Proceedings of the 7th International Conference on Asian and Pacific Coasts
(APAC 2013) Bali, Indonesia, September 24-26, 2013

WAVE-CURRENT FORCE ON BRIDGE FOUNDATION

J. N. Pan [1], D. T. Wang [1], X. G. Wang [1], F. Nie [1]

ABSTRACT : Many cross-sea bridges have been constructed or in construction in the Chinese coastal areas, such as Hangzhou Bay Bridge, East China Sea Bridge and Hong Kong-Zhuhai-Macau Bridge, etc. Usually the foundations of cross-sea bridges will bear the load of wave and current simultaneously, the extreme value of which is an important factor for bridge design. It is quite difficult to estimate the extreme wave-current forces on bridge foundations due to their complex form, for example a polygonal platform pier shape on piles. In this paper, we use a combination method to calculate wave-current force on such complex bridge foundation. The Morison equation is applied to calculation wave-current load on piles with small diameters (D/L<0.2) and the effect of wave phase difference between piles is considered when calculating the total force on all piles. For bridge foundation strctures of large diameters (D/L>0.2), wave force and curent force are calculated seperately and then added together, i.e. wave force is calculated by Boundary Element Method based on wave diffraction theory, while current force is calculated by the drag force formula. Some physical model tests are conducted for different types of bridge foudations, and wave-current forces are measured by force sensors. The calculation method is verified by the physical model and good agreement is obtained between calculation results and model tests data. It is revealed that the nonlinear wave-current interaction effect on force is not significant in large diameter stucture case, but the wave parameters used in the wave force model should be the ones affected by the current.

Keywords: Wave, current, force, bridge foundation

INTRODUCTION

Usually cross-sea bridges are constructed across estuary, bay or strait with formidable natural conditions including deepwater, rough sea and strong currents, such as Donghai Bridge, Hangzhou Bay Bridge and Hong Kong-Zhuhai-Macau Bridge, etc. Since the foundations of cross-sea bridges will bear considerable wave-current load, exactitude calculation of wave-current is the important guarantee for the safety, economical efficiency and rationality of the engineering design. However, the foundations of most cross-sea bridges are of irregular composite structure, such as the composite foundation of polygonal pile cap in combination with pile foundation, so it is quite different to make exactitude calculation of the wave-current force on it.

Many studies have shown that Morison equation is applicable to calculation of wave-current load on piles with small diameters, and wave diffraction theory is applicable for calculating wave load on large bodies (Sarpkaya and Isaacson, 1981). For the load of wave and weak current on large structures, Boundary Element Method (BEM) can be applied to obtain approximate solution based on potential flow theory (Tao and Liu, 1993; Teng and Eatock, 1995). For a complex structure, the wave-current force is usually determined through physical model experiment instead of numerical model, especially for the composite foundation structure with piles and pile cap like Hangzhou Bay Bridge (Pan and Wang 2003) and Donghai Bridge (Lan et al. 2005).

To estimate wave-current load on bridge foundation simply and practically, a combination calculation method is adopted in this paper, and its results are verified by physical model tests. Regarding different bridge foundation structures, physical model tests have been carried out on circular cylinder, square and rectangular piles and a composite foundation structure, and the wave-current force was measured by force sensors. In the combination calculation method, Morison equation is applied to calculation of the wave-current force on piles with small diameter (D/L<0.2), and the effect of wave phase difference between piles is considered when calculating the total force on all piles. For bridge foundation structures of large diameters (D/L>0.2), wave force and current force are calculated separately and then added together, i.e. wave force is calculated by boundary element method based on wave diffraction theory, while current force is calculated by drag force formula.

