

NONLINEAR EVOLUTION OF WAVE GROUPS IN DIRECTIONAL SEA

N.N. Pujianiki ¹ and W. Kioka ²

ABSTRACT: Nonlinear wave-wave interaction behavior in deep and intermediate water depths and also on a sloping beach are investigated using third-order Zakharov equation which is known as a superior model to predict the evolution of wave group without restriction on spectra width. Transfer energy occurs between the waves components when resonant conditions satisfy. It has been found that nonlinear transfer of energy controls the shape of directional spectrum, including development of the peak and wave group evolution for wave steepness $ak_p \geq 0.2$. The comparison of wave group evolutions on directional spectra with unidirectional spectra indicates that evolution of wave groups in deep water and at intermediate water depths are significantly affected by nonlinear interactions between directional components. When directional effect is considered, transformation of wave groups in deep water is much more pronounced at $ak_p = 0.2$. The effects of wave interaction are enhanced in relatively shallow water; however, is reduced on a sloping beach, which decreases the maximum wave height.

Keywords: Nonlinear wave, wave-wave interaction, wave groups, directional spectra.

INTRODUCTION

Ocean waves have a complex pattern and are random in amplitude, period and direction. According to Goda (2009) although sea waves may look random, inspection of wave records indicates that high waves fall into groups rather than emerge individually. Wave grouping and associated nonlinear effects play an important role for some coastal issues such as wave overtopping, wave run-up and sedimentation. However, the number of works dealing with nonlinear aspects of directional wave group transformation is still limited.

The nonlinear interaction of gravity waves has been a subject of interest for many years. The interaction produces only a small modification to the motion in the second-order, which remains bounded in time. In the third approximation, it is possible for a transfer of energy to take place from three primary waves to a fourth wave, in such a way that the amplitude of the fourth wave increases linearly with time (Longuet-Higgins 1961).

According to Shemer et al. (2001), the third-order Zakharov equation is generally accepted as a superior model for the description of the evolution of nonlinear water waves. This equation has been examined by some investigators, for instance Kit et al. (2000), Kit and Shemer (2002), Stiassnie and Shemer (1984, 2005), Kioka et al. (2005, 2011), Janssen (2003), Janssen and Onorato (2007), Stiassnie and Gramstad (2009). Unfortunately, the previous Zakharov models use a very narrow band and do not include the effect of directionality, which is possibly significant for wave-wave interaction. In the present study, nonlinear aspects

of directional spectra are first investigated at constant depths and on a sloping beach and then the transformation of wave groups are analyzed. The initial conditions for the numerical simulations are characterized by a Gaussian spectrum for several values of wave steepness and relative water depths. Wave group evolutions of directional spectra are compared with the results from unidirectional spectra to investigate the directional effect.

According to Shemer et al. (2001), the initial-phase in the complex wave spectra is essential in determining the eventual shape of the surface elevation variation. To eliminate the effects of initial random phases, the comparison of wave group evolutions in the current study is conducted using the same initial random phases.

To demonstrate the transformation of wave groups in nature, field observation data at Akabane beach were used. The characteristic of nonlinear spectral evolution propagating over the continental shelf toward the coastal region are investigated using the directional components from field observations, and are then related to the evolution of the amplitude.

THEORETICAL MODELS

The Third-order Zakharov Equation

The third-order Zakharov integral equation which describes slow temporal evolution of gravity waves in water of infinite depth firstly was derived by Zakharov

¹ Department of Civil Eng., Udayana University, Kampus Bukit, Jimbaran, Bali INDONESIA

² Department of Civil and Environmental Eng., Nagoya Institute of Technology, Gokiso-cho, Showa-ku, Nagoya JAPAN

(1968). In order to describe the spatial evolution of gravity waves in water of infinite/finite depth, Shemer et al. (2001) have modified the third-order Zakharov model, into the form

$$ic_g \nabla_h B = \iint \int_{-\infty}^{\infty} T(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) B_1^* B_2 B_3 \times \delta(\omega + \omega_1 - \omega_2 - \omega_3) \exp\{-i(\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3) \cdot \mathbf{x}\} d\omega_1 d\omega_2 d\omega_3 \quad (1)$$

where B denotes the complex amplitude, $*$ is the complex conjugate, δ is the Dirac δ -function, c_g is the group velocity, ∇_h is the horizontal gradient and the kernel $T(\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is given in Stiassnie and Shemer (1984) and corrected by Mase and Iwagaki (1986). This spatial Zakharov equation describes evolution of the complex amplitude B of each free wave in the spectrum due to four-wave interaction in mild slope ($|\nabla_h| \leq O(\varepsilon^2)$) space domain, which satisfies the near resonant condition:

$$\omega + \omega_1 - \omega_2 - \omega_3 = 0, |\mathbf{k} + \mathbf{k}_1 - \mathbf{k}_2 - \mathbf{k}_3| \leq O(\varepsilon^2) \quad (2)$$

where ε is a small parameter representing the magnitude of nonlinearity, and the wave vectors $\mathbf{k}, \mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3$ and the frequencies $\omega, \omega_1, \omega_2, \omega_3$ each satisfy the following dispersion relation, with h being the water depth:

$$\omega^2 = g|\mathbf{k}| \tanh|\mathbf{k}|h \quad (3)$$

The mode-coupled discrete Zakharov equation can be written as

$$ic_g \nabla_h B = T(\mathbf{k}_j, \mathbf{k}_j, \mathbf{k}_j, \mathbf{k}_j) |B_j|^2 B_j + \sum_{n \neq j} 2T(\mathbf{k}_j, \mathbf{k}_n, \mathbf{k}_j, \mathbf{k}_n) |B_n|^2 B_j + \sum_{\substack{p, q \neq j \\ 2\mathbf{k}_j = \mathbf{k}_p + \mathbf{k}_q}} T(\mathbf{k}_j, \mathbf{k}_j, \mathbf{k}_p, \mathbf{k}_q) B_j^* B_p B_q \times \exp\{-i(2\mathbf{k}_j - \mathbf{k}_p - \mathbf{k}_q) \cdot \mathbf{x}\} + \sum_{\substack{n, p, q \neq j \\ \mathbf{k}_j + \mathbf{k}_n = \mathbf{k}_p + \mathbf{k}_q}} T(\mathbf{k}_j, \mathbf{k}_n, \mathbf{k}_p, \mathbf{k}_q) B_n^* B_p B_q \times \exp\{-i(\mathbf{k}_j + \mathbf{k}_n - \mathbf{k}_p - \mathbf{k}_q) \cdot \mathbf{x}\} \quad (n, p, q = 1, 2, \dots, N) \quad (4)$$

