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CORRIGENDUM TO MINIMAL PRIME IDEALS OF ORE EXTENSION OVER A COMMUTATIVE DEDEKIND DOMAIN AND ITS APPLICATION

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Abstract

Let $R = D[x;\, \sigma,\, \delta]$ be an Ore extension over a commutative Dedekind domain D, where σ is an automorphism of D and δ is a left σ-derivation of D. In [1], minimal prime ideals of R were described, but the description

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carried a flaw in the proof and it did not describe all minimal prime ideals as pointed out by one of the authors in this paper. In this corrigendum, we provide a corrected proof and an application to the group of fractional *R*-ideals.

In this corrigendum, *D* denotes a commutative Dedekind domain except for Lemma 1 and Remark, and $R = D[x; \sigma, \delta]$ denotes an Ore extension over *D*, where σ is an automorphism of *D*, and δ is a left σ-derivation of *D*.

In [1, Theorem 4], it was shown that:

Let *P* be a prime ideal of *R* and $P \cap D = \mathfrak{p} \neq (0)$. Then *P* is a minimal prime ideal of *R* if and only if either $P = \mathfrak{p}[x; \sigma, \delta]$, where $\mathfrak{p}$ is a minimal (σ, δ)-prime ideal of *D* or (0) is the largest (σ, δ)-ideal of *D* in $\mathfrak{p}$. This result is not correct as it fails to describe all minimal prime ideals.

We now give a complete description of all minimal prime ideals of *R* as follows:

Lemma 1 [5, Proposition 3.5]**.** *Let P be a prime ideal of* $R = D[x; \sigma, \delta]$, *where D is a commutative Noetherian domain and let* $\mathfrak{p} = P \cap D$. *If* $\mathfrak{p}$ *is a prime ideal of D with* $\sigma(\mathfrak{p}) \neq \mathfrak{p}$, *Then P is not a minimal prime ideal of R.*

Proof. This implicitly follows from the proof of [5, Proposition 3.5]. However, we give the outline of the proof for reader's convenience by using Goodearl's notation: Let *y* be an indeterminate and $Y = \mathscr{C}^{\sigma}_{D[y]}(\mathfrak{p}[y])$. Set $D^{\circ} = D[y]Y^{-1}$ and $R^{\circ} = R[y]Y^{-1}$. Goodearl showed that $R^{\circ} = D[y]Y^{-1}[x; \sigma, \delta] = D^{\circ}[x^{\circ}; \sigma]$, where $x^{\circ} = x - b$ for some $b \in D^{\circ}$ and $P^{\circ} = \mathfrak{p}D^{\circ} + x^{\circ}R^{\circ}$ is a prime ideal of R° such that R°/P° is a commutative domain. Hence $P^{\circ} \cap R$ is a prime ideal with $\mathfrak{p} = P^{\circ} \cap D$ and so, by uniqueness, $P = P^{\circ} \cap R$. Put $P_1^{\circ} = x^{\circ}R^{\circ}$, a completely prime ideal of R° with $P^{\circ} \supsetneqq P_1^{\circ}$ and thus $P_1 = P_1^{\circ} \cap R$ is also a completely prime ideal of *R*, since $R/P_1 \subset R^{\circ}/P_1^{\circ}$. If $P_1 = P$, then $P_1 \supseteq \mathfrak{p}$ and so $P_1^{\circ} \supseteq \mathfrak{p}D^{\circ} + x^{\circ}R^{\circ} = P^{\circ}$ which is a contradiction. Hence *P* is not a minimal prime ideal. □

Let S be a ring with quotient ring Q and let $I(J)$ be a right (left) R-ideal of Q. We use the following notation:

$$(S : I)_l = \{q \in Q \mid qI \subseteq S\}, \quad (S : J)_r = \{q \in Q \mid Jq \subseteq S\},$$

and $I_v = (S : (S : I)_l)_r$ is again a right S-ideal containing I. If $I = I_v$, then it is called a *right v-ideal*. Similarly ${}_vJ = (S : (S : I)_r)_l$, and J is called *left v-ideal* if $J = {}_vJ$. An R-ideal A is said a *v-ideal* if $A_v = A = {}_vA$. We denote by $\mathrm{Spec}(S)$ the set of prime ideals of S.

Since $\mathrm{gl.dim}\, R \leq 2$, each v-ideal A is an invertible ideal by [4, p. 324]. We denote by $\mathrm{Spec}_0(R) = \{P \in \mathrm{Spec}(R) \mid P \cap D = (0)\}$.

