In general, the load acting on structures such as a breast wall or a quay wall is mainly the earth force. Meanwhile, the load on coastal structures such as a breakwater or a floating structure is mainly the wave force. The main difference between the earth and wave force is whether the pressure is a function of time or not. The earth force hardly changes in time except special situation like an earthquake or a landslide. However, the wave force changes in time. Therefore, the maximum force by statistical approach or empirical formula is used for the design wave force in general. Trætteberg (1968) studied the force reduction of a long structure due to phase lag of waves and he found that wave force decreases with the length of structure.

After a pioneering research of Trætteberg, several researchers have studied the wave force reduction on a long structure and found that the wave force decreases with the obliqueness of wave incidence. In particular, Battjes (1982) studied the wave force reduction of a long structure theoretically under directional random non-breaking wave conditions. Takahashi and Shimosako (1990) and Franco et al. (1996) studied the force

reduction of a long structure experimentally including breaking wave conditions. Martinelli et al. (2007) studied breaking and non-breaking wave force reductions for directional random waves theoretically using statistical method. Their results were only valid when relative length of structure ( $l/L$ ) is less than 1.0. Jung et al. (2011) studied reduced force ratio of a semi-infinite long structure considering diffracting wave forces.

In this study, we develop reduced force ratio on a long detached breakwater considering wave diffraction. Diffracting effect on the detached breakwater may be more dominant than the semi-infinite breakwater because wave diffraction occurs at both ends of the detached breakwater unlike the semi-infinite breakwater. The reduced force ratio on the long detached breakwater is developed separately for regular waves, uni-directional random waves and multi-directional random waves. The solution of Penney and Price (1952) is used to include the diffracting wave force. And, results with different directions of wave incidence and asymmetric parameters are given in detail.

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### REDUCED FORCE RATIO FOR REGULAR WAVES

The definition sketch of topography and variables is shown in Fig. 1. In this figure,  $L$  means the wavelength,  $\theta$  means the angle of wave ray from the normal direction of the structure,  $l$  means the structure length and  $h$  means the mean water depth.

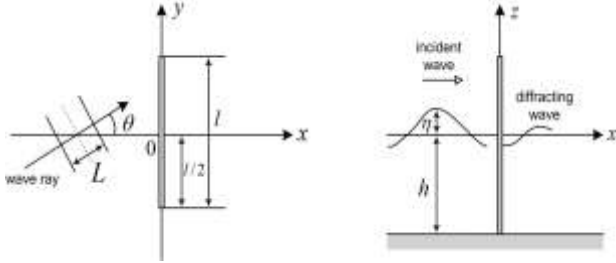


Fig. 1 Definition sketch for analysis of wave forces on a long detached breakwater

Before calculating the reduced force ratio for random waves, we first derive the reduced force ratio for regular wave. Velocity potentials of the incident and perfectly reflected waves can be expressed as

$$\phi_i = -\frac{ag}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t - kx \cos \theta - ky \sin \theta) \quad (1)$$

$$\phi_r = -\frac{ag}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t + kx \cos \theta - ky \sin \theta) \quad (2)$$

where  $a$  is the amplitude of incident waves,  $\omega$  is the angular frequency,  $k$  is the wave number and  $g$  is the gravitational acceleration. The velocity potential of the superposed waves in front of the structure is

$$\phi_f = \phi_i + \phi_r = -\frac{2ag}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t - ky \sin \theta) \quad (3)$$

Velocity potentials of diffracting waves at both ends of a long detached breakwater can be given by

$$\phi_{d1} = -\frac{b_1 g}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t - ky + \chi_1) \quad (4)$$

$$\phi_{d2} = -\frac{b_2 g}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \sin(\omega t + ky + \chi_2) \quad (5)$$

where  $b_1$  and  $b_2$  mean amplitudes of diffracting waves at both ends of the breakwater,  $\chi_1$  and  $\chi_2$  are the phase differences between diffracting waves and incident waves, respectively. Velocity potential of the diffracting waves at the back of the breakwater is given by

$$\phi_d = \phi_{d1} + \phi_{d2} = -\frac{g}{\omega} \frac{\cosh k(h+z)}{\cosh kh} \left[ \begin{array}{l} b_1 \sin(\omega t - ky + \chi_1) \\ + b_2 \sin(\omega t + ky + \chi_2) \end{array} \right] \quad (6)$$