The set of mode-coupled nonlinear complex ordinary differential equations is solved using the fourth-order Runge-Kutta method. When calculating the kernel in Eq. (1), we have introduced Stokes' corrections to remove

near-resonance singularities. Nevertheless, Eq. (1) is invalid for water of very shallow depth; the equation requires that the dispersion remain sufficiently strong (see Agnon 1993). The first-order free surface elevation $\eta(x, t)$ is related to the quantity B and computed through

$$\eta(x, t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left(\frac{\omega(\mathbf{k})}{2g} \right)^{1/2} [B(\mathbf{k}, t) \exp\{i(\mathbf{k} \cdot \mathbf{x} - \omega(\mathbf{k})t)\} + *] \quad (5)$$

Wave Group Structure

The structure of wave groups can be quantitatively described using a wave envelope. The wave envelopes of various frequency bands can be calculated using a Hilbert transform. If the sea surface elevation $\eta(t)$ is a stationary random function of time, then the Hilbert transform $\xi(t)$ is given by

$$\xi(t) = \frac{1}{\pi} P \int_{-\infty}^{\infty} \frac{\eta(t')}{t - t'} dt' \quad (6)$$

where P indicates the Cauchy value. With the Hilbert transform $\xi(t)$ of the function $\eta(t)$, the analytic function is given as

$$S(t) = \eta(t) + i\xi(t) = A(t) \exp\{i\varphi(t)\} \quad (7)$$

The wave envelope $A(t)$ can then be obtained by

$$A(t) = [\eta^2(t) + \xi^2(t)]^{1/2}, \quad (8)$$

The envelope $A(t)$ is always symmetrical with respect to the t -axis, as $\eta(t)$ is composed of only first-order free waves. Only the fundamental frequency band $0.5f_p \sim 1.5f_p$, which produces free waves only and does not include the bound waves, is considered and calculated. The amplitude A_{ave} denotes the average value of the envelope amplitude (see Fig. 1).

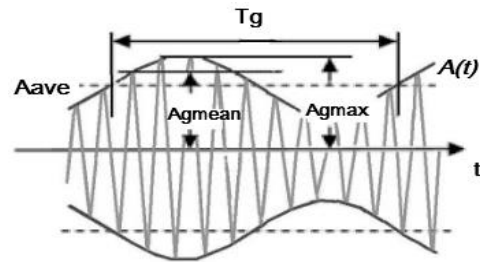


Fig. 1 Definition of wave group structures.

The zero-up cross method relative to A_{ave} is used to determine the wave group period Tg . The wave group amplitudes Ag_{mean} and Ag_{max} denote the average and maximum of the envelopes, respectively. The wave group period Tg_{mean} is the average value of Tg and Tg_{max} corresponds to the period of the wave group containing Ag_{max} .

NUMERICAL SIMULATIONS

The wave conditions for the numerical simulation, characterized by the peak period T_p , relative water depth $k_p h$, wave amplitude $a(\omega, \theta)$ and principal direction θ were defined for input in the nonlinear wave interaction modeling. The principal wave direction $\theta = 0$ was used for all the simulations except the field data. The wave model requires initial condition information, describing the initial state of the sea. In this study, the initial sea state was described as a Gaussian spectrum in the form

$$S(\omega, \theta) = \frac{m_0}{2\pi\sigma_\omega\sigma_\theta} \exp\left[-\frac{(\omega_i - \omega_0)^2}{2\sigma_\omega^2} - \frac{\theta^2}{2\sigma_\theta^2}\right] \quad (9)$$

where m_0 is the zero-th moment of the spectrum, ω is the angular frequency, and σ_ω and σ_θ are standard deviations for frequency and direction, respectively.

By taking a finite range of frequency ($\omega_{min}, \omega_{max}$) and direction ($\theta_{max}, \theta_{min}$), the initial amplitude $a(\omega, \theta) = (2S(\omega, \theta)d\omega d\theta)^{1/2}$ was determined for calculating the complex amplitude B , which is obtained by

$$B = \pi \left(\frac{2g}{\omega}\right)^{\frac{1}{2}} a(\omega, \theta) \exp(i\phi) \quad (10)$$

where ϕ is the random phase.

The wave steepness $ak_p = 0.07 \sim 0.2$ (a and k_p being the carrier wave amplitude and number) were used for simulation. The relative water depths, denoted by $k_p h$, in deep water and intermediate water depth are equal to 5.0 and 1.0, respectively. Directional spectra for sloping beach cases are studied by simulated numerical calculation from intermediate through shallow water depths. For modeling the field condition, the initial condition is specified at Station A, and further, the waves propagate to Station B with distance $30L_p$. The relative water depth on the sloping beach is $1.0 \geq k_p h \geq 0.5$, with slope calculation $k_p h(i) = k_p (26 - 13(i/z)^2)$, where z is the number of segments and $i = 1, 2, 3, \dots, z$.

At intermediate water depth $k_p h = 1.0$, we are not considering the adjustment of the spectrum as the effect of water depth, as in the Wallops spectrum. We just

assume that the same shape of the Gaussian spectrum is used in deep water and at intermediate water depth.

Directional spectra were simulated with 1050 components, which consisted of 50 components of frequency and 21 directional components. Additionally, refraction effects on sloping cases were calculated based on linear theory. The directional spectra were normalized by the peak of the initial directional spectrum $S_0(f_p, \theta_p)$. Finally, evolution of wave groups as a result of the directional spectrum was compared with unidirectional simulation, which consisted of 100 frequency components. The Runge-Kutta method, which solves a differential equation numerically, gives the integration of the spatial evolution of the nonlinear waves.