CALCULATION METHOD

Wave-current force on small structures

According to the Code of Hydrology for sea Harbor (JTJ/T213-98) (Ministry of Communications of the

[1] River and Harbor Department, Nanjing Hydraulic Research Institute, Hujuguan Road 34, Nanjing, 210024, CHINA

People's Republic of China 1998), Morison equation is applicable for calculation of the wave-current force on piles with small diameter (D/L<0.2, D is the diameter or the characteristic scale of piles, L is the wavelength). When wave and current coexist, wave will deform under the action of current, so wave parameters in current should be used to calculate the wave-current force. The wave-current force per unit length acting in forward direction at the height of z above sea bed comprises inertia force and drag force, which can be calculated with the following equation

$$f(z,t)=f_i(t)+f_d(t) = \frac{1}{4}C_M\left(\frac{\gamma}{g}\right)\pi D^2\dot{u}_l(z,t) + \frac{1}{2}C_D\left(\frac{\gamma}{g}\right)D\left[u_l(z,t)+U\right]\left|u_l(z,t)+U\right| \quad (1)$$

where, f_i and f_d are the inertia force and drag force on piles respectively, γ the gravity of water, D the diameter of piles, C_M and C_D coefficient of inertia force and drag force respectively, u_l and $\dot{u}_l$ velocity and acceleration of water particle respectively, and U the current velocity. The coefficient of drag force (C_D) and that of inertia force (C_M) under the combination action of wave and current are subject to Keulegan-Carpenter number (KC), and can be determined according to the Code of Hydrology for sea Harbor (JTJ/T213-98). u_l and $\dot{u}_l$ can be obtained with the following equation

$$\begin{aligned} u_l(z,t) &= \frac{\omega_r H}{2}\frac{\cosh kz}{\sinh kd}\cos(kx_l\cos\theta+ky_l\sin\theta-\omega t) \\ \dot{u}_l(z,t) &= -\frac{\omega_r^2 H}{2}\frac{\cosh kz}{\sinh kd}\sin(kx_l\cos\theta+ky_l\sin\theta-\omega t) \end{aligned} \quad (2)$$

where, ω_r represents the angular frequency of wave in current, H wave height in current, k wave number in current, and d water depth.

When wave and current in the same direction (including forward and reverse), wave parameters under the action of current can be calculated with the following equations

$$\omega_r=\omega-kU \quad (3)$$

$$\frac{H}{H_j}=\sqrt{\left(1-\frac{U}{C}\right)\frac{L_j}{L}\frac{A_j}{A}}\Bigg/\sqrt{1+\frac{U}{C}\frac{2-A}{A}} \quad (4)$$

$$A=1+\frac{2kd}{\sinh 2kd} \quad (5)$$

$$A_j=1+\frac{2k_jd}{\sinh 2k_jd} \quad (6)$$

$$\frac{L}{L_j}=\frac{C}{C_j}=\frac{\tanh kd}{\left(1-\frac{U}{C}\right)^2\tanh k_jd} \quad (7)$$

where, H_j means wave height in still water, U current velocity, C wave celerity in current, A and A_j transfer rate of wave energy in current and still water respectively, ω the angular frequency of wave in still water, L and L_j wave length in current and still water respectively, and $k=2\pi/L$ and $k_j=2\pi/L_j$ wave number in current and still water respectively.

For the total wave-current force on pile group, the wave phase difference between piles due to different pile locations should be considered. The total normal force per unit length on pile group acting at the height of z above sea bed will be obtained by superposition of wave forces on each pile at the same time, namely

$$f(z,t)=\sum_{l=1}^{N}\left[f_i(t)+f_d(t)\right]_l \quad (8)$$

where, N is the number of piles of the group.

The total horizontal force on pile group can be obtained by integration in vertical direction, namely

$$F(t)=\int_0^{Z_T} f(z,t)dz \quad (9)$$

$$M(t)=\int_0^{Z_T} f(z,t)zdz \quad (10)$$

where, z_T refers to the height of pile top from sea bed, and if the pile top is higher than the still water surface, then $z_T=d$.