The linear wave pressure at the back of the breakwater can be expressed as

$$\begin{aligned} p_d &= -\rho g z - \rho \frac{\partial \phi_d}{\partial t} \\ &= -\rho g z + \rho g \frac{\cosh k(h+z)}{\cosh kh} \left[ \begin{array}{l} b_1 \cos(\omega t - ky + \chi_1) \\ + b_2 \cos(\omega t + ky + \chi_2) \end{array} \right] \end{aligned} \quad (7)$$

Finally, the total pressure at the breakwater is given by

$$p_t = p_f - p_d = -\rho g \frac{\cosh k(h+z)}{\cosh kh} \left[ \begin{array}{l} 2a \cos(\omega t - ky \sin \theta) \\ - b_1 \cos(\omega t - ky + \chi_1) \\ - b_2 \cos(\omega t + ky + \chi_2) \end{array} \right] \quad (8)$$

The wave force per unit length of the structure can be derived as

$$F_u = \int_{-h}^0 p_t dz = \rho g 2ah \frac{\tanh kh}{kh} \left[ \begin{array}{l} \cos(\omega t - ky \sin \theta) \\ - \frac{b_1}{2a} \cos(\omega t - ky + \chi_1) \\ - \frac{b_2}{2a} \cos(\omega t + ky + \chi_2) \end{array} \right] \quad (9)$$

The total wave force on the whole length of the structure can be expressed as

$$F = \int_{-l/2}^{l/2} F_u dy = \rho g 2ah \frac{\tanh kh}{kh} r(kl, \theta) \sin(\omega t + \beta) \quad (10)$$

where

$$r(kl, \theta) = \sqrt{\begin{array}{l} \left( \begin{array}{l} A - B_1 \cos \chi_1 - C_1 \sin \chi_1 \\ - B_2 \cos \chi_2 + C_2 \sin \chi_2 \end{array} \right)^2 \\ + \left( \begin{array}{l} B_1 \sin \chi_1 - C_1 \cos \chi_1 \\ + B_2 \sin \chi_2 + C_2 \cos \chi_2 \end{array} \right)^2 \end{array}} \quad (11)$$

$$\beta = \sin^{-1} \frac{A - B_1 \cos \chi_1 - C_1 \sin \chi_1 - B_2 \cos \chi_2 + C_2 \sin \chi_2}{\left( \begin{array}{l} \left( \begin{array}{l} A - B_1 \cos \chi_1 - C_1 \sin \chi_1 \\ - B_2 \cos \chi_2 + C_2 \sin \chi_2 \end{array} \right)^2 \\ + \left( \begin{array}{l} B_1 \sin \chi_1 - C_1 \cos \chi_1 \\ + B_2 \sin \chi_2 + C_2 \cos \chi_2 \end{array} \right)^2 \end{array} \right)^{1/2}} \quad (12)$$

$$A = \frac{\sin\left(\frac{kl}{2}\sin\theta\right)}{\frac{kl}{2}\sin\theta} \quad (13)$$

$$B_1 = \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_1}{2a} \cos ky dy, \quad B_2 = \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_2}{2a} \cos ky dy \quad (14)$$

$$C_1 = \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_1}{2a} \sin ky dy, \quad C_2 = \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_2}{2a} \sin ky dy \quad (15)$$

where  $r(kl, \theta)$  means the reduced force ratio. The total force  $F$  is maximum when  $\sin(\alpha + \beta) = 1$ . The phase differences between incident and diffracting waves,  $\chi_1$  and  $\chi_2$ , can be calculated by matching the phases of incident and diffracting waves at  $y = -l/2$  and  $y = l/2$ , respectively, as

$$\chi_1 = -\frac{kl}{2}(1 - \sin\theta), \quad \chi_2 = -\frac{kl}{2}(1 + \sin\theta) \quad (16)$$

When  $b_2 = 0$ , Eq. (10) is reduced to Jung et al.'s (2011) wave force on a semi-infinite breakwater and, when  $b_1 = b_2 = 0$ , Eq. (10) is reduced to Battjes' (1982) wave force neglecting wave diffraction.