RESULTS AND DISCUSSION

Now we present the results of the simulations as well as an analysis of these results. Nonlinear wave interaction effects on the evolutions of directional spectra were analyzed to investigate the transformation of wave group structures. Attention is paid mainly to the transformation of directional spectra; then the evolution of the wave groups due to the nonlinear wave-wave interaction both for directional spectra and unidirectional spectra are compared.

Transformation of Directional Spectra

The transformations of directional spectra as the effects of wave steepness ak_p and water depths $k_p h$ are displayed in Figs. 2 and 3. Directional spectra transformation at a constant depth shows that nonlinear wave interaction more significant influence on the relative shallow water depth than in deep water as shown in Figs. 2 and 3 at the second row respectively. For $ak_p = 0.1$, directional spectra shows very small evolution, while the evolution of directional spectra for $ak_p = 0.2$ shows a significant transformation. By increasing the wave steepness, the directional spectrum in deep water grows near the peak until $x = 100L_p$, increases the energy which is absorbed more from higher frequencies than lower frequencies. Directional spreading occurs near the spectra peak. However in the relative shallow water depth, evolution of directional spectrum at $x = 50L_p$ indicates that distribution energy occurs near the spectra peak. This energy is absorbed more from lower frequencies than from higher frequencies. Dispersion spreading reduces as the water depth reduces. On sloping beach, evolution of directional spectra at Station B indicates that the distribution of energy dominant at the middle range of frequency. Transfer energy occurs from peak frequency of the spectrum to the lower frequency and higher frequency. At the lower frequency, the energy

is received from the peak frequency and absorbed, thus down shifting the peak frequency. Regarding the influence of shoaling, the low frequency part of the spectrum is affected more significantly than the high frequency part. As the waves propagate to the coast, the directional spreading becomes narrow owing to the wave refraction effect. Energy increased at the main direction which is caused by waves that propagate perpendicular to the coast.

Wave Group Evolutions

Wave group transformations of directional spectra at a constant depth and on sloping beach are expressed in Figures 4 and 5. Wave group structures are allocated using a wave envelope. The envelopes are formed only by free waves, not including the bound waves. The initial variation of the free surface elevation at $x = 0$ can be compared with the surface elevation at $x = 50L_p$ for $k_p h = 1.0$ and $x = 100L_p$ for $k_p h = 5.0$. For sloping cases, free surface elevation at Station A as the initial condition can be compared with free surface elevation at Station B.

Evolution of wave groups for $ak_p = 0.1$ shows that the group envelopes are almost same, which indicate that nonlinear effects are weak, only minor energy transfer occurs. By increasing the wave steepness, the shape of wave groups is significantly transformed. The maximum envelope fluctuates during evolution, reached a maximum and then subsided. The results were found to agree with Yuen and Lake (1982), that the evolution may be recurring or chaotic depending on the choice of modes.

Wave group transformations for the case of unidirectional spectra are presented in Figures 6 and 7. Wave group envelopes express that the group shape is almost the same for $ak_p = 0.1$, Fig.6 illustrates only a slight evolution of wave groups. At high steepness, the nonlinear effects are clearly pronounced and exhibit themselves in the evolution of the shape of wave groups, as shown in Fig.7. The shape of the wave groups is totally different.