Wave-current force on large structures

(1) Wave force

Potential flow theory is applied to calculating the wave force on large structures (D/L>0.2). The total velocity potential Φ in wave field can be expressed as

$$\Phi(x,y,z,t)=\left[\phi_I(x,y,z)+\phi_S(x,y,z)\right]e^{-i\omega t} \quad (11)$$

Where, ϕ_I and ϕ_S denotes the velocity potential of the incident waves and that of the scattered waves respectively.

According to the potential flow theory (Sarpkaya and Isaacson, 1981), the velocity potential of the scattered waves ϕ_S can be expressed as

$$\phi_S(x,y,z)=\frac{1}{4\pi}\iint_S f(\xi,\eta,\zeta)G(x,y,z;\xi,\eta,\zeta)ds \quad (12)$$

where, $f(\xi, \eta, \zeta)$ is the source strength distribution function on the structure surface, and $G(x, y, z; \xi, \eta, \zeta)$ is the Green's function of a point wave source of unit length at the point (ξ, η, ζ).

The boundary condition of zero fluid velocity normal to the body surface can be written as

$$-f(x,y,z)+\frac{1}{2\pi}\iint_S f(\xi,\eta,\zeta)\frac{\partial G}{\partial n}(x,y,z;\xi,\eta,\zeta)ds = -2\left.\frac{\partial \phi_I}{\partial n}\right|_S \quad (13)$$

Here, n is the distance normal to the body surface at the point (x, y, z).

BEM is applied to solve the above boundary integral equations, by dividing up the body surface into N small facets with the area of ΔS_j (j=1, 2…N). Selecting the centroid (x_j, y_j, z_j) of each facet as node, and assuming source strength f_j on each facet is constant, and then Eq. (13) after discretization may be written as below:

$$\alpha_{ij} f_j = b_i \quad (14)$$

Where, $\alpha_{ij}=-\delta_{ij}+\frac{1}{2\pi}\iint_{\Delta S_j}\frac{\partial G}{\partial n}(x_i,y_i,z_i;\xi,\eta,\zeta)ds$, δ_{ij} is Kronecker delta, and $b_i=-2\frac{\partial \phi_I(x_i,y_i,z_i)}{\partial n}$. Then ϕ_S can be determined by using a discrete version of Eq. (12) as

$$\phi_S(x_i,y_i,z_i)=\beta_{ij} f_j \quad (15)$$

in which, $\beta_{ij}=\frac{1}{4\pi}\iint_{\Delta S_j} G(x_i,y_i,z_i;\xi,\eta,\zeta)ds$. Wave force F_w is subsequently evaluated as below

$$F_w=-\iint_S \rho\frac{\partial \Phi}{\partial t}\vec{n}ds \quad (16)$$

in which ρ is water density.

(2) Current force

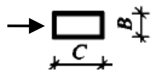
The flow drag force F_D can be evaluated by the following formula (JTS 144-1-2010, Ministry of Communications of the People's Republic of China 1998)

$$F_D=C_D\frac{\rho U^2}{2}A \quad (17)$$

where, C_D denotes the drag force coefficient, A is the projected area of the object on a plane normal to the flow, and U is the incident flow velocity undisturbed by the structure.

The values of coefficient C_D for pillars of some different shapes provided in the Code for Loads of Port (JTS 144-1-2010) are listed in Table 1:

Table 1 Flow Drag force Coefficient C_D of pillars

Shape	Diagram	C_D value			
Rectangle		C/B=1.0	1.5	2.0	≤3.0
		C_D=1.50	1.45	1.30	1.10
Round		C_D=0.73			

VERIFICATION FOR THE CALCULATION OF LARGE CYLINDER FOUNDATIONS

Physical model test results

A physical model test in wave flume has been adopted to verify the computing method of large-scale cylinder wave-current force. The test was carried out in the wave flume in Nanjing Hydraulic Research Institute, which can produce wave, current and wind. The flume has a length of 64m, a width of 1.8 and a depth of 1.8m.