Proposition 1. *Let $R = D[x; \sigma, \delta]$ be the Ore extension over a commutative Dedekind domain D. Then $\{\mathfrak{p}[x; \sigma, \delta], P \mid \mathfrak{p}$ is a non-zero (σ, δ)-prime ideal of D and $P \in Spec_0(R)$ with $P \neq (0)\}$ is the set of minimal prime ideals of R.*

Proof. Let $T = K[x; \sigma, \delta]$, where K is the quotient ring of D which is a field, and let $C = D - \{0\}$. Then C is a regular Ore set of R such that $T = R_C$. Hence there is a one-to-one correspondence between $\mathrm{Spec}_0(R)$ and $\mathrm{Spec}(T)$ (cf. [6, Theorem 9.22]) given by $P \to P' = PT$ and $P' \to P' \cap R$, where $P \in \mathrm{Spec}_0(R)$ and $P' \in \mathrm{Spec}(T)$. Let $P \in \mathrm{Spec}_0(R)$ with $P \neq (0)$. Then since R is Noetherian and T is a principal ideal ring, we have

$$P' = P'_v = (T : (T : P')_l)_r = (T : T(R : P)_l)_r = (R : (R : P)_l)_r T = P_v T$$

and so $P = P_v$ follows. Similarly, we have $P = {}_vP$ and hence P is a minimal prime ideal (cf. [7, Proposition 5.1.9]). Let P be a prime ideal of R with $\mathfrak{p} = P \cap D \neq (0)$. According to Goodearl's classification, there are two cases: either $\mathfrak{p}$ is a (σ, δ)-prime ideal or $\mathfrak{p}$ is a prime ideal with $\sigma(\mathfrak{p}) \neq \mathfrak{p}$. In the second case, P is not a minimal prime ideal by Lemma 1. In the first case, by [5, Proposition 3.3], $\mathfrak{p}[x; \sigma, \delta]$ is a prime ideal. Thus if P is a minimal prime ideal, then $P = \mathfrak{p}[x; \sigma, \delta]$. Conversely, let $\mathfrak{p}$ be a (σ, δ)-prime ideal of D. Then $\mathfrak{p}[x; \sigma, \delta]$ is a prime ideal and invertible, because $\mathfrak{p}$ is invertible. Hence $\mathfrak{p}[x; \sigma, \delta]$ is a v-ideal and so it is a minimal prime ideal. This completes the proof. □

As an application of Proposition 1, we have the following: we denote by $G(R) = \{A | A$ is an invertible R-ideal$\}$. It is well known that $G(R)$ is an abelian group generated by prime invertible ideals, i.e., P and $\mathfrak{p}[x; \sigma, \delta]$. Hence any invertible R-ideal is of the form:

$$(\mathfrak{p}_1^{e_1}[x; \sigma, \delta] \cdots \mathfrak{p}_k^{e_k}[x; \sigma, \delta])(P_1^{n_1} \cdots P_l^{n_l}),$$

where e_i and n_i are integers, $\mathfrak{p}_i$ are (σ, δ)-prime ideals, and $P_i \in \text{Spec}_0(R)$ with $P_i \neq 0$.

Similarly, let $G_{\sigma, \delta}(D) = \{\mathfrak{a} | \mathfrak{a}$ is an invertible D-ideal which is (σ, δ)-ideal$\}$. Then it is an abelian group generated by (σ, δ)-prime ideals. Hence, we have the following:

Proposition 2. $G(R) \cong G_{\sigma, \delta}(D) \oplus D(T)$. *The correspondence is given by*

$$(\mathfrak{p}_1^{e_1}[x; \sigma, \delta] \cdots \mathfrak{p}_k^{e_k}[x; \sigma, \delta])(P_1^{n_1} \cdots P_l^{n_l}) \to (\mathfrak{p}_1^{e_1} \cdots \mathfrak{p}_k^{e_k}) \oplus ((P_1')^{n_1} \cdots (P_l')^{n_l}),$$

where $P_i' = P_i T$ *for any* i, $1 \leq i \leq l$.

Remark. Let D be a Krull ring in the sense of [2] and $R = D[x; \sigma, \delta]$. In [3], the structure of v-ideals has been studied in the case either $\sigma = 1$ or $\delta = 0$.

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