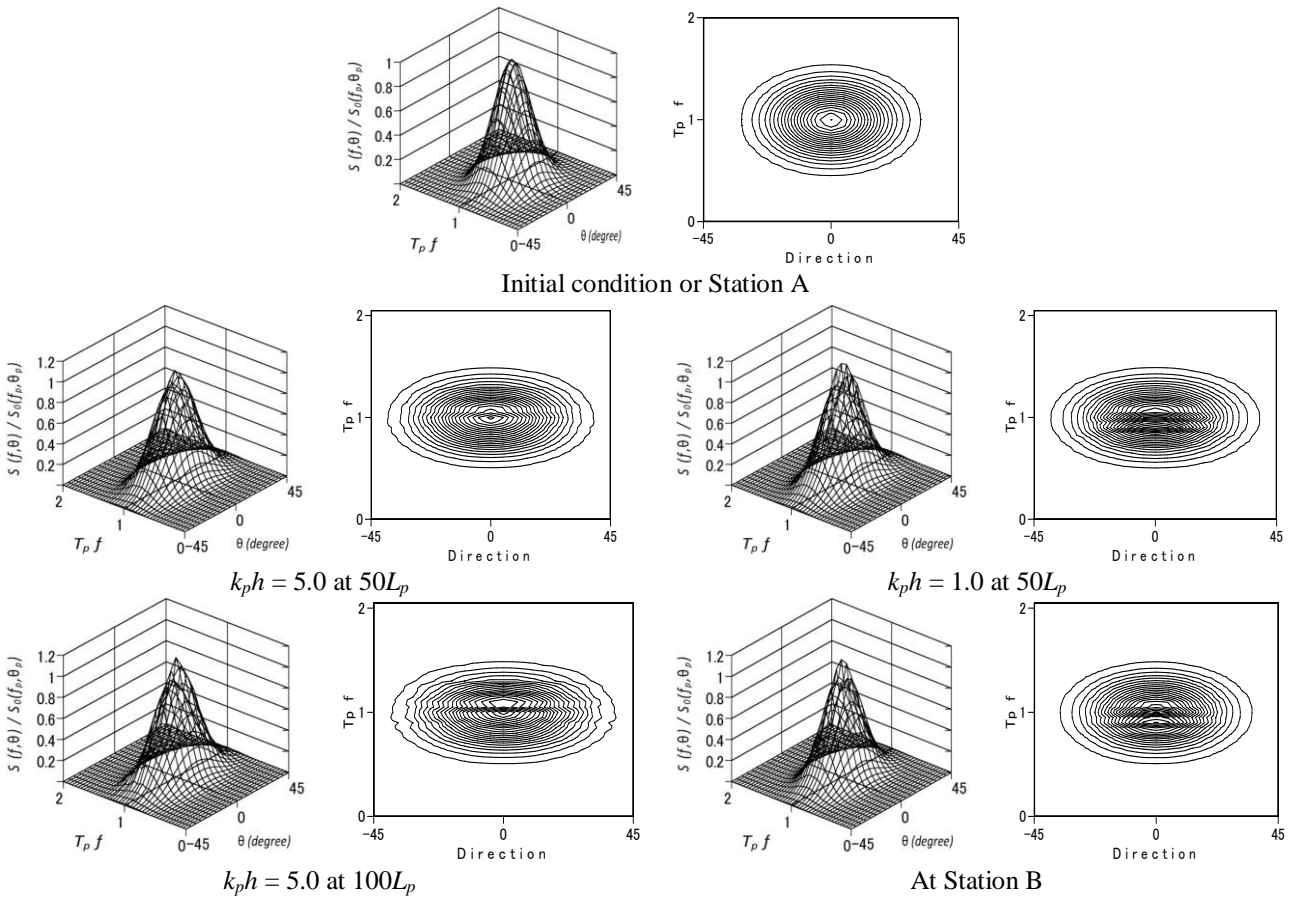


Fig. 2 Directional spectra evolution for $ak_p = 0.1$

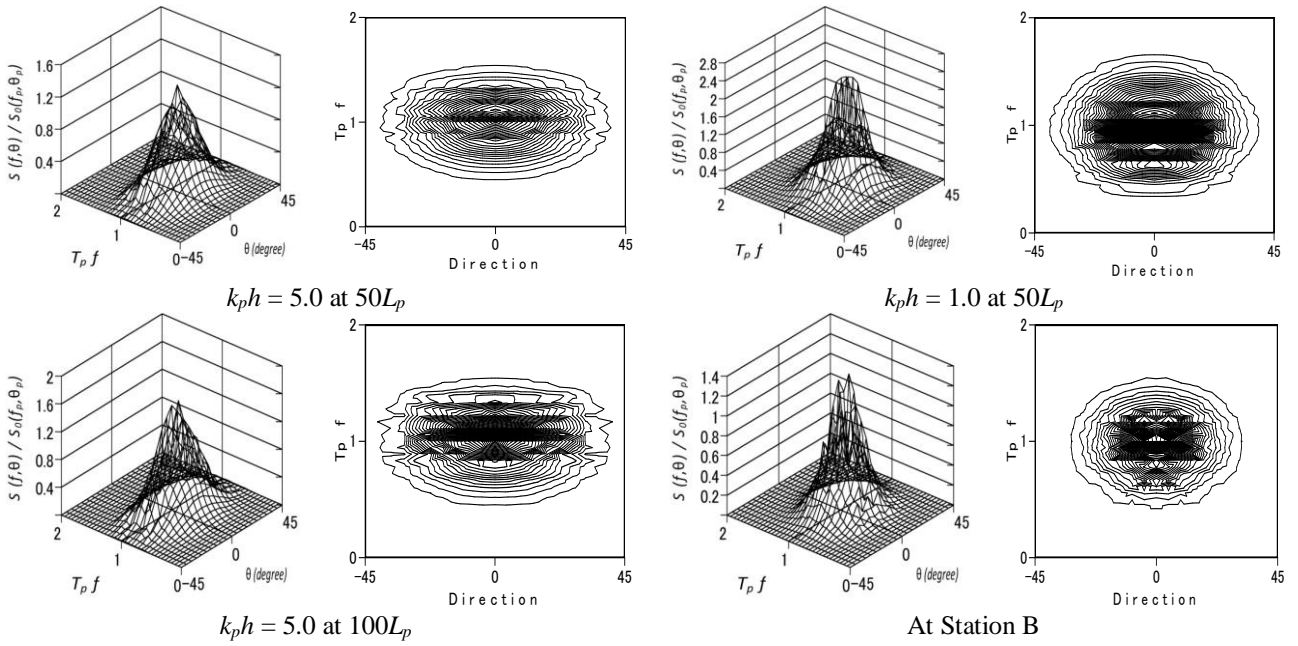
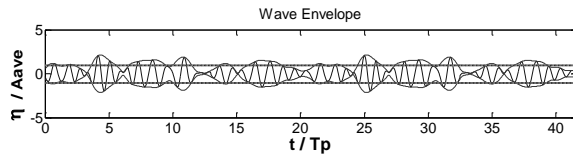


Fig. 3 Directional spectra evolution for $ak_p = 0.2$.



Initial condition or Station A

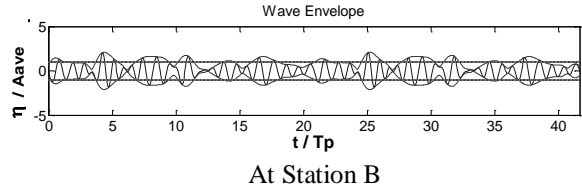
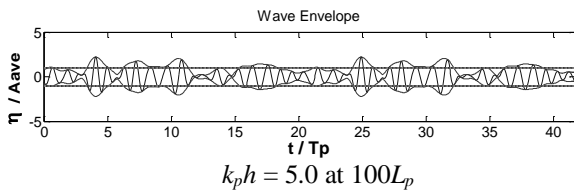
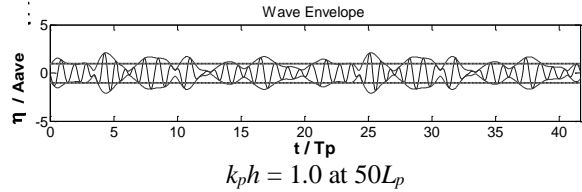
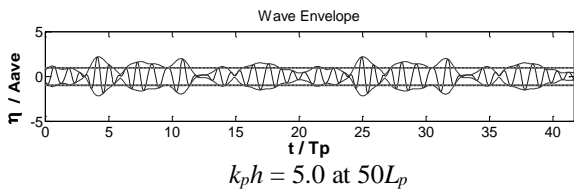
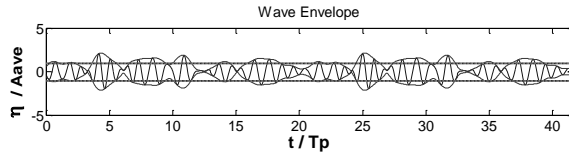


Fig. 4 Directional wave groups evolution for $ak_p = 0.1$.