#### AMPLITUDES OF DIFFRACTING WAVES

The amplitude of diffracting waves on a long detached breakwater can be calculated using solutions of Penney and Price (1952). The amplitude of waves diffracting through the incident end is given by

$$b_1 = K_{d1} \times a \quad (17)$$

$$K_{d1} = \left| 2f(\psi_1) \exp\left\{-ik\left(y + \frac{l}{2}\right)\cos\left(\frac{\pi}{2} - \theta\right)\right\} \right| \quad (18)$$

Where

$$f(\psi_1) = \frac{1+i}{2} \int_{-\infty}^{\psi_1} \exp\left(-\frac{1}{2}i\pi u^2\right) du, \quad (19)$$

$$\psi_1 = -2\sqrt{\frac{k(y+l/2)}{\pi}} \sin\left[\frac{1}{2}\left(\frac{\pi}{2} - \theta\right)\right]$$

In eq. (19),  $\psi_1 = 0$  when  $y = -l/2$  and the relation of  $f(0) = 1/2$  is true. Also, the relationship of  $K_{d1}(-l/2, \theta) = 1$  is true. The amplitude of waves diffracting in the opposite end is given by

$$b_2 = K_{d2} \times a \quad (20)$$

$$K_{d2} = \left| 2f(\psi_2) \exp\left\{-ik\left(y - \frac{l}{2}\right)\cos\left(\frac{\pi}{2} + \theta\right)\right\} \right| \quad (21)$$

where

$$f(\psi_2) = \frac{1+i}{2} \int_{-\infty}^{\psi_2} \exp\left(-\frac{1}{2}i\pi u^2\right) du,$$

$$\psi_2 = -2\sqrt{\frac{-k(y-l/2)}{\pi}} \sin\left[\frac{1}{2}\left(\frac{\pi}{2} + \theta\right)\right] \quad (22)$$

In eq. (22),  $\psi_2 = 0$  when  $y = l/2$  and the relations of  $f(0) = 1/2$  and  $K_{d2}(l/2, \theta) = 1$  are true. The amplitudes of diffracting waves at the back of the long detached breakwater are shown in Fig. 2. The amplitude of diffracting waves through the incident end increases with the angle of wave incidence. On the contrary, amplitude in the opposite end decreases with the angle of wave incidence. Amplitudes of diffracting waves are symmetric on the basis of  $y = 0$  in Fig. 2.

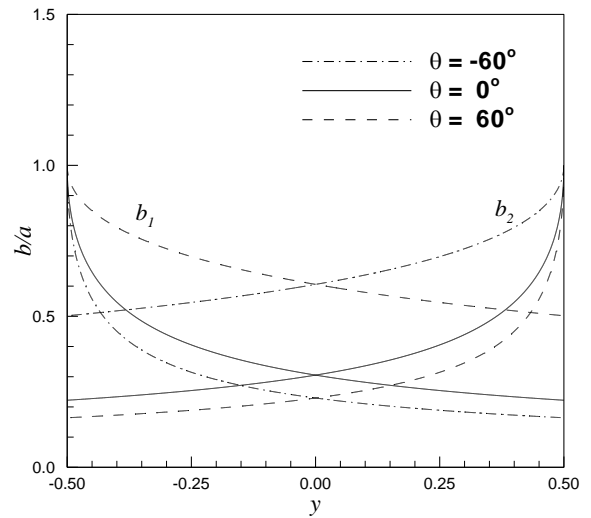


Fig. 2 Normalized amplitude of diffracting waves at the back of a detached breakwater ( $l = L$ )