Three kinds of cylinder structures have been adopted by the test: circular cylinder, square pillar and rectangular pillar. The diameter of circular cylinder is 25cm, the side length of the square pillar is 25cm and the dimensions of the rectangular column is 25cm×50cm (the short side length parallels with the wave crest line). The three cylinders have a same height of 85 cm. The water depth in the test is 50cm. The flow velocities are 10cm/s and15cm/s, respectively. The wave heights are H=2cm, 3cm and 4cm respectively and the wave periods are 0.6s, 0.7s and 0.8s respectively.

Before testing, the cylinder is installed in the test section of the flume (see Fig. 1). A force transducer is set on the front of the cylinder and two force transducers on the back, and some pulleys are installed between the cylinder bottom and the smooth flume bed.

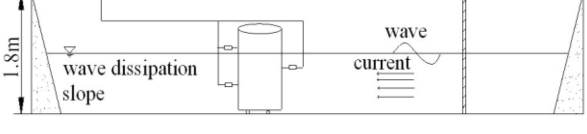


Fig.1 Layout of the flume

In the model tests, flow force, wave force and wave-current force have been measured successively. When measuring the wave force (F_W), although there's no flow, the wave parameters under the influence of flow have been adopted to ensure its wavelength be the same with that under the coexistence of wave and flow. It can be seen from the test results shown in Table 2 that the linear superposition value of wave force and flow force is 0.90~1.06 times of the wave-current force under the combined action of wave and flow, in which the average value for data less than 1.0 is about 0.96. That means the sum of wave force and flow force is slightly less than the actual wave-current force generally, which may be due to the nonlinear wave-current interaction. Therefore, a modification factor of 1.04 may be adopted in evaluating wave-current force for sake of engineering safety, i.e. the wave-current force can be obtained by multiplying the sum of wave force and flow force by 1.04.

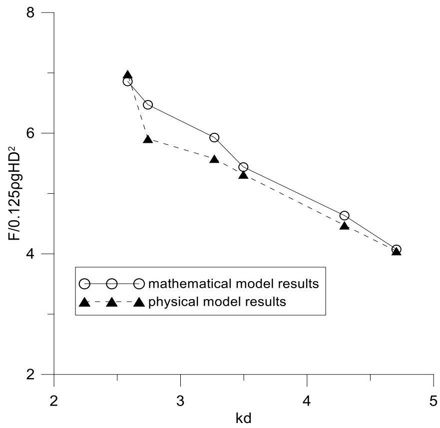
Table 2 Test results

Test conditions			$(F_W+ F_D)/F$		
U / $cm\cdot s^{-1}$	H /cm	T /s	A	B	C
10	2	0.6	1.02	0.99	0.97
10	3	0.6	1.00	0.95	0.95
10	4	0.6	0.98	0.95	0.93
10	2	0.7	1.02	1.06	0.94
10	3	0.7	0.95	0.95	0.98
10	4	0.7	0.96	0.96	1.02
10	2	0.8	0.99	0.92	0.96
10	3	0.8	0.96	0.96	0.95
10	4	0.8	1.03	0.97	0.99
15	2	0.6	1.05	0.93	0.92
15	3	0.6	1.00	0.99	0.94
15	4	0.6	1.00	1.04	0.94
15	2	0.7	1.05	0.98	0.97
15	3	0.7	1.01	1.01	1.00
15	4	0.7	1.01	0.96	1.04
15	2	0.8	0.95	0.99	0.98
15	3	0.8	0.91	0.98	0.99
15	4	0.8	0.90	0.96	0.98