Initial condition or Station A

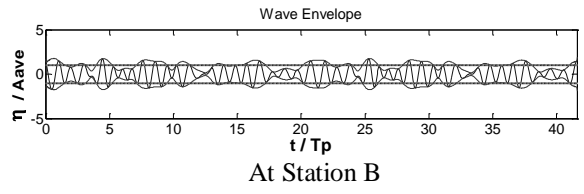
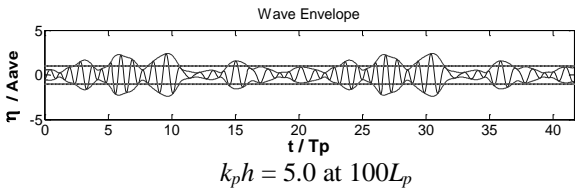
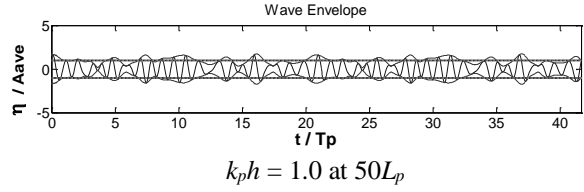
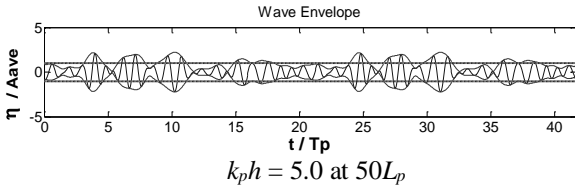
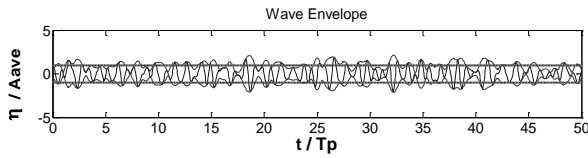


Fig. 5 Directional wave groups evolution for $ak_p = 0.2$.



Initial condition or Station A

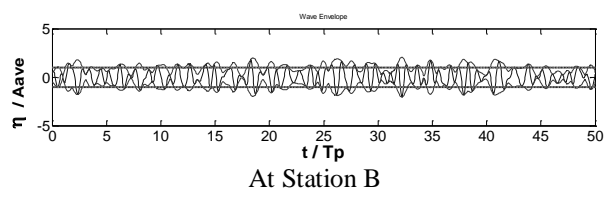
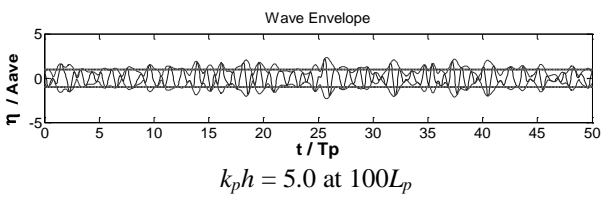
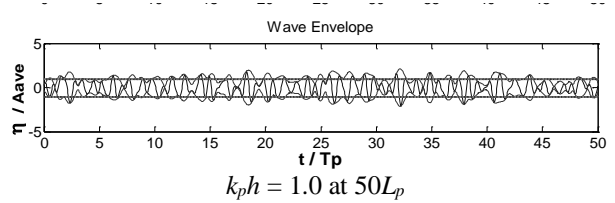
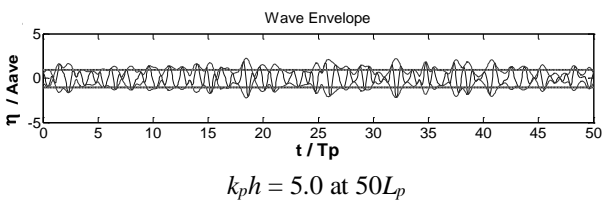


Fig. 6 Unidirectional wave groups evolution for $ak_p = 0.1$.