#### REDUCED FORCE RATIO FOR RANDOM WAVES

In this section, we calculate the reduced force ratio for random waves considering wave diffraction. The Reduction force ratios of uni- and multi-directional random waves are given, respectively, by

$$R_u = \frac{\int_0^{\infty} r^2(kl, \theta) S(f) df}{\int_0^{\infty} S(f) df} \quad (23)$$

$$R_m = \frac{\int_0^{\infty} \int_{-\pi}^{\pi} r^2(kl, \theta) S(f, \theta) d\alpha df}{\int_0^{\infty} \int_{-\pi}^{\pi} S(f, \theta) d\alpha df} \quad (24)$$

The reduced force ratios for incident waves, diffracting waves in the incident end in the opposite end can be defined as

$$r_i(kl, \theta) = A \quad (25)$$

$$r_{d1}(kl, \theta) = \sqrt{\frac{(-B_1 \cos \chi_1 - C_1 \sin \chi_1)^2}{(B_1 \sin \chi_1 - C_1 \cos \chi_1)^2}} \quad (26)$$

$$r_{d2}(kl, \theta) = \sqrt{\frac{(-B_2 \cos \chi_2 + C_2 \sin \chi_2)^2}{(B_2 \sin \chi_2 + C_2 \cos \chi_2)^2}} \quad (27)$$

When a length of long detached breakwater is convergent to 0,  $\chi_1$  and  $\chi_2$  are equal to 0 and the following relations is reduced

$$\lim_{l \rightarrow 0} A = \lim_{l \rightarrow 0} \frac{\sin\left(\frac{kl}{2} \sin \theta\right)}{\frac{kl}{2} \sin \theta} = 1.0 \quad (28)$$

$$\lim_{l \rightarrow 0} B_1 = \lim_{l \rightarrow 0} \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_1}{2a} \cos ky dy = 0.5 \quad (29)$$

$$\lim_{l \rightarrow 0} C_1 = \lim_{l \rightarrow 0} \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_1}{2a} \sin ky dy = 0 \quad (30)$$

$$\lim_{l \rightarrow 0} B_2 = \lim_{l \rightarrow 0} \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_2}{2a} \cos ky dy = 0.5 \quad (31)$$

$$\lim_{l \rightarrow 0} C_2 = \lim_{l \rightarrow 0} \frac{1}{l} \int_{-l/2}^{l/2} \frac{b_2}{2a} \sin ky dy = 0 \quad (32)$$

When the length of the structure is reduced to 0, the relationships of  $r_i(0, \theta) = 1.0$ ,  $r_{d1}(0, \theta) = r_{d2}(0, \theta) = 0.5$  and  $r(0, \theta) = 0$  are valid.

Both regular wave and random waves are considered for incident waves. We use the JONSWAP frequency spectrum (Hasselmann et al. 1973) and the directional spreading function of Lee et al. (2010) which considers directional asymmetry.

The total frequency spectrum is decomposed into 45 components from  $f = 0.02$  to  $0.25\text{Hz}$  and the directional spreading function is decomposed into 181 components from  $-\pi/2$  to  $\pi/2$ . So totally 8,145 components of waves are calculated and the results are superposed in order to get the total wave energy. The conditions are  $h = 10\text{m}$ ,  $H_{1/3} = 5.0\text{m}$  and  $T_{1/3} = 10\text{sec}$

### CHARACTERISTICS OF WAVE FORCES ON A LONG DETACHED BREAKWATER

The reduced force ratio on the long detached breakwater is calculated using eq. (11). The reduced

force ratios are shown in Fig. 3 considering the phase difference between the incident and diffracting waves. However, Jung et al. (2011) calculated the reduced force ratio neglecting the phase difference and they selected  $\chi_1$  and  $\chi_2$  which give the maximum total wave force. The reduced force ratio in this study is 0 when the relative length of the structure is close to 0, but the result of Jung et al. (2011) is 2.0. When the relative length of the structure is greater than unity, the reduced force ratio in this study is almost equal to that of Jung et al. (2011) because the effects of diffracting wave decrease with the relative length of the structure.