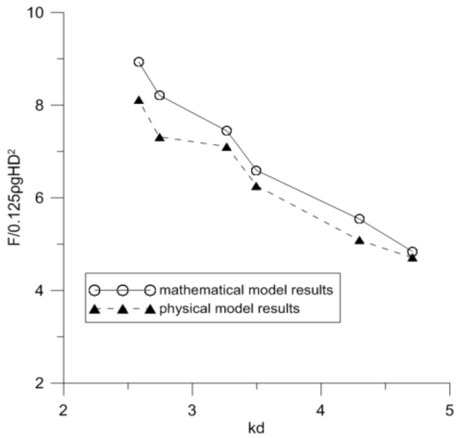
Notes: A- Circular Cylinder, B- Square pillar, C- Rectangular pillar

Comparison between the results of calculation and model test

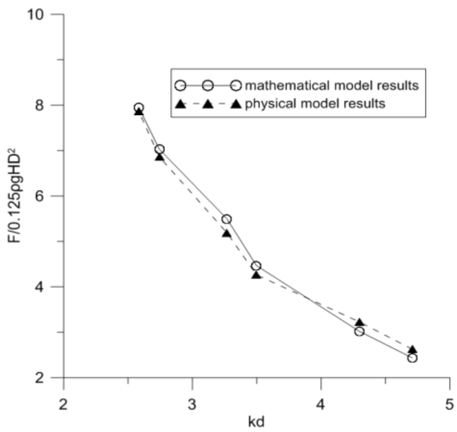
The numerical calculation method for large structures was adopted to calculate wave-current force on the pillar foundations under the test conditions and calculation results were compared with the test data. The computed value of the wave-current force was gained by adding wave force with flow force then multiplying by a modification factor of 1.04. Fig 2 shows both the results of mathematical model and those of physical model with wave height H=4cm for circular cylinder, square pillar and rectangular pillar respectively. In the figures, k indicates wave number and d indicates water depth. It can be seen from the Fig 2 that, the results of the calculation method coincides with the test results generally and is slightly conservative for circular cylinder and Square pillar.



(a) Circular Cylinder



(b) Square pillar



(c) Rectangular pillar

Fig.2 Comparison between the results of mathematical model and the model test data

VERIFICATION FOR THE CALCULATION OF COMPLEX BRIDGE FOUNDATION

The calculation method has been applied to evaluate the wave-current force on a complicated bridge foundation, on which physical model test has also been conducted to measure the wave-current load. The structure pattern of the bridge foundation is a composite of piles and a pile cap. The cap is 38.5m wide and 59.5m long. The bottom elevation of the cap is -4.2m and the crest level is +3.8m with a thickness of 8.0m. The pile foundation under the cap consists of 36 piles each with a diameter of 2.8m, located at the point of -20m water bottom elevation. The plane layout of the bridge foundation is shown in Fig 3.

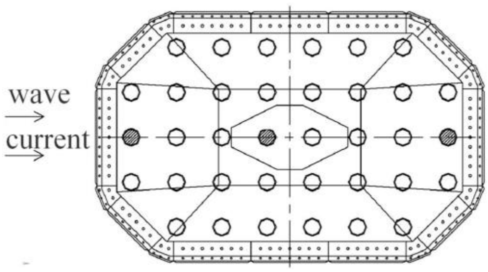


Fig.3 Plane layout of the bridge cap and pile foundation

The physical model test has been carried out in a wave basin of Nanjing Hydraulic Research Institute, which is 50m in length, 17.5m in width and 1.2m in height (refer to Fig 4) and can make wave and current simultaneously. The model scale is 1:50 and water level, and wave and current are generated in the same direction from left to right. Wave parameters in still water and current velocities adopted in the test are shown in Table 4.