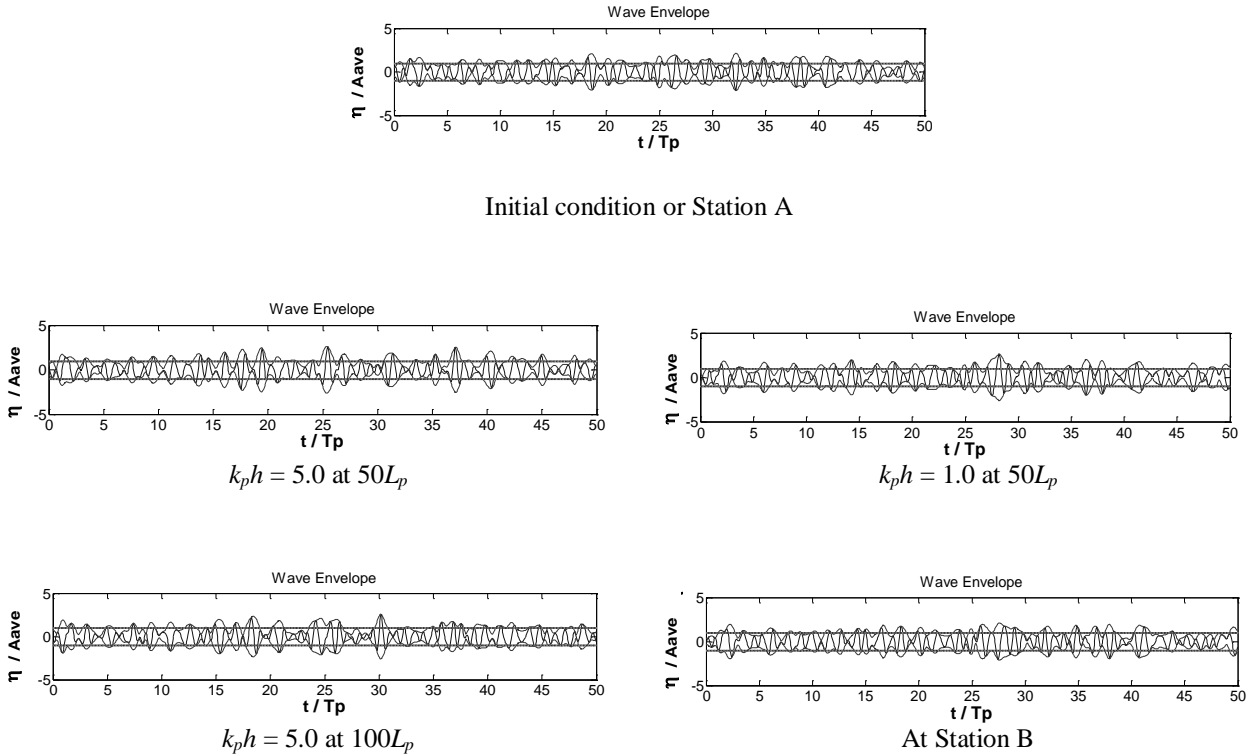


Fig. 7 Unidirectional wave groups evolution for $ak_p = 0.2$.

Comparison of the evolution of the wave group structures based on directional simulation and unidirectional simulation is presented in Table 1. The values of wave groups' structure are determined based on the average value of the three times simulation with different initial random phases; meanwhile the A_{gmax} is obtained from the highest value. As shown before that effect of wave-wave interaction is weak for small wave steepness, therefore only the wave groups' structures for higher wave steepness are compared.

The maximum value of the wave groups' amplitude A_{gmax} slightly increased in deep water, accompanied by reduction in the wave group period (Tg). At the relatively shallow water, nonlinear effects are stronger, the average of envelope amplitude A_{ave} and the maximum wave group amplitude A_{gmax} increase as the spectral peak become sharp, and the Tg_{max} becomes longer for directional case. On sloping beach, the wave groups become stretched due to wave-wave interaction. A_{gmax} decreased followed by an increase of Tg_{max} .

Comparison of evolutions of wave groups on directional spectra with those on unidirectional spectra indicates that evolution of the maximum wave groups' parameter in deep water and at intermediate water depth significantly affects nonlinear interaction in directional simulations.

Table 1 Comparison of the evolution of wave groups structures for directional spectra with that for unidirectional spectra.

Wave group structures		A_{gmean} / A_{ave}	A_{gmax} / A_{ave}	Tg_{mean} / T_p	Tg_{max} / T_p
$k_p h = 5.0$	Directional				
	$ak_p = 0.2$ $x = 0$	1.45	2.63	5.21	5.93
	$x = 100L_p$	1.64	2.70	4.17	2.22
	Unidirectional				
	$ak_p = 0.2$ $x = 0$	1.37	2.30	3.13	7.73
	$x = 100L_p$	1.45	2.55	2.94	6.11
$k_p h = 1.0$	Directional				
	$ak_p = 0.2$ $x = 0$	1.45	2.64	5.21	5.93
	$x = 50L_p$	1.54	2.70	5.21	6.85
	Unidirectional				
	$ak_p = 0.2$ $x = 0$	1.37	2.31	3.13	7.73
	$x = 50L_p$	1.41	2.60	3.33	7.73
Sloping	Directional				
	$ak_p = 0.2$ St. A	1.45	2.64	5.21	5.93
	St. B	1.34	2.05	5.21	7.22
	Unidirectional				
	$ak_p = 0.2$ St. A	1.37	2.31	3.13	7.73
	St. B	1.40	2.30	3.57	8.64

The effects of wave interaction are enhanced in relatively shallow water; however, the nonlinear interaction is reduced on a sloping beach, which decreases the maximum wave height.

The wave profiles have almost the same period but gradually varying amplitudes. This is caused by the energy of the wave spectrum, which is concentrated within a narrow range of frequency. The variability of the characteristic wave height increases as the spectral peak becomes sharp. However, if the frequency spectrum gets narrower, the envelope becomes longer, and if the directional spread decreases, the wave crest widens.

Field Observation

The wave data were collected 6km offshore (Station A; 26m deep) and 1km offshore (Station B; 13m deep) in an extension line perpendicular to the coastal line at Akabane in the Atsumi Peninsula, Japan on the Pacific coast. The bathymetry is nearly uniform, and the bottom slope changes gradually from 1/400 to 1/100. The measurements were performed by 2 wave gauges located at intermediate water depths, with $1.0 \geq k_p h \geq 0.5$. The sea surface elevation and bottom velocities were recorded every two hours for one hour long at a sampling data rate of 0.5 s for the observation period (Kioka et al. 2007).

Using the measurements from field observations at Akabane Beach, data on August 8, 2006 were calculated. The initial condition of the directional spectrum was determined by the incident wave spectrum which was obtained at Station A, as shown in Fig.8. The initial directional spectrum at Station A indicates that the spectrum centered on peak frequency $f_p = 0.084$ and $\theta = -9^\circ$. Waves propagated from Station A to Station B with a relative water depth of $1.0 \geq k_p h \geq 0.5$. An angle of 0 indicates a line perpendicular to the shoreline.

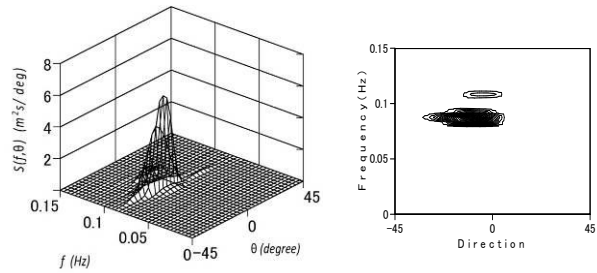
The directional spectrum of field data at Station B was used to verify the numerical result, and it indicates that the spectrum increases the peak frequency. The directional spectrum of field data was calculated using the EMLM estimation method (Johnson 2006). All available theories are based on linear theory, therefore we admit the linear theory analysis, but only the fundamental frequency band is considered, and that composes the free wave only.