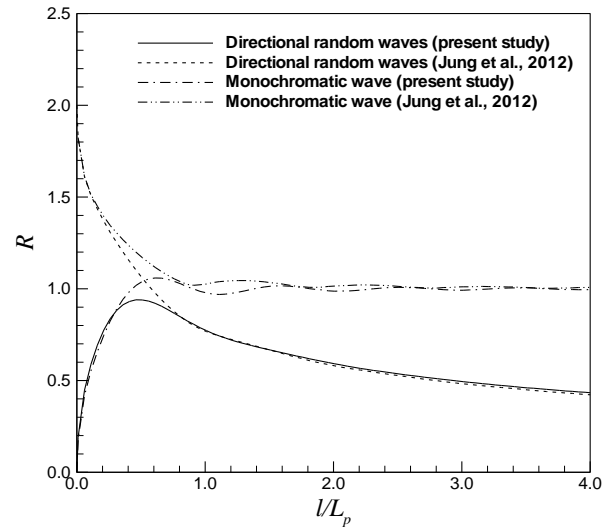


Fig. 3 Comparison of reduced force ratio between the present study and Jung et al.'s (2012) study ( $\theta_p = 0^\circ$ ,  $\gamma = 3.3$ ,  $s_{\max} = 10$ )

The reduced force ratios when incident wave angle is 0 and 60 degree are shown in Fig. 4. The reduced force ratio of the long detached breakwater is 0 when the relative length of structure is 0. This is because incident wave force is equal to the diffracting wave force. The Reduced force ratio for long-crested waves converges to 1.0 when the relative length of the structure is significantly long. However, the reduced force ratio for short-crested waves decreases as the relative length of the structure becomes longer.

The reduced force ratios for incident wave angle of multi-directional random waves are shown in Fig. 5. The reduced force ratios decrease as the magnitude of incident wave angle increases. And the reduced force ratios for directional asymmetric waves are shown in Fig. 6. The reduced force ratios increase when incident wave angle is positive and the reduced force ratios decrease when incident wave angle is negative as the asymmetry parameter decreases.

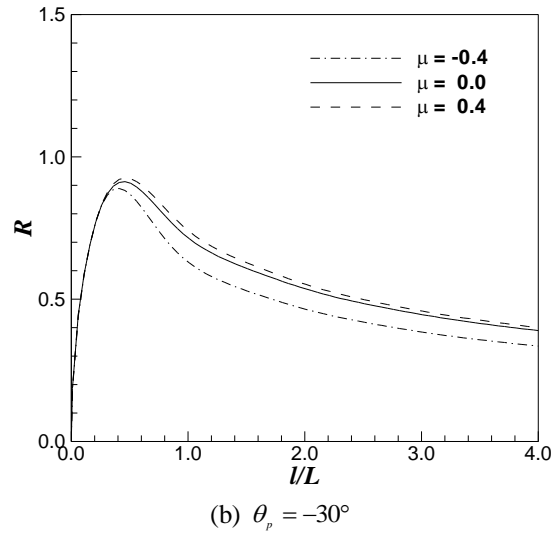
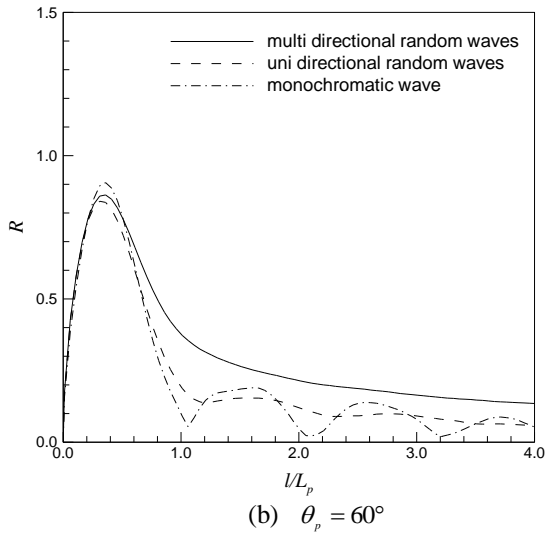
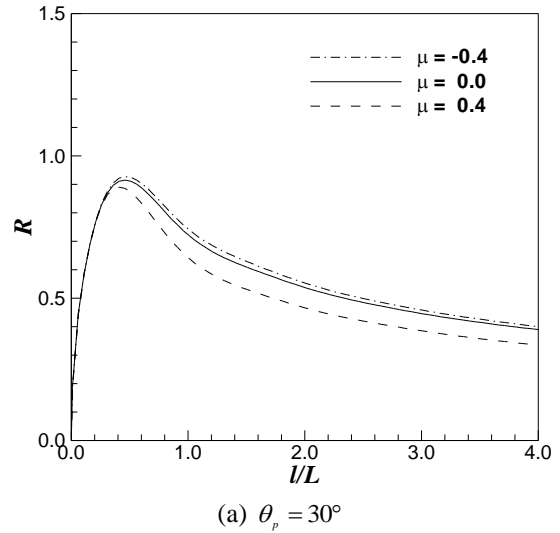
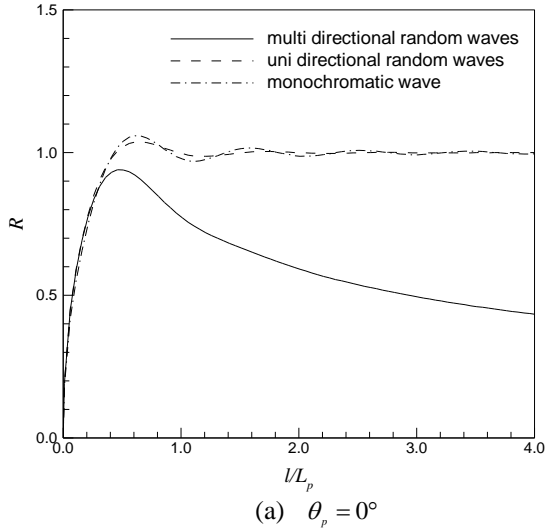