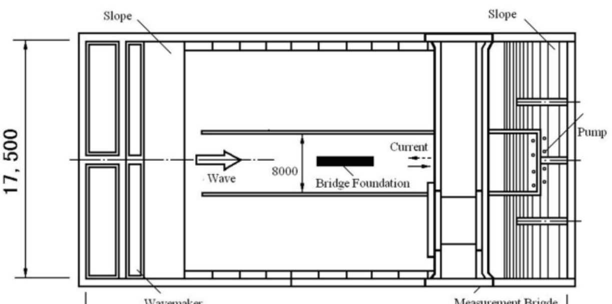


Fig. 4 Schematic diagram of layout of wave tank

Table 4 lists wave-current loads on the cap and the piles of the bridge foundation measured by force transducers in the model test and those evaluated by the calculation method. In calculation, the wave parameters in current are calculated by Eqs. (3) ~ (7) firstly. The cap is treated as large structure as $D/L>0.2$, and the calculated wave-current force on the cap is the result by multiplying the sum of wave force calculated by Eq. (16) and flow force calculated by Eq. (17) with a correction factor of 1.04. The piles are treated as small structure, and the wave-current force on piles is calculated by Eqs. (8) and (9). It can be seen from the data in Table 4 that: the calculated data coincides with the physical model results generally, the maximum difference between the calculated data and the physical model results is within 17%.

Table 4 comparison between the computed wave-current forces and the test values

Structure	Water level /m	H /m	T /s	U /m•s^{-1}	Test value /kN	Calculated value /kN
Cap	1.65	4.54	10.2	1.68	10025	10595
	1.65	4.50	9.3	1.57	8749	10324
Piles	1.65	4.54	10.2	1.68	3472	3696
	1.65	4.50	9.3	1.57	3017	3173

CONCLUSIONS

Generally, the wave-current force on the cross-sea bridge foundation with a complicated structure should be measured by physical model tests. This paper attempts to adopt a combined method to estimate the wave-current force in the cap-pile composite foundation. Morison equation is applied to calculate the wave-current force on the pile foundation (D/L<0.2) and take the influence of phase difference of waves between each pile into consideration; as for the cap structure (D/L>0.2), the boundary element method is adopted to calculate the wave force according to the potential flow theory and Morison equation is applied to calculate the flow force, then add the two together.

Physical model tests on large-scale cylinder structure with different shapes and typical cap-pile composite foundation have been carried out with a combined action of wave and current and force transducers have been adopted to measure the wave-current force. The wave-current force value calculated by the method in this paper can comparatively coincide with the test results. The research result shows that there's slightly influence of nonlinear wave-current interaction on larger structures. The ratio between the sum of pure wave force and pure flow force and the wave-current force is about 0.96. In practical application, one can multiply the sum of wave force and flow force by the correction factor of 1.04 to evaluate the wave-current load. As wave parameters in current can be changed by flow, the influence of flow to wave height, wave length and wave frequency should be taken into consideration in the calculation model.

The experimental groups of this research are limited and it's necessary to carry out more experimental researches in the future, especially tests under the condition of strong waves and strong currents to further verify and improve the calculation method.

REFERENCES

Lan, Y.M., Liu, H., Huang, F.X., Xue, L.P., Chen, G. (2005). Experimental studies on hydrodynamic loads on piles and slab of Donghai bridge, Part II: hydrodynamic forces on pile array and slab in wave-current combinations. Journal of Hydrodynamics. 20(3): 332-339.

Ministry of Communications of the People's Republic of China (1998). Code of Hydrology for sea Harbor (JTJ/T213-98). China Communications Press, Beijing.

Ministry of Communications of the People's Republic of China,.(2010). Load code for harbor engineering (JTS 144-1-2010). China Communications Press, Beijing.

Pan, J., Wang, D.T. (2003). Model test research on wave-current force on the foundations of Hangzhou Bay Bridge. Research Report, Nanjing Hydraulic Research Institute, China.

Sarpkaya, T. and Isaacson, M. (1981). Mechanics of wave Forces on Offshore Structures. Van Nostrand Reinhold Co., New York.

Teng, B., Eatock, T. R. (1995). New higher-order boundary element methods for wave diffraction/ radiation. Applied Ocean Research. 17:71-77.

Tao, J., Liu, L. (1993). Wave-Current Forces on a Large Vertical Cylinder. Journal of Hydrodynamics. 8(3): 265-272.