The numerical results of the spatial evolution directional spectrum at Station B indicate that the energy transfer occurs from the peak frequency to the lower frequency and higher frequency; therefore, the spectrum slightly widens in frequency and narrows in direction. The results of numerical simulation at Station B show a trend close to the directional spectrum of field data;

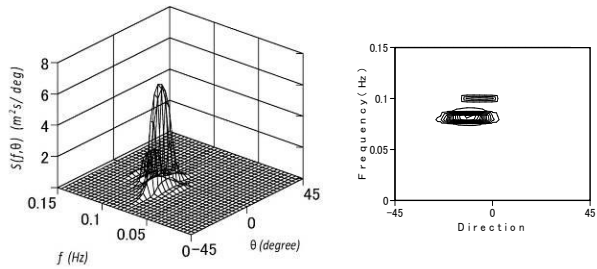
however, the energy at peak frequency looks smaller than in the field data. This discrepancy is caused by the effects of the linear calculation on the field data.

Lin and Lin (2004) introduce a new wave-breaking function to calculate the wave breaking as a result of white-capping at intermediate depths. We have used this formulation to see effects of the white-capping on the nonlinear directional spectra; however, the result only gives minor evolution. Therefore, the effect of white-capping in this simulation is weak.

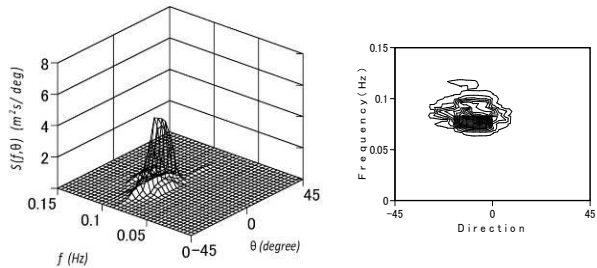
Wave group envelopes from measurement data at Station A and Station B are shown in Fig.9. In this figure, the wave group envelopes were governed only by the fundamental frequency, without the bound waves. Using the Hilbert transform wave group structure at Station A, the values $A_{ave} = 1.08m$, $A_{gmean} = 1.31m$, $A_{gmax} = 2.16m$, $T_{gmean} = 80s$ and $T_{gmax} = 72s$ were obtained; further at Station B, $A_{ave} = 1.03m$, $A_{gmean} = 1.25m$, $A_{gmax} = 2.15m$, $T_{gmean} = 80s$ and $T_{gmax} = 147s$.



(a) Field data of Station A



(b) Field data of Station B



(c) Simulation result of Station B

Fig. 8 Directional spectra of the field data.

Wave groups from the simulation results at Station B are presented in Fig.10. As discussed previously, the initial random phase that affects the energy distribution is different, which will affect the wave profiles and, further, the wave groups. Therefore, we calculated the directional spectrum at Station A three times, as shown in Fig.10.

The envelopes of the wave group were also formed by free waves only. By the Hilbert transform calculation, wave groups' structures at Station A were obtained: $A_{ave} = 1.07m$, $Ag_{mean} = 1.30m$, $Ag_{max} = 2.12m$, $Tg_{mean} = 80s$ and $Tg_{max} = 72s$ and the wave groups' structures as a result of directional simulation at Station B: $A_{ave} = 1.13m$, $Ag_{mean} = 1.38m$, $Ag_{max} = 2.10m$, $Tg_{mean} = 80s$ and $Tg_{max} = 150s$.

Wave group structures from the simulation results are in good agreement with the field data; therefore, this model could be used to predict the evolution of a wave group. Although the field data and simulation results have different shapes of directional spectra and wave group envelopes, they produce almost the same wave group structures.

CONCLUSIONS

The transformation of wave groups has been investigated by numerical simulation based on the third-order Zakharov equation. The main conclusions can be summarized as follows.

The third-order Zakharov equation model is able to predict the effect of the nonlinear interaction on the transformation of directional spectra for constant depth and for a sloping beach. The nonlinear transfer of energy was found to control the shape of the spectrum, including the development of the peak and the wave groups.

In relatively shallow water, nonlinear wave interactions appear to have a more significant effect than in deep water. The low-frequency part of the spectrum is affected more significantly than the high-frequency part; however, in deep water the high-frequency part is affected more significantly than the low-frequency part.

On the sloping beach, the transformation of directional spectra indicates that the lower frequencies are enhanced more than the higher frequencies. This results in a higher energy in the principal direction, and the peak of the spectrum slightly shifts to the lower frequency.

By increasing the wave steepness, the effects of the nonlinear wave interaction become stronger. The evolution of the directional spectrum is much more pronounced at a high steepness than at a lower steepness.

Wave groups can be characterized by the wave envelopes. The Zakharov equation, which contains initial-phase information, can be advantageous for prediction of the evolution of the wave groups' envelope. Initial random phases significantly affect the distribution of energy on the spectrum and the eventual shape of wave groups; however, they produce almost the same wave group structures.

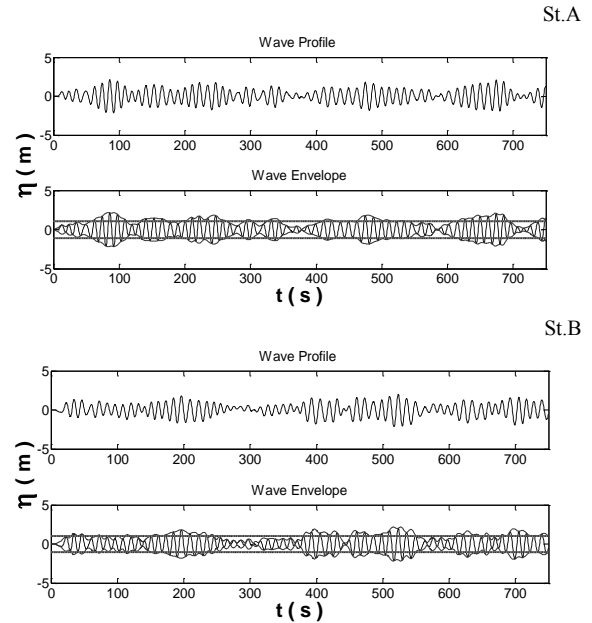


Fig. 9 Wave groups' evolution based on field data from St. A to St. B.