Fig. 4 Reduced force ratios with different wave conditions ( $\gamma = 3.3$ ,  $s_{\max} = 10$ )

Fig. 6 Reduced force ratios with different directional asymmetries ( $\gamma = 3.3$ ,  $s_{\max} = 10$ )

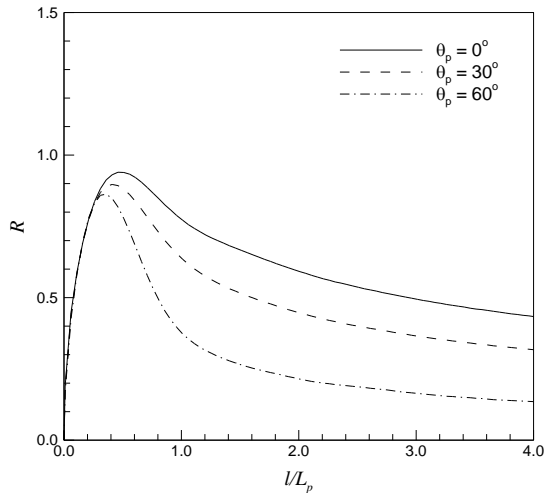


Fig. 5 Reduced force ratios with different angles of wave incidence ( $\gamma = 3.3$ ,  $s_{\max} = 10$ )

**CONCLUDING REMARKS**

In this study, the wave force on a long detached breakwater is investigated by considering phase difference between incident and diffracting waves. We study for incident waves which are regular waves, uni-directional random waves and multi-directional random waves. Amplitude of diffracting waves is calculated using solutions suggested by Penney and Price (1952).

When relative length of structure compared to the wavelength is 0, the reduced force ratio of a detached breakwater is 0. As the relative length of the structure increases the reduced force ratio increases rapidly and approach the peak value (about 0.9~1.0) And, when the relative length of the structure increases over about 0.5, the reduced force ratio decreases. The reduced force ratios decrease as the magnitude of incident wave angle increases. And the reduced force ratios increase when

incident wave angle is positive and the reduced force ratios decrease when incident wave angle is negative as asymmetry parameter decreases.

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