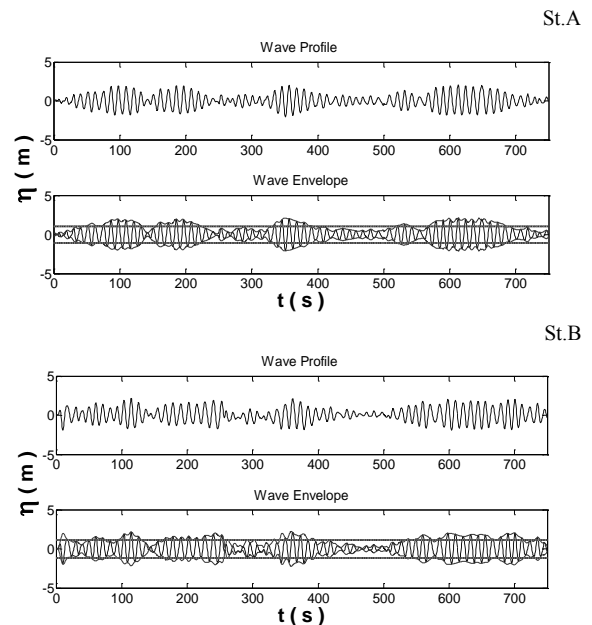


Fig. 10 Wave groups' evolution based on simulation results at St. A and St. B.

The transformation of the wave groups is in accordance with the evolution of the directional spectrum. If the energy of the wave spectrum is concentrated within a narrow range of frequency, the wave profiles have almost the same period, but gradually varying amplitudes.

The variability of the characteristic wave height increases as the spectral peak becomes sharp. However, if the frequency spectrum gets narrower, the envelope becomes longer, and if the directional spread decreases, the wave crest widens.

The comparison of the wave group evolutions on directional spectra to those on unidirectional spectra indicates that evolutions of wave groups in deep water and at intermediate water depths are significantly affected by nonlinear interactions between directional components. When a three-dimensional model is considered, transformation of wave groups in deep water is much more pronounced at $ak_p = 0.2$. The effects of wave interaction are enhanced in relatively shallow water; however, the nonlinear interaction is suppressed on a sloping beach, which decreases the maximum wave height.

ACKNOWLEDGMENT

The first author is greatly thankful to the Government of Indonesia (Ministry of National Education of the Republic of Indonesia) for providing a scholarship, and to Katoh H. for the data from field observation and some source codes for simulation. A part of this study was supported by a Grant-in-Aid for Scientific Research from the MEXT of Japan.

REFERENCES

- Agnon, Y. (1993). On a uniformly valid model for surface interaction, *J. Fluid Mech.*, Vol. 247:589-601.
- Goda, Y. (2009). *Random Seas and Design of Maritime Structure*. World Scientific.
- Janssen, P. A. E. M. (2003). Nonlinear four-wave interactions and freak waves, *Journal of Physical Oceanography*, Vol. 33:863-884.
- Janssen, P. A. E. M. and Onorato, M. (2007). The intermediate water depth limit of the Zakharov equation and consequences for wave prediction. *Journal of Physical Oceanography*, Vol. 37:2389-2399.
- Johnson, D. (2006). *Diwasp user manual version 1.2*, Coastal Oceanography Group Centre for Water Research University of Western Australia, Perth.
- Kioka, W., Hayashi, N. and Kitano, T. (2005). Transformation of strongly nonlinear wave group over sloping beach, *Proc. of the third International Conference on Asian and Pacific Coasts*:90-99.
- Kioka, W., Katoh, H. and Kitano, T. (2007). Field observations of wave group transformation during high waves, *JSCE-B2*, Vol. 54-1:161-165.
- Kioka, W., Okajima, M., Pujianiki, N. and Kitano, T. (2011). Reflection of strongly nonlinear waves from a vertical wall on sloping beach, *JSCE-B2*, Vol. 67-2: I_1-I_5.
- Kit, E. and Shemer, L. (2002). Spatial versions of the Zakharov and Dysthe evolution equations for deep water gravity waves, *J. Fluid Mech.*, Vol. 450:201-205.
- Kit, E., Shemer, L., Pelinovsky, E., Talipova, T., Eitan, O. and Jiao, H. (2000). Nonlinear wave group evolution in shallow water, *Journal of Waterway, Port, Coastal and Ocean Engineering*, September / October:221-228.
- Lin, L. and Lin, R. Q. (2004). Wave breaking function, 8th International Workshop on Wave Hindcasting and Forecasting, North Shore, Oahu, Hawaii.
- Longuet-Higgins, M. S. (1962). Resonant interaction between two train of gravity waves. *J. Fluid Mech.*, Vol. 12:321-332.
- Mase, H. and Iwagaki, Y. (1986). Wave group analysis of natural wind waves based on modulational instability theory, *Coastal Engineering*, Vol. 10:341-354.
- Shemer, L., Jiao, H., Kit, E. and Agnon, Y. (2001). Evolution of a nonlinear wave field along a tank: experiments and numerical simulations based on the spatial Zakharov equation, *J. Fluid Mech.*, Vol. 427:107-129.
- Stiassnie, M. and Gramstad, O. (2009). On Zakharov's kernel and the interaction of non-collinear wave trains in finite water depth, *J. Fluid Mech.*, Vol. 639:433-442.
- Stiassnie, M. and Shemer, L. (1984). On modifications of the Zakharov equation for surface waves, *J. Fluid Mech.*, Vol.143:47-67.
- Stiassnie, M. and Shemer, L. (2005). On the interaction of four water-waves, *Wave Motion*, Vol. 41, No. 4:307-328.
- Yuen, H.C. and Lake, B.M. (1982). Nonlinear dynamic of deep water gravity waves. *Adv. Appl. Mech.* 22: 67-229
- Zakharov, V. (1968). Stability of periodic waves of finite amplitude on the surface of a deep fluid. *J. Appl. Mech. Tech. Phys.*, Vol. 9:190-194.